TREND AND CYCLE IN THE PORTUGUESE *OUTPUT*

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It is common practice in Macroeconomics to decompose the real GDP or GNP, and many other time series, into a secular or permanent component, named Trend, and a transitory or cyclical component, named Cycle. The traditional view assumes that the cyclical fluctuations dissipate quickly and that the series reverts to the Trend, which moves slowly and smoothly over time.The method used to «detrend» the series is a regression on a time trend (or a polynomial in time). The residuals are interpreted as the Cycle.

Nelson and Plosser (1982) (hereafter N-P) question this traditional approach and suggest that many macroeconomic time series are non-stationary stochastic processes with highly permanent fluctuations and no tendency to return to a linear time trend, i. e., they have a stochastic trend. The «detrend» method is, therefore, the differentiation of the series. To decompose the series into Trend and Cycle, N-P use the Beveridge-Nelson Decomposition [Beveridge and Nelson (1981)] where the Trend is defined as a random walk and the Cycle as a stationary process. Furthermore, they conclude that the issue here is whether or not the series contain a unit root in their autoregressive representation. Dickey and Fuller (1979) first developed tests of the unit root hypothesis. Ever since, much research has been carried out, particularly by Phillips and Perron.

This paper is organized as follows: in subsection 1.1 we discuss the definitions of the deterministic and the stochastic trend, and the two decompositions that support them (deterministic and Beveridge-Nelson); in subsection 1.2 we review the main tests of the unit root hypothesis, and some of its late developments; in subsection 1.3 we refer recent developments in this area; in section 2 we apply this methodology to the study of Portuguese real per capita GDP; finally, in section 3 we present our conclusions, namely that we cannot reject the unit root in the Portuguese output, and give some perspectives of future developments of this study.

1 — Stochastic trend and unit root tests

1.1 — Stochastic trend

Traditionally, time series are decomposed into the sum of a smooth Trend, a deterministic function of time, and a Cycle that incorporates all the fluctua-

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tions of the series around the Trend. This decomposition originates the class of processes defined by N-P as Trend-Stationary (TS):

$$Y_t = [\alpha + \beta t] + c_t \tag{1.1a}$$

$$\phi(B)c_t = \theta(B)\varepsilon_t \quad , \quad \varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2) \tag{1.1b}$$

where Y_t is the natural log of the series, c_t is the Cycle, B is the lag operator, and $\phi(B)$ and $\theta(B)$ are polynomials in B (1) that satisfy the stationarity and invertibility conditions (all its roots are outside the unit circle). The deterministic character of this type of processes is clear: in the long run the only available information on Y_t is given by its expected value $(\alpha + \beta t)$.

Alternatively, N-P define a class of processes which they call Difference-Stationary (DS):

$$(1 - B)Y_t = \beta + d_t \tag{1.2a}$$

$$\delta(B)d_t = \lambda(B)\varepsilon_t \ , \ \varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2) \tag{1.2b}$$

where $\delta(B)$ and $\lambda(B)$ are also stationary and invertible polynomials in B. If a shock occurs, it will persist over time and nothing will make Y_t return to its previous path: the fluctuations of the series are permanent.

The Beveridge-Nelson Decomposition can now be used to decompose the series into the sum of the Trend, $\overline{Y}_t$, and the Cycle, c_t, perfectly correlated. Illustrating it for the MA (1) model, often identified as the generator process of $(1 - B)Y_t$ using the Box-Jenkins Methodology [Box and Jenkins (1976)],

$$(1 - B)Y_t = Y_t - Y_{t-1} = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} \ , \ |\theta_1| < 1$$

and letting $Y_0 = \varepsilon_0 = \mu = 0$, we have:

$$Y_t = Y_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1} = \sum_{r=1}^{t} \varepsilon_r - \theta_1 \sum_{r=1}^{t-1} \varepsilon_r$$

$$= \left[(1 - \theta_1) \sum_{r=1}^{t} \varepsilon_r \right] + \theta_1 \varepsilon_t = \overline{Y}_t + c_t \tag{1.3}$$

where the Trend is defined as a random walk and the Cycle as a white noise. The innovations in both components are perfectly correlated since they are both proportional to ε_t.

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(1) The Decomposition Theorem of wold is used throughout the paper. It says that any stationary stochastic process can be modelled as a MA model $\omega_t = \mu + \varepsilon_t + \lambda_1 \varepsilon_{t-1} + \ldots$, where μ is the long run mean of the w series, λ_t are constants and ε's are the uncorrelated random disturbances with $(0, \sigma_\varepsilon^2)$, often referred to as innovations.

The fundamental difference between the TS and the DS processes (2) is well described in terms of the roots of the AR and MA polynomials of (1.1) and (1.2). The differentiation of the TS process (1.1a) gives:

$$(1 - B)\, Y_t = \beta + (1 - B)\, c_t$$

and replacing c_t for its expression from (1.1b) we have:

$$\phi(B)\, [(1 - B) Y_t] = \beta' + (1 - B)\, \theta\, (B)\, \varepsilon_t \qquad \textbf{(1.4)}$$

meaning that there is a unit root in the MA polynomial of the ARMA (non-invertible) representation of $(1 - B)Y_t$, whereas its DS process [given by (1.2)] is stationary and invertible. For the DS process (1.2), the representation of Y_t in the levels is:

$$[\delta(B)\, (1 - B)]\, Y_t = \beta^* + \lambda(B)\, \varepsilon_t \qquad \textbf{(1.5)}$$

which contains an unit root in the AR polynomial and is non-stationary. On the contrary, the TS representation of Y_t is both stationary and invertible.

Put this way, the distinction between series generated by TS or DS processes is on the exact «location» of the unit root on their ARMA representations. As Dickey, Bell and Miller (1986) say, «the critical issue in choosing between differencing and fitting polynomial trends is not whether Y_t in fact follows a polynomial trend, since both approaches allow for this, but whether the deviations of Y_t from the polynomial require differencing». Therefore, we have to investigate whether the series contains a unit root or not. Dickey and Fuller (1979) first developed tests for the DS process as the null (unit root in the AR polynomial) and the TS as the alternative (stationary representation). In next subsection we will survey the most important and widely used tests of the unit root hypothesis.

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(2) Chan, Hayya and Ord (1977) and Nelson and Kang (1981 and 1984) have studied by simulation the consequences of the types of misspecification that arise from inappropriate detrending of time series. If the series is generated by a TS model, the use of first differences for its detrending does not produce a stationary series; rather, it induces a negative spurious level at the first lag of the ACF (autocorrelation function), attenuates the low and exaggerates the high frequencies of the spectrum. On the contrary, when the series is generated by a model of the DS class, the regression on a time trend originates, on average, an R^2 of 0.443; the value of the ACF for the first lag is approximately equal to $(1 - 10/n)$, where n is the sample size. In the spectrum, the low frequencies are now exaggerated. The spurious regressions thus obtained can be «uncovered» through the use of the Durbin-Watson statistic or the Durbin h statistic (when lagged variables are present).

1.2 — Unit roots tests

Let

$$Y_t = c + \phi Y_{t-1} + \varepsilon_t \Leftrightarrow (1 - \phi B)Y_t = c + \varepsilon_t \qquad \textbf{(1.6)}$$

be an AR (1) model where $\varepsilon_t \sim iid(0, \sigma^2)$. The root of its characteristic polynomial, $(1 - \phi B)$, is $B = 1/\phi$. If the absolute value of the root is outside the unit circle, $|B| > 1 \Leftrightarrow \phi < 1$, then Y_t is said to be stationary; when $|B| < 1$, Y_t is called explosive; finally, when $|B| = 1 \Leftrightarrow \phi = 1$ the polynomial has a unit root, i. e., (1.6) is a random walk (with drift if $c \neq 0$), or as Granger puts it, (1.6) is integrated of order one.

Dickey and fuller tests

It is known that the traditional OLS test statistics do not have the same distributions, even asymptotically, when $|B|$ is smaller, equal or bigger than one [see, for example, Fuller (1985)]. In the case of interest here (unit root), Dickey Fuller (hereafter D — F) show that the distributions of the test statistics $n(\hat{\phi} - 1)$ (3) and $\hat{\tau} = (\hat{\phi} - 1)/\hat{\sigma}_{\hat{\phi}}$ are strongly biased to the left, and have calculated their percentiles [tables in Fuller (1976)]. Using these tables it is possible to test the unit root hypothesis in autoregressive models (4).

To test the unit root hypothesis in AR (1) models, D — F (1979) define

$$H_0: Y_t = Y_{t-1} + \varepsilon_t \Leftrightarrow \phi = 1$$

$$H_1: Y_t = \phi\, Y_{t-1} + \varepsilon_t \qquad \textbf{(1.7)}$$

$$H_2: Y_t = c + \phi\, Y_{t-1} + \varepsilon_t \qquad \textbf{(1.8)}$$

$$H_3: Y_t = c + \phi\, Y_{t-1} + \beta t + \varepsilon_t \qquad \textbf{(1.9)}$$

where, for simplicity, we consider only the case $\phi > 0$. Under H_1 we use the tables for $n(\hat{\phi} - 1)$ and $\hat{\tau} = (\hat{\phi} - 1)/\hat{\sigma}_{\hat{\phi}}$; under H_2 we use the ones for $n(\hat{\phi}_\mu - 1)$ and $\hat{\tau}_\mu$ (5); finally, under H_2 we use the tables for $n(\hat{\phi}_\tau - 1)$ and $\hat{\tau}_\tau$, all in Fuller (1976) (6). D — F expand their results to the test of a unit root in AR (p) models.

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(3) When $\phi < 1$, $\sqrt{n}\hat{\phi}$ has an asymptotic normal distribution. However, when $\phi = 1$, it is $n(\hat{\phi} - 1)$ that has a non-degenerate limiting distribution.

(4) Bhargava (1986) and Ahtola and Tiao (1987) take a different approach to deduce tests of the unit root hypothesis in pure autoregressive models. In empirical studies the conclusions obtained using theirs and the D — F tests do not differ.

(5) The indices of $\hat{\phi}$ and $\hat{\tau}$ denote the exogenous variables present in the model under the alternative, where μ stands for the intercept and τ for the time trend, according to Fuller's notation.

(6) Out of practical considerations, D — F suggest the use of regressions of the type $\nabla Y_t = c + \alpha Y_{t-1} + Bt + \varepsilon_t$, where the t value given by the packages corresponds to the value of $\hat{\tau}_\tau$ since $\alpha = (\phi - 1)$.

D — F (1981) extend their procedures to the test of joint hypothesis, tabulating F tests for the three alternative models considered (7). For the test of H_0 $(c\ ,\ \phi\ ,\ \beta) = (0\ ,\ 1\ ,\ 0)$ against H_1: $(c\ ,\ \phi\ ,\ \beta) = (c\ ,\ \phi\ ,\ 0)$, they obtain a likelihood ratio that is a monotone function of

$$\Phi_2 = \frac{1}{3S^2_{0\tau}} [n\hat{\sigma}^2_0 — (n—3)S^2_{0\tau}] \qquad \textbf{(1.10)}$$

where $S^2_{0\mu}$ is the estimated variance of the OLS residuals under the alternative. The percentiles are given in their table IV. For testing H_0 against H_2, i. e., the joint test of $(c, \phi, \beta) = (0, 1, 0)$ on (1.8), the likelihood ratio is a monotone function of

$$\Phi_2 = \frac{1}{3S^2_{0\tau}} [n\hat{\sigma}^2_0 — (n—3)S^2_{0\tau}] \qquad \textbf{(1.11)}$$

with percentiles given in table V. Finally, to test $H_{2'}$: $(c, \phi, \beta) = (c, 1, 0)$ against H_1, the likelihood ratio is a monotone function of

$$\Phi_3 = \frac{1}{2S^2_{0\tau}} \{n[\hat{\sigma}^2_0 — (\bar{y}_{(0)} — \bar{y}_{(-1)})^2] — (n—3)S^2_{0\tau}\} \qquad \textbf{(1.12)}$$

independent of c (table VI). D — F also tabulate the percentiles for the t statistics of $\hat{c}$ and $\hat{\beta}$ in models (1.8) and (1.9') (tables I, II and III).

The power of all the D — F tests is rather small (8), which is not surprising since it is difficult to distinguish between a unit root and a root close but smaller than one using finite samples.

When chosing which alternative model (1.7)-(1.9′) to use, it should be taken into consideration that as D — F (1979) say: «The limiting distributions of $\hat{\phi}_\mu$ and $\hat{\tau}_\mu$ are obtained under the assumption that the constant term μ is zero. Likewise, the limiting distributions of $\hat{\phi}_\tau$ and $\hat{\tau}_\tau$ are derived under the assumption that the coefficient for time, β, is zero. The distributions of $\hat{\phi}_\tau$ and $\hat{\tau}_\tau$, are unaffected by the value of μ. If $\mu \neq 0$ [under (1.8)] or $\beta \neq 0$ [under (1.9)], the limiting distributions of $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$ are normal. Thus, if the maintained model (has a constant term) and the statistic $\hat{\tau}_\mu$ is used to test the (unit root) hypothesis, it will be accepted with probability greater than the nominal level where $\mu \neq 0$.» However, Hylleberg and Mizon (1989) simulation study shows that in small samples «one may be better off using the Dickey-Fuller tables».

Evans and Savin (1981 and 1984) and Nankervis and Savin (1985) study the problem from the point of view of the invariance of the test statistics to μ, β, and Y_0. Perron (1988) discusses their results and proposes a test strategy to limit its pernicious effects. For instance, if we suspect the series

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(7) D — F (1981) introduce a correction in the definition of the time trend in (1.9): instead of βt, they define $\beta(t—n/2)$. We denote the new specification of the model as (1.9′) and will use it from now on.

(8) Dickey and Fuller (1979 and 1981) and Dickey, Miller and Bell (1986) present studies of the power of these tests.

has a non-null mean (which is a common feature of macroeconomic time series), we should use model (1.9′) and its test statistics instead of (1.8), although those tests have less power than the ones based on the last model.

The D — F tests procedures ar not valid unless the residuals from the OLS estimation are *i.i.d.* Often this is not the case (9), and D — F use a parametric (autoregressive) correction for the autocorrelation present. They define the Augmented Dickey-Fuller model (ADF):

$$Y_t = c + \phi Y_{t-1} + \beta t + \sum_{j=1}^{s} \phi'_j \nabla Y_{t-j} + \varepsilon_t \qquad \textbf{(1.9′)}$$

where $\nabla \Leftrightarrow (1 - B)$ and the number of lags «s» is chosen in a way the residuals are no longer serially dependent (using the Ljung-Box Q or the Durbin h tests). This procedure has the disadvantage of implying the estimation of a number of nuisance parameters with all its consequences.

Perron and Phillips tests

Perron and Phillips (hereafter P — P) have made a decisive contribution to tests of the unit root hypothesis. Unlike D — F, they use a non-parametric correction of the autocorrelation and allow for very general conditions on the error structure [see Perron and Phillips (1988), for example]. They prove that their test statistics incorporating the non-parametric correction are transformations of the D — F ones, and converge to the same limiting distributions.

To test $H_0{:}(c, \phi, \beta) = (c, 1, 0)$ in the (1.9′) model the P — P test statistics are:

$$Z(\hat{\phi}_{\tilde{\tau}}) = n(\hat{\phi}_{\tilde{\tau}} - 1) - \left(\frac{n^6}{24D_x}\right)(S^2_{nl} - S^2_\varepsilon) \qquad \textbf{(1.13)}$$

$$Z(\hat{\tau}_\tau) = \left(\frac{S_\varepsilon}{S_{nl}}\right)\hat{\tau}_\tau - \left(\frac{n^3}{4\sqrt{3D_x}\,S_{nl}}\right)(S^2_{nl} - S^2_\varepsilon) \qquad \textbf{(1.14)}$$

$$Z(\Phi_3) = \left(\frac{S_\varepsilon}{S_{nl}}\right)^2\Phi_3 - \left(\frac{1}{2S^2_{nl}}\right)(S^2_{nl} - S^2_\varepsilon)\left[n(\hat{\phi}_{\tilde{\tau}} - 1) - \left(\frac{n^6}{48D_x}\right)(S^2_{nl} - S^2_\varepsilon)\right] \qquad \textbf{(1.15)}$$

where D_x is the determinant of $X'X$ under H_1, S^2_ε and S^2_{nl} are consistent estimators of σ^2_ε and σ^2, and $\hat{\tau}_\mu$ and Φ_3 are defined as in D — F. S^2_{nl} is the non-parametric counterpart of the autoregressive correction of D — F. The complete list of the P — P test statistics for the three alternatives can be found in Perron (1988).

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(9) Many macroeconomic time series are best modelled as MA or ARMA models. Said and Dickey (1984 and 1985) developed tests of the unit root hypothesis in ARMA (p, q) models. In the (1984) paper they show that ARMA (p, q) models can be well approximated by a pure AR (l) with $l = O(n^{1/3})$. In the (1985) paper they test an ARMA (p, 1, q) model against an ARMA ($p+1$, q) alternative.

The consistent estimation of σ^2 proves to be rather complicated. Phillips suggests the use of the Newey and West consistent and non-negative (by construction) estimator of the variance, which takes the form

$$S_{nl}^2 = \frac{1}{n} \sum_{t=1}^{n} \hat{\varepsilon}_t^2 + \frac{2}{n} \sum_{\tau=1}^{l} \omega_{\tau l} \sum_{t=\tau+1}^{n} \hat{\varepsilon}_t \hat{\varepsilon}_{t-\tau} \quad \textbf{(1.16)}$$

where l is the truncation lag parameter, $\omega_{\tau l} = 1 - \tau/(l+1)$ is the Bartlett window, and $\hat{\varepsilon}_t$ are the estimated residuals under the relevant alternative.

Perron (1988) concludes that in general the properties of the estimators are more influenced by the choice of the truncation lag than by the choice of the window (10) and gives clues to the choice of l. Furthermore, taking into account the poor power of these tests, he proposes a test strategy (11) that we will follow in our empirical application. In the published empirical studies, the P — P tests generally confirm the results of the D — F tests (non-rejection of H_0).

Hall tests

When the errors are generated by a MA process, the P — P tests have particular low power [see Schwert (1987)]. Hall (1989) suggests new tests using the same general conditions as P — P for the errors structure, and the Instrumental Variables method of estimation (IV).

Let Y_t be a time series generated by

$$Y_t = \phi Y_{t-1} + \varepsilon_t \; (\phi = 1) \quad \textbf{(1.17)}$$

and

$$\varepsilon_t = u_t - \theta_1 u_{t-1} - \ldots - \theta_q u_{t-q} \quad \textbf{(1.18)}$$

where an intercept can be included in (1.18), and the models (1.7)-(1.9′) are estimated consistently using the IV with instruments Y_{t-k} for Y_{t-1} $(k > q)$. Hall takes $k = q + 1$, where q is the order of the MA model in (1.18) identified by the Box-Jenkins Methodology, but advices a conservative choice of k, in order to avoid any correlation between Y_{t-k} and ε_t. Next, he proves that the limiting distributions of $n(\hat{\phi}_{IV} - 1)$, $n(\hat{\phi}_{\mu IV} - 1)$ and $n(\hat{\phi}_{\tau IV} - 1)$ are the ones tabulated

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(10) Stock and Watson (1986) suggest a test also making a non-parametric correction and using the Tukey-Hanning window. However, the test is only valid for null drift series which strongly limits its popularity.

(11) Start with the estimation of model (1.9′) and use $Z(\hat{\phi}_\tau)$, $Z(\hat{\tau}_\tau)$ and $Z(\Phi_3)$ (only the last one is invariant in β) to test H_0. If we reject H_0, the test is over; otherwise, providing $Z(\Phi_2)$ suggests that $c = 0$, we use the test statistics based on model (1.8) — $Z(\hat{\phi}_\mu)$, $Z(\hat{\tau}_\mu)$ and $Z(\Phi_1)$ — which have more power than the previous ones but are not invariant in c.

by Dickey whereas the $\hat{\tau}$ statistics are transformations of the D — F statistics. For models (1.8) and (1.9′) we have:

$$H(\hat{\tau}_\mu) = \left(\frac{S_\varepsilon}{S}\right) \hat{\tau}_\mu \qquad \textbf{(1.19)}$$

$$H(\hat{\tau}_\tau) = \left(\frac{S_\varepsilon}{S}\right) \hat{\tau}_\tau \qquad \textbf{(1.20)}$$

where S_ε^2 and S^2 are consistent estimators of σ_ε^2 e σ^2. Hall defines them as

$$S_\varepsilon^2 = \left(\sum_{t=0}^{q} \hat{\theta}_t^2\right) S_u^2 \qquad \textbf{(1.21)}$$

where S_u^2 is a consistent estimator of the variance of u_t and

$$S^2 = S_\varepsilon^2 + 2\left(\sum_{j=1}^{q} \sum_{t=0}^{q-j} \hat{\theta}_{j+t}\, \hat{\theta}_t\right) S_u^2 \qquad \textbf{(1.22)}$$

By simulation, Hall concludes that his tests improve substantially the power of the P — P tests for the IMA (1,1), in particular for positive parameters. When $\theta = 0$, they are inferior, thus suggesting a complementary utilization of these tests for the unit root hypothesis.

Solo test

For testing the unit root hypothesis in ARMA models, Solo (1984) deduces a Lagrange Multiplier test (LM) defining the alternative as an ARMA $(p+1,\ q)$ stationary model. Under the null, from the minimisation of the Lagrangean, Solo obtains

$$X_{t-1} = -\frac{\delta \varepsilon_t}{\delta \alpha_1} = -\frac{\delta \varepsilon_t}{\delta \phi} = [\theta(B)]^{-1} Y_{t-1} \qquad \textbf{(1.23)}$$

To perform the LM test, we adjust an ARMA $(p,\ q)$ to the differenced series (i. e., under H_o) and estimate (1.23) using the Box-Jenkins method [see Box and Jenkins (1976)]. We then regress $\hat{\varepsilon}_t$ on $\hat{X}_t$, thus obtaining the auxiliary regression of this test. The test statistic is the usual nR^2, where R^2 is the coefficient of determination of the auxiliary regression. Its limiting distribution, as Solo shows, is $\hat{\tau}^2$ (the square of the $\hat{\tau}$ distribution) (12) and has a bilateral critical region.

As Dickey, Bell and Miller (1986) admit, this test is much easier to implement than the other tests for unit roots in ARMA models. However, its power remains unknown.

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(12) As macroeconomic time series often have a non-null mean, we should substract it in (1.23). The limiting distribution of nR^2 becomes $\hat{\tau}_\mu^2$.

We consider a final test for the existence of a multiple unit root in the series. This test was proposed by Dickey and Pantula (1987) (hereafter D — P) after realizing that when the series contains a multiple unit root, the existing tests tend to reject the null at a higher level of significance than α (13). On the contrary, their test has a probability of rejection of the null that converges to the significance level as $n \to \infty$. The strategy is the following: never test the existence of a unit root before having tested, and rejected, the existence of a double unit root, i. e., «the order of testing should begin with the highest (practical) degree of differencing and work down toward a test on the series level», exactly the opposite of what is usually done.

To test a triple unit root in an AR (3), we define $Z_t = \nabla Y_t$, $W_t = \nabla^2 Y_t$, and $X_t = \nabla^3 Y_t$ which allow us to rewrite the AR (3) as

$$Y_t = \xi_1 Y_{t-1} + \xi_2 Z_{t-1} + \xi_3 W_{t-1} + \varepsilon \qquad \textbf{(1.24)}$$

Denoting the usual t statistic to test $\xi_i = 0$ as $t_i(3)$, $i = 1, 2, 3$, D — P show that an appropriate consistent test is obtained considering the t statistic $t^*_3(3)$ in the regression of Y_t, only on W_{t-1} where, implicitly, $\xi_1 = \xi_2 = 0$. Then, if $t^*_3(3) \leq \hat{\tau}$, we reject the null of a triple unit root. To test the null of a double unit root, we regress X_t *on* Z_{t-1} and W_{t-1}; if $t^*_2(3) \leq \hat{\tau}$ in addition to $t^*_3(3) \leq \hat{\tau}$, we reject it. Finally, to test a single unit root against a stationary alternative, we regress X_t *on* Z_{t-1}, W_{t-1} and Y_{t-1} and reject it if $t^*_i(3) \leq \hat{\tau}$, $i = 1, 2, 3$. D — P prove that the limiting distribution of t^* is $\hat{\tau}$ (14).

The poor power of the tests of the unit root hypothesis, already mentioned, is well known [see, for example, Schwert (1987)]. Perron (1988) shows that when there is a change in the sample period, more observations do not mean a more powerful test; Shiller and Perron (1985) show that the power of the tests of the unit root against both stationary and explosive alternatives depends more on the time interval of the data than on the actual number of observations available. This result is coherent with the nature of the phenomenon under study — trend reversion — which, if it does, occurs in the long run.

1.3 — Recent developments

We must take into account that long series are bound to include trand breaks (possibilities are 1929 — the Great Crash — and 1973 — the oil price shock). Rappoport and Reichlin (1989) define a class of processes with a Segmented Trand (ST) which moves away from the traditional dichotomy TS/DS

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(13) Hasza and Fuller (1979) develop a test for a double unit root, which behaves the same way when the series contain a triple unit root.

(14) The inclusion of an intercept in the various regressions implies that this limiting distribution is $\hat{\tau}_\mu$.

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processes by the consideration of a Trend that changes irregularly and infrequently over time, and is well characterized by different segments limited by breaks. Perron (1989) introduces dummies for the modelling of the presence of a one-time change in the level or in the slope of the trand caused by the shocks, taken as exogenous ([15]) and not as a realization of the underlying data-generating mechanism.

Perron defines three different models: the «crash model» (model A) where the null is characterized by a dummy wich takes the value one at the time of the break; the «changing growth model» (model B) where, under the alternative, a change in the slope of the trend function without any sudden change in the level is allowed at the time of break; and model C which allows for both effects, i. e., a sudden change in the level followed by a different growth path of the trend. To test these three models, Perron uses three regressions that extend directly from the D — F test procedures, and are constructed nesting the null and the alternative hypothesis ([16]):

$$A: Y_t = \hat{\mu}^A + \hat{\theta}^A\ DU_t + \beta^A t + \hat{d}^A D(TB)_t + \hat{\alpha}^A Y_{t-1} + \sum_{i=1}^{k} \hat{c}_i \nabla Y_{t-i} + \varepsilon_t \qquad \textbf{(1.25)}$$

$$B: Y_t = \hat{\mu}^B + \beta^B\ t + \hat{\gamma}^B DT^*_t + \hat{\alpha}^B Y_{t-1} + \sum_{i=1}^{k} \hat{c}_i \nabla Y_{t-i} + \varepsilon_t \qquad \textbf{(1.26)}$$

$$C: Y_t = \hat{\mu}^C + \hat{\theta}^A\ DU_t + \beta^C t + \hat{\gamma}^C DT_t + \hat{d}^C D(TB)_t + \hat{\alpha}^C Y_{t-1} + \sum_{i=1}^{k} \hat{c}_i \nabla Y_{t-i} + \varepsilon_t \qquad \textbf{(1.27)}$$

where TB is the time of the break, and the dummies are defined as

$DU_t = 1$, $DT_t = t$ and $DT^*_t = t - TB$, if $t > TB$
$D(TB)_t = 1$, if $t = TB + 1$ 0 , otherwise

Under the null, we have for the three models $\alpha^A = \alpha^B = \alpha^C = 1$, $\beta^A = \beta^B = \beta^C = 0$, $\theta^A = \theta^C = 0$, $\gamma^B = \gamma^C = 0$, and d^A, d^C substantially different from zero. Under the alternative we have $\alpha^A, \alpha^B, \alpha^C < 1$, β^A , β^B , $\beta^C \neq 0$, $\theta^A, \theta^C \neq 0$, $\gamma^B, \gamma^C \neq 0$, and d^A, d^C close to zero. Via simulation, Perron has calculated the critical values for $\hat{\tau}$ the three models and for different values of $\lambda = TB/n$ (though they do not seem to vary much with λ). These critical values are larger (in absolute value and considering the left tail) than the Dickey-Fuller ones. The application of these tests leads to the rejection of the unit root null in most US macroeconomic time series.

([15]) «The exogeneity assumption is not a statement about a descriptive model for the time series representation of the variables. It is used here as a device to remove the influence of these shocks from the noise function [. . .] into the trend function without specific modeling of the stochastic nature of the behaviour of the constant term and slope of the trand.» [Perron (1989)].

([16]) To choose the value of k, the number of extra lags in the ADF type of models (1.25)–(1.27), Perron takes a «liberal» procedure. He choses $k = k^*$ if the t statistic on $\hat{c}_t$ is greater than 1.60 in absolute value, and the t statistic on $\hat{c}_t$ is less than 1.60, with a maximum of $k = 8$.

Christiano (1988) is very skeptical about this rejection and shows that it is spurious and due to the non-consideration of the use of «a priori» information in the selection of the hypothesized date for the break in the critical values of the test statistics. Using bootstrap methods, Christiano shows that there is no statistical evidence for a trend break in the post-war quarterly USA GNP. Andrade and Proença (1990) discuss the evidence for the Portuguese GDP.

Diebold and Rudebusch (1989) define the ARFIMA (Fraction Integrated ARMA) models through the generalization of the FARMA models of Hosking, and Granger and Joyeux, where the degree d of differentiation of a series is defined as a real number. Studying the USA GNP, they conclude that the usual $d=0$ or $d=1$ values implicit in the unit root tests are too restrictive, and estimate $d=0.7$ for that annual series.

The most influential development in this area is probably the definition of the Co-integrated Models of Engle and Granger (1987) and the test of common trends among time series.

An alternative and totally different approach to the decomposition of a series into Trend and Cycle is developed by Harvey (1985), Watson (1986), and Clark (1987) using Unobserved Components models. Their results are highly contradictory to those obtained with the Beveridge-Nelson Decomposition, concluding, in general, that the series are trend-reverting and that the shocks dissipate rather quickly. Another approach is proposed by Campbell and Mankiw (1987), and Cochrane (1988). They define measures of the persistence (both parametric and non-parametric) of the shocks to the output, and, thus, are able to estimate the importance of the permanent component in the output.

2 — Trend and Cycle in the Portuguese output

2.1 — Stochastic trend

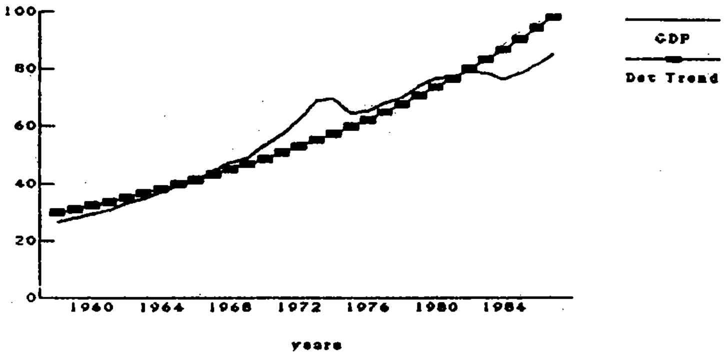
We study the Portuguese real annual per capita GDP 1958-1987 (17) (18). To start with, we perform the traditional decomposition of the output in its deterministic Trend and Cycle. We regress the (logarithm) real GDP on an intercept and time:

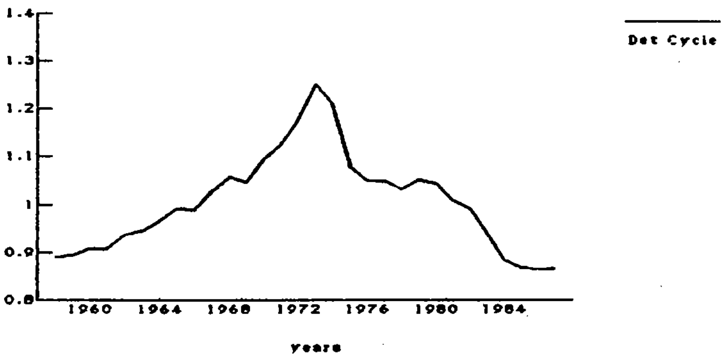
$$\hat{Y}_t = 3.352{,}339 + 0.041{,}145{,}55\, t$$
$$t \quad 87.693{,}99 \quad 19.107{,}98$$
$$R^2 = 0.928{,}774 \quad DW = 0.1304$$

(17) Real GDP is the most comprehensive measure of the macroeconomy. Nevertheless we focus our attention on real per capita GDP because the movements induced in the aggregate output by a varying population may obscure the persistence intrinsic to the economy.

(18) For the period 1958-1985, we use the series of annual GDP at 1977 prices of Cartaxo and Rosa (1986) updated with the growth rates in volume published in the annual reports of the Banco de Portugal. For the population, we use the series of Cónin (1979) and Carrilho (1985) updated with values taken from «Estatísticas Demográficas» and «Anuários Estatísticos» of INE.

which is a spurious regression. The Trend is defined as $\hat{Y}_t = 3.352,339 + 0.041,145,55\, t$, a deterministic function of time, and the Cycle as the series of the estimated residuals of the regression. In the figures (original non-logarithmised values) we see the exponential trend around which the real GDP oscillates, and a «hump shaped» (non-stationary) Cycle which incorporates all the fluctuations of the GDP:





The application of the Beveridge-Nelson decomposition and the identification of the stochastic Trend and stationary Cycle is much more complex. Using the Box-Jenkins Methodology and the Automatic Criteria ([19]), we identified an MA (1) as the generating process of the first differences of the real per capita GDP, i. e., an ARIMA (0, 1, 1) for the level of the series [see Andrade (1990)

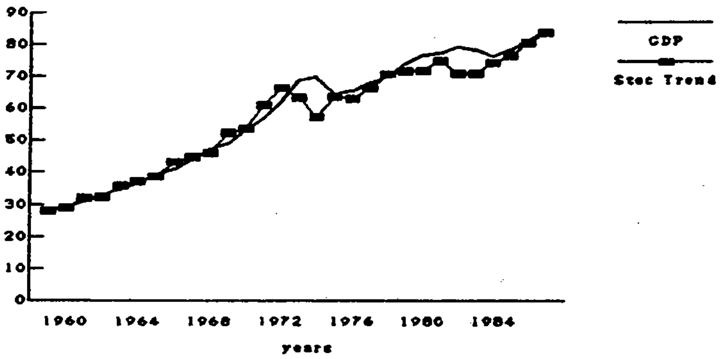
([19]) We used the criteria defined by Akaike (AIC and BIC), Schwartz (SC), Hurvich and Tsai (AICc), and Geweke and Meese (BEC).

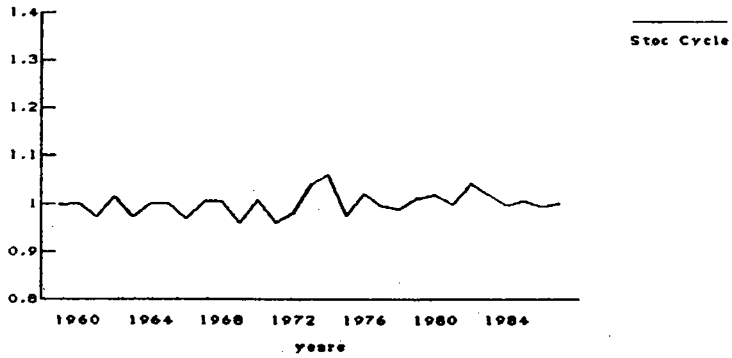
for details]. The estimation of the MA (1) by the Box-Jenkins method of estimation produced

$$\hat{X}_t = 0.040,100,83 + \hat{\varepsilon}_t + 0.788,818\ \varepsilon_{t-1}$$
$$\quad 4.005,749 \qquad\qquad 7.340,924 \qquad\qquad Q(7) = 10.1972$$

(t statistics under the estimates) which is stationary and invertible: the root of the characteristic polynomial, $B = 1.267,719,5$, is outside the unit circle. Since the estimated AR (1) is still close to the MA (1), we tested the two models against each other using a non-nested models test developed by McAleer, McKenzie and Hall (1988) which uses auxiliary regressions to calculate the test statistics (very similar to the ones used in the Lagrange Multiplier tests). We are not able to reject one of the models at the 5 % level, but at 25 % we are able to reject the AR (1) against the null MA (1). We also checked the properties of the estimated residuals of the MA model. We could neither reject their normality at 5 % level using the skewness and kurtosis measures [tables for small samples in White and MacDonald (1980)], nor their homoscedasticity using the ARCH test of the residuals developed by Engle (1982).

The application of the Beveridge-Nelson decomposition is now simple. The stochastic Trend is given by $\overline{Y}_t = \overline{Y}_{t-1} + 0.040,100,83 + 1.788,818\ \hat{\varepsilon}_t$, a random walk with drift, and the Cycle is $\hat{C}_t = -0.788,818\ \hat{\varepsilon}_t$, a white noise:





A substantial part of the fluctuations of the data is affected to the Trend, whereas the Cycle oscillates very little around its mean.

2.2 — Unit root tests

We now apply the unit root tests to confirm the existence of a unit root in the data, and therefore, the stochastic Trend we have estimated. We start with the D — F tests (1979 and 1981). We employ the test h of Durbin [20] to determine the number s of lags to be included in the ADF model:

	$s=0$	$s=1$
$\hat{\Phi}$	0.954,78	0.961,30
$n(\hat{\Phi}_\mu - 1)$	—1.311,49	-
$\hat{\tau}_\mu$	—2.671,43	—2.029,84
h	1.689,56	1.291,46
$\hat{\Phi}$	0.959,92	0.912,28
$n(\hat{\Phi}_\tau - 1)$	—1.162,44	-
$\hat{\tau}_\tau$	—0.609,77	—1.302,90
h	1.839,18	1.010,22
Φ_3	3.307,94	2.313,91
Φ_2	2.599,56	3.514,97
Φ_1	0.155,81	0.253,56

We can never reject H_0 at the 5 % level. The estimates of the intercept and the time trend (not included) are never significant also at 5 %. The non-rejection of H_0 in Φ_2 test means that we cannot reject the joint hypothesis $(c,\phi,\beta) = (0, 1, 0)$. Next, we implement the P — P tests considering $l = 0, \ldots, 3$ and the Bartlett window:

	$l=0$	$l=1$	$l=2$	$l=3$
$Z(\hat{\Phi}_\tau)$	—0.974,24	—1.469,09	—1.469,51	—1.391,23
$Z(\hat{\tau}_\tau)$	—0.570,08	—0.712,79	—0.712,93	—0.687,75
$Z(\Phi_3)$	3.627,99	3.154,97	2.925,25	3.009,73
$Z(\Phi_2)$	2.858,42	2.468,04	2.287,50	2.356,71
$Z(\hat{\phi}_\mu)$	—1.297,44	—1.334,89	—1.335,61	—1.330,44
$Z(\hat{\tau}_\mu)$	—2.509,91	—3.056,64	—3.069,29	—2.977,74
$Z(\hat{\Phi}_1)$	0.098,12	0.237,54	0.239,74	0.223,48

The conclusion is the same — non rejection of H_0 — for any test statistic and truncation lag l. Since we do not reject H_0 using the test statistic

(20) The h statistic has an asymptotic normal distribution when we test H_0: no first order autocorrelation in the residuals against H_1: positive or negative autocorrelation.

$Z(\Phi_2)$, indicating that $c = 0$, we can use the more powerful test statistics based on model (1.8). The Hall tests confirm the non-rejection of H_0:

	$q = 1$	$q = 2$
$n(\hat{\phi}_{\mu IV} - 1)$	— 1.509,70	— 1.545,08
$H(\hat{\tau}_\mu)$	— 2.095,93	— 1.962,07
$n(\hat{\phi}_{\tau IV} - 1)$	— 2.343,81	— 2.637,32
$H(\hat{\tau}_\tau)$	— 0.941,89	— 0.792,18

as well as the Solo test, where we have $nR^2 = 1.077,05$ for the MA (1) null.

We now check that the GDP contains a single unit root with the D-P test. First, we reject a triple and a double unit root at 5 % level because $t^*_3(3) = — 5.934,19$ and $t^*_2(3) = — 3.029,93$; next we cannot reject a simple unit root, as before, because $t^*_1(3) = — 2.236,47$.

Finally, we perform the trend break test of Perron (1989) considering the trend break, TB, at 1974 (and not 1973 as Perron), simultaneously close to the date of the oil price shock and the year of the Portuguese Revolution, and the «changing growth» model (1.26). The estimation of this model produce

$$\hat{Y}_t = 2.803,5 + 0.054,141\, t — 0.038,386\, DT^*_t + 0.110,84\, Y_{t-1} + 0.598,53\, \nabla Y_{t-1}$$
$$\quad 4.939,7 \quad 4.647,9 \qquad —4.535,5$$

where $\hat{\tau} = —4.842,08$ allows us to reject the null [the critical values for $\lambda = 0.6$ are —3.95 at 5 % level and —4.57 at 1 %, from table V.B in Perron (1989)]. Also, we have $\beta \neq 0$, $\gamma \neq 0$ highly significant. This result means that our previous non-rejection of the null was probably due to confusion between a one time innovation, the break trend, and the yearly innovations, making these seem more lasting than they actually are. However, as Perron admits, the rejection of the null does not mean that the trend function including its changes are deterministic, but rather that the timing of the occurrence of the shocks is rare relative to the sequence of innovations.

3 — Conclusion

In this study we cannot reject that the Portuguese output in the period 1958-1987 contains a unit root using the Dickey and Fuller, Perron and Phillips, Hall, Solo, and Dickey and Pantula tests. According to Nelson and Plosser, and now using the Beveridge-Nelson decomposition, this means that the Portuguese output has a stochastic Trend and a small stationary Cycle, and that we can reject its traditional deterministic decomposition, and the hump shaped Cycle which had been taken as «one of the few undisputed facts in macroeconomics» [Blanchard (1981)] for a long time. Nevertheless, these conclusions must be taken carefully. First of all, it is very difficult to distinguish between trend-reverting series and series with a stochastic trend based on finite samples. Second, we may be ignoring a trend break and take its permanent effect as if it were provoked by the yearly innovations, as Perron recent unit root test points out. We should also take into consideration that univariate models have a limited capacity to interpret reality.

The economic implications of the existence of the unit root are enormous. Following Nelson and Plosser (1982), we can say that the real shocks drive the output fluctuations, and thus find empirical support for the Real Business Cycle Theory. We can also conclude that the stabilization policies, in particular the «stop and go» policies implemented in Portugal under the IMF supervision during the last years of the period studied, have not driven the output back to its smooth long run trend simply because that does not exist. On the contrary, the effects of those policies will be persistent over time, and will affect the structure of the Portuguese economy itself. Finally, the fluctuations of the output are largely structural and the Business Cycle is not very significant.

This work may be extended in two directions. One is to use the same techniques to study more and longer Portuguese macroeconomic time series, namely quarterly series that, so far, are not available. Another, and more decisive one, is to use multivariate models, such as the Co-Integrated models, to study Portuguese macroeconomic time series.

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