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Abstract

Despite their sensitivity to misspecification, parametric binary choice models are widely used in practice. Here we present the results of a small simulation study on the finite sample performance of parametric and semiparametric specification tests for this kind of models.

Keywords: HH-test; Logit; Score tests.

JEL Classification codes: C12, C14, C25, C52.

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1 INTRODUCTION

Parametric binary choice models are widely used in applied econometrics. Examples of application of these models can be found in areas such as travel demand, credit scoring, firm failures and participation in the job market. See Amemiya (1981) for many other examples.

Because parametric binary choice models are generally not robust to misspecification, it is important to test for departures from the null hypothesis. Davidson and MacKinnon (1984) introduced a simple way to construct regression-based score tests of the null against parametric alternatives. More recently, Horowitz and Härdle (1994) have developed a new approach to the construction of specification tests for binary choice models. If under the null the model is of the single index type, then it can be tested against a model with the same index but with an unspecified link function, using simple semiparametric techniques. The attractive feature of the test against the semiparametric alternative is that it may have power against a wider range of departures from the null hypothesis than the simpler score tests. However, the semiparametric test is computationally much more demanding.

In this paper we present the results of a small simulation exercise comparing the finite sample performance of the semiparametric test with that of several specification tests based on the more traditional score test approach. Throughout, the popular logit model is taken as the null hypothesis.

2 MODEL AND TESTS

Consider a binary random variable Y_i such that

$$P\left(Y_i = 1|x_i\right) = E\left(Y_i|x_i\right) = F\left(x_i^T\beta\right) \quad i = 1, \cdots, n$$

where x_i is a vector of covariates, β is a conformable vector of parameters and $F(\cdot)$ is a non-decreasing continuous link function such that $0 \square F(-\infty) < F(\infty) \square 1$. Given $F(\cdot)$, $\hat{\beta}$, the maximum likelihood estimates of β, can be obtained as described in any standard textbook (see for example Davidson and MacKinnon, 1993). In what follows, we concentrate on specification tests to detect departures from the null hypothesis $H_0 : E\left(Y_i|x_i\right) = F_0\left(x_i^T\beta\right) = \left\{1 + \exp\left(-x_i^T\beta\right)\right\}^{-1}$.

2.1 Score tests

A simple way to check the correct specification of $E(Y_i|x_i)$ is to test the null against a more general model that has $F_0\left(x_i^T\beta\right)$ as a special case. In this paper we consider score tests against the three following generalizations of the logit model:

a) $E(Y_i|x_i) = \left\{1+\exp\left(-\,\theta\left(\rho x_i^T\beta\right)/\rho\right)\right\}^{-1}$, where ρ is a parameter and $\theta(\cdot)$ is a function such that $\theta(0) = 0$, $\theta'(0) = 1$ and $\theta''(0) \neq 0$ (see Davidson and MacKinnon, 1993, p. 527);

b) $E(Y_i|x_i) = \int\left\{1+\exp\left(-x_i^T\beta-\varepsilon_i\right)\right\}^{-1} g(\varepsilon|x_i)\, d\varepsilon$, where ε_i is a random variable with conditional density $g(\varepsilon|x_i)$ and $E(\varepsilon_i|x_i) = 0$ and $E(\varepsilon_i^2|x_i) = \omega$;

c) $E(Y_i|x_i) = \left\{1+\exp\left(-x_i^T\beta/\sigma\left(z_i^T\delta\right)\right)\right\}^{-1}$, where $\sigma(\cdot) > 0$ is a function such that $\sigma(0) = 1$ and $\sigma'(0) \neq 0$, and z_i denotes a vector containing the i-th observation of all explanatory variables except the intercept.

Tests against these alternatives can be performed as score tests for the omission of certain test variables, as described in Davidson and MacKinnon (1984) and Orme (1988). Specifically, the test for $\rho = 0$ in a) is a RESET-type test of the kind advocated by Pagan and Vella (1989) that checks for the omission of the additional regressor defined by $\left(x_i^T\hat\beta\right)^2$; the test against b) can be interpreted as an information matrix test (Chesher, 1984) for which the test variable is $1-2F_0\left(x_i^T\hat\beta\right)$; finally the score test against c) is a test against an heteroskedastic logit of the type discussed by Davidson and MacKinnon (1984) in which the test regressors are given by $\left(x_i^T\hat\beta\right) z_i$. Under the null, the RESET-type and the information matrix test statistics follow an asymptotic $\chi^2_{(1)}$ distribution, while the statistic for the test against the heteroskedastic logit has an asymptotic $\chi^2_{(k-1)}$ distribution, where k is the dimension of x_i.

2.2 The semiparametric test

The motivation behind the semiparametric test is that the adequacy of the parametric link can be assessed by evaluating the weighted difference between the parametric estimate, $F_0\left(x_i^T\hat\beta\right)$, and the nonparametric estimate of $F(\cdot)$, given $x_i^T\hat\beta$. To perform the semiparametric test in the context of binary choice models, the test procedure of Horowitz and Härdle (1994), also known as the HH–test, can be used. The HH-statistic conveniently standardized is asymptotically distributed as a standard normal variate. However, simulations in Proença (1995) revealed that the distribution of the HH statistic in finite samples is bandwidth dependent and has a negative bias, which affects the performance of the test. These problems led Proença and Ritter (1994) to propose a modified test, the MHH-test, which is less bandwidth dependent than the HH-test, and has the same asymptotic distribution (see Härdle, Mammen and Proença, 1999). Moreover, they have deduced analytical corrections to the bias and variance that make the asymptotic critical values more accurate in finite samples. This corrections make the calculations of the test statistic more burdensome, but the development of computational capacities makes it possible to apply the MHH-test to problems with reasonably large data sets.

The MHH–statistic is based on $T_n = \sqrt{h}\hat{r}^T W \hat{r}$, where $\hat{r}$ is a vector of residuals with typical element $\hat{r}_i = Y_i - F_0\left(x_i^T\hat{\beta}\right)$ and W is a $n \times n$ matrix, the smoothing matrix, corresponding to leave-one-out kernel smoothing of the residuals with bandwidth h. The elements of W are defined by

$$w_{ij} = \begin{cases} \frac{K\left[\left(x_j^T\hat{\beta} - x_i^T\hat{\beta}\right)/h\right]}{\sum_{j\neq i} K\left[\left(x_j^T\hat{\beta} - x_i^T\hat{\beta}\right)/h\right]} u\left(x_i^T\hat{\beta}\right) & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$

where $u\left(x_i^T\hat{\beta}\right)$ is a weight function that equals 1 for the 90% central values of the ordered $x_i^T\hat{\beta}$, being zero otherwise.[1]

Let X denote the design matrix obtained by stacking the vectors x_i^T, and define $D = (I - H)^T W (I - H)$, where $H = VX\left(X^T V X\right)^{-1} X^T$, and V is a diagonal matrix with diagonal elements defined as $v_i = F_0\left(x_i^T\hat{\beta}\right)\left[1 - F_0\left(x_i^T\hat{\beta}\right)\right]$. Denoting by d_i the i-th diagonal element of D, the estimates for the bias correction and corrected variance, when the null is the logit, are given respectively by $bc = \sqrt{h}\ \sum_{i=1}^n d_i v_i$ and $\hat{\sigma}_n^2 = 2\,\mathrm{tr}\,(DVDV) + \sum_{i=1}^n d_i^2\left[v_i - 6v_i^2\right]$. Finally, the test statistic can be computed as $T_n^{MHH} = (T_n - bc)/\sqrt{\hat{\sigma}_n^2}$, which under the null has an asymptotic standard normal distribution.

[1]Different cut-off points can be used for these weights (see Horowitz and Härdle, 1994).

3 SIMULATION RESULTS

In this section we present the results of a small simulation study comparing the empirical size and power of the score and semiparametric specification tests described above. The null hypothesis is the logit model. Besides $F_0\left(x_i^T\beta\right)$, three other link functions were used to generate the data:

$$F_1\left(x_i^T\beta\right) = 1 - \exp\left\{-\exp\left(x_i^T\beta\right)\right\}, \quad (1)$$

$$F_2\left(x_i^T\beta\right) = \left\{1 + \exp(-x_i^T\beta)\right\}^{-1} - \frac{5(x_i^T\beta)}{9}\phi\left(\frac{(x_i^T\beta)}{1.5}\right), \quad (2)$$

$$F_3\left(x_i^T\beta\right) = \left\{1 + \exp\left(-x_i^T\beta/\sqrt{0.5 + |x_i^T\beta|}\right)\right\}^{-1}, \quad (3)$$

where $\phi(\cdot)$ denotes the standard normal density. Model (1) is the classic complementary log-log. Model (2) is a logit model perturbed by a bump. The bump was chosen so that the conditional expectation of Y_i is non-decreasing with $x_i^T\beta$. Finally, model (3) is a logit with heteroskedasticity. When data are generated by $F_0\left(x_i^T\beta\right)$ the empirical size of the tests is assessed, while for data generated by the other models the percentage of rejections gives the empirical power of the tests.

For all experiments the index function was assumed to be $1 + x_{i1} + x_{i2}$, where x_{i1} comes from a standard normal and x_{i2} from a Bernoulli distribution with parameter 0.75. The response was generated from a Bernoulli distribution with probability of success given by the four models described above. The variables were generated using the same random seed for each model and the regressors were newly drawn in each replication. The sample sizes considered were 250, 500 and 750. For each experiment 5000 replications were performed.

The score tests were computed using the artificial regression approach suggested by Davidson and MacKinnon (1984). In the semiparametric test the bandwidth for kernel estimation was set to $h = cn^{-0.2}$ with $c = 0.6$, $c = 1.5$ and $c = 2.4$. These choices were made after a graphical inspection of the kernel regression estimate. The larger h considered slightly over-smooths the data whereas the smaller h somewhat under-smooths it. The semiparametric test was performed using a one-sided critical region of the type $\left\{T_n^{MHH} > z_\alpha\right\}$, where z_α is the $(1-\alpha)$ percentile of the standard normal (see Horowitz and Härdle, 1994). Tables 1 and 2 give the percentage of rejections of the null at the nominal 5 and 10% levels, respectively.

Table 1: Percentage of rejections at the nominal 5% level

Sample size	True model	Score tests against: a)	b)	c)	Semiparametric test $c=0.6$	$c=1.5$	$c=2.4$
250	$F_0\left(x_i^T\beta\right)$	0.0394	0.0502	0.0466	0.0648	0.0696	0.0686
	$F_1\left(x_i^T\beta\right)$	0.5396	0.8706	0.8276	0.2484	0.3570	0.4422
	$F_2\left(x_i^T\beta\right)$	0.2504	0.2308	0.2328	0.1668	0.2474	0.2606
	$F_3\left(x_i^T\beta\right)$	0.1414	0.1410	0.1208	0.1028	0.1372	0.1290
500	$F_0\left(x_i^T\beta\right)$	0.0414	0.0458	0.0466	0.0684	0.0692	0.0644
	$F_1\left(x_i^T\beta\right)$	0.7408	0.9952	0.9854	0.3366	0.5468	0.7174
	$F_2\left(x_i^T\beta\right)$	0.4896	0.4842	0.4440	0.2630	0.4072	0.4862
	$F_3\left(x_i^T\beta\right)$	0.2298	0.2446	0.2008	0.1356	0.2074	0.2494
750	$F_0\left(x_i^T\beta\right)$	0.0488	0.0464	0.0506	0.0646	0.0642	0.0644
	$F_1\left(x_i^T\beta\right)$	0.9314	1.0000	0.9996	0.4118	0.6936	0.8646
	$F_2\left(x_i^T\beta\right)$	0.6814	0.6806	0.6344	0.3512	0.5528	0.6572
	$F_3\left(x_i^T\beta\right)$	0.3254	0.3450	0.2906	0.1578	0.2712	0.3456

Table 2: Percentage of rejections at the nominal 10% level

Sample size	True model	Score tests against: a)	b)	c)	Semiparametric test $c=0.6$	$c=1.5$	$c=2.4$
250	$F_0\left(x_i^T\beta\right)$	0.0738	0.1022	0.0984	0.1018	0.1042	0.0954
	$F_1\left(x_i^T\beta\right)$	0.6390	0.9436	0.9028	0.3038	0.4226	0.5258
	$F_2\left(x_i^T\beta\right)$	0.3834	0.3642	0.3508	0.2236	0.3060	0.3174
	$F_3\left(x_i^T\beta\right)$	0.2106	0.2176	0.2028	0.1458	0.1816	0.1750
500	$F_0\left(x_i^T\beta\right)$	0.0782	0.1006	0.0924	0.1038	0.1020	0.0942
	$F_1\left(x_i^T\beta\right)$	0.8752	0.9986	0.9952	0.4138	0.6286	0.7860
	$F_2\left(x_i^T\beta\right)$	0.6370	0.6260	0.5762	0.3352	0.4738	0.5576
	$F_3\left(x_i^T\beta\right)$	0.3284	0.3424	0.2954	0.1844	0.2642	0.3064
750	$F_0\left(x_i^T\beta\right)$	0.0898	0.0990	0.1024	0.1010	0.0936	0.0932
	$F_1\left(x_i^T\beta\right)$	0.9882	1.0000	1.0000	0.5052	0.7744	0.9112
	$F_2\left(x_i^T\beta\right)$	0.7992	0.7954	0.7472	0.4314	0.6148	0.7084
	$F_3\left(x_i^T\beta\right)$	0.4310	0.4550	0.4040	0.2198	0.3298	0.4100

Under the null, all tests perform reasonably well even in samples of size 250, but the semiparametric test has a slight tendency to overreject at the 5% level. Under-smoothing in the semiparametric test clearly damages its power and some over-smoothing appears to be desirable. However, in these experiments, the semiparametric test is never clearly superior to the score tests, and most of the times it is out-performed by them. It is interesting to notice that the capacity to detect this form of heteroskedasticity is relatively poor for all procedures, even in samples with 750 observations. On the other hand, the tests revealed good performance when the alternative is the complementary log-log, even for samples with size 250.

4 CONCLUSIONS

In the cases considered here, there is no clear superiority of one test procedure over its competitors. Moreover, although constructed with a specific alternative in mind, the score tests studied here are powerful against a wide range of alternatives and are not systematically outperformed by the semiparametric test. In view of these results, the computational burden of the semiparametric test is hard to justify, at least for routine use.

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