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High-speed rail transport valuation

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In this paper, the optimal timing for investing in high-speed rail projects under uncertainty in relation to the utility provided to railway users was investigated. To accomplish this, a continuous time real options analysis framework using a stochastic demand model was developed to determine the optimal time to invest. Uncertainty upon investment expenditures was also added in an extended framework. The value of the option to defer and the investment opportunity value were also assessed.

Keywords: real options; uncertainty; timing; waiting; investment; high-speed rail

JEL Classification: D81; D83; D92

1. Introduction

Under uncertainty, it is important for a firm to be flexible with the products it is creating, to accommodate for technological changes and competition shifts. Flexibility is also crucial to limit potential losses related to unexpected adverse scenarios (Trigeorgis 1996).

Real option analysis (ROA) emerged in the academy to value investments in real assets under uncertainty (Brennan and Schwartz 1985; McDonald and Siegel 1986; Dixit 1989; Pindyck 1991; and Dixit and Pindyck 1994, amongst others). Nowadays ROA has already made an impact in the business world, since an increasing number of companies and managers are adopting a real options perspective. This new paradigm has been used in the area of capital budgeting analysis and in the assessment of strategic positioning and competitiveness (Paddock, Siegel and Smith 1988; Nichols 1994; Kallberg and Laurin 1997; Moel and Tufano 2002; Smit 2003).

Transport infrastructures are critical for sustainable growth and the development of an economy. According to Wilson (1986), since the year 1870, economists have drawn their attention to transportation, where rail transport assumes an important role. The same author suggested that poor transportation policies and investment mistakes in transport infrastructures may compromise economic growth. To prevent this compromise in growth, it is important to develop and apply suitable decision criteria based on a thorough cost/benefit analysis.

Infrastructure investments, such as in seaports, airports, railways, energy networks, and road systems, have provided huge economic benefits and have leveraged economic growth. The size, budget and impact of these investments on the global economic activity led transportation investments to assume the role of strategic options. Almost all transportation investments include a portfolio of options to protect the enormous funds required to implement the investments.

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Rose (1998) valued the concession of a toll road in Australia, considering the existence of two options interacting with each other. Similarly, Brandão (2002) applied the Copeland and Antikarov (2001) framework to value options embedded in a highway investment project in Brazil. Two other empirical ROA studies focused on the valuation of structural investments in the transportation sector: Smit (2003) and Bowe and Lee (2004). The first analyzed the expansion of a European airport. Smit (2003) combined ROA and game theory, in a discrete time framework, to fill a gap in the real options literature regarding competition effects. The second analyzed a railway transportation investment. Bowe and Lee (2004) applied binomial analysis to evaluate high-speed train investment in Taiwan.

The valuation framework proposed in this paper was inspired by a set of investments in high-speed rail (HSR) across Europe. The structural nature of the HSR investments for the countries involved; the need to renew the railway sector; the huge amounts of money required; the uncertainty about the timings to invest; and the economic challenge inherent in developing a conceptual setting for a decision that needs to consider the interest of all European taxpayers, all play a part in providing relevance to the study of the ideal time to invest and the embedded options to be deferred.

The framework proposed draws on the work of Salahaldin and Granger (2005) on the valuation of sustainable systems of urban transport aimed at relieving air pollution. Like their framework, ours will be comprised of a unique change, from an inactive to an active state; it will also consider a single stochastic variable, extended afterwards to two uncertainty factors. However, our framework is distinct, because it incorporates the time to build, combining it with the benefits of travel time saved. In addition, fixed and variable operating costs will be incorporated.

Transportation investment analysis rarely incorporates real option theory. As a result, this paper will introduce the transportation investment analysis of the HSR investment valuation in continuous time, providing some closed form solutions. Although Pereira *et al.* (2006) studied these issues, their work focused on airport construction. Our ROA framework will support the utility balance for the user between different rail speed services.

Compared to classical works in the economic literature, such as McDonald and Siegel (1986), we will extend the methodology regarding closed form solutions for the value of waiting to invest. We also broaden the applications of ROA to HSR investments and incorporate issues related to elasticity in ROA frameworks. We hope insights provided by this new valuation framework will be useful in many areas of the transportation industry. For example, it may help improve the investment flexibility in order to reduce the delay in the optimal time to invest.

The rest of the paper will be structured as follows. In Section 2, we develop the valuation framework. In Section 3, we provide the numerical results. In Section 4 uncertainty upon investment expenditures is added in an extended framework. The paper's primary conclusions and recommendations for future studies are presented in Section 5.

2. Investment valuation using a real options framework

In HSR investment, the owner of the investment holds the possibility of acquiring the future cash flow generated, at any moment in time, in exchange for the payment of the corresponding implementation costs. Thus, we will be investigating an option to invest. Considering the investment in HSR as an optimal stopping problem permits us to determine the value of the embedded option to defer. Following the work of McDonald and Siegel (1986) and Salahaldin and Granger (2005), our model will allow us to determine the optimal time period to invest. The model considers an *a priori* dimensioned HSR project.

In this paper, we will assume that the option to defer is perpetual in nature ($T = \infty$), but also that, once implemented, the investment will produce perpetual benefits. Without major technological changes, the impact of these assumptions in the global valuation should not be unreasonable for two reasons. First, because the present value of the more remote cash flows tends naturally to zero. Second, because maintenance and conservation expenses tend to restore the operational aptitude of the assets and the corresponding flow of benefits.

2.1 Optimal timing to invest

The decision to implement an investment in a non-optimal way implies the destruction of value. Finding the optimal timing to invest provides us with the possibility to value the ability to delay the project, as well as also its corresponding impact on the investment opportunity value. The optimal timing to invest may be given by a demand threshold supporting, in a rational way, the decision of implementing the investment. Once implemented, the investment expenditures become sunk costs, since there is no other use for railways.

Because investment in infrastructures, like HSR, will affect the economic and social conditions of future generations, it should be assessed in terms of economic welfare. The value per railway user is calculated based on utility theory, namely on the consumption capacity of each user. Investing in a moment other than the optimal timing implies a reduction in the global level of utility achieved by the users, compromising the HSR investment success. In such circumstances, any potential user may maintain his/her current level of utility, choosing to travel on the conventional railway, rather than in the new HSR. If a suboptimal investment timing is chosen, the ability of the HSR to attract users will be strongly distressed.

The framework does not explicitly account for the competition between railway transportation and other alternative transport modes. Implicitly competition effects are incorporated in railway demand stochastic process parameters. Indeed, any user from other alternative transport modes, such as road or air flights, is a potential railway user. However, facing two similar railway travel services, users will decide for HSR if at least utility remains. Note that competition effects from the conventional railway or other alternative transport modes through its fare, frequency or better service improvements should also be implicitly incorporated in the HSR demand stochastic process parameters.

At any moment, users may choose to travel by the conventional railway, without any constraints. Consequently, to maintain the users' level of utility, the fraction of the new investment supported by each one must be identical to the sum of the benefits earned from the reduction in travel time and the conventional railway travel fare saved, net of the variable and fixed operational costs upheld. Given fixed investment expenditures, the higher the demand, the higher the expected net benefit per capita. Consequently, higher levels of demand tend to lead to the anticipation of the optimal invest timings. The main source of uncertainty is derived from the level of future HSR demand.

We will consider that the demand for the new HSR, x_t , follows a geometric Brownian motion process:

$$dx_t = \mu_x x_t dt + \sigma_x x_t dw_t, \quad (1)$$

where μ_x represents the growth rate and σ_x represents the standard deviation of HSR demand. We assume that both parameters are constant in time. The Wiener process, w_t , has a zero mean and standard deviation of $\sigma_x \sqrt{dt}$. Under these circumstances, it is reasonable to expect that, in the future, the HSR demand will reach a level capable of providing a rational reason to invest.

Similar assumptions were found in Rose (1998), who modeled highway traffic, Salahaldin and Granger (2005), who modeled the dynamics of a city's population, and Marathe and Ryan (2005) and Pereira *et al.* (2006), who modeled airline demand. Emery and McKenzie (1996), on the other hand, implicitly assumed that income from the railway followed a geometric Brownian motion process.¹ Bowe and Lee (2004) implicitly assumed the discrete time analogue of a geometric Brownian motion for the operational cash flows of an HSR investment.

Assuming that each user will face a cost for travel between two cities, ψ , that is a function of the total value of travel time for the user, η , and the travel fare, p . According to the literature, both of these variables exhibit a relationship to railway demand (*vide* Owen and Phillips 1987; Wardman 1994; and Wardman 1997).

The following functional form illustrates the relationship between the total value of travel time and the demand for faster railway travel (Owen and Phillips 1987; Wardman 1994):

$$\eta(x_t) = \beta x_t^{\delta_\beta}, \quad (2)$$

where δ_β represents the elasticity between the total value of travel time, η , and the HSR demand, x . Consequently, β is the scale parameter between HSR demand, x , and the total value of travel time, η .

The relationship between the travel fare and the HSR demand is given by the following functional form (Owen and Phillips 1987):

$$p(x_t) = \alpha x_t^{\delta_\alpha}, \quad (3)$$

where the elasticity between the fare value, p , and the HSR demand, x , is represented by the parameter δ_α . The scale parameter α relates HSR demand, x , and the travel fare, p .

We now assume a risk neutral user with a utility function $U(c) = c$. This utility is solely the function of the mean consumption per user, in which c represents the mean consumption of all users. The budget constraint is given by

$$m_t = c_t - \psi(x_t) \quad (4)$$

where ψ represents the travel cost and m the individual disposable income by unit of time.

Replacing the level of consumption in the utility function, we obtain the following value function, V , representative of the value that each user confers to a railway trip:

$$V(x_t) = U(c_t) = m_t - \psi(x_t). \quad (5)$$

The relationship between HSR demand, x_t , and the total value of travel time, (i) in the period of time that precedes the investment, η_0 ; (ii) during the investment's implementation, η_1 ; and (iii) after the investment's implementation, η_2 , is represented, respectively, by β_0 , β_1 and β_2 . Since the HSR will save travel time and, consequently, will reduce the total value of travel time from η_0 to η_2 , it will be reasonable to expect that from the pre-investment period to the operational period, β_0 will become β_2 , with $\beta_0 > \beta_2$. The difference between β_0 and β_2 reflects the decrease in travel time.

Large investments need time to be implemented. Thus, it is crucial to incorporate the time-to-build in the ROA framework. During the building period, n , the cost of travelling is given by ψ_0 and the relationship between demand and travel costs, β_1 , remains equal to β_0 ($\beta_1 = \beta_0$). When the HSR begins to operate, the cost of travelling will change to ψ_2 and β_2 reflects the decrease in travel time. Analytically, the cost of travelling by a conventional railway, ψ_0 , and the cost of

travelling by the HSR, ψ_2 , will be represented by the following equations:

$$\psi_0(x_t) = \beta_0 x_t^{\delta_\beta} + \alpha_0 x_t^{\delta_\alpha}, \quad (6)$$

$$\psi_2(x_t) = \beta_2 x_t^{\delta_\beta}, \quad (7)$$

where the conventional railway travel cost, ψ_0 , includes both the total value of the travel time lost and the travel fare. In contrast, the HSR travel cost function, ψ_2 , is not affected by the corresponding travel fare, p_2 , because the current valuation framework implicitly assumes that each user will bear his/her part of the investment expenditure plus the corresponding operating costs per user. Hence, a socially acceptable HSR travel fare is already implicitly considered in the valuation framework. Consequently, it does not make sense to duplicate it.

The conventional railway with a travel fare p_0 enables us to identify the relationship between HSR demand, x_t , and the price of a substitute service (Owen and Phillips 1987; Wardman 1997) given by Equation (3).

Until the HSR begins to operate, the value function per user will be given by

$$V_0(x_{t+n}) = m_{t+n} - \beta_0 x_{t+n}^{\delta_\beta} - \alpha_0 x_{t+n}^{\delta_\alpha} \quad (8)$$

After the investment is implemented, the users will continue to face a value for travel time, but it will be a smaller travel time. However, since the analysis performed here takes into consideration all costs and benefits induced by the HSR investment (including the investment expenditure, fixed and variable operating costs), the new value function per user will be given by

$$V_2(x_{t+n}) = m_{t+n} - \beta_2 x_{t+n}^{\delta_\beta} - \omega - \frac{\varphi}{x_{t+n}} - \frac{\rho\gamma e^{\rho n}}{x_{t+n}}, \quad (9)$$

where γ represents the capital investment expenditure, ρ the discount rate, ω the variable operating costs and φ the fixed operating costs. Notice that φ/x_t and $\rho\gamma/x_t$ represent the fixed operating costs and the investment expenditure per unit of time for each HSR user. We implicitly assume that the HSR investment cash flows will last for an unlimited time horizon.

Using the objective function of Ramsey–Koopmans to compute the net benefits generated by the HSR investment, according to the appendix, the investment opportunity value, here denoted by $v(x)$, is given by

$$v(x) = \int_0^\infty e^{-\rho(t+n)} E[V_2(x_{t+n}) - V_0(x_{t+n})] dt. \quad (10)$$

The purpose of the model is to calculate the optimal timing to invest preserving utility function balance. For that, it is necessary to locate the HSR demand threshold, x^* , above which it will be optimal to invest. Thus one wants to find the optimal value x^* such that

$$v(x^*) = \sup_x v(x). \quad (11)$$

The investment opportunity value is determined through the maximization of the following equation (appendix):

$$v(x^*) = A_{tc}(x^*)^{\theta_\beta} + B_{tc}(x^*)^{\theta_\alpha} + F_{tc}(x^*) + C_{tc} + D \quad (12)$$

with

$$A_{tc} = \frac{2(\beta_0 - \beta_2)e^{(\mu_x\theta_\beta + (1/2)\theta_\beta(\theta_\beta - 1)\sigma_x^2 - \rho)n}}{2\rho - 2\mu_x\theta_\beta - \theta_\beta^2\sigma_x^2 + \theta_\beta\sigma_x^2} \quad (13)$$

$$B_{tc} = + \frac{2\alpha_0 e^{(\mu_x\theta_\alpha + (1/2)\theta_\alpha(\theta_\alpha - 1)\sigma_x^2 - \rho)n}}{2\rho - 2\mu_x\theta_\alpha - \theta_\alpha^2\sigma_x^2 + \theta_\alpha\sigma_x^2} \quad (14)$$

$$C_{tc} = - \frac{\varphi e^{-\rho n}}{\rho} \quad (15)$$

$$F_{tc} = - \frac{\omega e^{(\mu_x - \rho)n}}{\rho - \mu_x} \quad (16)$$

$$D = -\gamma \quad (17)$$

To include economic intuition, note that: A_{tc} reflects the present value of travel time savings; B_{tc} reflects the present value of conventional railway travel fare; C_{tc} reflects the present value of fixed operating costs; D represents the present value of investment expenditures; and F_{tc} represents the present value of variable operating costs. The subscript tc , used above, indicates the time-to-build effect.

Now, as the investment opportunity function, $v(\cdot)$, is a function of the demand process $\{x_t\}$ that follows a geometric Brownian motion, if we apply Ito's lemma to $v(x_t)$, we end up with the following ordinary differential equation:

$$\frac{1}{2}\sigma_x^2 x^2 v''(x) + \mu_x x v'(x) - \rho v(x) = 0 \quad \text{for } x \neq x^* \quad (18)$$

subject to the boundary equations:

$$v(0) = 0 \quad (19)$$

$$v(x) = A_{tc}x^{\theta_\beta} + B_{tc}x^{\theta_\alpha} + F_{tc}x + C_{tc} + D \quad \text{with } x = x^* \quad (20)$$

$$v'(x) = \theta_\beta A_{tc}x^{\theta_\beta - 1} + \theta_\alpha B_{tc}x^{\theta_\alpha - 1} + F_{tc} \quad \text{with } x = x^* \quad (21)$$

Note that the first condition means that the process is absorbing when the HSR demand is 0; the second is the value-matching condition; and the third is the smooth-pasting condition.

Therefore the investment opportunity function, $v(\cdot)$, considering the current HSR demand, is given by the supremum of Equation (11), that satisfies the differential Equation (18).

Since Equation (18) is a *Cauchy–Euler* second order homogeneous differential equation, the solution may be written as

$$v(x) = a_1 x^{r_1}, \quad (22)$$

where r_1 is the positive root of the quadratic equation:

$$\frac{1}{2}\sigma_x^2 r(r - 1) + \mu_x r - \rho = 0 \quad (23)$$

given by

$$r_1 = \frac{((1/2)\sigma_x^2 - \mu_x) + \sqrt{(\mu_x - (1/2)\sigma_x^2)^2 + 2\rho\sigma_x^2}}{\sigma_x^2} \quad (24)$$

Using the condition $v(x^*) = A_{tc}x^{*\theta_\beta} + B_{tc}x^{*\theta_\alpha} + F_{tc}x^* + C_{tc} + D$, we calculate the coefficient $a_1 = A_{tc}x^{*\theta_\beta-r_1} + B_{tc}x^{*\theta_\alpha-r_1} + F_{tc}x^{*1-r_1} + C_{tc}x^{*-r_1} + Dx^{*-r_1}$, concluding that the solution of Equation (18) is

$$v(x) = [A_{tc}x^{*\theta_\beta-r_1} + B_{tc}x^{*\theta_\alpha-r_1} + F_{tc}x^{*1-r_1} + C_{tc}x^{*-r_1} + Dx^{*-r_1}]x^{r_1} \quad (25)$$

For a given value of x in $t = 0$, the value of x^* that maximizes $v(x)$ is given by the numerical solution of the equation:

$$A_{tc}x^{*\theta_\beta-r_1}(\theta_\beta - r_1) + B_{tc}x^{*\theta_\alpha-r_1}(\theta_\alpha - r_1) + F_{tc}x^{*1-r_1}(1 - r_1) - C_{tc}x^{*-r_1}r_1 - Dx^{*-r_1}r_1 = 0 \quad (26)$$

with r_1 given by Equation (24).

The HSR demand threshold, x^* , may only be found through a numerical solution of Equation (26), except if two assumptions are made. The first assumption, related to the equality between the total value of travel time/HSR demand elasticity and the conventional railway travel fare/HSR demand cross elasticity, equaling $\theta_\beta = \theta_\alpha = \theta$. This assumption means that the conventional railway travel fare and the value of travel time have a similar trend. The second assumption comes from the possibility of neglecting the operational variable costs, $F_{tc} = 0$, considering the operational characteristics of the HSR investment. Operational variable costs include essentially those related to ticket printing, since for an *a priori* dimensioned HSR project with a pre-established operation schedule, all the major operational costs tend to be fixed.

Considering these two assumptions, the HSR demand threshold, x^* , obtains the following closed form solution:

$$x^* = \exp \left[\frac{\ln((-r_1(C_{tc} + D))/((A_{tc} + B_{tc})(r_1 - \theta)))}{\theta} \right] \quad (27)$$

When the HSR demand threshold, x^* , is reached, it justifies (becomes optimal) an immediate implementation of the HSR investment, which will begin to operate n periods afterwards. This solution preserves the utility balance for users between the HSR and the conventional railway, making the optimal solution independent of the original income, m , and the initial HSR demand, x_0 . Because the framework deals with an economic welfare issue, based on the utility balance for users between two similar transportations, this framework is especially adequate to analyze governmental scale investment decisions.

Using traditional capital budgeting analysis, based on the net present value (NPV), the rationale for making the decision would be similar. The investment should only be implemented when the reduction in the cost of travelling provided by the HSR and measured by the difference between ψ_0 and ψ_2 is enough to cover the investment expenditure plus the operating costs. Analytically, for $\theta = \theta_\beta = \theta_\omega$, $F_{tc} = 0$ and any $n \geq 0$, we have

$$\beta_0 x_{t+n}^\theta + \alpha_0 x_{t+n}^\theta > \beta_2 x_{t+n}^\theta + \varphi + \rho \gamma e^{\rho n} \quad (28)$$

Considering $x_{t+n}^\theta \equiv x_t^\theta e^{\theta \mu n}$, it would only become optimal to invest when the HSR demand reaches the threshold \hat{x} ,

$$x_t > \hat{x} = \left[\frac{\varphi + \rho \gamma e^{\rho n}}{(\beta_0 - \beta_2 - \alpha_0)e^{\theta \mu n}} \right]^{1/\theta} \quad (29)$$

with \hat{x} representing the HSR demand threshold given by the traditional capital budgeting analysis.

The comparison between the optimal decision to invest, given by ROA Equation (27) and by the traditional capital budgeting analysis, becomes evident if in an investment implemented in one single period of time, we consider $\theta = 1$, as well as no fixed and variable costs ($\varphi = \omega = 0$).

In this case, Equations (27) and (29) would become:

$$x^* = \frac{-r_1 D}{(r_1 - 1)(A_{tc} + B_{tc})} \quad (30)$$

$$\hat{x} = \frac{\rho\gamma}{(\beta_0 - \beta_2 + \alpha_0)} \quad (31)$$

Equations (30) and (31) illustrate that $x^* > \hat{x}$. Thus, when $\hat{x} < x_t < x^*$, the decision to invest based on a traditional capital budgeting analysis results in a value reduction for the investment. In this situation, the investment opportunity value will be smaller than the sum of the investment expenditure and the value of the (sacrificed) option to defer. The ability to delay has value, because it allows the uncertainty resolution.

2.2 Valuation of an HSR investment using an ROA framework

Consider the investment value function given by Equation (25), for a given HSR demand, x , with $t = 0$. The investment opportunity value when $x < x^*$ is given by

$$v(x) = \left(\frac{x}{x^*}\right)^{r_1} [A_{tc} x^{*\theta_\beta} + B_{tc} x^{*\theta_\alpha} + F_{tc} x^* + C_{tc} + D] \quad (32)$$

while for $x \geq x^*$, the investment opportunity value is given by

$$v(x) = [A_{tc} x^{\theta_\beta - r_1} + B_{tc} x^{\theta_\alpha - r_1} + F_{tc} x^{1 - r_1} + C_{tc} x^{-r_1} + D x^{-r_1}] x^{r_1} \quad (33)$$

Assuming $\theta = \theta_\beta = \theta_\alpha$ and $F_{tc} = 0$, we may replace the HSR demand threshold, x^* , given by Equation (27) in the second part of the RHS of Equation (32). After simplifying, the investment opportunity value may be rewritten in the following terms:

$$v(x) = \begin{cases} \left(\frac{x}{x^*}\right)^{r_1} \left[\frac{\theta(C_{tc} + D)}{\theta - r_1} \right] & \text{for } x < x^* \\ (A_{tc} + B_{tc})x^\theta + C_{tc} + D & \text{for } x \geq x^* \end{cases} \quad (34)$$

with C_{tc} , D , r_1 , A_{tc} and B_{tc} given by Equations (15), (17), (24), (13) and (14).

In accordance to previous studies (vide McDonald and Siegel 1986; and Dixit and Pindyck 1994), from the moment τ , in which the HSR demand threshold is reached, x^* , the value of the option to defer is zero. As a result, it is always better to invest and receive in exchange the NPV – given by $A_{tc}x^{\theta_\beta} + B_{tc}x^{\theta_\alpha} + F_{tc}x + C_{tc} + D$ – of the expected decrease in the cost of travelling.

As long as the optimal timing to invest has not been reached, $t < \tau$, there is always an inherent value of waiting for new information about the HSR demand. In this case, the value of the option to defer is given by the difference between the investment opportunity value, $v(x)$, and the NPV calculated, using the expected HSR demand at that moment. In addition, for allowing the inclusion of the (i) time-to-build, (ii) fixed operating costs and (iii) variable operating costs, in the investment opportunity value, these developments take into consideration the elasticity between the value of travel time and demand. As the framework is developed, considering the users utility balance between the HSR and conventional railway travel, factors other than those related to travelling (e.g. income) are assumed to be constant and do not influence the outcome.

The HSR transport uses clean energy. However the impact of the fuel price expectations on fuel dependent transport modes is captured indirectly through the HSR demand stochastic process parameters.

Whenever the elasticity between the total value of travel time and the HSR demand is null ($\delta_\beta = 0 \Rightarrow \theta_\beta = 1$), we are implicitly assuming no real changes in the value of travel time. Real changes imply positive levels of elasticity. Similarly, the conventional railway travel fare remains constant in real terms whenever $\delta_\alpha = 0 \Rightarrow \theta_\alpha = 1$. If $\theta_\beta > 1$, increases in the value of travel time will be directly related to the demand growth rate. The demand behavior for faster rail transportation when the value of travel time rises is economically rational, as supported by Owen and Phillips (1987) and Wardman (1994). Therefore, it is acceptable that increases in the HSR demand are, at least partially, due to increases in the value of travel time. When $\theta_\alpha > 1$, the cross elasticity between the conventional railway travel fare and the HSR demand is positive. According to Owen and Phillips (1987) and Wardman (1997), an increase in the travel fare of substitute services justifies increases in the railway demand.

The investment opportunity value determined by this ROA framework includes the ability to wait for an uncertainty resolution, provided by the option to defer. When the ability to delay does not exist, as in the traditional capital budgeting decision analysis, this component is not taken into consideration, underestimating the investment opportunity fair value. The value of the option to postpone the investment comes from the incorporation of the “good tail” of the HSR demand uncertainty. The “bad tail” of demand uncertainty is limited by the option to defer the HSR investment until the situation becomes attractive enough (McDonald and Siegel 1986; Dixit and Pindyck 1994).

3. Numerical illustration

Assume a project for the construction of an HSR connecting two cities. The basic parameters values will include those in Table 1, supported by the released Portuguese Government data on the HSR investment. The construction period is 5 years and the investment expenditure's present value is 15 billion Euros. According to HSR demand studies provided, the actual HSR demand is 3 million passengers and should rise 3.5% per year with 20% standard deviation. However, the expected HSR demand growth rate and standard deviation could be estimated by the mean and variance of demand instantaneous growth rate upon historical data.

Table 1. Base-case parameters for the HSR investment.

Parameter	Value
x – HSR demand at the actual moment	3 M
γ – Present value of the investment expenditures	€5000 M
η_0 – Total value of travel time by the conventional railway	€30
η_2 – Total value of travel time by the HSR	€10
p_0 – Conventional railway travel fare	€25
ω – Variable operating costs	€1
φ – Fixed operating costs	€90 M
ρ – Discount rate	0.09
μ_x – HSR demand expected growth rate	0.035
σ_x – HSR demand standard deviation	0.20
n – Time-to-build (years)	5
δ_β – Elasticity between the total value of travel time and the HSR demand	0.60
δ_α – Cross elasticity between the conventional railway travel fare and the HSR demand	0.40

Note: M, millions.

Table 2. HSR investment valuation results.

Output	Value
x^* – HSR demand threshold	10.777 M
$v(x)$ – Investment opportunity value	€3743.3 M
npv – Net present value	€254.2 M
vod – Value of the option to defer	€3489.1 M

The conventional railway operates between the same two cities. The new HSR will reduce the travel time to one third comparative to the conventional railway travel. Therefore, a 3-hour journey by the conventional railway will be around 1 hour by HSR. Using official data provided by the EU guide to appraise infrastructural investments, the estimated value of travel time per hour in Portugal is 10 Euros. It is also considered that 65% of the passengers travel on non-working time and 35% on working time. The total value of travel time by the conventional railway and HSR is given by the multiplication of the value of travel time per hour and the travel time spent for each railway service.

Table 2 presents the HSR investment valuation results for the base-case parameters.

According to the results (Table 2), the HSR construction should only begin when the demand reaches 10.777 million passengers. Because investment expenditures and operational costs were considered as *a priori* known, the demand threshold obtained is therefore suitable only for an *a priori* dimensioned HSR project. Although the HSR investment has a slightly positive NPV, it should not be implemented at the current time. The HSR demand uncertainty forces a delay in the HSR investment. Maintaining “alive”, this investment opportunity has a value of 3743 million Euros. The value of the option to defer the investment represents 93.21% of the investment opportunity value.

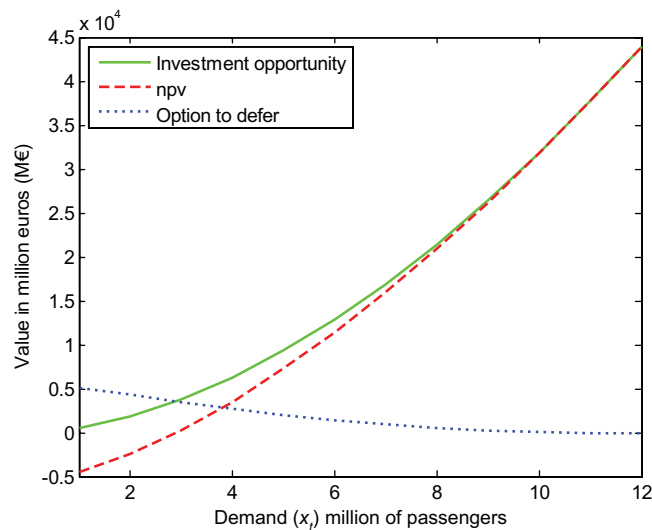


Figure 1. Investment opportunity value, NPV and value of the option to defer.

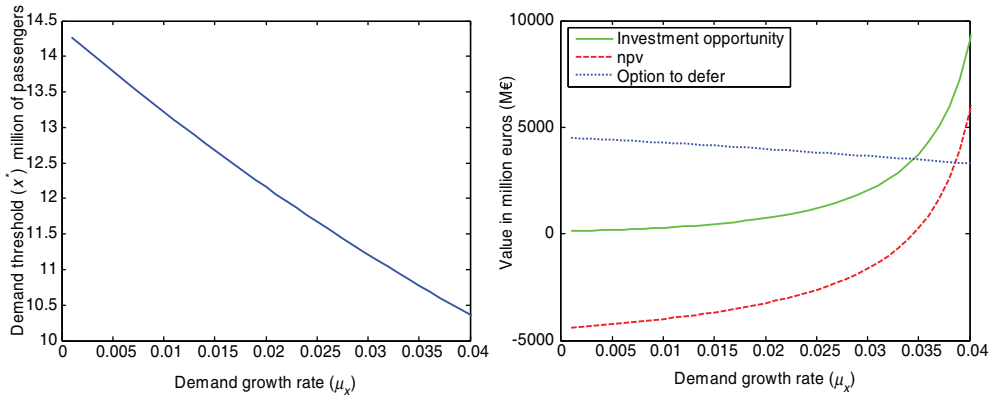


Figure 2. The impact of the growth rate.

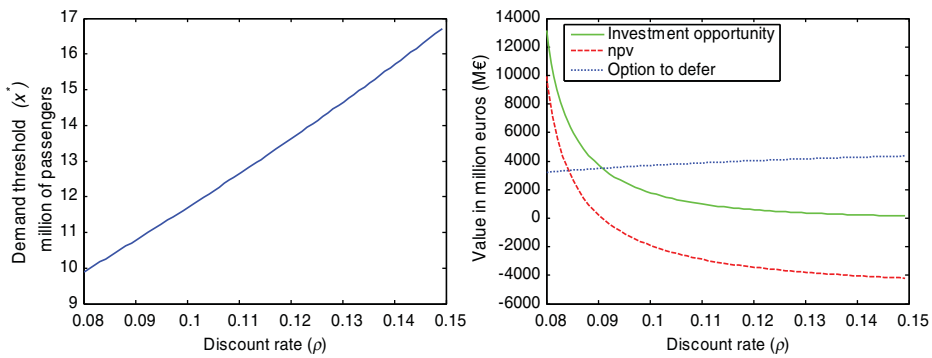


Figure 3. The impact of the discount rate.

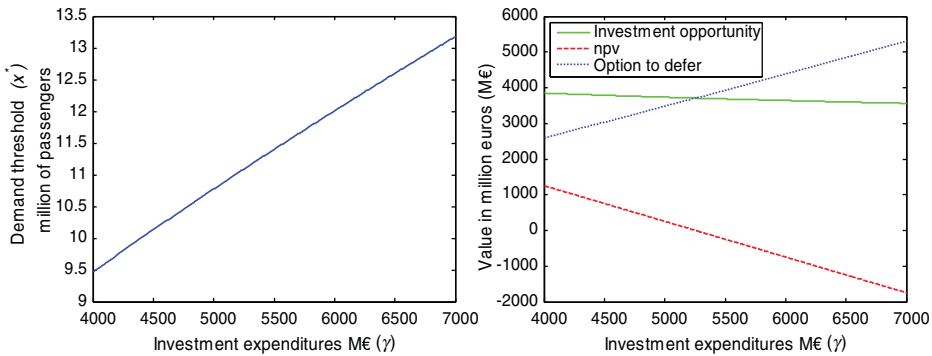


Figure 4. The impact of the investment expenditures.

Figure 1 illustrates the investment opportunity value, the NPV and the option to defer when the HSR demand increases over time. As we may observe, if the demand exceeds 10.777 million passengers, the option to defer the investment no longer has a value. Thus, from this point on, the decision to immediately invest is the one which maximizes the investment value for its owners.

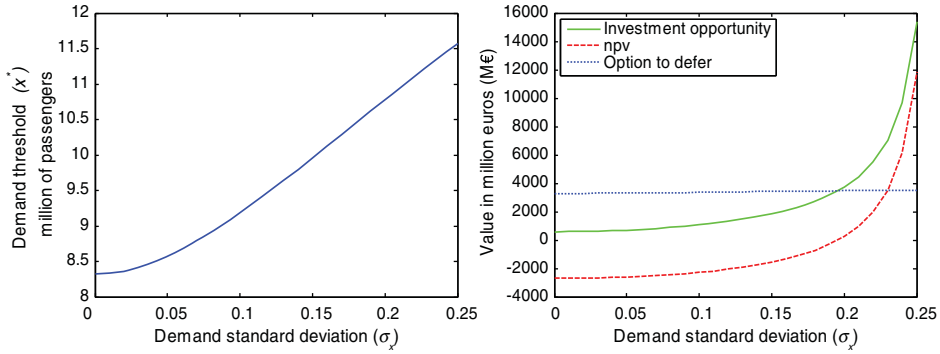


Figure 5. The impact of the HSR demand volatility.

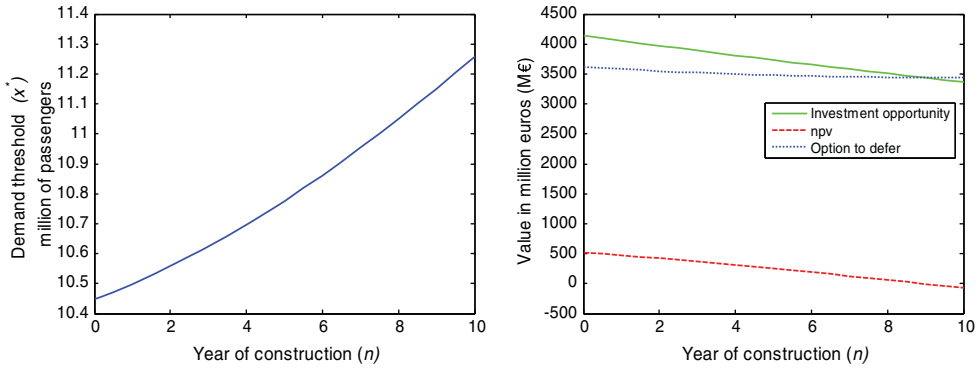


Figure 6. The impact of the time-to-build.

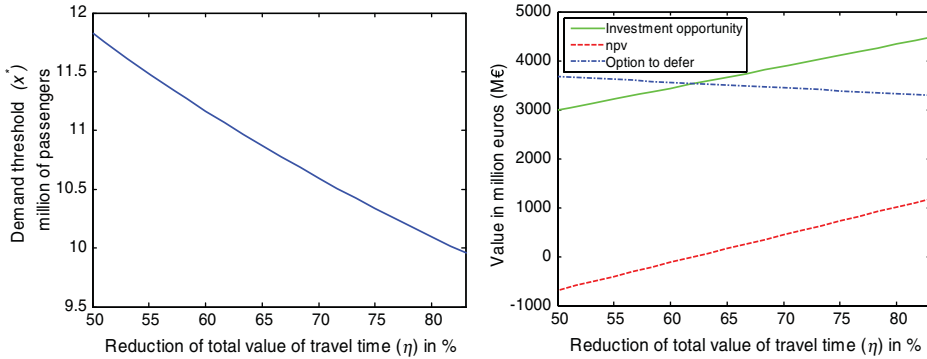


Figure 7. The impact of the total value of travel time savings.

Figure 2–7 illustrate the sensibility of the valuation results regarding the variation of various input parameters.

Thus, the HSR demand threshold, x^* , varies inversely with the HSR demand growth rate, μ_x (Figure 2), and with the total value of travel time savings given by $(\eta_0 - \eta_2)/\eta_0$ (Figure 7). For

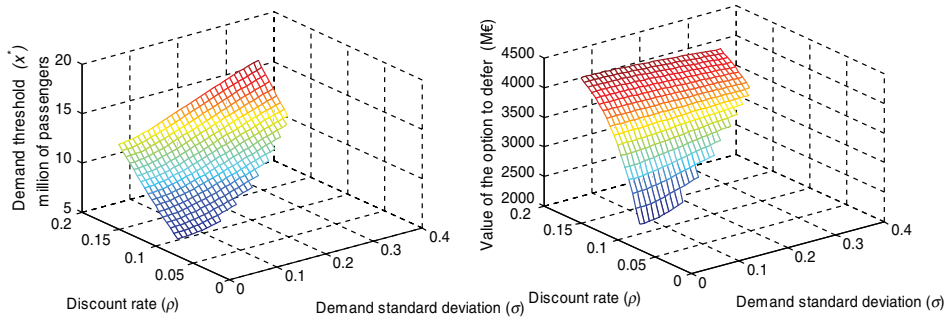


Figure 8. The impact of both the HSR demand volatility and the discount rate.

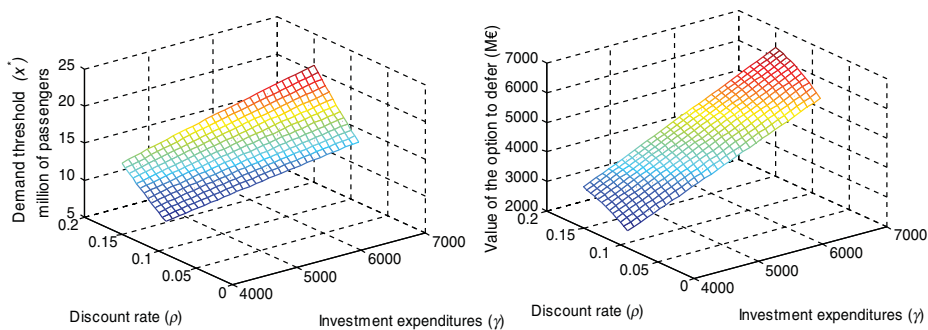


Figure 9. The impact of both the investment expenditures and the discount rate.

higher HSR demand growth rates, μ_x , and with a major total value of travel time savings, the present value of the HSR benefits increases, justifying investment anticipation.

The other parameters analyzed assume a direct relationship with the HSR demand threshold, x^* . Larger discount rates (Figure 3), larger investment expenditures (Figure 4), larger HSR demand volatility (Figure 5) or more construction time needed (Figure 6) instigates significant delays in the optimal time to invest.

Behind variations in any of the input parameters, the investment opportunity value and the NPV have the same trend, for each one of the parameters, although with different drifts. Figure 5 illustrates that the NPV increases with the increase in uncertainty. This finding is related to the elasticity between the total value of travel time and HSR demand and the cross elasticity between the conventional railway travel fare and HSR demand. This specificity of the framework results in a value of the option to defer that slightly diminishes with an increase in uncertainty. These findings are also revealed in Figure 8, where it is assumed that the discount rate remains unchanged when volatility changes.

If a larger time-to-build is required, the increase in uncertainty throughout time and the delay in the HSR operation benefits reduce the investment opportunity value and the NPV (Figure 6).

Figure 9 illustrates the joint impact of both the discount rate ρ and the investment expenditures γ on the HSR demand threshold, x^* , and on the value of the option to defer. In both cases, there is a direct relationship between these two input parameters, turning the option to defer more valuable when the value of these input parameters increases. As illustrated in Figures 3 and 4, this is due to a deeper decrease in NPV than the one registered in the investment opportunity value.

4. Extensions

Let us also consider that investment expenditures follow a geometric Brownian motion process (McDonald and Siegel 1986):

$$d\gamma_t = \mu_\gamma \gamma_t dt + \sigma_\gamma \gamma_t dw_t \quad (35)$$

where μ_γ represents the growth rate and σ_γ represents the standard deviation of investment expenditures. We assume that both parameters are constant in time. The Wiener process, w_t , has a zero mean and standard deviation of $\sigma_\gamma \sqrt{dt}$.

Adding uncertainty upon investment expenditures to the previous framework, the optimal timing to invest is given by

$$q^* = \left[\frac{(l+1)}{(A_{lc} + B_{lc})} \right] \left[\frac{s_1}{s_1 - 1} \right] \quad (36)$$

Note that in order to get a closed form solution, we must set:

1. The fixed operating costs as a proportion (l) from the investment expenditures; and
2. The optimal decision dependent on the threshold q^* , which represents the optimal ratio between HSR demand and the investment expenditures, given by x^θ/γ .

In Equation (36), A_{lc} and B_{lc} are given by Equations (13) and (14). The positive root, s_1 , of the quadratic equation similar to (23), is given by

$$s_1 = \frac{((1/2)\sigma_q^2 - \mu_q) + \sqrt{(\mu_q - (1/2)\sigma_q^2)^2 + 2\sigma_q^2(\rho - \mu_\gamma)}}{\sigma_q^2} \quad (37)$$

with, $\sigma_q^2 = \sigma_x^2 \theta^2 - 2\sigma_x \sigma_\gamma \text{corr}_{x,\gamma} \theta + \sigma_\gamma^2$ and $\mu_q = \mu_x \theta + (1/2)\sigma_x^2 \theta(\theta - 1) - \mu_\gamma$.

Under the same assumptions, this optimal timing threshold is consistent with the one obtained when only HSR demand is stochastic. The difference between Equations (27) and (36) reflects the additional impact from investment expenditure uncertainty.

If a positive growth in the investment expenditures is expected, this extended framework supports an anticipation of the optimal timing to invest, regarding the optimal timing to invest when the uncertainty comes only from HSR demand. The increase in investment expenditures under uncertainty justifies an anticipation of the HSR investment implementation taking advantage from lower investment expenditures (McDonald and Siegel 1986).

With $v(x, \gamma) = \gamma f(q)$, the investment opportunity value can be computed if at any moment of time, the investment expenditure, γ , is known. Hence, we have

$$v(x, \gamma) = \begin{cases} \left(\frac{q}{q^*} \right)^{s_1} \left[\frac{(l+1)}{(s_1 - 1)} \right] \gamma & \text{for } q < q^* \\ [(A_{lc} + B_{lc})q - l - 1] \gamma & \text{for } q \geq q^*. \end{cases} \quad (38)$$

In the continuation region, while threshold q^* is not reached, Equation (38) incorporates the value of the option to defer, which represents the value of waiting for new information about demand and investment expenditures. When the threshold q^* is reached, the HSR investment should be implemented immediately.

In the stopping region, $q \geq q^*$, the investment opportunity value is given by NPV.

This extended framework assesses simultaneously the impact of investment expenditures and HSR demand uncertainties on the optimal timing to invest and on the investment opportunity value.

5. Conclusions

This paper developed a framework to determine the optimal timing to invest in HSR, in an uncertain environment. We introduced several extension adjustments to the original option to the defer valuation framework by McDonald and Siegel (1986) and to the optimal stopping framework of Salahaldin and Granger (2005). Those extensions were made, given the need to design an adequate framework for HSR investments in an environment of stochastic demand, combined afterwards with stochastic investment expenditures. An ROA closed form solution to value railway investments has never been conducted previously.

The HSR investment analysis was incremental regarding conventional railways. The users' utility balance between the HSR and the conventional railway quantified the benefits to the HSR users. The optimal timing to invest was calculated with the HSR demand threshold model. The developments regarding the optimal timing to invest and the investment opportunity value present the advantage of offering a clear way to evaluate the HSR investment opportunity at each moment in time, for the set of potential users. The numerical illustration and simulation of some important input parameters demonstrated the consistency of the framework.

We recommend that future research enriches the framework to include more uncertainty factors, such as travel fare and demand random shocks. Additionally, we expect to extend the empirical application and use it to carry out additional improvements in the structure of the valuation framework.

Acknowledgement

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Note

1. It is easy to illustrate that if the travel fare is non-stochastic, the number of passengers follows a geometric Brownian motion process.

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Appendix

Note that the entire framework might be understood as an intergenerational welfare problem. Because of this situation, we may use the objective function of Ramsey and Koopmans, adopted by Salahaldin and Granger (2005). Analytically, we have

$$\sup_{x^*} E_x \left(\int_{-n}^{\tau} x_{t+n} e^{-\rho n} V_0(x_{t+n}) e^{-\rho t} dt + \int_{\tau}^{+\infty} x_{t+n} e^{-\rho n} V_2(x_{t+n}) e^{-\rho t} dt \right), \quad (\text{A1})$$

where

- τ is the moment of time at which the optimal value is achieved;
- $V_0(x_{t+n})$ is the value function per user per unit of time until the HSR begins to operate;
- $V_2(x_{t+n})$ is the value function per user per unit of time after the HSR begins to operate;
- n is the time-to-build (construction); and
- x_t is the HSR demand, given by Equation (1).

Considering the global value of all users before and after the HSR begins to operate, and replacing V_0 and V_2 in Equation (A1) for (8) and (9), we obtain

$$\sup_{x^*} E_x \left(\int_{-n}^{\tau} e^{-\rho t} \left[(m_{t+n} x_{t+n} - \beta_0 x_{t+n}^{\theta_\beta} - \alpha_0 x_{t+n}^{\theta_\alpha}) e^{-\rho n} \right] dt + \int_{\tau}^{+\infty} e^{-\rho t} \left[(m_{t+n} x_{t+n} - \beta_2 x_{t+n}^{\theta_\beta}) e^{-\rho n} - \omega x_{t+n} e^{-\rho n} - \varphi e^{-\rho n} - \rho \gamma \right] dt \right) \quad (\text{A2})$$

assuming $\theta_\beta = 1 + \delta_\beta$ and $\theta_\alpha = 1 + \delta_\alpha$.

Simplifying and excluding the components that do not depend on τ and x^* , it is possible to obtain the following objective function:

$$\sup_{x^*} E_x \left(\int_{\tau}^{+\infty} e^{-\rho t} [(\beta_0 - \beta_2)x_{t+n}^{\theta_\beta} e^{-\rho n} + \alpha_0 x_{t+n}^{\theta_\alpha} - \omega x_{t+n} e^{-\rho n} - \varphi e^{-\rho n} - \rho \gamma] dt \right) \quad (A3)$$

Equation (A3) maximizes the net benefits generated by the HSR investment. The decision to implement the investment requires n building periods before the HSR begins to operate.

With v denoting the investment opportunity value, let

$$v(x^*) = E_x \left(\int_{\tau}^{+\infty} e^{-\rho t} [(\beta_0 - \beta_2)x_{t+n}^{\theta_\beta} e^{-\rho n} + \alpha_0 x_{t+n}^{\theta_\alpha} - \omega x_{t+n} e^{-\rho n} - \varphi e^{-\rho n} - \rho \gamma] dt \right) \quad (A4)$$

Using the strong Markov property from Oksendal (2003) on the RHS, we obtain

$$\begin{aligned} E_x \left[\int_{\tau}^{+\infty} e^{-\rho t} [(\beta_0 - \beta_2)x_{t+n}^{\theta_\beta} e^{-\rho n} + \alpha_0 x_{t+n}^{\theta_\alpha} - \omega x_{t+n} e^{-\rho n} - \varphi e^{-\rho n} - \rho \gamma] dt \right] \\ = E_{x^*} \left[\int_0^{+\infty} e^{-\rho t} [(\beta_0 - \beta_2)x_{t+n}^{\theta_\beta} e^{-\rho n} + \alpha_0 x_{t+n}^{\theta_\alpha} - \omega x_{t+n} e^{-\rho n} - \varphi e^{-\rho n} - \rho \gamma] dt \right] \end{aligned} \quad (A5)$$

Under the dominated convergence theorem, we have

$$\begin{aligned} E_{x^*} \left(\int_0^{+\infty} e^{-\rho t} [(\beta_0 - \beta_2)x_{t+n}^{\theta_\beta} e^{-\rho n} + \alpha_0 x_{t+n}^{\theta_\alpha} - \omega x_{t+n} e^{-\rho n} - \varphi e^{-\rho n} - \rho \gamma] dt \right) \\ = \int_0^{+\infty} e^{-\rho t} [(\beta_0 - \beta_2)E_{x^*}(x_{t+n}^{\theta_\beta})e^{-\rho n} + \alpha_0 E_{x^*}(x_{t+n}^{\theta_\alpha})e^{-\rho n} - \omega E_{x^*}(x_{t+n})e^{-\rho n} - \varphi e^{-\rho n} - \rho \gamma] dt \end{aligned} \quad (A6)$$

Since HSR demand, x_t , follows a geometric Brownian motion described by Equation (1), then

$$E_{x^*}(x_{t+n}^{\theta}) = (x^*)^{\theta} e^{(\theta\mu_x + (1/2)\theta(\theta-1)\sigma_x^2)(t+n)} \quad (A7)$$

To assure the optimal timing to invest, the condition $\rho - \theta\mu_x - (1/2)\theta(\theta-1)\sigma_x^2 > 0$ is required. This condition also imposes the HSR demand growth rate to be lower than the discount rate, thus providing a rational economic interpretation to the mathematical developments. Simplifying again, we have

$$\begin{aligned} \int_0^{+\infty} e^{-\rho t} [(\beta_0 - \beta_2)E_{x^*}(x_{t+n}^{\theta_\beta})e^{-\rho n} - \omega E_{x^*}(x_{t+n}^{\theta_\omega})e^{-\rho n} - \varphi e^{-\rho n} - \rho \gamma] dt \\ = \frac{2(\beta_0 - \beta_2)(x^*)^{\theta_\beta} e^{(\mu_x\theta_\beta + (1/2)\theta_\beta(\theta_\beta-1)\sigma_x^2)n} e^{-\rho n}}{2\rho - 2\mu_x\theta_\beta - \theta_\beta^2\sigma_x^2 + \theta_\beta\sigma_x^2} + \frac{2\alpha_0(x^*)^{\theta_\alpha} e^{(\mu_x\theta_\alpha + (1/2)\theta_\alpha(\theta_\alpha-1)\sigma_x^2)n} e^{-\rho n}}{2\rho - 2\mu_x\theta_\alpha - \theta_\alpha^2\sigma_x^2 + \theta_\alpha\sigma_x^2} \\ - \frac{\omega(x^*)e^{(\mu_x-\rho)n}}{\rho - \mu_x} - \frac{\varphi e^{-\rho n}}{\rho} - \gamma \end{aligned} \quad (A8)$$

Rewriting Equation (A3) after simplifying, we obtain

$$\begin{aligned} v(x^*) = \frac{2(\beta_0 - \beta_2)(x^*)^{\theta_\beta} e^{(\mu_x\theta_\beta + (1/2)\theta_\beta(\theta_\beta-1)\sigma_x^2-\rho)n}}{2\rho - 2\mu_x\theta_\beta - \theta_\beta^2\sigma_x^2 + \theta_\beta\sigma_x^2} + \frac{2\alpha_0(x^*)^{\theta_\alpha} e^{(\mu_x\theta_\alpha + (1/2)\theta_\alpha(\theta_\alpha-1)\sigma_x^2-\rho)n}}{2\rho - 2\mu_x\theta_\alpha - \theta_\alpha^2\sigma_x^2 + \theta_\alpha\sigma_x^2} \\ - \frac{\omega(x^*)e^{(\mu_x-\rho)n}}{\rho - \mu_x} - \frac{\varphi e^{-\rho n}}{\rho} - \gamma \end{aligned} \quad (A9)$$