5-2000

Universidade Técnica de Lisboa
INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

CADERNOS DE ECONÓMICAS

DOCUMENTO DE TRABALHO Nº 5/2000

AN EMPIRICAL TEST ON THE FORECAST ABILITY OF. THE BAYESIAN AND BLUME TECHNIQUES FOR INFREQUENTLY TRADED STOCKS

JOÃO DUQUE
GUALTER DO COUTO

ISSN nº 0874-8470

DEPARTAMENTO DE GESTÃO

R. Miguel Lupi, 20 – 1249-078 Lisboa – Fax: 21.392.28.08 – Telf. 21.392.28.12/23

AN EMPIRICAL TEST ON THE FORECAST ABILITY OF THE BAYESIAN AND BLUME TECHNIQUES FOR INFREQUENTLY TRADED STOCKS

Gualter do Couto

Assistant
Departament of Economics and Business Administration
Universidade dos Açores
Rua da Mãe de Deus, 9500 Ponta Delgada, Azores, Portugal
Email: gcouto@notes.uac.pt

and

João Duque
Associate Professor
Universidade Técnica de Lisboa
Instituto Superior de Economia e Gestão
Rua Miguel Lupi, 20, 1200 LISBOA, Portugal
Email: jduque@iseg.utl.pt

AN EMPIRICAL TEST ON THE FORECAST ABILITY OF THE BAYSIAN AND BLUME TECHNIQUES FOR INFREQUENTLY TRADED STOCKS

Gualter do Couto and João Duque[1]

ABSTRACT

This paper tests the forecast ability of different methods to estimate systematic risk. We address the issue in a small market where stocks are infrequently traded.

We used the Blume technique and the Vasicek technique compared with two naïve techniques for different sample time periods (sample sizes) and different frequencies for data collection. We tested all the models using standard betas and betas adjusted for infrequently traded stock according to Scholes and Williams methodology. This study was carried on single stocks listed in the Portuguese stock exchange (BVL) instead of stock portfolios.

We concluded that adjusted betas using either the Baysian model or the Blume technique produce a better result than unadjusted betas, but it is not clear whether the former produces consistently better results than the latter.

We also found empirical support for the convergence phenomenon of betas of individual stocks towards one when they are either unadjusted or adjusted for infrequent trading.

Key words: betas, systematic risk, Blume technique, Vasicek Technique, infrequently traded stocks, stock pricing.

JEL Classification: G12 - Asset Pricing

I – Introduction

[1] The authors are grateful to BVL – Lisbon Stock Exchange for generous data supplying that supported this research study. The authors are also grateful to Julia Sawiki and to the attendees of the 1st Finance Conference of the Portuguese Finance Network.

Several studies revealed that beta coefficients were relatively stationary along time, particularly for stock portfolios (see Blume [1971]). Even so, it is documented a consistent tendency for a portfolio with a low (high) historical beta calculated for a given time period, to show a higher (lower) value for the subsequent time period. As high and low betas are defined in relation to the market beta (one), betas seem reveal a convergence tendency to one. This tendency seems to be particularly significant in the case of stock portfolios.

If this tendency is stationary then future betas could be forecasted with some confidence. Blume [1971] and [1975] and Vasicek [1973] presented two different techniques for estimating future betas based on historical coefficients for systematic risk.

We are interested in testing if the observed tendency of betas towards one is still observable for individual stocks traded in a small market characterised by thin trading. In addition we are interested in the forecast ability of Blume [1971] and [1975] and Vasicek [1973] techniques to predict betas along time, when compared with two naive forecasting techniques.

Although many studies on stock betas forecast used only monthly returns, as daily data becomes available other empirical tests are possible. In order to check whether the frequency of stock data collection has a significant impact on the forecast ability of stock betas, we compare the results achieved with daily, fortnightly and monthly returns.

Unfortunately, serious problems may occur when applying the market model in infrequently traded stocks. This is particularly relevant in small capital markets, where trading is thin. As we used the Portuguese capital market, where only some stocks are effectively frequently traded unsynchronous observations for stock prices and the market portfolio turns to be a significant problem. Sometimes infrequency is so strong that for the least traded stocks we get missing observations for a significant number of days. Therefore, completely synchronous returns for stocks and for the market portfolio are virtually impossible to get. This introduces an econometric source of error when the market model is applied. In these cases several methodologies were suggested to treat infrequently traded stocks among which we considered the Scholes' and the Williams' methodology [1977].

We start this paper by briefly describing the literature concerning the beta tendency and its forecast ability. Next we present the data and the methodology in use. Then we present the empirical results and conclusions.

II –Literature Review

Assuming that betas are a valuable instrument in finance for different purposes such as for stock valuation or portfolio optimal composition, where only future values are relevant, the forecast of systematic risk becomes a significant issue. Therefore, stationary characteristics of stock and portfolio betas turn to be a researchable question. Is it possible to observe any pattern on stocks' systematic risk in order to increase its forecast ability?

According to Blume [1971], the systematic risk of stock portfolios tends to show relatively stable characteristics. However, he observed a tendency of betas to converge towards the mean of all betas (1.0). Other authors (see Levy [1971]) also confirmed this empirical finding. However Kolb and Rodriguez [1989] support that two different movements may be found. While extreme high (low) betas tend to move towards the mean, as observed by Blume, betas near the mean in one period tend to move away from the mean. This would, however, keep the distribution of betas reasonably stationary over time.

Gooding and O'Malley [1977] who developed an empirical test on both adjusted and unadjusted betas rejected beta stationarity. They found that well-diversified portfolios of extreme betas are significantly nonstationary. They also found that major market trends are associated with these nonstationaries. Therefore they conclude that in order to improve performance on beta forecasts, adjustments should be made not only to take into consideration the regression tendencies but the market trends too.

Why does beta converge towards the mean for stocks and portfolios? Different explanations have been suggested for this phenomenon. Goldberg [1981] suggests that in the US economy the monetary policy of the Federal Reserve System is the main cause. According to the authors, the observed tendency is a result of the monetary policy, which result in a relationship between real interest rates and inflation rates that may explain the regression movement towards the unity. Moreover, as these factors would apply to all firms, this would be reflected on all equity securities.

Another source for explaining this beta tendency is the firm size. As firms grow in assets size its reasonable that its assets become more diversified and therefore the operational risk tends "to suffer" the diversification effect, seeming possible to approach the overall mean (unity). Burnett, Carroll and Thistle [1996] introduced this variable as a possible explanation for changes in the market model parameters although these changes seemed to follow gradual transitions, rather than abrupt shifts in figures. However, Murray [1995] denies such influence at least on the bias of the estimated betas. In a different perspective, Goldberg and Heflin [1995] found that betas of international companies, tend to present lower systematic risk than domestic firms, although their total risk may increase as a result of their exposure to exchange risk. Associating size with international diversification, this finding may confirm the firm size explanation to betas tendency towards one.

Last but not the least, we find in the IPO literature some hints on why should firms systematic risk decrease over time. After the findings of Ibbotson [1975] on the short term underpricing for stocks that went public but with a opposite behaviour on the long term found in Ritter [1991] or in Loughran e Ritter [1995], several hypothesis tried to justify these findings. Possible explanations found in Mauer and Senbet [1992], Rock [1986] or in Beatty and Ritter [1986] are associated with t*he market incompleteness hypothesis* or with *the adverse selection hypothesis.* The supporters of these hypothesis presume that investors ask a higher risk premium for holding stocks placed from an IPO when compared with other listed stocks. After a while this premium vanishes, returns adjust to the equilibrium and the systematic risk tends to drop, converging to the unity.

To correct the tendency towards one, two main models were suggested in the literature: the autoregressive nonweighted Blume's model and the autoregressive weighted Vasicek's model. Both models start with the assumption that betas may change over time converging to the unity. Therefore, $\beta_{i,t}$ (the systematic risk of stock i for the time period t) depends on the same parameter estimated for the previous time period ($\beta_{i,t} = f(\beta_{i,t-1})$):

Blume [1971] suggested that a general equilibrium autoregressive equation could be achieved from historical data to get better forecasts for future betas. Using an historical data set from July 1954 to June 1961 he proved empirically that estimated betas for individual stocks, as well as for portfolios of two or more stocks had a superior forecast capacity than the *"naive"* estimation of betas ($\beta_{i,t} = \beta_{i,t-1}$). Therefore, adjusting the historical betas could be a significant improvement on the estimation of future beta distributions, even if this parameter may not be considered strictly stationary.

Vasicek [1973] proposed the second model. It starts from the same empirical evidence found in Blume [1971] that betas tend to converge towards the unity over time. In order to get an estimate for the future beta Vasicek's model proposes a weighted average between the overall mean (one) and the stocks' historical beta. The weights are a function of the quality of the historical regression when estimating the parameters of the market model. Therefore, as the variance of the errors increases the quality of the historical beta decreases and weight should be given to the overall mean. If the opposite is true, then weight should be placed on the stock historical beta[2]. This technique should be consistent with the tendency of betas to converge to one. The speed of adjustment depends on the weights that are given to each of the components.

According to Vasicek [1973] this Bayesian approach minimises losses due to weak estimations (while the estimation obtained trough the sample theory minimises the sample error), and considers possible losses of a previous distribution adding new information to the sample.

Which model is preferable, if any, in forecasting betas? Several papers present evidence that adjusted betas tend to outperform unadjusted betas (see among others Klemkosky and Martin [1975], Murray [1995], Hawawini, Michel and Corhay [1985], Luoma, Martikainen and Perttunen [1996] or Elton Gruber and Urich [1978]). Although, there is no consensus on which model is preferable, there is a general tendency to point out the Vasicek Bayesian technique as preferable.

Klemkosky and Martin [1975] tested both adjustment techniques and concluded that both increase the beta forecast capacity when compared with naive forecasting of betas. They also supported the Bayesian technique once it shows a slightly better result on forecasting future betas. Nevertheless the difference is small: while for some time periods the Blume technique presented a superior forecast ability when compared to the Bayesian, the opposite was true for other time periods.

Luoma, Martikainen and Perttunen [1996], studying the Finnish market found that the Vasicek [1973] technique would be the most appropriate. This conclusion was achieved after regressing the difference between the alternative beta estimates minus the traditional market model, against several factors. They found that those differences are best explained in the case of Vasicek's betas under thin market factors.

But as we observed previously, there is no unanimous opinion on what model is preferable. Elton, Gruber and Urich [1978] found some time periods where, with statistical significance, the Blume technique outperformed the Vasicek technique on forecasting future betas. But the answer to which is the best, should be a result of the goal for which betas are being computed.

Eubank and Zumwalt [1979] came with a different conclusion. Firstly these adjustment techniques seem to show better results when applied to singular stocks than to stock portfolios. Second, they concluded that the Blume [1971] technique show superior forecasting ability for the short term, but Vasicek [1973] technique seemed to

---

[2] This model would be an application of Bayes' theory and this is the reason why it is also called the Bayesian model.

present better results for long term forecasts. They also observed that differences between both methods become irrelevant when betas are quite close to one. Thirdly, they suggested that when betas are high the random error term becomes irrelevant in terms of adjustment purpose.

We also find in the literature some concern on how do these models work in small capital markets where thin trading is a relevant issue. Hawawini, Michel and Corhay [1985], Murray [1995] and Luoma, Martikainen and Perttunen [1996] studied, respectively, the Finnish, Irish and the Belgian markets and found that adjusted betas seem to outperform unadjusted betas.

As a conclusion we may state that after the Blume's observation that betas tend to converge towards the unity over time, some discussion followed. While some authors confirmed the pattern (Levy [1971]), others rejected the findings in general terms. Kolb and Rodriguez [1989], for instance, observed that while the findings seemed to be true for extreme betas (extremely high or extremely low seem to converge to unity) the pattern seems to reverse for betas close to the unity. Gooding and O'Malley [1977] show empirical evidences of the influence of the market trend on the nonstationarity of well-diversified extreme beta portfolios.

The reasons pointed out in the literature for this convergence pattern are monetary policy (Goldberg [1981]) and firm size. But, while Burnett, Carroll and Thistle [1996] support the hypothesis that size would affect beta changes over time, Murray [1995] denies such influence. The firm size present some appealing rationality, since as firms increase size, if this size is a result of diversification in assets and investments, operational risk tend to dilute and systematic risk should tend to the market risk. The findings of Goldberg and Heflin [1995] supporting that betas of internationalised firms tend to present lower systematic risk seems to support the firm size explanation if we associate international diversification with size. The reverse would apply too. Another hypothesis also derived from the literature is based on the observation that when firms go public through an IPO we tend to observe the well-known short term underpricing effect. This effect is associated with a market penalisation of the stock, increasing the risk premium (*the market incompleteness hypothesis* supported by Mauer and Senbet [1992] or *the adverse selection hypothesis* presented by Rock [1986] and empirically tested by Beatty and Ritter [1986] are two possible explanations). But as time goes by the market would tend to reduce the risk premium and, ceteris paribus, systematic risk should reduce.

When testing the forecast ability of the market model parameters, two main techniques were found in the literature: the Blume [1971] and the Vasicek [1973] techniques. There is a general consensus that adjusted betas result in a better forecast than unadjusted betas (see among others Klemkosky and Martin [1975], Murray [1995], Hawawini, Michel and Corhay [1985] or Luoma, Martikainen and Perttunen [1996]). Although consensus could not be found among researchers whether one of the methods is preferable, there is a general opinion that the Vasicek model outperforms the Blume model. While Klemkosky and Martin [1975] and Luoma, Martikainen and Perttunen [1996] seem to show some superior ability in the Vasicek Bayesian model, Elton Gruber and Urich [1978] found periods where the Blume technique outperformed the former. In other paper Eubank and Zumwalt [1979] show empirically that the model depends on the time horizon under scope: while Blume [1971] technique show superior forecasting ability for the short term, the Vasicek [1973] technique seemed to present better results for long term forecasts.

Additionally we may found several concerns in the literature for beta patterns in markets where securities are thinly traded. Murray [1995] and Luoma, Martikainen and Perttunen [1996] support that Vasicek [1973] technique is the most appropriated method in thin markets. Hawawini, Michel and Corhay [1985] have also studied a small market (the Belgian securities market) and also found that even in that small market adjusted betas would still be preferable to unadjusted betas.

II –Methodology

We start this study by calculating the betas of a series of securities using the market model. Following we tested if the documented tendency of betas to converge to unity is observable in the Portuguese capital market. Then we compared and tested different forecasting models for stocks systematic risk traded in a thin market. Therefore we adapted the standard methodologies in order to cover the thinly traded market.

As we referred earlier we started by computing betas for each security using the market model as in Sharpe [1963]:

$$R_{i,t} = \alpha_i + \beta_i Rm_t + e_{i,t} \qquad \textbf{Eq. 1}$$

where $R_{i,t}$ represents the return of security i for the time period t and Rm_t the market return during the same time period. Log price differences were used as measures for market and stock returns in all calculations.

In order to test the hypothesis of betas' convergence to unity, we estimated a regression equation as in Blume [1971] (equation 2 bellow) and tested whether the slope contains one within a 95% confidence interval. We tested different models according to different sample sizes and data collection frequencies as well as before and after correcting the beta calculations for thin trading. Several authors, such as MartiKainen [1991], Dimson and Marsh [1983], studied the effect of data frequency, thin trading and betas. They concluded that less frequent transaction effect is stronger when the returns are calculated daily and that betas tend to be higher when they are estimated on a weekly or monthly basis.

According to Blume [1971] betas tend to converge to unity through a general equilibrium autoregressive equation:

$$\beta_{i,t} = b_0 + b_1 \beta_{i,t-1} + e_i \qquad \textbf{Eq. 2}$$

where $\beta_{i,t}$ represents the systematic risk of stock i for the time period t. Using an historical data set from July 1954 to June 1961 he found the estimated Eq. 2 to be:

$$\beta_{i,t} = 0.343 + 0.677\beta_{i,t-1} + e_i$$

For individual stocks, as well as for portfolios of two or more stocks, the method suggested by Blume [1971] proved to have a superior forecast capacity than the "*naive*" estimation of betas: $\beta_{i,t} = \beta_{i,t-1}$.

We also used the model described by equation 2 in order to estimate the general equilibrium model for the Portuguese market. Then we calculated the limits to which the autorregressive model would lead us. Assuming that the second parameter estimated from Equation 1 falls within the interval $-1 \langle\, b_1 \,\langle\, 1$ as empirically has been founded, it can be proved (see Appendix 1) that:

$$\lim_{m\to\infty} \beta_{i,t} = b_0\left[1+\frac{b_1}{1-b_1}\right] \qquad \textbf{Eq. 3}$$

Then we estimated a 95% confidence interval for the observed empirical tendency. Assuming:

$$\beta^-_{i,t} = \lim_{m\to\infty} \beta_{i,t} - t_{\alpha/2,n-2}\, s\sqrt{\frac{1}{n}+\frac{\left(\beta_{i,t}-\overline{\beta}_{im}\right)^2}{SS_{\beta_{i,t}}}}$$

and

$$\beta^+_{i,t} = \lim_{m\to\infty} \beta_{i,t} + t_{\alpha/2,n-2}\, s\sqrt{\frac{1}{n}+\frac{\left(\beta_{i,t}-\overline{\beta}_{im}\right)^2}{SS_{\beta_{i,t}}}}$$

where s stands for the sample standard deviation and $SS_{\beta_{i,t}}$ stands for the sum of the squares of the betas, the interval becomes:

$$\beta^-_{i,t} < \lim_{m\to\infty} \beta_{i,t} < \beta^+_{i,t} \qquad \textbf{Eq. 4}$$

If 1 falls within the interval we cannot reject the null hypothesis, accepting that betas tend to converge towards unity.

Then we used different models to forecast future betas. In addition to the Blume model, just described, we used three other models: the Vasicek [1973] model, and two naive models.

The Vasicek [1973] also known as the Bayesian model also uses the empirical findings of Blume [1971] anchoring betas around the unity. The forecasted beta of firm i for the time period m, becomes the result of a weighted average of the historical beta observed in the time period m-1 and the average beta (that should be around unity). Let $\overline{\beta}_t$ represent the average beta estimated in period t ($\cong 1$), let $\sigma^2\overline{\beta}_{t-1}$ stand for the variance of the distribution of the historical estimates of beta across the sample, let

$\sigma_{\beta_{i,t-1}}$ stand for the standard error of the historical beta estimated for the time period t-1 for each stock *i* using *T* observations, and let $e_{\tau,i}$ stand for the error of observation τ when estimating the beta of stock *i*:

$$\overline{\beta}_t = \frac{1}{N}\sum_{ii=1}^{N}\beta_{i,t}$$

$$\sigma^2_{\overline{\beta}_{t-1}} = \frac{\sum_{i=1}^{N}\left(\beta_{i,t} - \overline{\beta}_t\right)^2}{N-1}$$

$$\sigma^2_{\beta_{i,t-1}} = \frac{\frac{1}{T-2}\sum_{\tau=1}^{T} e^2_{\tau,i}}{\sum_{\tau=1}^{T}\left(R_{m_\tau} - R_m\right)^2}$$

Therefore, the Vasicek [1973] forecast for the beta of stock i for the time period t, becomes:

$$\beta_{i,t} = \frac{\sigma^2_{\beta_{i,t-1}}}{\sigma^2_{\overline{\beta}_{t-1}} + \sigma^2_{\beta_{i,t-1}}}\overline{\beta}_{i,t} + \frac{\sigma^2_{\overline{\beta}_{t-1}}}{\sigma^2_{\overline{\beta}_{t-1}} + \sigma^2_{\beta_{i,t-1}}}\beta_{i,t-1} \qquad \textbf{Eq. 5}$$

As we may observe, as the standard error of the estimated beta increases the weight given to the average beta increases, which means that as the quality of the estimated beta decreases we increase the weight on the average beta.

Additionally we computed two other estimates for beta that we called naive techniques. We are using these naive techniques as a benchmark for testing the quality of the previous beta estimators. The first naive technique is simply the assumption that the future systematic risk of security *i* equals the past systematic risk for the same security using the same time length period to compute it:

$$\beta_{i,t} = \beta_{i,t-1} \qquad \textbf{Eq. 6}$$

Therefore, if we were interested in forecasting the future beta for a two years time period we used a two years time period to compute the historical beta.

The second naive technique to be considered uses a simple average of historical betas computed for a series of historical periods with the same time length as the time length considered for forecasting:

$$\beta_{i,t} = \overline{\beta}_{i,t_0 - t_1} = \sum_{t=t_0}^{t_1} \frac{1}{t-1} \beta_{i,j} \qquad \textbf{Eq. 7}$$

According to this equation, the beta of a specific stock equals the average beta estimated for a series of time intervals between t_0 and t_1.

As some of the stocks traded in the Lisbon Stock Exchange, are quite unfrequently traded, we additionally used the Scholes and Williams [1977] methodology in order to correct for thin trading.

$$\beta_i = \frac{\beta_i^{-1} + \beta_i^0 + \beta_i^{+1}}{1 + 2\rho_m} \qquad \textbf{Eq. 8}$$

Where the beta estimated for an historical time period (beta i) is the average of three different computed betas using leads, simultaneous and lags series of the market returns. Therefore: β_i^{-1} stands for the beta estimated with a regression of security returns on market returns for day t-1, β_i^0 stands for the beta estimated with synchronous observations and β_i^{+1} stands for the beta computed with a regression of security returns on market returns for day t+1. ρ_m stands for the market return autocorrelation coefficient. Although some stocks trade infrequently, when fortnight or monthly data is used unobserved trades within these time ranges stop to occur. Stocks are sometimes infrequent but not so infrequent! Therefore, we only applied the Scholes and Williams [1977] methodology to daily data.

In order to test the forecast ability of each methodology, we computed several indicators comparing the forecasting error e_i for each stock i:

$$e_i = \beta_i - \beta_i^m$$

The observed error equals the difference between the observed beta and the forecasted beta according to method *m*. We used four different error indicators:

1. The mean square error indicator (MSE):

$$MSE = \frac{\sum_{t=K}^{T} e_t^2}{T - K + 1} \qquad \textbf{Eq. 9}$$

2. The mean absolute error indicator (MAE):

$$MAE = \frac{\sum_{t=k}^{T} |e_t|}{T - k + 1} \qquad \textbf{Eq. 10}$$

3. The relative mean absolute error (RMAE):

$$RMAE = \frac{\sum_{t=k}^{T} \frac{|e_t|}{\beta_{im}}}{T - k + 1} \qquad \textbf{Eq. 11}$$

4. The relative mean error (RME):

$$RME = \frac{\sum_{t=k}^{T} \frac{e_t}{\beta_{im}}}{T - k + 1} \qquad \textbf{Eq. 12}$$

III - Data

In order to develop the empirical tests we used closing prices for 32 Portuguese stocks traded in BVL (the Lisbon Stock Exchange), from January 1st, 1993 and June 30, 1998.

In order to compute historical betas defined by Equation 1, we estimated daily, fortnightly and monthly returns after correcting stock price series for dividends, stock splits and other effects. As a proxy for the Portuguese stock market we used BVL-Geral a broad index composed from a data set of companies listed in the main market of Lisbon Stock Exchange (the Market of Official Quotations). The composition of the index as well as the market capitalisation and the annual turnover in terms of number of shares and volume for each constituent is presented in Annex 1. Betas calculated according to different methodologies are shown in Annexes 2 to 10.

Calculations used TSP and the regressions with serious problems of autocorrelation detected after using the Durbin-Watson statistic were corrected using Cochrane and Orcutt [1949] procedure.

Regression equations were tested for any problem of heteroscedasticity and it was corrected when found.

We start computing 143 betas with daily observations for a one-year time period (with approximately 250 observations for each) and repeated the computations for fortnightly observations also for a one-year time period (with 26 observations each). This enabled us to compute a maximum of 6 betas for each company[3].

Then we computed 63 betas with a two-year time period of data (500 daily observations, 52 fortnightly observations and 24 monthly observations) for 3 different time periods: 1993-1994, 1995-1996 and 1997-1998.

In order to check if the data collection period could change our findings we also computed betas for a three-year time period (750 daily observations, 78 fortnightly observations and 36 monthly observations). With this methodology we could only compute a total of 40 betas.

After this simple calculation we recomputed betas correcting for the thin trading bias. However only betas computed with daily data were recalculated.

Table 1 summarises the number of betas computed, as well as number of observations used for all calculations.

PLEASE INSERT TABLE 1

IV – Empirical Results

We started this empirical investigation by calculating the betas on individual stocks using different window time periods and different data observation frequencies. The

[3] Some companies were not listed for the entire sample period, which unable us to calculate 6 betas for some of the companies.

result[4] shows that estimated betas tend to be statistically significant as the time window used to the estimation increases, particularly with high frequency data. Therefore we found a great number of insignificant betas when using daily data during one year to estimate them, and a small number of insignificant betas when estimating betas with a three years time period and daily data. This seems to contradict some opinions that express concerns on daily observations for beta estimation, since daily data would add too much noise to the calculations.

After estimating the betas for individual stocks according to Equation 1, using daily, fortnightly and monthly data, we estimated a general equilibrium autoregressive equation as defined in Equation 2 and supported by Blume [1971] and [1975]. For each data type and time interval we could estimate a different general equilibrium model. The results are presented in Table 2.

PLEASE INSERT TABLE 2

As we may observe, most of the equations fail to be significant at a 5% significance level. Assuming that for each model the slope parameter (b_1) lies between -1 and +1, the value to which the general equilibrium equations tend, converge to the value expressed by Equation 3. These values are presented in Table 3.

PLEASE INSERT TABLE 3

As we may observe, results may vary widely being closer to one as the time window used to estimate betas increase and data frequency decreases. The best results (meaning that the tendency approaches to unity) are obtained with monthly data and with a 2-years time period. The tendency to unity seems to fade as data frequency increases and the time period used for estimating betas reduces. This first approach is entirely confirmed with the estimated confidence intervals for the estimated central tendency of forecasted betas. As Table 4 documents based on the calculations of the limits of the intervals expressed by Equation 4, we can only accept the general beta tendency to unity for betas calculated upon monthly data. As a general rule daily data

---

[4] The full set of results is reported in Couto [1998] and may be supplied upon request to the authors.

tends to show too much noise and results are only significant when Scholes and Williams [1977] methodology to correct for infrequency data observation was applied. However, as the quality of the equations presented in Table 2 is low for almost the cases, we must be cautious when generalising our results. We think we should emphasise that results seem to be greatly dependent on the methodology in use.

PLEASE INSERT TABLE 4

As stated earlier, we found that betas estimated with a narrow time interval and high frequency data tend to show higher levels of statistical insignificance. Therefore we suspected that some of the findings that we have been reporting could be affected by these econometric problems. In order to overcome this problem we decided to recompute the first two regression equations (2.a and 2.b in Table 2) which results are shown in Tables 5.

PLEASE INSERT TABLE 5

The procedures followed in order to obtain Tables 3 and 4 were then repeated for these new equations and the results are presented in Tables 6 and 7.

PLEASE INSERT TABLE 6 AND TABLE 7

The results seemed to be improved when we dropped betas with low statistical quality and the general tendency of betas to converge to unity seemed to arise even when daily data is used. But the low quality of the regressions obtained as general equilibrium models does not bring additional confidence to our previous findings.

Having observed some signs that Portuguese betas may converge to unity, although those signs seemed to be dependent of the methodology in use, we wonder if a particular methodology is preferable when forecasting future betas.

Future realised betas were estimated using data of 1997 and 1998 and were compared with forecasted betas in order to check the forecast ability of different methodologies.

As the Bayes - Vasicek method requires the calculation of the average and standard deviation for all betas previously estimated, we calculated two models for forecasting based on this methodology. The first model applies all the betas previously estimated while the second forecast is based only on the sample of betas that were found statistically significant. Using daily data observations, Table 8 and Table 9 present the results of the forecasting errors for betas estimated for 1997, while Table 10 and Table 11 present the results of the forecasting errors for betas estimated for 1998.

PLEASE INSERT TABLE 8 AND TABLE 9

In 1997 adjusted betas (Blume and Bayes - Vasicek) shown a lower forecasting error than unadjusted (naive) betas and the Bayes - Vasicek technique presented better forecasting results for the majority of the tracking errors statistics. However, when we applied the same tests to 1998, the results were not so clear.

PLEASE INSERT TABLE 10 AND TABLE 11

In average, the forecasting error obtained by using an adjusted method is lower when compared with the forecasting error obtained with the naive techniques, but the best results were almost always obtained using a naive technique. It was not also clear whether the Vasicek technique beat the forecasting error of the Blume's technique. Therefore, it seems that when forecasting betas using daily data, adjusted betas lead us to lower forecasting errors than unadjusted betas and in the majority of the forecasts, the Vasicek technique beat the Blume technique.

When testing similar forecasts but with betas obtained by using fortnightly data either for 1997 and 1998, the results seemed to confirm some preference for adjusted betas. However, in terms of preference for a specific technique the results were dubious.

PLEASE INSERT TABLE 12 AND TABLE 13

As a general conclusion we would say that adjusted betas tend to perform better than unadjusted betas, but although the Vasicek technique tend to show better results, these conclusions tend to depend on time and data frequency for calculating betas. When using naive techniques to forecast future betas, which sometimes result in lower errors than adjusted betas, the simple average of past betas tends to present the best results. These findings seem to support the conclusions pointed out by Elton, Gruber and Urich [1978] and Eubank and Zumwalt [1979].

V – Conclusions

This paper tests whether the observed tendency of betas towards one is still observable for individual stocks traded in a small market characterised by thin trading. In addition we are interested in the forecast ability of Blume [1971] and [1975] and Vasicek [1973] techniques for forecasting betas along time, when compared with other forecasting techniques called *naive*.

After the Blume's observation that betas tend to converge towards the unity over time, some discussion followed. While some authors confirmed the pattern, such as Levy [1971], others rejected the findings in general terms. Kolb and Rodriguez [1989] observed that while the findings seemed to be true for extreme betas (extremely high or extremely low seem to converge to unity) the pattern seemed to be the reverse for betas close to the unity. Gooding and O'Malley [1977] show empirical evidences of the market trend on the nonstationarity of well-diversified extreme beta portfolios.

This regression tendency may well be a result of a statistical phenomenon. Betas estimated with historical data are subject to sampling error. High (low) beta estimates are likely to be affected by positive (negative) sampling error. When they are estimated in successive time periods they will tend to converge to one. But they also may be a result of fundamentals.

The reasons found in the literature for this convergence pattern are monetary policy (Goldberg [1981]) and firm size. But, while Burnett, Carroll and Thistle [1996] support the hypothesis that size would affect beta changes over time, Murray [1995] denies such influence. The firm size presents some appealing rationality. If firms increase in size and if this increase is the result of a diversification of assets and investments, operational risk tend to dilute and systematic risk should tend to the market risk. The findings of Goldberg and Heflin [1995] supporting that betas of internationalised firms tend to present lower systematic risk seems to support the firm size explanation if we associate international diversification with size.

Another hypothesis may be found in the IPO´s literature. When firms go public the well-known short term underpricing effect described in the literature is associated with a penalisation of the market, increasing the risk premium. But as time goes by the market would tend to reduce the risk premium and, ceteris paribus, systematic risk should reduce.

We started by computing betas on individual stocks using different window time periods and different data observation frequencies. The result show that estimated betas tend to increase statistical significance as the time window used to the estimation increases, particularly with high frequency data. A great number of insignificant betas were found when using daily data during one year of data and a small number of insignificant betas when estimating betas with a three years time period particularly if daily data was used. This seems to contradict some opinions that express concerns on daily observations for betas estimation. According to these opinions daily data would add too much noise to the calculations.

Then we tested the tendency of betas to converge to unity. We found that betas based on less frequent observations (monthly data) and based on widen sample time periods (two and three years time instead of one year) tend to show stronger signs of this tendency than betas based on more frequent observations (daily observations) and shorter sample time periods (one year time). However, these results show strong weaknesses from an econometric point of view. The exception was found when betas calculated with daily data were adjusted for the infrequency based on Scholes and Williams [1977] methodology. Therefore, if this tendency exists in terms of betas estimated for singular stocks, our findings are weak and surely dependent on the methodology in use.

Additionally, as in the literature we were concerned with the effects that thin trading could have on our conclusions. Murray [1995] and Luoma, Martikainen and Perttunen [1996] support that Vasicek [1973] technique is the most appropriated method for thin markets. Hawawini, Michel and Corhay [1985] have also studied a small market (the Belgian securities market) and also found that even in that small market adjusted betas would still be preferable to unadjusted betas. In this paper we also found more robust results when correcting for infrequency when testing the general tendency of betas to converge to unity.

When testing the forecast ability of the market model parameters, two main techniques were found in the literature: the Blume [1971] and the Vasicek [1973] techniques. There is a general consensus that adjusted betas result in a better forecast than unadjusted betas (see among others Klemkosky and Martin [1975], Murray [1995], Hawawini, Michel and Corhay [1985] or Luoma, Martikainen and Perttunen [1996]). Although consensus could not be found among researchers whether one of the methods is preferable, there is a general opinion that the Vasicek model outperforms the Blume model. While Klemkosky and Martin [1975] and Luoma, Martikainen and Perttunen [1996] seem to show some superior ability in the Vasicek Bayesian model, Elton Gruber and Urich [1978] found periods where the Blume technique outperformed the former and Eubank and Zumwalt [1979] show empirically that the model depends on the time horizon under scope: while Blume [1971] technique show superior forecasting ability for the short term, the Vasicek [1973] technique seemed to present better results for long term forecasts.

As a result of the empirical investigation developed on Portuguese data the results are quite strongly dependent on the methodology in use. There is no permanent patter for a specific method to show a systematically better forecasting power than others. However, we seem to detect that forecasts based on adjusted methods for future beta estimations result on lower errors than naive or unadjusted techniques. In average, we observe lower forecasting errors if adjusted betas are used, but for some of the occasions the best result was obtained with a simple rule of forecasting the future based on averaging the past. This means that our second naive technique was preferable when comparing both naive techniques. Our findings tend to support previous research on small markets namely Murray [1995], Luoma, Martikainen and

Perttunen [1996] and Hawawini, Michel and Corhay [1985] for choosing adjusted betas with powerful forecast results.

The quality of these forecasts based on averaging past betas, associated with our previous findings of some weakness of the general tendency of betas to converge to unity lead us to speculate on the hypothesis of betas of singles stocks to present similar patterns as volatility: mean reverting to a long term mean?

References

Beatty, R. e Ritter, J. (1986) "Investment banking, reputation and the underpricing of initial public offering", *Journal of Financial Economics*, 15, pp. 213-232.

Blume, Marchall (1971) "On The Assessment of Risk", *Journal of Finance*, 6, No. 1, March, pp. 1-10.

Blume, Marchall (1975) "Betas and Their Regression Tendencies", *Journal of Finance*, 10, No. 3, June, pp. 785-795.

Burnett, John E., Carolyn Carroll and Paul Thistle (1996) "The Detection of Nonstationarity in the Market Model", *Quarterly Journal of Business and Economics*, 35, 1, Winter, pp. 36-50.

Cochrane, D. and G. Orcutt (1949) "Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms", *Journal of American Statistical Association*, 44, pp. 32 - 61.

Couto, Gualter M. M. (1998) "Estimação Temporal dos Betas: Um Teste ao Modelo de Blume e Vasicek. Uma Aplicação ao Mercado Português", Master Dissertation, ISEG - Instituto Superior de Economia e Gestão, UTL - Universidade Técnica de Lisboa, Lisboa, Portugal.

Dimson, E. e P. R. Marsh (1983) "The Stability of Risk Measures and the Problem of Thin Trading", *Journal of Finance*, 38, No. 3, June, pp. 753-783.

Elton, Edwin J., Martin J. Gruber and Urich (1978) "Are Betas Best?", *Journal of Finance*, 13, 5, December, pp. 1375-1384.

Eubank Jr., A. A. and J. K. Zumwalt (1979) "An Analysis of the Forecast Error Impact of Alternative Beta Adjustment Techniques and Risk Classes", Journal of Finance, 34, June, pp. 761-776.

Goldberg, Michael (1981) "An Explanation of Security Beta Regression Tendencies", Southern Economic Journal, 47, 3, January, pp. 809-815.

Goldberg, Stephen R and Frank L Heflin (1995) "The Association Between the Level of International Diversification and Risk", *Journal of International Financial Management & Accounting*, 6, pp. 1-25.

Gooding, Arthur E. and Terence P. O'Malley (1977) "Market Phase and the Stationarity of Beta", Journal of Financial and Quantitative Analysis, 12, 5, December, pp. 833-857.

Hawawini, Gabriel [a], Pierre [a] Michel and Albert Corhey (1985) "New Evidence on Beta Stationarity and Forecast for Belgian Common Stocks", *Journal of Banking and Finance*, 9, 4, December, pp. 553-560.

Ibbotson, R. (1975) "Price performance of common stock new issues", *Journal of Financial Economics*, 2, pp. 235-272.

Klemkosky, R. C. e J. D. Martin (1975) "The Effect of Market Risk on Portfolio Diversification", *Journal of Finance*, X, No. 1, March, pp. 147-153.

Kolb, Robert W. and Ricardo J. Rodriguez (1989) "The Regression Tendencies of Betas: A Reappraisal", Financial Review, 24, 2, May, pp. 319-334.

Levy, Robert (1971) "On the Short-Term Stationarity of Beta Coefficients", *Financial Analysts Journal*, 27, No. 5, December, pp. 55-62.

Loughran, T. e Ritter, J. (1995) "The New Issues Puzzle", *Journal of Finance*, 50, pp. 23-51.

Luoma, Martti, Martikainen Teppo and Jukka Perttunen (1996) "A Pseudo Criterion for Security Betas in the Finnish Stock Market", *Applied Economics*, 28, 1, January, pp. 65-69.

Martikainen, Teppo. (1991), "The Impact of Infrequent Trading on Betas Based on Daily, Weekly and Monthly Return Intervals: Empirical Evidence with Finnish Data". *Finnish Economic Papers*, Vol. 4, No. 1.

Mauer, David C. and Lemma W. Senbet, (1992) "The Effect of the Secondary Market on the Pricing of Initial Public Offerings: Theory and Evidence", *Journal of Financial and Quantitative Analysis*, Mar, Vol. 27, 1, pp. 55-25.

Murray, Louis (1995) "An Examination of Beta Estimation Using Daily Irish Data", *Journal of Business Finance and Accounting*, 22, pp. 893-906.

Ritter, J. (1991) "The Long-run Performance of Initial Public Offerings", *Journal of Finance*, 46, pp. 3-27.

Rock, K. (1986) "Why New Issues Are Underpriced", *Journal of Financial Economics*, 15, pp. 187-212.

Sharpe, W. F. (1963), "A Simplified Model for Portfolio Analysis", *Management Science*, Vol. 9, January, pp. 277-293.

Scholes, M. e J. Williams (1977), "Estimating Betas from Non-Synchronous Data", *Journal of Financial Economics*, 5, No. 3, December, pp. 309-328.

Vasicek, Oldrich. (Dezembro de 1973), "A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Betas", *Journal of Finance*, 8, No. 5, pp. 1233-1239.

TABLE 1

Betas sample size

Window Time Period	**Data Frequency**		
	Daily	Fortnightly	Monthly
1 year	≈ 250 days 143 betas	≈ 26 fortnights 141 betas	
2 years	≈ 500 days 63 betas	≈ 26 fortnights 63 betas	≈ 24 months 63 betas
3 years	≈ 750 days 40 betas	≈ 26 fortnights 40 betas	≈ 36 months 40 betas

The table presents the number of betas used in the sample for testing the forecast ability of beta estimations showing the number of data points in use for computing betas, according to data frequency and the window time period considered.

TABLE 2

General Equilibrium Autoregressive Equations Estimated Using Individual Stocks in the Portuguese Stock Market

Betas / Data	$\beta_{im} = b_0 + b_1 \beta_{im-1}$	**Equation**
1-Year Betas/ Daily data	$\beta_{im} = 0.625338^* + 0.217276\ \beta_{im-1} + ei$	(2.a)
1-Year Betas/ Fortnightly data	$\beta_{im} = 0.834330^* + 0.058684\ \beta_{im-1} + ei$	(2.b)
2-Year Betas/ Daily data	$\beta_{im} = 0.546325^* + 0.265338^*\ \beta_{im-1} + ei$	(2.c)
2-Year Betas/ Fortnightly data	$\beta_{im} = 0.885776^* - 0.016467\ \beta_{im-1} + ei$	(2.d)
2-Year Betas/ Monthly data	$\beta_{im} = 0.910567^* + 0.067181\ \beta_{im-1} + ei$	(2.e)
3-Year Betas/ Daily data	$\beta_{im} = 0.613431^* + 0.233589^*\ \beta_{im-1} + ei$	(2.f)
3-Year Betas / Fortnightly data	$\beta_{im} = 0.722570^* + 0.147355\ \beta_{im-1} + ei$	(2.g)
3-Year Betas / Monthly-data	$\beta_{im} = 0.781651^* + 0.092615\ \beta_{im-1} + ei$	(2.j)
1-Year Betas S-W/ Daily data	$\beta_{Im} = 0.724303^* + 0.222118^*\ \beta_{im-1} + ei$	(2.k)

* - Parameters statistically significant at a 5% significance level.

TABLE 3

Beta Limits of Convergence when Following the Estimated General Equilibrium Autoregressive Equations

Betas / Data	$\lim_{m\to\infty} \beta_{im} = b_0\left[1+\frac{b_1}{1-b_1}\right]$	Equation
1 Year Betas/ Daily data	0.798925	(3.a)
1 Year Betas/ Fortnightly data	0.886344	(3.b)
2 Year Betas/ Daily data	0.743641	(3.c)
2 Year Betas/ Fortnightly data	0.871426	(3.d)
2 Year Betas/ Monthly data	0.976145	(3.e)
3 Year Betas/ Daily data	0.800394	(3.f)
3 Year Betas / Fortnightly data	0.847445	(3.g)
3 Year Betas / Monthly-data	0.861432	(3.j)
1 Year Betas S-W/ Daily data	0.931122	(3.k)

TABLE 4

Confidence Intervals for Betas Estimated According to the General Equilibrium Autoregressive Equations

Betas/ Data	$\hat{\beta}_0 + \hat{\beta}_1\beta_{im-1} \pm t_{\alpha/2} S\sqrt{\frac{1}{n}+\frac{(\beta_{im}-\overline{\beta_{im-1}})^2}{\sigma^2\beta_{im-1}}}$	Equation
1 Year Betas/ Daily data	$0.760375 \leq \hat{\beta}_{im} \leq 0.837475$	(4.a)
1 Year Betas/ Fortnightly data	$0.760942 \leq \hat{\beta}_{im} \leq 1.011746$	(4.b)
2 Year Betas/ Daily data	$0.683951 \leq \hat{\beta}_{im} \leq 0.803331$	(4.c)
2 Year Betas/ Fortnightly data	$0.765951 \leq \hat{\beta}_{im} \leq 0.976901$	(4.d)
2 Year Betas/ Monthly data	$0.673375 \leq \hat{\beta}_{im} \leq 1.278915$	(4.e)
3 Year Betas/ Daily data	$0.758057 \leq \hat{\beta}_{im} \leq 0.842731$	(4.f)
3 Year Betas / Fortnightly data	$0.757478 \leq \hat{\beta}_{im} \leq 0.937412$	(4.g)
3 Year Betas / Monthly-data	$0.706010 \leq \hat{\beta}_{im} \leq 1.016854$	(4.j)
1 Year Betas S-W/ Daily data	$0.775928 \leq \hat{\beta}_{im} \leq 1.086316$	(4.k)

TABLE 5

General Equilibrium Autoregressive Equations Estimated Using Statistically Significant Stock Betas

Betas*/ Data	$\beta_{im} = b_0 + b_1 \beta_{im-1}$	Equation
1 Year Betas/ Daily data	$\beta_{im} = 0.799406^* + 0.183375\ \beta_{im-1} + ei$	(2.a2)
1 Year Betas/ Fortnightly data	$\beta_{im} = 1.13192^* + 0.054255\ \beta_{im-1} + ei$	(2.b2)

* - parameters statistically significant at a 5% significance level.

TABLE 6

Beta Limits of Convergence when Following the Estimated General Equilibrium Autoregressive Equations and Using Statistically Significant Stock Betas

Betas/ Data	$\lim_{m \to \infty} \beta_{im} = b_0 \left[1 + \frac{b_1}{1 - b_1} \right]$	Equation
1 Year Betas/ Daily data	0.978914	(3.a2)
1 Year Betas/ Fortnightly data	1.196855	(3.b2)

TABLE 7

Confidence Intervals for Betas Estimated According to the General Equilibrium Autoregressive Equations

Betas/ Data	$\hat{\beta}_0 + \hat{\beta}_1 \beta_{im-1} \pm t_{\alpha/2} S \sqrt{\frac{1}{n} + \frac{(\beta_{im} - \overline{\beta_{im-1}})^2}{\sigma^2 \beta_{im-1}}}$	Equation
1 Year Betas/ Daily data	$0.935554 \leq \hat{\beta}_{im} \leq 1.022274$ (5.35)	(4.a2)
1 Year Betas/ Fortnightly data	$1.099711 \leq \hat{\beta}_{im} \leq 1.293999$ (5.36)	(4.b2)

TABLE 8

Forecasting errors for beta forecasts on 1997 using betas estimated with 1-year data and daily observations

Forecasting Model	Data	N	MSE	MAE	RMAE	RME
Blume´s technique	1993 - 1996	24	0.1476	0.3288	53.18%	-10.12%
Vasicek's technique	1996	24	0.1693	0.3128	42.49%	3.91%
Naive's technique I [1]	1996	24	0.2539	0.3784	60.79%	20.11%
Naive's technique II [2]	1993 - 1996	19	0.1708	0.3325	50.28%	-16.81%

[1] $\hat{\beta}_{im} = \beta_{im-1}$

[2] $\hat{\beta}_{im} = \overline{\beta}_{im-j} = \sum_{j=1}^{m-1} \frac{1}{m-1} \beta_{ij}$

TABLE 9

Forecasting errors for beta forecasts on 1997 using betas estimated with 1-year data and daily observations and excluding statistically insignificant betas

Forecasting Model	Data	N	MSE	MAE	RMAE	RME
Blume´s technique	1993 - 1996	15	0.2300	0.3406	79.97%	-77.53%
Vasicek's technique	1996	15	0.1597	0.3089	41.31%	-8.37%

TABLE 10

Forecasting errors for beta forecasts on 1998 using betas estimated with 1-year data and daily observations

Forecasting Model	Data	N	MSE	MAE	RMAE	RME
Blume´s technique	1993 - 1997	28	0.1784	0.3022	80.44%	-46.39%
Vasicek's technique	1997	28	0.1891	0.3522	72.14%	-35.61%
Naive's technique I [1]	1997	28	0.6225	0.4425	77.92%	-19.92%
Naive's technique II [2]	1993 - 1997	19	0.1587	0.3269	63.37%	-33.97%

[1] $\hat{\beta}_{im} = \beta_{im-1}$

[2] $\hat{\beta}_{im} = \overline{\beta}_{im-j} = \sum_{j=1}^{m-1} \frac{1}{m-1} \beta_{ij}$

TABLE 11

Forecasting errors for beta forecasts on 1998 using betas estimated with 1-year data and daily observations and excluding statistically insignificant betas

Forecasting Model	Data	N	MSE	MAE	RMAE	RME
Blume´s technique	1993 - 1997	25	0.2982	0.4494	94.29%	-90.56%
Vasicek's technique	1997	25	0.1414	0.3057	54.21%	-30.52%

TABLE 12

Forecasting errors for beta forecasts on 1997 using betas estimated with 1-year data and fortnightly observations

Forecasting Model	Data	N	MSE	MAE	RMAE	RME
Blume´s technique	1993 a 1996	24	0.5695	0.4603	63.28%	-23.22%
Vasicek's technique	1996	24	0.7747	0.5992	77.67%	-34.28%
Naive's technique I [1]	1996	24	1.4262	0.8196	113.71%	-26.90%
Naive's technique II [2]	1993 a 1996	19	0.2837	0.4200	69.39%	-25.36%

[1] $\hat{\beta}_{im} = \beta_{im-1}$

[2] $\hat{\beta}_{im} = \overline{\beta}_{im-j} = \sum_{j=1}^{m-1} \frac{1}{m-1} \beta_{ij}$

TABLE 13

Forecasting errors for beta forecasts on 1998 using betas estimated with 1-year data and fortnightly observations

Forecasting Model	Data	N	MSE	MAE	RMAE	RME
Blume´s technique	1993 - 1997	25	0.1731	0.3362	39.96%	-5.94%
Vasicek's technique	1997	25	0.1831	0.3466	37.95%	-2.25%
Naive's technique I [1]	1997	25	0.2356	0.3976	43.73%	-1.67%
Naive's technique II [2]	1993 - 1997	19	0.3364	0.4848	66.46%	-23.37%

[1] $\hat{\beta}_{im} = \beta_{im-1}$

[2] $\hat{\beta}_{im} = \overline{\beta}_{im-j} = \sum_{j=1}^{m-1} \frac{1}{m-1} \beta_{ij}$

ANNEX 1

Listed Companies	Market Capitalisation (1)	Annual Turnover N. Shares (2)	Annual Turnover Volume (3)
A. Silva & Silva	34.867	2.643.988	11.178
BCP	5.206.147	172.173.905	4.630.430
BES	3.160.184	58.401.138	1.549.399
BPI-SGPS	2.194.796	61.327.503	1.784.151
C. S. Império	334.756	29.025.569	229.295
C. S. Mundial confiança	1.379.346	37.314.271	946.565
C. S. Tranquilidade	537.829	6.406.483	169.718
Caima	25.890	848.576	7.625
Cimpor-SGPS	2.061.812	73.092.106	1.991.773
Cofina	60.604	1.960.643	30.024
Corticeira Amorim	179.034	6.660.808	87.108
EDP	5.389.369	231.346.476	4.637.891
Espart	61.053	536.080	2.866
Estoril-Sol	52.154	2.955.490	35.928
Inapa	153.889	9.786.394	94.263
Inparsa	1.110.433	38.902.347	1.062.443
Investec	129.687	1.586.391	55.173
ITI	29.861	909.755	6.080
Lisnave	12.030	110.969	942
Lusomundo	110.135	4.717.225	50.411
Mague	129.902	943.831	27.555
Modelo-Continente	2.017.154	81.384.724	1.670.039
Portugal Telecom	5.358.715	234.239.922	9.700.389
Reditus	4.858	1.353.553	3.560
Semapa	421.431	12.661.258	228.348
Somague	88.706	9.423.285	54.362
Sonae Imobiliária	588.083	10.293.450	155.878
Sonae Investimento	1.636.456	27.203.612	1.007.323
Unicer	384.685	4.472.696	90.714

Source: BVL, "Sociedades Cotadas, 1998".
(1) 10^3 euros in 30/11/1998.
(2) Between 01/June/1997 and 31/05/1998.
(3) 10^3 euros between 01/June/1997 and 31/05/1998.

ANNEX 2 - Historical Betas: 1 Year Betas / Daily Data

Betas	β_{93}	β_{94}	β_{95}	β_{96}	β_{97}	β_{98}
A. Silva & Silva				0,451988	0,725226 *	0,551000 *
BCP	1,220880 *	1,253310 *	0,951053 *	1,199370 *	1,204050 *	1,046130 *
BES	0,541805 *	0,995716 *	0,664059 *	0,503686 *	0,956369 *	0,999343 *
BPI	1,060780 *	1,758210 *	1,345420 *			
BPI-SGPS				0,762708 *	1,237230 *	1,045410 *
C. S. Império	-0,069969	0,516551 *	1,493650 *	0,321399	1,045560 *	1,208800 *
C. S. Mundial confiança	0,813711	0,602789 *	0,579520	0,514343 *	1,564060 *	1,073060 *
C. S. Tranquilidade	1,196550	0,366991 *	0,190535	0,376948 *	0,208681 *	0,770809 *
Caima	1,521570 *	0,385070	0,972508 *	0,404978	0,315811 *	1,140920 *
Cimpor		0,355135 *	0,378384 *	0,939087 *	0,913061 *	
Cimpor-SGPS					0,967697 *	0,684866 *
Cofina						0,960132 *
Corticeira Amorim	1,526110 *	1,506270 *	0,759917 *	1,313660 *	1,111280 *	0,866813 *
EDP					0,885013 *	1,005560 *
Espart	0,045222	0,739153 *	1,210650 *	0,725879	0,886969 *	1,182250 *
Estoril-Sol	0,490365	0,216063	0,541823	-0,346622	0,972359 *	0,161301
Gestnave					-2,752680	0,867033 *
Inapa	1,147300 *	0,976877 *	0,422355 *	0,638278 *	0,386586 *	0,423289 *
Inparsa					1,646570 *	2,306410 *
Investec					0,268815	0,062336
ITI	0,663193 *	0,705677 *	0,528754	0,546150	0,321291 *	0,791733 *
Lisnave	3,036810 *	0,091185	-0,600480	-0,333502	0,153194	
Lusomundo				-0,024440	0,610335 *	0,498282 *
Mague	0,499744	0,239870	-0,061147	1,124070 *	0,397416 *	0,299373
Modelo-Continente	0,572319 *	1,006760 *	0,931707 *	0,569466 *	1,297020 *	0,964932 *
Portugal Telecom			0,536592 *	1,337620 *	0,992930 *	0,904762 *
Reditus	0,596320	1,506040 *	1,301800	0,368258	0,646883 *	0,842491 *
Semapa			0,866589 *	0,811434 *	1,127330 *	0,890628 *
Somague		1,589920 *	0,633254 *	1,262870 *	1,248830 *	0,992694 *
Sonae Imobiliária					0,275687	1,088700 *
Sonae Investimento	2,936910 *	2,015180 *	1,175420 *	1,119890 *	1,216800 *	0,781355 *
Unicer	0,817618 *	0,828578 *	0,763321 *	0,819668 *	0,784330 *	0,316428 *

Obs: * Parameters statistically significant at a 5% significance level.

ANNEX 3 - Historical Betas: 1 Year Betas / Fortnightly Data

Betas	β_{93}	β_{94}	β_{95}	β_{96}	β_{97}	β_{98}
A. Silva & Silva				1,169670	0,538699 *	1,311270 *
BCP	1,030910 *	1,215470 *	1,015990 *	0,936165 *	1,124860 *	1,425540 *
BES	0,281277	0,770649 *	0,993026	0,705922 *	1,140870 *	1,618350 *
BPI	0,918476 *	1,821990 *	2,002980 *			
BPI-SGPS				0,545414	1,214790 *	1,760710 *
C. S. Império	0,377903	0,448970	2,371130 *	0,062148	1,134200 *	0,923149 *
C. S. Mundial confiança	1,904100 *	0,372684	-0,915253	0,149948	1,628760 *	1,653200 *
C. S. Tranquilidade	1,507180	0,419496 *	0,472767	0,375735	0,309782	1,238160 *
Caima	2,321950 *	0,902008 *	1,468140	1,181310	0,582233 *	1,480480 *
Cimpor		0,562845	0,178950	2,043830 *	1,318940 *	
Cimpor-SGPS					0,893361 *	0,670263 *
Cofina						-0,313772
Corticeira Amorim	1,945390 *	1,376360 *	-0,361901	2,072470 *	1,301210 *	0,742764
EDP					0,661873 *	0,864061 *
Espart	-0,426923	0,636385 *	1,000220	-0,580807	0,764667 *	0,633648
Estoril-Sol	0,908503 *	0,730644 *	-0,715205	0,647803	0,750012 *	0,959403 *
Gestnave						1,536070 *
Inapa	1,561640 *	0,736534 *	1,056200	0,228092	0,742101 *	0,397581
Inparsa					1,663410 *	1,156350 *
Investec						0,583559 *
ITI	0,922864 *	0,878196	-0,431336	1,808160	0,190494	0,313200
Lisnave	1,901780	0,626534	-1,662680	-0,432076	3,833240	
Lusomundo				-0,291584	0,193230	1,207890 *
Mague		0,417259	0,231896	2,704020 *	0,797193 *	0,947859
Modelo-Continente	0,886676 *	0,949472 *	1,609480 *	0,835832	1,154700 *	0,853918 *
Portugal Telecom			0,438003	1,546320 *	1,102470 *	0,500369
Reditus	0,792663	1,327640	-2,298280	0,081779	0,339320	0,936890
Semapa			0,909604	0,989001	1,081170 *	0,976935 *
Somague		1,892860 *	1,158450 *	1,034060	1,193850 *	1,082510 *
Sonae Imobiliária						0,662334
Sonae Investimento	2,900880 *	1,754490 *	0,969254 *	1,919020 *	1,091640 *	0,645265 *
Unicer	1,173430 *	1,199270 *	2,070300 *	1,248190 *	0,549031 *	0,394262

Obs: * Parameters statistically significant at a 5% significance level.

ANNEX 4 - Historical Betas: 2 Year Betas / Daily Data

Betas	$\beta_{93/94}$	$\beta_{95/96}$	$\beta_{97/98}$
A. Silva & Silva			0,666650 *
BCP	1,249150 *	1,027830 *	1,132200 *
BES	0,800257 *	0,608797 *	0,973165 *
BPI	1,479140 *		
BPI-SGPS			1,154270 *
C. S. Império	0,275234 *	1,023870 *	1,099640 *
C. S. Mundial confiança	0,604270 *	0,677181 *	1,330070 *
C. S. Tranquilidade	0,714005 *	0,269873 *	0,475920 *
Caima	0,859946 *	0,699266 *	0,693663 *
Cimpor		0,621328 *	
Cimpor-SGPS			0,827296 *
Corticeira Amorim	1,512700 *	0,986092 *	0,987959 *
EDP			0,939645 *
Espart	0,478612 *	0,883734 *	0,968704 *
Estoril-Sol	0,347363 *	0,313802	0,549714 *
Gestnave			
Inapa	1,087770 *	0,549560 *	0,401291 *
ITI	0,634777 *	0,672737	0,566265 *
Lisnave	1,246960 *	-0,465059	
Lusomundo			0,555189 *
Mague	0,254110	0,415664	0,342112 *
Modelo-Continente	0,866566 *	0,791862 *	1,161650 *
Portugal Telecom		0,973926 *	0,951901 *
Reditus	1,114940 *	1,010940	0,743327 *
Semapa		0,802828 *	1,023910 *
Somague		0,899108 *	1,129370 *
Sonae Investimento	2,369520 *	1,185090 *	1,008060 *
Unicer	0,830531 *	0,764658 *	0,620916 *

Obs: * Parameters statistically significant at a 5% significance level.

ANNEX 5 - Historical Betas: 2 Year Betas / Fortnightly Data

Betas	$\beta_{93/94}$	$\beta_{95/96}$	$\beta_{97/98}$
A. Silva & Silva			0,922358 *
BCP	1,181100 *	0,840443 *	1,252050 *
BES	0,535379 *	0,880963 *	1,320600 *
BPI	1,470580 *		
BPI-SGPS			1,425920 *
C. S. Império	0,341832	1,066750 *	1,120880 *
C. S. Mundial confiança	0,991096 *	0,304589	1,620010 *
C. S. Tranquilidade	0,775122	0,381567 *	0,676105 *
Caima	1,503640 *	0,992074	1,015480 *
Cimpor		1,022870 *	
Cimpor-SGPS			0,804301 *
Corticeira Amorim	1,607130 *	0,993612 *	1,087600 *
EDP			0,788928 *
Espart	0,147767	0,569148	0,709594 *
Estoril-Sol	0,853311 *	0,741558	0,851720 *
Inapa	1,045460 *	0,781591 *	0,567833 *
Inparsa			1,404820 *
ITI	0,724014 *	1,519200	0,278411
Lisnave	1,338550 *	-0,526002	
Lusomundo			0,674652 *
Mague	0,474471	1,473100	0,680466 *
Modelo-Continente	0,920809 *	1,166740 *	1,034780 *
Portugal Telecom		1,241590 *	0,820218 *
Reditus	0,824682 *	0,047377	0,614956 *
Semapa		0,827879	1,017850 *
Somague		1,457060 *	1,056940 *
Sonae Investimento	2,229130 *	1,546290 *	0,899893 *
Unicer	1,142870 *	1,394000 *	0,515484 *

Obs: * Parameters statistically significant at a 5% significance level.

ANNEX 6 - Historical Betas: 2 Year Betas / Monthly Data

Betas	$\beta_{93/94}$	$\beta_{95/96}$	$\beta_{97/98}$
A. Silva & Silva			1,046670 *
BCP	1,380590 *	1,079790 *	1,220600 *
BES	0,154261	0,280197	1,330650 *
BPI	1,169060 *		
BPI-SGPS			1,744830 *
C. S. Império	0,137237	0,311486	1,238330 *
C. S. Mundial confiança	1,339920 *	1,271620	1,544320 *
C. S. Tranquilidade	0,196752	0,368548	0,422482
Caima	1,999370 *	0,082991	0,920629 *
Cimpor		1,022760 *	
Cimpor-SGPS			0,891771 *
Corticeira Amorim	1,718160 *	1,354500 *	0,774820 *
EDP			0,818644 *
Espart	0,329128	1,048330	0,987970 *
Estoril-Sol	1,058840 *	1,521990	0,550198
Inapa	1,384000 *	0,488858	-0,020789
Inparsa			2,131700 *
ITI	0,571220	2,864390 *	0,838714 *
Lisnave	2,137950 *	-0,325368	
Lusomundo			0,708025
Mague	0,744729	2,797440 *	0,931076 *
Modelo-Continente	1,258970 *	2,055930 *	1,409710 *
Portugal Telecom		1,493780 *	0,717425 *
Reditus	0,651799	-0,774465	0,862955 *
Semapa		0,754725	0,968970 *
Somague		0,883210	1,153790 *
Sonae Indústria	1,987010 *	2,541270 *	0,987815 *
Sonae Investimento	2,331210 *	1,590760 *	0,658509 *
Unicer	1,514620 *	0,761882 *	0,717216 *

Obs: * Parameters statistically significant at a 5% significance level.

ANNEX 7 - Historical Betas: 3 Year Betas / Daily Data

Betas	$\beta_{93/95}$	$\beta_{96/98}$
A. Silva & Silva		0,642900 *
BCP	1,185570 *	1,142590 *
BES	0,774740 *	0,954748 *
BPI	1,442740 *	
BPI-SGPS		1,159420 *
C. S. Império	0,480020 *	1,064497 *
C. S. Mundial confiança	0,698634 *	1,280180 *
C. S. Tranquilidade	0,612634 *	0,468239 *
Caima	0,827299 *	0,687837 *
Corticeira Amorim	1,389620 *	1,018800 *
Espart	0,571359 *	1,001460 *
Estoril-Sol	0,407071 *	0,523374 *
Inapa	0,948802 *	0,418199 *
ITI	0,635389 *	0,585086 *
Lisnave	0,881590 *	
Lusomundo		0,538557 *
Mague	0,164083	0,391725 *
Modelo-Continente	0,881755 *	1,117030 *
Portugal Telecom		0,972729 *
Reditus	1,137240 *	0,713644 *
Semapa		1,004230 *
Somague		1,125270 *
Sonae Investimento	2,166380 *	1,014220 *
Unicer	0,822151 *	0,634076 *

Obs: * Parameters statistically significant at a 5% significance level.

ANNEX 8 - Historical Betas: 3 Year Betas / Fortnightly Data

Betas	$\beta_{93/95}$	$\beta_{96/98}$
A. Silva & Silva		0,892194 *
BCP	1,135030 *	1,258210 *
BES	0,590339 *	1,284900 *
BPI	1,464570 *	
BPI-SGPS		1,385640 *
C. S. Império	0,593437 *	1,087280 *
C. S. Mundial confiança	0,887392 *	1,516360 *
C. S. Tranquilidade	0,684585	0,642025 *
Caima	1,494240 *	0,969617 *
Corticeira Amorim	1,535690 *	1,156460 *
Espart	0,324643	0,606130 *
Estoril-Sol	0,785482 *	0,794489 *
Inapa	1,099720 *	0,558459 *
ITI	0,744114 *	0,133401
Lisnave	1,166510 *	
Lusomundo		0,467684
Mague	0,404422	0,807746 *
Modelo-Continente	1,027100 *	1,025940 *
Portugal Telecom		0,854856 *
Reditus	0,760836	0,492180
Semapa		0,972480 *
Somague		0,987008 *
Sonae Investimento	2,150320 *	0,927796 *
Unicer	1,178220 *	0,568733 *

Obs: * Parameters statistically significant at a 5% significance level.

ANNEX 9 - Historical Betas: 3 Year Betas / Monthly Data

Betas	$\beta_{93/95}$	$\beta_{96/98}$
A. Silva & Silva		0,962831 *
BCP	1,335540 *	1,262300 *
BES	0,389321	1,254910 *
BPI	1,095330 *	
BPI-SGPS		1,712130 *
C. S. Império	0,326150	1,176130 *
C. S. Mundial confiança	1,189160 *	1,489010 *
C. S. Tranquilidade	0,251714	0,462930 *
Caima	1,832830 *	0,747865 *
Corticeira Amorim	1,682110 *	0,977878 *
Espart	0,494621	0,863901 *
Estoril-Sol	1,134590 *	0,457200
Inapa	1,281050 *	0,220906
ITI	0,694160	0,596320
Lisnave	1,844400 *	
Lusomundo		0,531224
Mague	1,211980	1,052320 *
Modelo-Continente	1,523640 *	1,349260 *
Portugal Telecom		0,770661 *
Reditus	0,560805	0,516320
Semapa		0,871797 *
Somague		0,909698 *
Sonae Indústria	2,388970 *	1,311110 *
Sonae Investimento	2,281640 *	0,693827 *
Unicer	1,411590 *	0,707241 *

Obs: * Parameters statistically significant at a 5% significance level.

ANNEX 10 - Historical Betas Scholes-Williams: 1 Year Betas /Daily Data

Betas	$\beta_{S\text{-}W93}$	$\beta_{S\text{-}W94}$	$\beta_{S\text{-}W95}$	$\beta_{S\text{-}W96}$	$\beta_{S\text{-}W97}$	$\beta_{S\text{-}W98}$
A. Silva & Silva				3,055985	0,467412	0,226823
BCP	1,145314	1,174690	1,319938	1,295330	1,192002	0,575292
BES	0,598156	1,236252	0,714612	0,579956	0,741489	0,930511
BPI	1,208153	2,615950	1,487771			
BPI-SGPS				1,099692	1,308674	0,919488
C. S. Império	0,327248	0,525590	2,131932	0,442260	1,228242	0,222805
C. S. Mundial confiança	2,309668	1,449000	0,638421	0,491092	1,716393	1,252088
C. S. Tranquilidade	1,061277	0,430366	0,232938	0,740245	0,306717	0,937842
Caima	2,118087	0,844926	2,217468	0,964838	0,798498	0,926398
Cimpor		0,665090	0,550813	1,272220	1,545167	
Cimpor-SGPS					0,846018	0,708063
Cofina						0,496145
Corticeira Amorim	1,595315	2,246911	1,066687	1,520328	1,218110	0,594595
EDP					0,584519	0,917565
Espart	0,601501	1,516471	2,404333	0,201333	0,833779	0,891598
Estoril-Sol	1,057539	0,990938	1,059768	0,063151	0,811901	0,424481
Gestnave					-2,295376	0,325045
Inapa	1,242209	1,250536	0,952744	1,174816	0,412008	0,226623
Inparsa					0,901209	1,349476
Investec						0,594258
ITI	1,132658	1,342659	0,704864	0,423676	0,302337	1,015135
Lisnave	2,047017	0,571719	-0,818184	-0,802691	0,154661	
Lusomundo				1,131386	0,757978	0,834957
Mague		0,116800	1,248570	2,285776	1,164225	0,101579
Modelo-Continente	0,777220	1,830167	1,791673	1,684902	1,326509	0,734992
Portugal Telecom			0,550970	1,400413	0,767502	0,695003
Reditus	0,607093	2,389432	0,584975	-2,083320	0,657324	0,827995
Semapa			1,601192	1,130881	1,110108	0,928701
Somague		2,916410	1,036145	1,381551	1,052548	0,892534
Sonae Imobiliária						0,884971
Sonae Investimento	2,733354	2,478110	1,297038	1,343620	1,232446	1,073731
Unicer	0,918207	1,779716	1,414060	0,792767	0,872721	0,122675

APPENDIX A

$$\beta_{im} = b_0 + b_1 \beta_{im-1}$$

$$\beta_{im} = b_0 + b_1 \left[b_0 + b_1 \beta_{im-2} \right]$$

$$\beta_{im} = b_0 + b_1 b_0 + b_1^2 \beta_{im-2}$$

$$\beta_{im} = b_0 \left[1 + b_1 \right] + b_1^2 \beta_{im-2}$$

$$\beta_{im} = b_0 \left[1 + b_1 \right] + b_1^2 \left[b_0 + b_1 \beta_{im-3} \right]$$

$$\beta_{im} = b_0 \left[1 + b_1 + b_1^2 \right] + b_1^3 \beta_{im-3}$$

$$\beta_{im} = b_0 \left[1 + b_1 + b_1^2 + \ldots + b_1^{m-2} \right] + b_1^{m-1} \beta_{i1}$$

$$\beta_{im} = b_0 \left[1 + \sum_{k=1}^{m-2} b_1^k \right] + b_1^{m-1} \beta_{i1}$$

As we empirically observe that:

$$-1 \langle b_1 \langle 1 , \; \lim_{m \to \infty} b_1^{m-1} = 0$$

Therefore:

$$\lim \left[b_1^{m-1} \beta_{i1} \right] = 0$$

Hence, the starting value for β_{i1}, is irrelevant. Therefore:

$$\lim_{m \to \infty} \beta_{im} = \lim_{m \to \infty} b_0 \left[1 + \sum_{k=1}^{m-2} b_1^k \right] = b_0 \left[1 + \lim_{m \to \infty} \sum_{i=1}^{m-2} b_1^i \right]$$

As

$$\lim_{m \to \infty} \sum_{i=1}^{m-2} b_1^i = \lim_{m \to \infty} \sum_{i=1}^{m-2} b_1 \times b_1^{i-1} = \frac{b_1}{1 - b_1}$$