REM WORKING PAPER SERIES

(De) industrialization in the Von Thünen's economy

José Pedro Pontes, Armando J. Garcia Pires

REM Working Paper 0141-2020

September 2020

REM – Research in Economics and Mathematics
Rua Miguel Lúpi 20,
1249-078 Lisboa,
Portugal

ISSN 2184-108X

REM – Research in Economics and Mathematics

Rua Miguel Lupi, 20
1249-078 LISBOA
Portugal

Telephone: +351 - 213 925 912
E-mail: rem@iseg.ulisboa.pt

https://rem.rc.iseg.ulisboa.pt/

https://twitter.com/ResearchRem

https://www.linkedin.com/company/researchrem/

https://www.facebook.com/researchrem/

(De)industrialization in the Von Thünen's economy

by
José Pedro Pontes[1] and Armando J. Garcia Pires[2]

Date: 27/09/2020

Abstract: In the Von Thünen (1826)'s economy, manufacturing decentralization is viewed as the refining of an agricultural commodity near the cultivation site, which substitutes for its transport to an industrial mill located in the Town. As Friedrich List (1841) added, this substitution is economically feasible only if the savings in transport costs following from in site refining cover the increase in fixed costs associated with a second industrial plant. In market equilibrium terms, this happens when the decentralized machine is provided collectively by the landowners, who fund it through the proceeds of the rise in total land rent following from the industrial investment. This condition will be satisfied more likely in a large economy with high average transport costs and where manufacturing specializes in relatively weight losing activities.

If industrial decentralization is feasible, then the new factories will prefer to locate outside the Town, in formerly rural areas endowed with an intermediate degree of centrality. Their distance to the Town will be directly related with the intensity of input refining that they are able to carry out. This model appears to account for main stylized trends of manufacturing relocation nowadays, which are jointly labeled as "(de)industrialization".

JEL Classification: O12, O14, R12, B20

Keywords: Von Thünen, Manufacturing Location, Industrialization, External Economies of Scale.

Acknowledgements: The authors wish to thank participants in the seminar at Faculdade de Economia of Universidade do Porto, which took place in the 7^{th} February 2020. The usual disclaimer applies.

This article was funded by UECE-REM (Research Unit on Complexity and Economics - Research in Economics and Mathematics), a research center of ISEG/Universidade de Lisboa. The authors acknowledge financial support from FCT - Fundação para a Ciência eTecnologia (Portugal), national funding through grant UIDB/05069/2020.

[1] Corresponding author. Instituto Superior de Economia e Gestão, Universidade de Lisboa and UECE-REM, Rua Miguel Lupi, 20, 1249-078 Lisboa. Email: ppontes@iseg.ulisboa.pt

[2] Centre for Applied Research at NHH (SNF), Norwegian School of Economics (NHH), Helleveien 30, 5045 Bergen. Email: armando.pires@snf.no

1 Introduction

In spite of sharp variations across countries, the average degree of industrialization in Europe, as measured by the share of manufacturing value added in GDP, seems to have been increasing moderately since the beginning of the century, a trend that accords with the picture drawn by RODRIK (2016) for the main regions of the world economy.[3]

It has been widely admitted for some time that the variation in industrialization across countries and regions can be accounted for by two major causal factors (see, among others, SPILIMBERGO, 1998). The first main determinant is the general trend of transport and communication costs to fall. Until recently, the improvement of transportation has been matched by a similar trend of trade costs, namely ad valorem tariffs and other non-tariff barriers to trade. Although some change to an opposite course of action has been taking place recently, there is no reason to believe that a sharp and general reversal of the trend to free trade will occur in the future. The second major cause of regional asymmetries in industrialization lies in the fast growth in productivity in manufacturing, mainly associated with the automation of increasingly complex tasks. Such gains in industrial efficiency clearly outpace the progress found in non-manufacturing activities.

Some authors have established a causal link between these factors and the geographical variation in industrialization through the international trade theory based on the Ricardian comparative advantage, which assumes zero factor mobility between countries or regions and complete international mobility of products. For instance, RODRIK (2016) explains the intensity of manufacturing growth in a country by the change in relative unit production costs of manufacturing and non-manufacturing activities, using the world mean evolution of relative costs as a benchmark.

Other approaches based on the comparative advantage concept use instead the Heckscher-Ohlin framework, which is founded on differences in relative factor abundance across countries. According to this view, the fall in transport costs gives birth to comparative advantages that were previously hidden. Labor intensive manufacturing operations are moved to low wage countries or, by contrast, automated industrial processes return to core, capital abundant countries.

We depart here from the comparative advantage approach since we deal with the spatial differences in manufacturing development using the economic geography model of VON THÜNEN (1826). The crucial difference between the latter model and the Ricardian comparative advantage lies in two opposite assumptions (see SAMUELSON, 1983; VENABLES and LIMÃO, 2002). Although production still requires an immobile factor (namely, land), other factors such as labor are freely mobile. Indeed, the theory explains the equilibrium location of workers. In addition, it is presupposed that commodities bear positive transport costs, which are product specific.

[3] This picture would be much different if the share of manufacturing in overall employment would be used instead.

Is VON THÜNEN (1826)'s economic geography adequate to rationalize contemporary changes in the spatial distribution of manufacturing? Several issues should be handled. At least since HARRIS (1954) it is generally agreed that the "market" for a given manufacturer is made by a set of centers, whose relative importance (if they are similar in size) depends inversely on how far away they are placed from the industrialist. The assumption of a single and given center of activity, which is fundamental in the Thunian framework, seems at odds with reality. However, as FUJITA (2012) noted, the withdrawal of the assumption of a single market center renders the model non-competitive and requires that it is set in terms of monopolistic competition and increasing returns technology. FUJITA and KRUGMAN (1995) performed this task at the price of a rising complexity of the analysis and the removal of the equivalence between market equilibrium of locations and the social optimum. As such an equivalence is crucial for our analysis, we will keep ourselves within the Thunian boundaries of a single center of activity.

Another factor behind the choice of the VON THÜNEN's framework, with its emphasis on transport costs of the commodities, is the increasing awareness that it is a useful tool to analyze economic development in backward countries and regions. Gravitational models show that trade flows decrease dramatically with transport costs, the elasticity might reaching 2 (EATON and KORTUM, 2002: LIMÃO and VENABLES, 2001). According to STOREYGARD (2016), this harmful influence accounts for the fact that, in Sub-Saharan African countries where the capital city is also the major seaport, the economic size and growth of secondary cities is explained by the transport costs to the primate city. Natural factors of access to trade, such as a coastal location, appear to be a more important cause of economic density than the availability of fertile land in developing countries (HENDERSON et Al., 2018).Manufacturing activity seems to be rather concentrated in major urban areas in developing countries, a pattern that is reminiscent of VON THÜNEN (1826)'s Isolated State in the beginning of the nineteenth century. Nowadays, manufacturing in Europe is much more decentralized. There are two ways to deal with this apparent contradiction between VON THÜNEN (1826)'s theory and the reality of contemporary industrial Europe.

The first kind of approach consists in integrating both approaches (i.e. "comparative advantage" and "geographical barriers to trade") within the description of a spatial economy. This path of analysis may consist in generating a "hybrid" theory of location and trade, as EATON and KORTUM (2002) and VENABLES and LIMÃO (2002) did, but we can object that the most basic assumptions on VON THÜNEN and RICARDO/HECKSHER-OHLIN are utterly opposed.

An alternative option is to assign the two theories to different geographical scales, as we find in COSAR and FAGELBAUM (2016). While VON THÜNEN (1826)'s theory would be fit to explain the internal geography of a large country, such as India or China,which is spatially organized around a small set of transport hubs (like seaports), the comparative advantage framework could account for the nature of trade flows across these "international gates". This research seems to be more solid than the former one.

Our approach builds on the industrial relocation analysis made by VON THÜNEN in the chapter on "Distilling". There he deals with the productive location of a crop, such as wheat. In an initial stage, this crop is raised and then carried as a raw material to the Town. Here it is refined (or "distilled") into alcohol as a final output by means of a fixed equipment. The author assumes in the beginning that there are "restrictive regulations" that constrain the distilling activity to be concentrated in the Town. Under this constraint, the cultivation of wheat must take place not too far away from the activity center, since wheat is heavy and difficult to transport.

However, if it happens that, in a second stage, the administrative constraints on the distilling location are removed, then a decentralized refining machine can be installed aside the wheat field. Since alcohol has a much lower weight per unit of value than the grains, the raising of this kind of crop becomes profitable in areas that are much more distant from the Town than before.

Hence, VON THÜNEN's model includes both a theory of industrialization of peripheral areas, which were formerly purely agrarian, and the "deindustrialization" of the Town and its suburbs, which lose a considerable share of its initial manufacturing output.

However, the insight by the great German economist does not contain yet an economic model of manufacturing decentralization, since it completely depends on a switch of political regulations on manufacturing activities. By reading carefully MILLS (1970; 1972, chapter 5), who attempted to formalize the Thunian insight, we can understand why it is so. Concerning this issue, an important assumption is that both primary and manufacturing take place under a technology where the proportions between factors, including land and labor, are fixed. This assumption allows us to reduce all the costs incurred by the producers simply to transport costs. Hence, if manufacturing is weight losing, then the relocation of industrial plants to outer areas is always profitable, and we have to resort to exogenous factors, such as "politicy restrictions", in order to explain its timing.

In this context, LIST (1841) offered a crucial insight by stressing that manufacturing should be regarded as an increasing returns activity, which contrasts with the mostly constant returns to scale nature of agriculture. In order to set up a second decentralized industrial plant, an additional fixed cost must be borne. Such a fixed outlay should be covered by the decrease in aggregate transport costs caused by the industrial investment.

With this change, the economy still operates under perfect competition because farmers use jointly the refining equipment, so that economies of scale are external to each individual producer. LIST (1841) assumes that the second refining machine is provided collectively by the landowners, who use for that purpose the proceeds of the rise in total land rent that derives from the industrial investment. In a competitive economy where all factors of production are used in fixed proportions with land, the variation in total land rent is coincident with the decrease in aggregate transport costs. Hence, when the installation of a decentralized refining machine is profitable from the landowners' private viewpoint, it is also socially optimal as it minimizes total production and transport costs.

This article contains two more parts. Section 2, a formal model is presented in detail. Section 3 discusses the results, draws the main conclusions and indicates likely paths for additional examination.

2 A model of resource based industrialization in a core-periphery economy

2.1 The economy in time period 0 with manufacturing concentration in a single factory

VON THÜNEN (1826)'s economy is usually displayed in a homogeneous plane where transport activity is feasible in every each direction at a constant unit cost, a framework which he labels as an Isolated State. However, it can be cast in alternative as a linear economy, which stands as a fairly good approximation of the homogeneous surface. In this setting, the line is viewed as a transport lane, along which the unit transport cost is arbitrarily small when compared with the cost of carrying goods along other directions. Then, every each agent in the plane can be assigned an address on the line, which expresses the transport cost between his production site and the Town.

VON THÜNEN mentions the case of a navigable river that goes through the Town.

> Once we know the relative cheapness of water as compared with land freights, it will be easy to determine the economic situation of a farm, which sends its grain to market by water.
>
> Suppose that freight rates by water are one-tenth those by land. On a farm on a river, 100 miles from the Town, the value of grain (and all values deriving from this) will be the same as on a farm in the Isolated State at 10 miles from the Town.
>
> A farm 5 miles from the river incurs the same costs on 5 miles transport by land and 100 miles by water, and is in the same economic position as a farm in the Isolated State, which lies at 15 miles from the Town. (VON THÜNEN, 1966: 172)

Hence, we assume that $2x$ farmers are distributed along a line without fixed boundaries , where a Town is placed at the origin. Each farmer produces a unit of a raw material labeled as "commodity 1" using one unit of land, under a constant returns technology. The total area of fertile land available in each point is one unit of extent.

In order to be sold and consumed, the agricultural raw material must be industrially transformed into a final consumer good, named as "product 2". A crucial assumption of this approach to manufacturing growth is that the transformation has a weight losing nature. If one unit of input 1 is transformed into one unit of product 2, then the latter should be lighter and easier to carry than the former, so that the inequality $t_2 < t_1$ is always satisfied. Instances of this kind of industrial process are the "refining" of agricultural commodities. A grain such as wheat can be either "milled" into flour, or "distilled" into alcohol.

This economy works with two vertically related operations. The first one consists in the production of commodity 1 and its workings follow the well-known decentralized VON THÜNEN (1826)'s process. Each farmer bids a rent for every each point of space assuming that its profit is normal (or zero). Then, he is assigned the lot where the rent he offers is maximal. The market land rent curve is the upper envelope of the bid rent curves by all producers. In this specific case, where farmers produce the same crop, the individual bid rent curves are identical and are coincident with the market land rent curve.

During the second operation, the agricultural commodity 1 is transformed into a consumer good 2 in the context of an increasing returns operation that uses only a "machine" with a fixed cost F. Nevertheless, the whole productive process remains perfectly competitive since the farmers share the fixed input, which is provided collectively by the landowners. Each farmer carries its output to the "machine" in order to be refined and then delivers the final consumer good in the Town, where it is sold at the parametric full price p.

We assume that each unit lot is owned by a different individual $i = 1, 2, \ldots, 2x$, who receives the market land rent R_i, if the two stage productive process is completed, and 0 otherwise.

The former possibility occurs if the set of landowners collectively provide and install an industrial "machine" and such option will be profitable if the aggregate land rent exceeds the machine fixed cost, i.e. if

$$\sum_{i=1}^{2x} R_i \geq F \tag{1}$$

By contrast, if $\sum_{i=1}^{2x} R_i < F$, industrial plant will not be set up and the economy will produce nothing. Each landowner will receive a zero payoff.

We will concentrate on the former possibility. It is clear that the two-stage economy with manufacturing behaves in spatial terms in a way that is similar to the purely farming economy described in VON THÜNEN (1826).

It is well known that the competitive nature of the VON THÜNEN (1826)'s economy implies some welfare properties. The decentralized workings of the land market maximizes total land rent. Since the latter is the surplus of total revenues over aggregate transport costs, we can assess the pattern of equilibrium producer locations by finding out the aggregate transport cost-minimizing configuration.

The workings of this economy can be described as a two-stage game, where firstly the set of landowners decide cooperatively whether to set up an industrial plant. If the machine is not installed, nothing is produced and the game ends. Otherwise, the producers (farmers and industrialist) select non-cooperatively their locations.

We will solve this game backwardly by computing firstly the equilibrium locations of producers, and then assessing whether the industrial plant is economically feasible.

For this purpose, we define δ as the "refining intensity", i.e. the relative gain in product transportability due to industrial transformation as

$$\delta \equiv \frac{t_1 - t_2}{t_1} \tag{2}$$

In what follows, we will assume that agriculture is the unique activity that uses significant areas of land. Both the Town and the factory uses arbitrarily small lots, which can be approximated by zero.

We start by proving the following proposition.

Proposition 1 *If an industrial machine is set up in the VON THÜNEN (1826)'s economy and the refining rate* δ *exceeds* $\frac{1}{2}$*, then the unique equilibrium pattern of productive locations is unique and consists in the location of the factory at the Town and the placement of farmers along a connected interval centered around the origin of the line* $[-x, x]$.

Proof. Given the welfare properties of the model, we will assess the pattern of equilibrium locations by finding out the spatial configuration that minimizes aggregate transport costs.

Since they are self-evident, we will assume without proof the following assertions.

1. In a transport cost-minimizing arrangement, the farming area should be connected, without "holes" (vacant land).

2. In a transport cost-minimizing pattern, the locations of both the Town and the factory should be interior points to the farming area.

We plot the VON THÜNEN (1826)'s economy in Figure 1. Each place outside the Town is labeled by a distance to the origin and by a sign. We name as s the industrial unit place and by $\underline{r}$ the left hand side boundary of the farming area. We presuppose without loss of generality that the factory is placed on the right hand side of the Town, so that $s \geq 0$. Since in Figure 1 there is no ambiguity, each point will be labeled by a positive number corresponding to a distance to the origin.

We now write the aggregate transport cost function in relation to two arguments, s and $\underline{r}$, which includes both the total transport cost of the raw a material at a rate t_1, $TC_1(\underline{r}, s)$ and the total transport cost of the finished product at the rate t_2, $TC_2(s)$. Thus, we have

$$TC(\underline{r}, s) = TC_1(\underline{r}, s) + TC_2(s) \tag{3}$$

where

$$TC_1 = t_1 \left\{ \int_0^{\underline{r}} (\underline{r} - r)\, dr + \int_0^{s} (s - r)\, dr + \int_s^{2x - \underline{r}} [(2x - \underline{r}) - r]\, dr \right\} \tag{4}$$

$$TC_2 = t_2 (2xs) \tag{5}$$

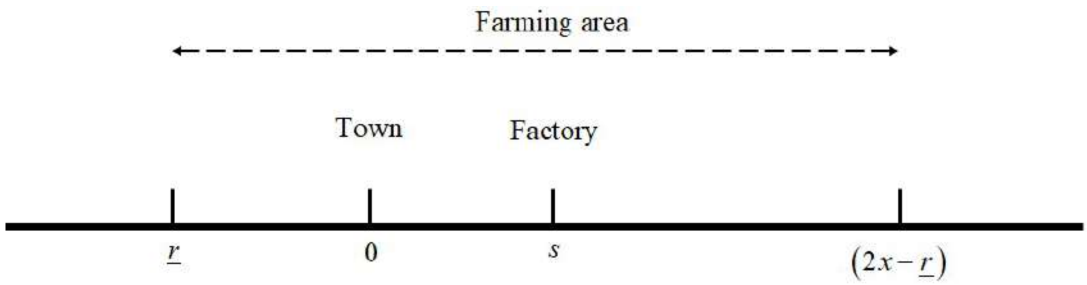

Figure 1: Von Thunen's linear economy with a factory

Hence, the aggregate transport cost is

$$TC(\underline{r}, s) = t_1 \left\{ \int_0^{\underline{r}} (\underline{r} - r)\, dr + \int_0^s (s - r)\, dr + \int_s^{2x-\underline{r}} [(2x - \underline{r}) - r]\, dr \right\} + t_2 (2xs) \tag{6}$$

The optimal spatial pattern of the productive activity minimizes function (6) in relation to $\underline{r}$ and s, subject to the constraints

$$0 \leq \underline{r} \leq 2x \tag{7}$$

$$0 \leq s \leq 2x - \underline{r} \tag{8}$$

The first partial derivatives of (6) are

$$\frac{\partial TC}{\partial \underline{r}} = \frac{\partial TC_1}{\partial \underline{r}} = t_1 [s + 2(\underline{r} - x)] \tag{9}$$

$$\frac{\partial TC}{\partial s} = t_1 [\underline{r} + 2(s - x)] + 2t_2 x \tag{10}$$

The Hessian matrix of function (6) is

$$\begin{bmatrix} \frac{\partial^2 TC}{\partial \underline{r}^2} & \frac{\partial^2 TC}{\partial \underline{r} \partial s} \\ \frac{\partial^2 TC}{\partial s \partial \underline{r}} & \frac{\partial^2 TC}{\partial s^2} \end{bmatrix} = \begin{bmatrix} 2t_1 & t_1 \\ t_1 & 2t_1 \end{bmatrix} \tag{11}$$

which is clearly positive definite. Hence, function (6) is a strictly convex function. Consequently, the necessary conditions of a local minimum subject to constraints (7) and (8), the so-called Kuhn-Tucker conditions, are also necessary and sufficient conditions of a unique minimum.

We check now whether the point $\underline{r} = x$ and $s = 0$ meets the first order conditions. Since, $\underline{r} = x$ is an interior point, the Kuhn-Tucker condition is just $\frac{\partial TC}{\partial \underline{r}}(\underline{r}, s) = 0$. By contrast, $s = 0$ is a boundary point, so that the first order condition is $\frac{\partial TC}{\partial s}(\underline{r}, s) \leq 0$.

It is clear that the conditions

$$\begin{aligned} \frac{\partial TC}{\partial \underline{r}} &= t_1 \left[s + 2 \left(\underline{r} - x \right) \right] = 0 \\ \frac{\partial TC}{\partial s} &= t_1 \left[\underline{r} + 2 \left(s - x \right) \right] + 2t_2 x \leq 0 \end{aligned}$$

are met by the point $(\underline{r}, s) = (x, 0)$ if and only if

$$x \left(2t_2 - t_1 \right) \leq 0 \Leftrightarrow \delta \geq \frac{1}{2} \tag{12}$$

where $\delta \equiv \dfrac{t_1 - t_2}{t_1}$ is the degree of weight loss during transformation, i.e. the relatively gain in product transportability. ■

Proposition 1 ensures that if a machine is provided and the industrial process is enough weight losing, then the equilibrium locations of both kinds of producing units, farmers and factory, will be symmetrical in relation to the Town. It remains to show that such refining machine will be indeed provided by the set of landowners.

Under equilibrium/socially optimal locations of producing units, the landowners will collectively provide the refining machine as long as total market land rent exceeds the fixed cost F of the industrial equipment. The total land rent is the surplus of the farmers' revenues over the aggregate transport cost. While the total revenues of the farmers is simply $p\,(2x)$, the aggregate transport cost can be easily computed from (6) performing the substitutions of variables $\underline{r} = x, s = 0$. Hence, we obtain

$$TC\,(x, 0) = 2t_1 \int_0^x (x - r)\, dr = t_1 x^2 \tag{13}$$

Hence, total market land rent is

$$TLR = p\,(2x) - t_1 x^2 \tag{14}$$

It remains to determine the competitive price of the finished product p, which we will do assuming that the land rent in the boundary $\underline{r} = x$ of the farming area is zero. Since all farmers raise a single crop, the bid rent $\Upsilon\,(r)$ will equal the market rent $R\,(r)$ in each point and it will be given by difference between the price of the final consumer good and the transport cost of the raw material to the factory sited in the Town, i.e.

$$R\,(r) = \Upsilon\,(r) = p - t_1 r \tag{15}$$

Hence, the competitive delivered price of the finished product is given by

$$R\,(\underline{r}) = R\,(x) = 0 \Leftrightarrow p = t_1 x \tag{16}$$

Substituting p from (16) into (14), the total market land rent is simply $t_1 x^2$. Thus, the set of landowners will install a refining machine if and only if

$$\begin{aligned} t_1 x^2 &\geq F \text{ or} \\ t_1 x &\geq \frac{F}{x} \end{aligned} \tag{17}$$

Inequality (17) has a straightforward interpretation, namely the transport cost over whole producing area should be high in relation to the burden of scale economies implied by industrialization, which is expressed by the per capita fixed cost of the refining machine.

We can summarize the results obtained thus far by means of the following Proposition.

Proposition 2 *A VON THÜNEN (1826)'s economy with manufacturing will attain an equilibrium monocentric pattern, with a set of farmers symmetrically distributed around a factory, which is sited at the Town, if and only if two conditions are satisfied. Firstly, the degree of input refining in the industrial process should be high enough, i.e.* $\delta \geq \frac{1}{2}$. *Secondly, transport costs over the whole farming area should be high in relation to the per capita manufacturing fixed costs, i.e.* $t_1 x \geq \frac{F}{x}$.

2.2 The economy in time period 1 with two industrial plants

Let us assume now that as the economy moves from time period 0 to period 1, either the overall size of the economy, measured by x, or the technological level, as expressed by the intensity of input refining δ, increase significantly. Then, we try to find out whether the entry of a second industrial plant becomes feasible in economic terms. We will continue to assume that a second industrial "machine" will be provided if and only if the associated increase in total land rent will suffice to cover the additional fixed cost F. The incumbent manufacture sited in the Town can be freely relocated following the entry of a second industrial plant. The equivalence between market equilibrium and social optimum is maintained since the collective of landowners will set up a second "machine" if and only if the entry minimizes aggregate transport costs.

Furthermore, we presuppose that the set of landowners firstly decide whether to set up a second industrial plant and then they will select the pattern of locations of both factories. Hence, we solve this model backwards. Firstly, assuming that a second refining machine has entered, we find out the aggregate cost minimizing pattern of locations for both factories. Then, we assess whether the entry is economically feasible.

In this section, we will assume that farmers distribute symmetrically around the Town thus forming a connected cultivation area without "holes" (vacant land). As before, both the Town and the factories require lots of arbitrarily small size. Then we prove the following proposition.

Proposition 3 *If two factories enter the VON THÜNEN (1826)'s economy and the "refining rate" satisfies the condition $\delta > 1 - x$, then they will adopt equilibrium symmetrical locations at a distance $r^* = \frac{1}{2}(x + \delta - 1)$ from the Town.*

Proof. We label the factories as A and B and the distances between each plant and the Town as r_a and r_b, respectively. It is clear that any aggregate transport cost minimizing plant locations will belong to the farming area $[-x, x]$, each "machine" being placed in a different side of the Town. The spatial economy with two industrial plants is depicted in Figure 1, where each location is labeled through a distance to the Town without ambiguity. The location $\frac{r_b - r_a}{2}$ is the middle point between the factories locations, which we assume to be positive without loss of generality.

Since equilibrium locations are also necessarily socially optimal, the first step is to write down the aggregate transport cost function $TC(r_a, r_b)$. Let $C_a(r_a, r_b)$ (respectively, $C_b(r_a, r_b)$) be the total transport cost resulting from the operation of factory A (respectively, B). Then, $TC(r_a, r_b)$ can be written

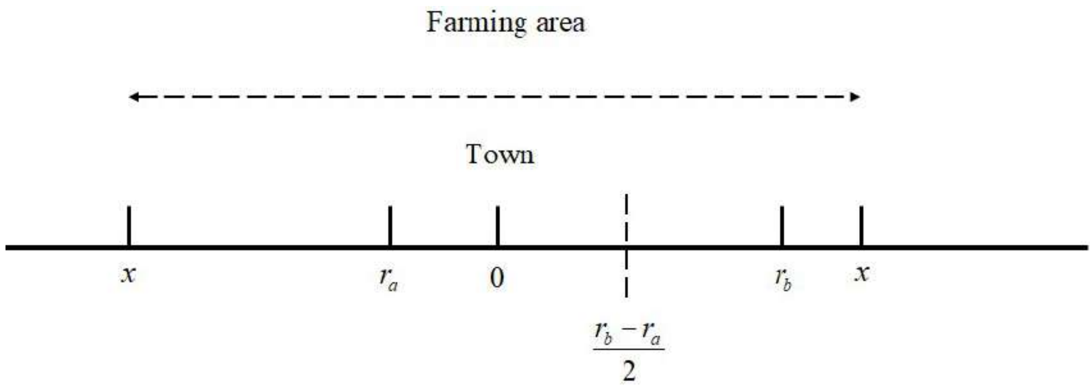

Figure 2: VON THUNEN's economy with two factories

as

$$TC\left(r_a, r_b\right) = C_a\left(r_a, r_b\right) + C_b\left(r_a, r_b\right) \tag{18}$$

where

$$C_a\left(r_a, r_b\right) = t_1 \left[\begin{array}{c} \int_0^{\frac{r_b - r_a}{2}} \left(r + r_a\right) dr + \int_0^{r_a} \left(r_a - r\right) dr + \\ + \int_{r_a}^{x} \left(r - r_a\right) dr \end{array} \right] + t_2 r_a \tag{19}$$

$$C_b\left(r_a, r_b\right) = t_1 \left[\int_{\frac{r_b - r_a}{2}}^{r_b} \left(r_b - r\right) dr + \int_{r_b}^{x} \left(r - r_b\right) dr \right] + t_2 r_b \tag{20}$$

The partial derivatives of $TC\left(r_a, r_b\right)$ can be easily computed as

$$\frac{\partial TC}{\partial r_a} = t_2 + t_1 \left(\frac{3r_a}{2} + \frac{r_b}{2} - x \right) \tag{21}$$

$$\frac{\partial TC}{\partial r_b} = t_2 + t_1 \left(\frac{r_a}{2} + \frac{3r_b}{2} - x \right) \tag{22}$$

We can conclude that $TC\left(r_a, r_b\right)$ is a strictly convex function as the Hessian matrix is positive definite.

$$\left[\begin{array}{cc} \frac{\partial^2 TC}{\partial r_a^2} & \frac{\partial^2 TC}{\partial r_b \partial r_a} \\ \frac{\partial^2 TC}{\partial r_a \partial b} & \frac{\partial^2 TC}{\partial r_b^2} \end{array} \right] = \left[\begin{array}{cc} \frac{3t_1}{2} & \frac{t_1}{2} \\ \frac{t_1}{2} & \frac{3t_1}{2} \end{array} \right]$$

Consequently, the necessary first order conditions of a local minimum of $TC\left(r_a, r_b\right)$ are also necessary and sufficient conditions of a unique minimum.

Formally, we minimize $TC(r_a, r_b)$ subject to the constraints

$$\begin{aligned} 0 &\leq r_a \leq x \\ 0 &\leq r_b \leq x \end{aligned} \tag{23}$$

We search an interior solution, i.e. one that solves the linear equation system

$$\begin{cases} \dfrac{\partial TC}{\partial r_a} = t_2 + t_1 \left(\dfrac{3r_a}{2} + \dfrac{r_b}{2} - x \right) = 0 \\ \dfrac{\partial TC}{\partial r_b} = t_2 + t_1 \left(\dfrac{r_a}{2} + \dfrac{3r_b}{2} - x \right) = 0 \end{cases} \tag{24}$$

whose solution is

$$r_a^* = r_b^* = \frac{1}{2} \left(x - \frac{t_2}{t_1} \right) = \frac{1}{2} (x + \delta - 1) \tag{25}$$

Locations of industrial plants given by (25) are indeed interior to the set defined by constraints (23). ■

If the economy accommodates two factories, then Proposition 1 tells that there will be "deindustrialization" of the Town, which loses entirely its manufacturing basis while remaining an export terminal. The proposition tells us further that the degree of decentralization of manufacturing away from the Town is directly related with the intensity of weight loss or increase in product transportability during industrial transformation.

We now tackle the economic feasibility of the entry of a second factory in this economy. Let $r^* \equiv r_a^* = r_b^*$ as defined in (25). Then, the aggregate transport cost given by (18) simplifies as

$$\begin{aligned} TC(r^*) &= 2 \left\{ t_2 r^* + t_1 \left[\int_0^{r^*} (r^* - r)\, dr + \int_{r^*}^{x} (r - r^*)\, dr \right] \right\} \\ &= 2t_2 r^* + t_1 \left[r^{*2} + (x - r^*)^2 \right] \end{aligned} \tag{26}$$

The rise in total land rent due to the entry of a second industrial plant ΔTLR equals the decrease in aggregate transport cost between time periods 0 and 1. If we recall that the aggregate transport cost in time 0 TC_0 is $t_1 x^2$, we can write taking into account (26)

$$\begin{aligned} \Delta TLR &= TC_0 - TC_1 \\ &= t_1 x^2 - \left\{ 2t_2 r^* + t_1 \left[r^{*2} + (x - r^*)^2 \right] \right\} \\ &= 2r^* \left[t_1 (x - r^*) - t_2 \right] \end{aligned} \tag{27}$$

Hence, the collective of landowners will install a second factory if and only if

$$2r^* \left[t_1 (x - r^*) - t_2 \right] \geq F$$

Substituting r^* from (25), the condition $\Delta TLR \geq F$, can be written as

$$\frac{t_1}{2}\left(x+\delta-1\right)^2 \geq F$$

Consequently, both the expansion of the economy (higher x) and a higher weight loss rate δ foster the entry of a second industrial plant VON THÜNEN's economy. Since it is clear that $t_1 x^2 > \frac{t_1}{2}\left(x+\delta-1\right)^2$, we can summarize our results as follows.

Let us assume that the technology of refining a raw material is such that the resulting gain in transportability in relative terms δ is higher than $\max\left\{\frac{1}{2}, 1-x\right\}$. Then, the development of this economy, related with the increase in both x and δ, involves a sequence of three stages.

1. When $F > t_1 x^2$, nothing is produced.

2. When $t_1 x^2 \geq F > \frac{t_1}{2}\left(x+\delta-1\right)^2$, a consumer good is produced. Farmers symmetrically distributed around the town raise a raw material that is transformed by a "machine" located in the Town and exported from there.

3. When $t_1 x^2 > \frac{t_1}{2}\left(x+\delta-1\right)^2 \geq F$, the raw material becomes transformed in two industrial plants symmetrically located around the Town at a distance $r^* = \frac{1}{2}\left(x+\delta-1\right)$. The refining rate δ influences directly both the emergence of a "deindustrialized" Town and the range of manufacturing relocation. Thus, the Town is now bound to trade the final consumer good.

3 Discussion and concluding remarks

The model presented in the previous section allows us to describe the geographical pattern of a resource based industrialization process in the context of a core-periphery economy. In an homogeneous space, a set of competitive farmers raise a specific crop using land in fixed proportions with other primary production factors. Then they carry the raw material to an industrial plant in order to be transformed into a final consumer good. Although such a transformation takes place under increasing returns to scale, the economy remains competitive, since the associated economies of scale are external to the individual farmers, who share the fixed input of "machine". Eventually, the manufactured good is brought to a market center (a Thunian Town), where it is sold either to local consumers or is exported. We describe the industrial evolution of this spatial economy along two time periods.

In time period 0, we address the issue of the entry of a single industrial plant. Since only the final manufactured good is valued by the consumers, this economy produces nothing unless at least one factory is installed. Such a "machine" will be provided by the set of landowners if the total land rent exceeds the factory related fixed cost. It is clear that this condition means that the transport cost over the whole productive area far outweighs the fixed cost implied by an industrial investment. This is the same as a break even condition, so that a minimum number of farmers is a precondition for the entry of a factory producing a final consumer good.

Now, we assume that the industrial process has a "refining" or "distilling" nature, so that its output is much more transportable than its input, an assumption made by VON THÜNEN (1826). Then, if the installation of a single "distillery" is economically feasible, then total land rent is maximized (or the aggregate transport cost is minimized) if the farmers distribute themselves symmetrically in relation to the Town, where the single factory is located. Hence, the Town is both a trading center and a manufacturing site.

Assume now that the economy size (as measured by the number of farmers) expands and technology (expressed by the refining intensity) improves. Then, a second factory might enter, with two factories occupying locations away from the Town but at an identical distance from it. The causal factors that allow the entry of a second manufacture, i.e. the number of farmers and the refining intensity of the input, also influence positively the distance between each factory and the Town.

The center of activity of the economy loses its industrial basis and thus becomes only an export terminal, in the context of a process usually labeled as "deindustrialization". It should be remarked that the industrial activity leaves the Town but it remains sited in points endowed with intermediate accessibility, so that remote points are discarded as feasible manufacturing places.

Our inquiry has a weakness in that it handles the reduction in output weight relative to the input for given amounts of either good. This was done for simplicity and it follows the framework put forward by VON THÜNEN (1826) and MILLS (1970; 1972, chapter 5).

Nevertheless, as LIST (1841), LAUNHARDT (1885) and DOS SANTOS FERREIRA and THISSE (1996) noticed, the decentralization of manufacturing not only reduces the output transport cost in relation to the input for fixed quantities of both goods, but it also expands significantly the demand addressed to the producers. Since the transformed product becomes lighter and easier to carry, it is no longer bounded to be sold near the production site and it can be exported at a wider range.

The inclusion of the demand enhancing effect of industrial decentralization is left for future research. For that purpose, the VON THÜNEN (1826)'s economy should contain demand by consumers in the context of a general equilibrium model in the line of approaches such a as those by SAMUELSON (1983) and NERLOVE and SADKA (1991).

References

[1] COSAR, A. Kerem and Pablo D. FAJGELBAUM (2016), "Internal geography, international trade and regional specialization", American Economic Journal: Microeconomics, 8(1), pp. 24-56.

[2] DOS SANTOS FERREIRA, Rodolphe and Jacques-François THISSE (1996), "Horizontal and vertical differentiation - the Launhardt model", International Journal of Industrial organization, 14, pp. 485-506.

[3] EATON, Jonathan and Samuel KORTUM (2002), "Technology, geography and trade", Econometrica, 70(5), pp. 1741-1779.

[4] FUJITA, Masahisa and Paul KRUGMAN (1995), "When is the economy monocentric? Von Thünen and Chamberlin unified", Regional Science and Urban Economics, 25, pp. 505-528.

[5] HARRIS, Chauncy D. (1954), "The market as a factor in the localization of industry in the United States", Annals of the Association of American Geographers, 44(4), pp. 315-348.

[6] HENDERSON, J. Vernon, Tim Squires, Adam STOREYGARD and David WEIL (2018), "The global distribution of economic activity: nature, history and the role of trade", Quarterly Journal of Economics, 133(1), pp. 357-406.

[7] LAUNHARDT, Wilhelm (1885), Mathematical Principles of Economics, London, Edward Elgar (English translation in 1993 of the first German edition).

[8] LIMÃO, Nuno and Anthony VENABLES (2001), "Infrastructure, geographical disadvantage, transport costs and trade", World Bank Economic Review, 15(3), pp. 451-479.

[9] LIST, Friedrich (1841), National System of Political Economy, Volume 2: The Theory, New York Cosimo (English translation of the first German edition in 2005).

[10] MILLS, Edwin S. (1970), "The efficiency of spatial competition", Papers and Proceedings of the Regional Science Association, 25, pp. 71-82.

[11] MILLS, Edwin S. (1972), Studies in the Structure of the Urban Economy, Baltimore and London, The John Hopkins Press.

[12] NERLOVE, Marc L. and Efraim SADKA (1991), "Von Thünen's model of the dual economy", Journal of Economics - Zeitschrift fur Nationalokonomie, 54(2), pp. 97-123.

[13] RODRIK, Dani (2016), "Premature deindustrialization", Journal of Economic Growth, 21(1), pp. 1-33.

[14] SAMUELSON, Paul A. (1983), "Thünen at two hundred", Journal of Economic Literature, 21(4), pp. 1468-1488.

[15] SPILIMBERGO, Antonio (1998), "De-industrialization and trade", Review of International Economics, 6(3), pp. 450-460.

[16] STOREYGARD, Adam (2016), "Farther on down the road: transport costs, trade and urban growth in Sub-Saharan Africa", Review of Economic Studies, 83, pp. 1263-1295.

[17] VENABLES, Anthony J. and Nuno LIMÃO (2002), "Geographical disadvantage: a Hecksher-Ohlin-von Thünen model of international specialization", Journal of International Economics, 58, pp. 239-263.

[18] VON THÜNEN, Johann Heinrich (1826), Von Thünen's Isolated State, Oxford Pergamon Press (English translation by Carla M. Wartenberg in 1966 of the first German edition).