DEMAND FOR MONEY AS FINANCIAL ASSET

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This paper is published here because it was developed into an article of a book, where its text is supposed to be known. The article is "Função de Procura de Moeda – uma Especificação Post-Keynesiana", authored by José Martins Barata & Cecília Campos in
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ABSTRACT

Tobin's seminal article (1958) derived the behaviour of money demand due to the speculative motive for liquidity preference, pioneering the idea of an existing financial investment motive behind holding money, which is supposed to be included in the investors' portfolios as a riskless financial asset. His objective was to find a consistent explanation for the liquidity preference function of Keynes (L_2). Tobin moved to conclude that every risk-averse rational individual optimising his expected utility always invests a part of his wealth in money and concluded that the keynesian L_2 was consistent. However, he did not derive a formula for money demand and did not find its elasticity. Moreover, he did not take into account that money is an asset yielding some income, since it includes time deposits earning interests. This paper generalises money demand on the basis of the theory of financial investments, derives overall money demand, and examines the problem of its elasticity, so as to recover the issue of liquidity trap existence. We discuss the similarity of this demand function with Keynes's, as well as its robusteness (and that of the underlying microeconomic model) to alternative scale factors, to the consideration that money yields a certain return, and to alternative concepts of uncertainty/risk, sheding some light on the Post Keynesian discussion on this matter.

1. INTRODUCTION

This paper generalises money demand on the basis of the theory of financial investments, and examines the problem of its elasticity, so as to re-discuss the issue of monetary policy effectiveness in areas of low interest rates. Our concern goes to knowing whether these areas exhibit the high elasticities some economists assume. The analysis is carried out in the light of both theoretical contributions and acknowledged econometric estimation.

In this context we resume the algebraic proof of the money demand function presented in Tobin (1958), since this author only presented the graphical solution. This methodology had been approached previously in other works (see Barata, 1998, pp.179-198; Barata and Variz, 2000). This paper prepares the theoretical background for econometric research as well as for a review of the most recent literature on demand for money. By the moment, we stay with a systematic presentation of the mathematical results we derived.

In his formulation of the theory of preference for liquidity, Keynes took the risk problem into account in its twofold form - market risk (i.e., the possibility of incurring in capital losses due to reductions in bond prices) and credit or insolvency risk (translated into the risk premium that is added to pure interest). However, only the former is relevant in the context of the optimal portfolio choice theory.

In the framework of traditional economic analysis, risk has no effect on the rational individual's behaviour. Neo-classical theory assumes that the behaviour of individuals is limited to the maximisation of utility associated to his consumption over distinct periods of time, subject to the market interest rate, constraints imposed by the initial wealth level and future income balances, which were taken as known. As a result, the financial market would play no role in determining the risk of financial assets, though in practice future income flows are indeed uncertain.

Breaking with the precedent theory, Keynes pioneered the distinction between saving formation and its application (i.e., financial investment): while funds saved are determined as a function of income, the investment of these funds depends on the interest rate and risk levels. As he wrote, «it should be evident that the interest rate cannot provide a reward for saving or abstinence as such. When an individual accumulates savings in the form of cash balances no interest earnings are raised. On the other hand, the interest definition *per se* indicates shortly that it is a reward for the renunciation to liquidity for a given period» (c.f. The General Theory..., Chapter 13, Sec. II).

Thus, the financial market takes the important role of allocating a certain part of total savings to investment, alternatively to holding them under the form of money. The fact that this allocation is made in this way exposes financial investments to the consequences of securities price changes, that is, the investment portfolios are affected by market risk and this influences the behaviour of investors, forcing them to forecast securities price changes in order to buy when prices are low and to sell later when they are high, as Keynes supposed.

Fellner (1946, pp. 145-151), criticised that view, arguing that it would not be reasonable to admit that the financial agents had expectations of interest rates rises just when they tumbled. If such expectations did exist, he argued, then other variables related with economic recovery - namely investment - would act on the interest rate time structure in such a manner that they would not fall.

Leontief, quoted by Tobin (1958), argued also that the speculative motive for liquidity preference is necessarily null in equilibrium situations, whatever the interest rate be (one must observe that Keynes has said the same, provided that the assumption of a world without uncertainty concerning interest rates holds). As a matter of fact, Leontief noted that the existing deviation between the observed interest rate and the expected one would be eliminated, since experience shows to investors how to build correct forecasts; even if an interest rate is persistently very low, one may accept it as "normal", if it remains with such values for a long time.

Later, also the monetarists contested the keynesian theory, denying the relevance of the speculative motive for liquidity preference as an explanation for money demand, hence merging the real and the monetary spheres of the economy (Friedman, 1956, p.259).

As a reply to all this criticism, Tobin wrote the article quoted above, where he derived the behaviour of money demand due to the speculative motive for liquidity preference, pioneering the idea of an existing financial investment motive behind holding money, as to cover market risk. This supposes that money is considered as one of the financial assets included in the investors' portfolios. The investor is supposed to be rational, in the sense that he aims to get an optimal combination of investments, in such a manner that the portfolio yield is maximised and its risk is minimised. However, he did not derive a formula for money demand and did not find its elasticity. Moreover, he did not take into account that money is an asset yielding some income, since it includes time deposits earning interests. This paper will restate this theory considering the missing issues just mentioned.

Another aspect in Tobin's model deals with the alleged disregard for uncertainty and its faithfulness to Keynes's interpretation of uncertainty. This point deserves also to be studied and we shall do it in this article.

One may say that after this summit of monetary theory, where we must include also the work of Patinkin (1956), we are unable to observe equally important theoretical developments, unless the inclusion of rational expectations in monetary theory is seen as a theoretical improvement. However, it seems that such a step was taken rather in order to get "stable" results for money demand, that is, it responds to an empirical need to suit econometric estimates, instead of providing a solid theoretical framework in the field of monetary theory.

After reading the book written by Laidler (1993) on demand for money, we think that this idea may be confirmed. A more recent work, whose author is S. Sriram (1999), allows us to maintain that statement. As matter of fact, the core of research on money demand has to do with empirical analysis, especially with the background of monetarism. Moreover, over the course of the last two decades - especially after the 1985 article by Rogoff - leading monetary literature was mainly concerned with the issue of credibility and independence of central banks.

One may put the question whether the last kind of literature, including also the developments on monetary rules, is not the unique way for pertinent research, within the framework of the Stability Treaty of the European Monetary Union, which practically abolished the economic policy in the countries involved. We do not think so, because most of that literature has to do with ergodic models rather than with economic theory. As a matter of fact, a macroeconomic theoretical background is indispensable to understand correctly and evaluate the results that occur in the economies.

For those reasons we think it is worthy to outline the theoretical work on money demand within the framework of the Keynesian theory and to develop empirical analysis on it, considering recent econometric techniques, like co-integration. Doing this, we do not intend to produce ergodic models. We just want to check how the theory fits to reality, considering past evidence.

2. THE PREFERENCE FOR LIQUIDITY IN THE PRESENCE OF RISK

2.1 - **The theory of Tobin**

As a starting point, Tobin (1958) built a formalisation for the reasoning of Keynes as regards the assumptions of money demand for speculation reasons. Take investment in consolidated bonds or perpetual rents, at an annual interest rate r (which is fixed at the issuing date). The interest rate that will be recorded in the market in a certain future moment is unknown, but can be estimated or anticipated by economic agents. We denote this rate by r_e.

At the date of issuing, the interest rate on bonds equals the market interest rate. Thus, if its interest takes the value of one currency unit, its current price shall be 1/r.

If the market interest rate always equalled r, the value of the capital generating one currency unit at the end of the following year would also equal 1/r and the investor would incur in no losses. Obviously, if the interest rate decreases, bond prices rise (hence the investor raises gains), falling otherwise. Nevertheless, these developments are unknown at present, therefore the different existing expectations regarding r_e. The rate of capital gains or losses, g, can therefore be written as follows:

$$g = \frac{1/r_e - 1/r}{1/r} = r/r_e - 1.$$

and the anticipated total rate of return is

$$r + g = r + \frac{r}{r_e} - 1. \quad (1)$$

This shows that the anticipation of interest rates is equivalent to projecting the bond price and yield. Money demand for speculation results from this appraisal by economic agents: "bulls", which are those foreseeing an interest rate fall (a rise in bond prices) purchase securities, looking forward to selling these at a higher price (hence showing a negative preference for liquidity); “bears”, which are those exhibiting opposite expectations (of interest rate increases, thus quotes tumble) sell bonds (prefer liquidity) as to avoid future capital losses, or if these agents hold no bonds – due, for instance, to previous disposal - they keep liquidity, to be used in periods of lowering prices these agents foresee.

In order to explain the negative slope of the function of preference for liquidity, and to counterattack the criticism of Leontieff and Fellner, Tobin made reference to the portfolio theory - introducing the concept of risk-less asset, as well as the conclusion that, in such situations, the optimal portfolio of each investor always encompasses part of the risk-less asset and part comprehending the risky asset. In the applications of this theory, money is of course the risk-free asset.

Consider two assets: asset 1 is money, x_1 representing its share in total portfolio value. This asset yields no return, hence bearing no risk[1]. Asset 2 stands for the risky asset, which yields a positive income and accounts for x_2 of the portfolio. Hence we have that $x_1+x_2=1$, each share falling between zero and one.

The starting point for Tobin is the Keynesian principle that asset 2 comprises consolidated bonds (i.e., bonds giving rise to a perpetual rent), which by assumption are issued by the government (hence the insolvency risk is null). The market risk exists because bond prices vary conversely to the market interest rate, as seen above. Therefore g (the rate of profits/losses) is a random variable.

With R_p standing for the average rate of return of portfolio p and R_2 denoting that of asset 2 in the portfolio, σ_p as the standard deviation of the rate of return of the portfolio and σ_2 the corresponding for asset 2, we have:

$$\text{I) } E(R_p) = x_2 \, E(R_2)$$

$$\text{II) } \sigma_p = x_2 \sigma_2$$

Rearranging equation II) yields

$$x_2 = \frac{\sigma_p}{\sigma_2}.$$

Since asset 2 is built from consolidated bonds at a fixed interest rate r, the following holds:

$$E(R_2) = E(g) + r;$$

$$\sigma_2 = \sigma_g .$$

Assuming that g is normally distributed with a null average and a standard deviation that equals σ_g, we have:

$$E(R_2) = r \;;$$

$$x_2 = \frac{\sigma_p}{\sigma_g} .$$

[1] Eventual risk associated with changes in money valuation other than those due to return fluctuations are disregarded in the analysis.

Substituting these expression in $E(R_2)$ and x_2 in equation I) gives rise to

$$E(R_p) = \frac{r}{\sigma_g} \sigma_p .$$

The latter equation defines the geometrical place of investment opportunities in the plan $E\sigma$, which in this case is a straight line that starts at the origin of the axis and passes by point (r,σ_g). The whole straight line segment from the origin up to this point provides the efficient frontier. The following step consists of determining the portfolio chosen by the investor, which shall fall over this segment.

Such choice is determined by the tangency point of an indifference curve - derived from the utility function the investor exhibits towards the return of the portfolio, which is denoted U(R).

Tobin assumes that function U(R) is quadratic, given that return is measured by the expected value of R and risk is measured by a second order moment (the variance, which in square roots gives the standard deviation). Hence, with the decision principle defined according to these two parameters, implicit is the acceptance of the existence of indifference curves based upon the same elements. Indeed, if $U(R) = R$ stood for the mathematical hope that utility were equal to the expected value of R (which is a constant μ) and from the viewpoint of economic behaviour this would mean that maximising the first would be the same as maximising return in an environment of certainty. Therefore, admitting the following

$$U(R) = (1+b) R + b R^2,$$

yields

$$E[U(R)] = (1+b) \mu + b (\sigma^2 + \mu^2),$$

where $-1 < b < 0$ for a risk-averse investor (i.e., an individual for whom marginal utility of return is always equal to or greater than zero at any pint, and is also a decreasing function in R).

Every time E[U(R)] is held constant an indifference curve is obtained. Plotting the family of indifference curves in the axis $E\sigma$, one of these shall be tangent to the efficient frontier, determining the point the investor chooses.

Therefore, every rational individual that is risk-averse and optimises his expected utility of R, invests a part of his wealth in money and another part in bonds, even if being sure that his interest rate forecast will not fail, as Leontief suggested. This is the well-known separation theorem. This holds provided that the existence of risk is recognised (i.e., even such sureness is always subject to the possibility of some error) and the behaviour of individuals is risk-averse.

The derivative of the indifference curves is

$$\frac{d\mu}{d\sigma} = \frac{\sigma}{-(1+b)/2b - \mu} = \frac{x_2\,\sigma_g}{-(1+b)/2b - x_2\, r}$$

At the tangency point, the angle coefficient of the straight line equals the derivative of the indifference curve,

$$\frac{r}{\sigma_g} = \frac{x_2\,\sigma_g}{-(1+b)/2b - x_2\, r}$$

thus

$$(2) \qquad x_2 = \frac{r}{r^2 + \sigma_g{}^2} \cdot [-(1+b)/2b].$$

It is concluded that, *ceteris paribus*, the proportion invested in bonds rises with the interest rate (since $dx_2/dr > 0$). If x_2 rises, then the share of money in the portfolio ($1-x_2$) decreases – i.e., money demand for speculative motives decreases.

When the interest rate is null, the amount invested in bonds is also zero (Keynes noted that in practice such never occurs, as r never falls below a certain minimum – an assumption that could be introduced in the model). In this case, $x_1=1$. When r reaches a certain high level such that the value of x_2 rises to 1, then x_1 becomes zero.

The expression of x_2 reveals that when r rises x_2 also increases, and therefore risk, $\sigma_p= x_2\sigma_g$, rises as well. This occurs since in addition to the income effect (an increase in r) there is a substitution effect with respect to risk (the disposal of some security as a counterpart of a higher return), which results from the existence of a utility function of order two.

Indeed, if r rises to r' with σ_g constant, the slope of the efficient frontier increases, reaching a higher indifference curve.

When r increases, bondholders incur in capital losses, but this is implicit in the investor's choice in terms of the structure of his portfolio, hence in the chosen value of σ_p. In turn, opting for higher risk, after r has increased, investors can calculate an adequate percentage of bonds in the portfolio. Due to the higher interest rate, this higher percentage allows to fully compensate for the capital losses occurred. The demonstration of this aspect is absent from the 1958 article of Tobin, but can be easily made.

Indeed, the rate of capital losses is $g = r/r' - 1$. The rate of increase of the income raised from the new portfolio is $x'_2\, r' - x_2\, r$. Rearranging and substituting gives rise to:

$$r/r' - 1 = (\sigma'_p\, r' - \sigma_p\, r)/\sigma_g \Rightarrow \sigma'_p = (r'-r)\sigma_g/rr' + \sigma_p r/r' \quad \text{and therefore } x'_2 = \sigma'_p/\sigma_g.$$

Tobin analyses the effects from the reduction of risk, everything else constant, which meets the argument of Leontief regarding investors' learning process. If the behaviour of monetary authorities – through declarations and market practices – reduces uncertainty as regards interest rates, then σ_g decreases, which has the same effect of an interest rate increase, that is, money demand decreases (see the expression of x_2).

Tobin generalises this theory to an environment of n assets. In this context, asset 2 now represents the set of risk-bearing assets (bonds and other credit instruments exhibiting distinct terms, borrowers and other features). The results above still hold, with money demand decreasing in the presence of interest rate increases.

The aggregation of individual demand functions gives rise to overall money demand. Though this function was not explicitly indicated in the article, it can be shown here based upon the elements contained in Tobin's article and taking into account his concept of wealth as a sum of monetary assets with other assets, subject to the so-called portfolio equilibrium principle (Tobin, 1955). This principle says that the community breaks down its wealth into real capital (possibly financed through bonds) and monetary assets in a way that this composition satisfies the community and, in a context of stable prices, no demand excess exists in any of these assets.

Let W stand for total wealth, i.e. the overall portfolio comprising immediate means of settlement (i.e., assets in the form of liquidity, denoted L_2) and bonds (B):

$$W = L_2 + B.$$

Then $W = L_2 + x_2 W$, and

$$(3) \qquad L_2 = (1 - x_2)\, W.$$

Replacing the percentage of securities in the portfolio by the respective value at the point where the efficient frontier is tangent to an indifference curve yields:

$$(4) \qquad L_2 = \{1 - \frac{r}{r^2 + \sigma_g^2} [-(1+b)/2b]\}\, W.$$

This function can be proven to be downward sloped and concave. This curve almost crosses the interest rate axis at $r = \sigma_g$ and crosses the money axis at L_2=W (when r equals zero). Therefore, this function differs from the Keynesian function L(i), which in at no point coincides with the money axis (instead, it tends asymptotically to the line of the minimum interest rate).

Function (4) is downward sloped, since

$$(4\text{-A}) \quad \partial L_2/\partial r = - \frac{\sigma_g^2 - r^2}{(r^2 + \sigma_g^2)^2} [-(1+b)/2b]\, W < 0,$$

whenever $\sigma_g > r$ (it should be noted that $g = r/r_e - 1$ and r_e is normally smaller than 1). Tobin considers that this feature renders the behaviour of the function somewhat ambiguous, especially for very high interest rates – the case where individuals attribute a low value to σ_g - since this variable derives from individual opinions or estimations (normally taking into account past data). In these cases, function L_2 can be increasing. However, in situations of sharp increases of interest rates investors incur in capital losses, which they will tend to annul by investing more in securities. This behaviour will in turn reduce - not increase – money demand. Making the calculations, it becomes evident that the second derivative is positive, which shows that we are in the presence of a convex function.

If instead of wealth account is taken of income Y as a factor of scale, function (4) can be quite easily adapted bearing that $W = Y/r$ (wealth is the present value of income for a rate of discount r, the general interest rate). For $Y=PX$, where P is the general price index and X denotes real output, the demand for real money for financial investment purposes can be written as follows:

$$(5) \qquad L_2/P = \{1/r - \frac{1}{r^2 + \sigma_g^2} [-(1+b)/2b]\} X.$$

If the nominal levels for this function are taken instead of real levels (Y thus serving as the scale factor), both the behaviour and the elasticity would remain unaltered.

The derivative of function (5) is

$$(6) \qquad \frac{\partial L2/P}{\partial r} = [-1/r^2 + 2ar/(r^2 + \sigma g^2)^2] X,$$

with $\quad a = -(1+b)/2b > 0.$

This derivative is negative when $2ar < 1$, implying that b cannot become too close to zero. Keeping the hypothesis of $r < \sigma_g$, the second derivative is positive, therefore real demand for money is similar to that of nominal demand.

It should be noted that the theory of Tobin was criticised as it assumed a concept of money as a non-income-raising asset, hence excluding quasi-money.

2.2 - **The missed liquidity trap principle**

As a foundation for the keynesian money demand for speculative motives, based in the portfolio choice principles, the theory of Tobin was incomplete, since it did not allow to study correctly the elasticities.

The elasticity of Tobin's money demand function with respect to the interest rate (recall that $L_2 = (1-x_2) W$) is

$$E_{L_2,r} = \partial L_2/\partial r \; r/L_2 = - \frac{\sigma_g^2 - r^2}{(r^2 + \sigma_g^2)^2} \left(-\frac{1+b}{2b}\right) W \frac{r}{(1-x_2)W}$$

from which results, given $r = x_2(r^2 + \sigma_g^2)[-2b/(1+b)]$ from equation (2),

$$(8) \qquad E_{L_2,r} = - \frac{\sigma_g^2 - r^2}{(r^2 + \sigma_g^2)} \frac{x_2}{1-x_2} .$$

Replacing x_2 by its value gives rise to the alternative expression

$$(9) \quad E_{L_2,r} = - \frac{(\sigma_g^2 - r^2)\, a\, r}{(r^2 + \sigma_g^2)(r^2 + \sigma_g^2 - a\, r)} ,$$

with $a = -(1+b)/2b > 0$.

Tobin did not study this elasticity, but calculated and analysed the elasticity of the demand for securities, with the following expression

$$([\sigma_g^2 - r^2]/[r^2 + \sigma_g^2]).$$

The latter form is simpler and does not depend on parameters or variables other than r and σ_g.

Let us assume that $r < \sigma_g$, which helps to determine the sign of elasticities. This assumption is quite realistic, as the volatility of nominal interest rates is generally high.

From here Tobin concluded that the demand for bonds is less elastic for higher interest rates than for lower ones. Indeed, this elasticity is always smaller than one and tends to 1 when the interest rate becomes close to zero. However, this does not seem to allow for conclusions on the validity of the keynesian principle that the preference for liquidity presents high elasticities in the low interest zone, which is contrary to Tobin's remarks at the end of his article.

Indeed, the analysis of the elasticity function, in its alternative forms ((8) or (9)) leads to the conclusion that when r tends to zero this elasticity also becomes close to zero. This is more easily conveyed by (9).

Therefore it becomes apparent that this reasoning will not provide a conclusive appraisal of the validity of the keynesian principle, according to which money demand elasticity in the zone of lower interest rates is very high.

When r tends to a given intermediate value, midway to its maximum value σ_g (or in its neighbourhood), elasticity converges to -1: this is shown by calculating the limit of (8) when r tends to $\sigma_g\sqrt{2x_2 - 1}$. Finally, when r tends to σ_g, elasticity tends again to zero.

In effect, the analysis of function (8) and its derivative (which requires some burdensome algebra we avoid here) shows that the absolute value of this elasticity, when r ranges between zero and σ_g, starts at zero, peaks for intermediate levels of interest rates (this peak may exceed 1, depending on the value of a), decreasing afterwards until zero, the latter being its trend value at the point r reaches the neighbourhood of σ_g. Its image resembles an inverted U over the r axis.

The problems raised by the behaviour described above are clearly overcome if income is used as a factor of scale, whether using nominal levels or in the real money demand function (5). The respective elasticity is

$$(10) \qquad \frac{\partial\ L_2/P}{\partial\ r}\ \frac{r}{L_2/P} = \frac{2ar^3/(r^2+\sigma_g{}^2)^{\ 2} - 1}{1 - ar/(r^2 + \sigma g^2)}.$$

When r tends to zero this elasticity approaches -1.

When r tends to a level such that $2ar^3=(r^2+\sigma g^2)^2$, the elasticity converges to zero. Its absolute value also exhibits an inverted U on the r axis, ranging from 1 to zero. It takes high absolute values for small and medium interest rates, and low values for high interest rates. This means that the demand for money for financial investment purposes, taking income as a scale factor, has a behaviour which is similar to that Keynes attributed to the preference for liquidity for speculation motives, which is not the case when the chosen scale factor is wealth.

Fig. 1 depicts the result of a typical simulation of the functioning of model (5) (real money demand with income as the scale factor). After several experiences with this model and also with model (4), we verified that both formulae show similar behaviours except as regards elasticities, as seen in the theoretical plan. We also observe that the typical case where b=-0,55 can be considered, which leads to interest rates raising above 2%. The chart lies upon this assumption. Some point elasticities are indicated above the curve.

Therefore, taking income as the scale factor, we conclude that the behaviour of elasticities is similar to the one Keynes attributed to L(i), although the virtually horizontal segment tending towards the minimum interest rate is lacking here.

As expressions (9) and (10) show, the absolute value of the money demand elasticity increased when a takes greater values – i.e., the closer b is to zero. This indicates that high values for the elasticities depend on the utility function defined in relation to the rates of return of financial assets. However, these arguments pose an obvious drawback, which is that of the viability of money demand: if a reaches a given level that is very high, it becomes negative, which is meaningless. Therefore, this is not the correct approach to elasticities growing *ad infinitum* in the zone of low interest rates that the traditional keynesian theory refers to and the liquidity trap principle is missed.

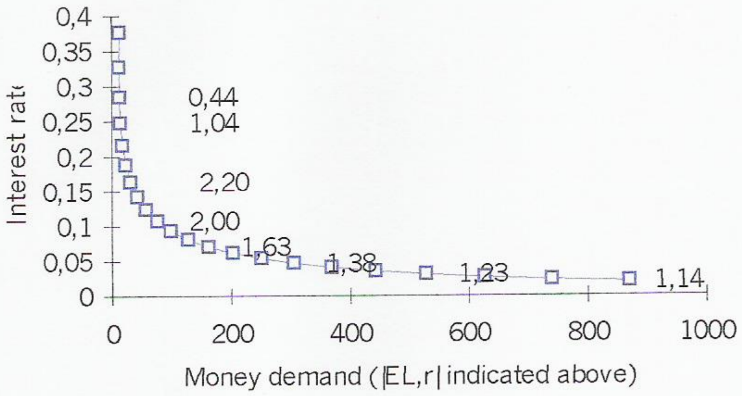

Fig. 1: Simulation of money demand model $L_2(r)$

3. THE DEMAND FOR MONEY AS FINANCIAL ASSET

3.1 - The liquidity preference above a minimal interest rate

First of all, one must observe that the concept of money *strictu sensu* (aggregate M_1, that is cash plus demand deposits) is the most obvious, but not necessarily the most adequate nor it is an imposition resulting from any sort of irrevocable theory. However the same applies much more to the enlarged concepts of money, since they include assets not immediately liquid and were introduced to meet monetarists' need to find variables that allow to support the "monetary variables stability" postulates in the context of econometric researches (this is why aggregate M_1 was replaced by M_2, since the latter delivered the results predicted by their theory).

Nevertheless, it could be argued that nowadays it is broader monetary aggregates that are most widely adopted by central banks. The crucial aspect, is that assets included in the money concept are risk-less, that is to say, they are subject to no capital losses due to interest rate changes. This is what currently occurs, even when adopting aggregate L, the most recently introduced among us.

So, to meet the present statistical practices of central banks, we have to use a broader monetary aggregate and this implies reformulating the Tobin's theory.

One must consider that in a modern economy there is an amount of financial funds, coming from savings and credit, that are applied to finance physical investment and financial investment. Financial investment is composed of several securities, bank deposits and cash. Some of these assets produce income (e.g. dividends, interests) and may have market risk (as a consequence of the volatility of their quotes) and other, such as cash an most of demand deposits have no income and no risk.

Let us take M_2 (cash plus total bank deposits) as the concept of money aggregate. Thus, we are dealing with a financial asset with some income, since time deposits yield interests, and it has no market risk. Part of this aggregate is destined to purchase goods and services (transactions in the economy) and other part is a financial investment, since its destination is to buy securities when this operation is considered worthy by investors or it is kept liquid until they judge the exact moment has come. They take this decision considering the level of interest rates in the market and their evaluation of the prices of securities and expectations of changes. The last part of the aggregate is the demand for money as financial asset. It is equivalent to the Keynes's speculative motive for liquidity preference.

We shall derive hereafter the function of the demand for money as financial asset. We consider all the financial assets in the economy grouped in three sets: money (M), equities (E) and bonds (B). Money has no market risk and yields some income, which is the interest of deposits. Equities and bonds have market risk. In the set of bonds, those issued by enterprises and the ones issued by the Government, as well as Treasury Bills, are included. Usually bonds have a smaller market risk (denoted σ_B) than equities and also lower rate of return (denoted R_B). Some have their interests indexed to a market interest rate and Treasury Bills have no risk. So, in the plan (E(R), σ) – where E(R) or $\overline{R}$ denotes the rate of return expected values – the locus of the investment opportunities in E and B is an hyperbole going from B to E (see Fig. 2), as it is known from the portfolio theory. As investors have also the possibility of investing their funds in time deposits, which compose M_2, there is an asset without risk. In this case, the portfolio theory shows also that the new efficient frontier is the segment of the straight line tangent to the hyperbole, since R_M until the efficient portfolio e. We assume a quadratic utility function, like that of Tobin, with the restriction that $R>i_m$, where i_m is a very low interest rate (to discuss in the next section). As it is known from the literature concerning the Tobin's separation theorem, the investors equilibrium portfolio (p) is given is given by the tangency of an iso-utility curve and the straight line efficient frontier in the point ($\overline{R}_p$, σ_p).

Let us study the problem algebraically. The equations that will determine the efficient frontier are:

$$1)\ \overline{R}_p = x_M R_M + (1-x_M)\ \overline{R}_e$$

$$2)\ \sigma_p = (1-x_M)\ \sigma_e,$$

where $\overline{R}_e$ and σ_e denote, respectively, the expected rate of return and the risk measure of the efficient portfolio e; $x_M+x_e=1$, with x_M the fraction of the total financial investment placed in money and x_e the fraction invested in the efficient portfolio e (composed of equities and bonds).

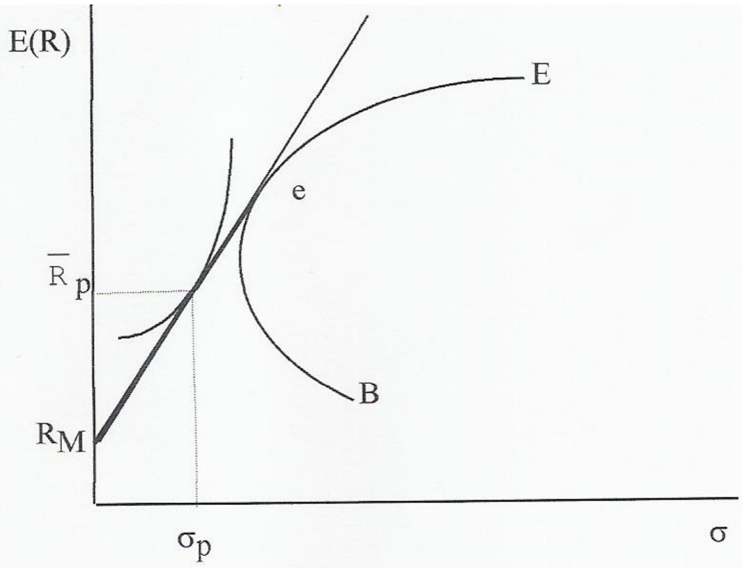

Fig. 2: Efficient frontier when money yields income

Taking equation 2) we derive

$$x_M = 1 - \frac{\sigma_p}{\sigma_e};$$

replacing this expression in equation 1), we obtain the straight line

$$\bar{R}_p = R_M + [(\bar{R}_e - R_M)/\sigma_e]\,\sigma_p.$$

Considering that the derivatives of the iso-utility curve and the efficient frontier are equal in the tangency point, we have

$$\frac{(1-x_M)\,\sigma_e}{-(1+b)/2b - x_M R_M - (1-x_M)\bar{R}_e} = \frac{\bar{R}_e - R_M}{\sigma_e}.$$

Let us denote with i the interest rate in the market, corresponding, for example to $\bar{R}_B$. Considering that $\bar{R}_e$ is higher than i, because it incorporates a risk premium (denoted r_p), we may write $\bar{R}_e = i + r_p$. In the other hand, as most assets included in M_2 do not yield interests and that the structure of this aggregate changes when interest rates rise, we consider that the relationship between its interest rate R_M and is not linear and is proportional to i: let us pose $R_M=\alpha i$, with $\alpha<1$. To simplify the resolution formula, let us denote $\bar{R}_e - R_M = u$ and

$(1+b)/2b=a$ (remember that $a<0$) . Note that $u=(1-\alpha)i+r_p$ and $du/di=1-\alpha$. . Thus, resolving the last equation, we obtain

$$x_M = \frac{\sigma_e^2 + a u + \bar{R}_e u}{\sigma_e^2 - R_M u + \bar{R}_e u},$$

hence

$$x_M = \frac{\sigma_e^2 + u\,(a+i+r_p)}{\sigma_e^2 + u^2}.$$

The derivative $dx_M/d\,i$ (with a too long formula) is negative as i increases.

Let us examine the following derivative:

$$\frac{d\,x_M}{d\,i} = \frac{[(1-\alpha)(a+i+r_p)+u](\sigma_e^2+u^2) - 2u(1-\alpha)[\sigma_e^2+u(a+i+r_p)]}{(\sigma_e^2+u^2)^2}.$$

This formula is negative if and only if

$$[(1-\alpha)(a+i+r_p)+u](\sigma_e^2+u^2) < 2u(1-\alpha)[\sigma_e^2+u(a+i+r_p)].$$

After a burdensome algebra, it is concluded that this inequality is true, since it is equivalent to

$$|a| > \frac{\bar{R}_e[(2-\alpha)-2u(1-\alpha)] - R_M - 2u(1-\alpha)\sigma_e^2}{(1-\alpha)(1-2u)}$$

and this is true because the denominator is positive (α and $2u$ are less than 1) and the numerator is negative – because, among other obvious reasons, $[(2-\alpha)-2u(1-\alpha)] < 0$ since $\alpha-2<0$, which guarantees $u>(\alpha-2)/2(1-\alpha)$.

With that derivative negative, the fraction of M_2, x_M, in the global portfolio of financial investors will decrease as i rises.

Let us examine graphically the effect of a rise in the interest rate of the market, i.

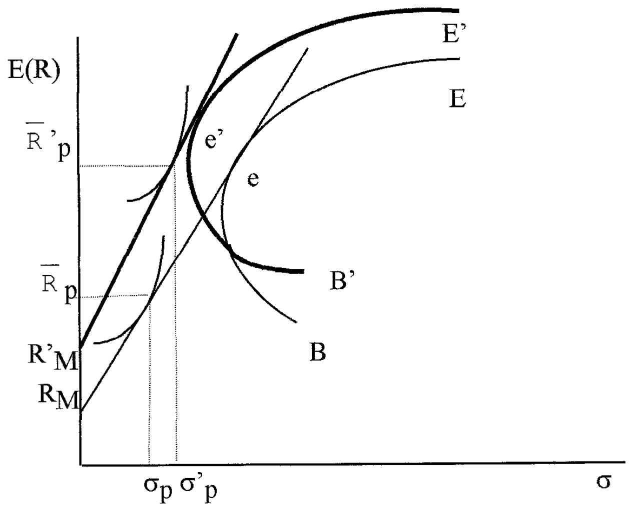

Fig. 3: The effects of a rise of the interest rates R_B and R_M

In Fig.3 it is supposed that an economic growth and productivity increase had made enterprise profits to rise and this made to jump $\bar{R}_E$. In order to slow this potential inflation situation, the central bank decided an interest rate increase, in order to "cool" the economy, which provoked a rise of $\bar{R}_B$. So, the frontier hyperbole moved upwards (to the thick shape). The interest rate of money increased also. The tangency with a higher iso-utility curve of investors determined a portfolio with a higher expected rate of return ($\bar{R}'_p$) and a higher risk too (σ'_p), as a consequence of effects of income and substitution.

So, considering that

$$x_M = 1 - \frac{\sigma_p}{\sigma_e},$$

in such a situation, with an increase of σ_p and a decrease of σ_e , x_M decreases, and the same happens with the demand for money as financial investment, denoted L_2.

Let us consider the national wealth, noted W, computed with the formula Y/i, where Y is the national income. Assuming that the investors place in financial investments a proportion ν of W, their global portfolio values ν W. Consequently, the demand for money as financial investment amounts

$$L_2 = x_M \nu W.$$

Replacing x_M by its value derived above,

$$L_2 = \frac{\sigma_e^2 + u(a + i + r_p)}{\sigma_e^2 + u^2} \nu W.$$

Replacing u and W by the variables they express, we obtain

$$(11) \quad L_2 = \frac{\sigma_e^2 + [(1-\alpha)i + r_p](a + i + r_p)}{\sigma_e^2 + [(1-\alpha)i + r_p]^2} \nu \frac{Y}{i}$$

As $d L_2/d i = \nu (d x_M/d i \,.\, Y/i + d(Y/i)/di \; x_M)$, this derivative is negative, which means that L_2 is decreasing. The second derivative is positive because the opposite hypothesis leads to a false conclusion, as we shall show hereafter. So the function is concave upward. As a matter of fact,

$$d^2L_2/d i^2 = \nu (d^2x_M/d i^2.Y/i + d(Y/i)/di.d x_M/d i + d^2(Y/i)/di^2.x_M + d x_M/d i.\, d(Y/i)/di)$$
$$= \nu Y (d^2x_M/d i^2.1/i - 2/i^2.d x_M/d i + 2/i^3.x_M), \text{ noting that}$$
$d(Y/i)/di = -Y/i^2$ and $d^2(Y/i)/di^2 = 2Y/i^3$.

Let us suppose that $d^2L_2/d i^2 < 0$, that is, L_2 is convex upward. As the second and third terms within the brackets are positive, d^2x_M/di^2 has to be negative and with an appropriate absolute value, in order this hypothesis be respected. Then, as $0 \leq x_M \leq 1$, resolving the inequality with respect to x_M,

$$d^2L_2/d i^2 < 0 \Rightarrow x_M < i/2(- d^2x_M/d i^2 \,.\, i + 2\, d x_M/d i),$$

with the first term within the brackets supposedly positive and the second negative.
When i becomes very low, approaching zero, we get $x_M = 1$.

Hence, $1 < i/2(- d^2x_M/d i^2 \,.\, i + 2\, d x_M/d i)$. In this situation, as L_2 was supposed convex upward, the absolute value of the negative dx_M/di becomes extremely high, the supposedly positive first term within the brackets becomes very low, with i approaching zero. The value within the brackets approaches zero. Multiplied by i/2 the whole approaches zero and it can not be higher than 1. So the conclusion is false and the departing hypothesis must be the opposite, that is, $d^2L_2/d i^2 > 0$, what implies that L_2 concave upward.

3.2 - The zone of the liquidity trap principle

Until now, we did not put any restriction to the value of the interest rate and did not question about the possibility whether it reaches zero. The portfolio theory ignores this point. So, uniquely based on this theory, as Tobin did, the L_2 curve may cross the abscises axle. This means to ignore the keynesian theoretical problem of the zone where elasticities of L_2 are infinite, that is, the liquidity trap zone.

Keynes argued that a minimal interest rate exists, because, when one lends money he has, at least, to be paid for the risk of default. Otherwise, nobody would lend money.

Beyond this argument, one must consider that it makes no financial sense to have interest rates decreasing until zero. Considering that a fixed-rate bond's price equals, *ceteris paribus*, its interest divided by the market interest rate, if the interest rate tends to zero, bond prices in the stock market boost immeasurably, tending to infinitum. No investor in such a situation would refrain from selling his securities to make fabulous profits, quite before the interest rate approaches zero. The only remaining issue would be to identify the purchaser, since hardly anyone would continue to forecast further interest rate reductions. This purchaser could eventually be the banking system, if the latter were devoted to lower the rates by purchasing bonds in open market operations. However, in this case, from a given interest rate level i_m, all investors would already have sold their bonds to the system.

This means that, as Keynes admitted, a minimum interest rate does in fact exist (noted i_m). It is that which speculators already consider being so low that that it becomes worthy to sell all securities in their portfolio and collect all capital gains. This is when x_e becomes zero and all financial wealth is kept in the form of money (equities would be no more a financial investment, they would be kept merely as representing the property of enterprises).

This issue implies that investors' utility function has to be re-formulated as to take into account the referred behaviour of selling all bonds when i_m is reached.

Indeed, expression (2) shows that, given a significant interest rate, when it equals a very low level i_m, x_e only becomes zero with b=-1, what is not possible because b was assumed to be smaller than 1. In that situation the expression of the quadratic utility function U(R) should equal zero and not only when R=0. Therefore, its form should be:

$$U(R) = \begin{cases} (1+b)\,R + b\,R^2, \text{ for } -1 < b < 0 \quad \text{and} \quad R > r_m; \\ 0, \text{ for } R \leq r_m \end{cases}$$

As for the demand for money for financial investment reasons (the speculation motive), taking income as the scale factor yields:

$$
(12)\, L_2 = \begin{cases} \dfrac{\sigma_e^{\,2} + [\,(1-\alpha)i + r_p]\,(\,a + i + r_p\,)}{\sigma_e^{\,2} + [\,(1-\alpha)i + r_p]^{\,2}} \; v\,\dfrac{Y}{i} & \text{for } i > i_m \\ vW & \text{for } i \le i_m \end{cases}
$$

This function does not tend asymptotically to the horizontal line starting at i_m: instead, it merges with this line straight from the point where the initial curve crosses it up to the vertical line beginning at vW. When the interest rate i_m is reached, all financial wealth is in the form of money and for any value of W it is $L_2 = vW$. Therefore, the demand curve has a section that is perfectly horizontal for $i = i_m$, as shown in Fig. 4.

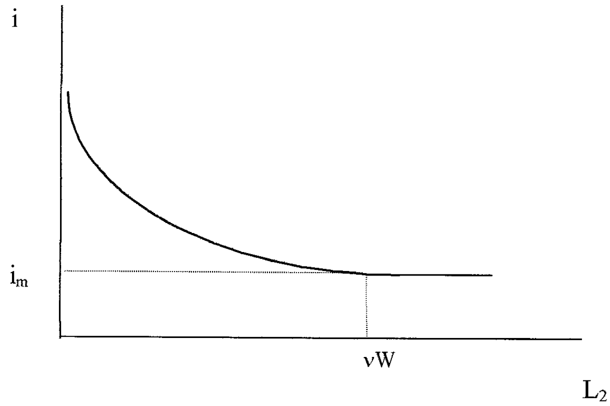

Fig. 4 : The demand for money –financial investment

This curve leads to a typically keynesian demand for money, because if money supply intersects this horizontal section, the liquidity trap will indeed occur. However, we must remember that Keynes wrote, in The General Theory, chapter 15,III, about the second reason for the existence of i_m (liquidity preference may become virtually as a consequence of too low interest rates), that "whilst this limiting case might become practically important in future, I know no example of it hitherto".

Nevertheless, the discussion whether it is realistic to consider that interest rates do approach, or not, i_m, nowadays in actual economies is beyond the purposes of this article.

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4. IS THE DERIVED L_2 AN ACCURATE INTERPRETATION OF KEYNES'S IDEAS?

The main point to discuss has to do with the relationship between keynesian uncertainty and the measure of risk we used above, following Tobin. One may ask whether the use of the standard deviation as a measure of risk, considering an underlying normal distribution of probabilities concerning the rate of return of financial investments respects the idea of keynesian uncertainty, as well as whether it is not an interpretation accepting the axiom of ergodicity, according to which the salient features of the socioeconomic realm "can be fully described by a set of unchanging conditional probability probability distribution functions" (Davidson, 1996, p. 479). If it is so, this theoretical view of the speculation motive demand for money, would infringe the Post Keynesian thought, mainly the philosophy of Paul Davidson, according to which the socioeconomic environment is transmutable or nonergodic, which means that it cannot be characterised in terms of probability distribution functions that remain invariant over time (Lewis and Runde, 1999, p.42).

Concerning nonergodicity, we accept this epistemological principle and think that it is respected when we use probability distributions and their statistical moments to describe the adherence of economic theory with past reality, as is our meaning when we use standard deviation as a measure of risk. This does not suppose that the probability distribution functions are unchanging. Contrary to many financial theorists, we think that it is not possible to forecast future events in the markets applying statistical methods to past data. Recent bad experiences with this, namely in USA, with models very famous in the main stream analysis, give support to our statement. We use models and statistical instruments as a method of describing reality and not to forecast it.

The issue of knowing whether the risk concept used in this work respects the keynesian uncertainty idea was discussed in Barata and Variz (2000), where was concluded that there is no incompatibility. As a matter of fact, considering a simple typology of uncertainty definitions, following closely Dequech (2000), we may distinguish between strong uncertainty and weak uncertainty.

Strong uncertainty comprises a range of alternative concepts with varying degrees of confidence, grouped into ambiguous and fundamental uncertainty. **Ambiguous uncertainty** corresponds to assuming that the agent lacks the existing information necessary to decide correctly. This is the case where economic agents are unable to link a unique function of probabilities to a set of events, but instead consider a family of functions, each of which bears an unambiguous (second-order) probability. In this environment, uncertainty as regards the probabilities may lead to non-action. It should be noted, however, that ambiguous uncertainty may give place to weak uncertainty (i.e. subjective expected utility becomes objective utility) as the full set of information becomes known to the decision-maker. The concept of **fundamental uncertainty** is based on the idea that the set of information never becomes fully known. According to Dequech (2000), because the future has not yet happened, it cannot be anticipated by a full reliable probabilistic estimate. This is said to invalidate any usage of probability distributions to mirror uncertainty. Shackle's uncertainty (as presented in Ford (1990)) provides an example of the latter, considering stages in ordering uncertain prospects: first, through a potential surprise function, a degree of belief is derived for every possible outcome, corresponding to the surprise its non-occurrence would rise. Second, through an ascendancy function decision-makers focus on the worst losses and the best gains that are feasible for each choice. Third, these pairs are ranked according to the so-called gambler preference function to select the optimal act. This rather broken-down scheme of the uncertainty-clearing process gives a detailed description of the decision-making psychology, but doubt is cast on its capacity to provide a guidance for decision-makers to perform.

Within **weak uncertainty**, investors attach a probability function to the occurrence of events. This corresponds to the traditional risk framework. The uncertainty concept used by Tobin falls in this case.

If Tobin is accused of completely ignoring uncertainty, critics may point to an alleged absence of strong uncertainty – which would be the Keynes's concept of uncertainty – since only weak uncertainty is present in his article. However, it should be noted that interpretations of Keynes's concept of uncertainty are far from consensus. This is partly explained by Keynes's blurred distinction between ambiguous and fundamental uncertainty, the shifting uncertainty concept over the course of his writing years, and by some apparently contradictory remarks on this issue; nonetheless, Keynes's writings do indeed seem to confirm that only interval probabilities can be used in case of ambiguity uncertainty (not point probabilities), and many argue that they deny the axiom of ergodicity. But Winslow (1995) recalls that in *A Treatise of Money* he admits that speculators can effectively engage in doing forecasting rationally based on expectations and past experience of sometimes irrational trends. Moreover, revisiting the chapter 13, II of *The General Theory*, when Keynes writes on the function of liquidity preference, that he denotes there M=L(r), we observe that he speaks of uncertainty about the interest rate as a necessary condition to hold liquid money, as well as of "risk of loss", when he gives the example of the possible change of the value $_{n}d_{r}$ (an accumulated capital between year n and year n+r, invested in debts, like bonds), if the investor needs liquidity before n years. He adds that the "mathematical expectation of gain, calculated in accordance with the existing probabilities – if it can be so calculated, which is doubtful – must be sufficient to compensate for the risk of disappointment". This supposes an underlying normal distribution function of probabilities, which allows to use standard deviation as a measure of risk. To know its future exact value is impossible and this makes the calculus "doubtful".

Thus, considering the arguments presented above, we think that the interpretation of Keynes's description of financial behaviour of investors (speculators) we made here, using statistical tools, is correct and more easily acceptable by most economists.

5. ELASTICITY, ECONOMETRIC MODELS AND FINDINGS

Finally, the last issue deals with knowing the effect of elasticities on overall money demand, $M_d = L_1 + L_2$, where L_1 is the transactions motive demand for money and L_2 is the financial investment motive demand for money.

It should be noted that, according to mathematics textbooks, the elasticity of M_d is

$$E_{Md,r} = E_{L1,r}\frac{L1}{Md} + E_{L2,r}\frac{L2}{Md}.$$

Bearing in mind the theory of money demand for transaction purposes of Baumol (1952) and the money demand function for speculation reasons in given by $L_2 = x_M \nu Y/i$, the Keynesian money demand in terms of real assets gains the following form:

$$(13)\quad \frac{M_d}{P} = k X^{0,5} r^{-0,5} + x_M \frac{\nu}{i} X,$$

where X=Y/P.

The elasticity of the function M_d/P is

$$\frac{d\,Md/p}{d\,i}\;\frac{i}{Md/p} = -0{,}5\,\frac{L_1/P}{Md/P} + \nu\left(\frac{d\,x_M}{di} - x_M\right)\frac{X}{Md/P}\,.$$

Examining this formula we conclude that its absolute value may not be usually very high. As a matter of fact the term X/Md/P is the velocity of circulation of money. It may be high, but it comes multiplied by values with an absolute value usually low ($\nu < 1$; $0 \leq x_M \leq 1$; $d\,x_M/di < 0$). The absolute value of the derivative becomes high only when i is high and in this case x_M approaches 0.

What kind of behaviour has this elasticity when i becomes very low? As x_M approaches 1 in this situation and the absolute value of the derivative of x_M becomes very low (it is concave), the elasticity becomes lower. So, this is the opposite of what Tobin expected and stated in his article (1958).

The monetarist money demand function takes the following form

$$M/P = f(r,p,X)$$

where p is the inflation rate. This variable is included as monetarists understand that it reflects the opportunity cost of money with respect to holding physical goods.

The remaining variables have the meanings that we have come to use. The inflation rate has been interpreted as an indication of inflationary expectations, more than as an information of the past. The utilisation of the instantaneous rate given by $\ln(Pt/Pt-1)$ for variable p simply corresponds to adopting a process of static adaptive expectations. The multiplicative form has provided the most used specification of the model in econometric estimations.

For the keynesian authors (cf. Goldfeld, 1973, reproduced by Thorn (1976, p.191)), that function has the form

$$M/P = f(r,X)\,.$$

No theoretical explanation for the inclusion of the inflation rate as an argument of the function exists for keynesians (cf. GOLDFELD, 1973, p.213).

From an econometric point of view, keynesians have in favour of them the fact that the inflation rate is already included in the nominal interest rate r, which indicates that the monetarist model bears multicolinearity problems.

Apart from the problem of the arguments – which does not appear to us as being a crucial issue – both functions are quite similar. Indeed, the common field of both currents seems quite vaster than a first approach could suggest, at least in what concerns strictly to this function. Obviously, in the theoretical stance, the principles of monetarists and keynesians as regards economic policy could not differ more. Disagreement starts at the principle of neutrality of money, ending at the issue of the stability of M_d and the elasticity at the minimum interest rate level.

Goldfeld (1973) used multiplicative models to specify these functions and presented a remarkable research containing several experiences of econometric estimations based on quarterly data for the USA from 1952 to 1972.

The estimations of Goldfeld made use of monetary aggregate M1, while taking income as the scale variable. For the Keynesian-type function, Goldfeld reached an elasticity of -0,019 with respect to the interest rate on commercial paper, and -0,045 for the interest rate on time deposits (cf. op. cit. p.193). These results were -0,015 and -0,038, respectively, in the case of the monetarist specification.

Goldfeld also carried out estimations using wealth as the scale variable. However, results proved less plausible. The elasticities with respect to interest rates shown no significant changes.

Judd and Scadding (1982), through a comprehensive revision of literature on this matter from 1973 onwards, give information on the results of numerous published works. Results are similar to those of Goldfeld. For instance, in the most recent research among those cited – the regression in the work of Kimball (1980) – the elasticity of M1 with respect to the interest rate on Treasury bills is -0,019, being -0,038 for the interest rate on time deposits (Judd e Scadding, pp.1016-1017).

Boughton (1982) estimated for the USA a multiplicative model of the kind of that of Goldfeld, adding to its arguments Tobin's "q " (i.e., the relative price of capital goods), with quarterly data running from 1952 up to 1978, and found an elasticity of -0,012 for commercial paper and -0,013 for time deposits.

Given this wide range of results, we may conclude that real money demand elasticities with respect to interest rates are quite smaller than one, the function being particularly inelastic – which conforms with the theory as presented above. It may be argued that these results are not reliable enough, since the model specification may not be the most correct one. Indeed, if taking stock of the Keynesian theory, it seems that alternatively the estimation of a model of the kind of identity (7) should be attempted instead of the multiplicative model.

Proving the relevance of other specifications are the results of Hunt & Volker (1981), obtained from a portfolio model structured in linear relationships. This research shows that this method yields quite robust parameters. However, no information on the elasticities or the data that could support the respective calculation is provided by the work.

Beyond the problem of model specification, the empirical results may also be doubtful owing to estimation methods. We have more literature to analyse and take into consideration the results of estimations respecting co-integration. In a later version of this paper our empirical results will be presented and more recent empirical analysis will be considered.

As for the monetarists, these always estimated models in logarithms. But this may have been mere empirical convenience, since these authors are not furnished with a theory that allows to detect whether that formula provides the best specification.

However, based on the theory we presented above, the commented empirical findings seem to us good indicators of the correct elasticities and point towards the rigidity of the function. In fact, that is a predictable conclusion in the light of the money demand theory that comprises the optimal portfolio choice theory.

6. CONCLUSION

This paper derived a money demand function in the light of the optimal portfolio choice theory. We found results that confirm the shape of the Keynes's L_2, but not the high elasticities in the zone of low interest rates. Instead of a curve of L_2 tending asymptotically to the straight line departing from i_m, the curve merges with this line at a certain point. The liquidity trap case would occur only on this straight line.

The problem of knowing if the risk measure used in this approach is a good interpretation of the keynesian uncertainty concept was studied. We noted that a theory based on the Tobin's principles does not disregard uncertainty, as any critics have argued. It encompasses a concept of uncertainty which appears to be at the same time compatible with Keynes's analysis of speculative behaviour, in a more operational context that one that fundamental uncertainty could offer, and most likely to provide theoretical framework more realistic.

We analysed the money demand elasticities considering the function derived as an achievement of Tobin's theory. The theoretical research showed that this function, in its nominal demand version, taking wealth as the scale factor, exhibits a zone of low elasticities in absolute terms, corresponding to the interest rates closer to zero; an intermediate zone of higher elasticities follows. These can be quite greater than 1. Finally a rigid zone corresponds to high interest rates.

However, if income is the chosen as a scale variable, elasticities are in absolute value higher than one in the low and intermediate interest rate zone, to which corresponds the typical Keynesian situation. Admitting adequate assumptions for investors' utility function, as to reflect their behaviour when interest rates are low and bond prices are very high, an infinite elasticity zone is established, to which corresponds a minimum interest rate. But immediately above this rate, the absolute values of elasticities are lower than 1 when the scale variable is wealth and slightly above 1 when income is chosen for that effect.

However, never or seldom is this minimum interest rate reached (as Keynes admitted). This is why its level has never been defined in precision, being only estimated to be somewhere close to zero. Thus, in general, admitting situations of interest rates below a given level – which in the simulation stands around 10% - the overall money demand function is inelastic. This is what several published empirical researches have shown.

Therefore, conversely to what has been reasoned by many economists, monetary policy is efficient even when lower interest rates prevail, and not only in the presence of high interest rates. In turn, according to exclusively theoretical results, such effectiveness may dwindle for intermediate interest rates.

REFERENCES

BARATA, José Martins - *Moeda e Mercados Financeiros*, 2nd Author's ed., Lisbon, 1998

BARATA, José Martins & VARIZ Paulo Eurico, «Demand for Money and Risk», paper for the METU Conference in Economics/IV, Ankara, 13-16 September 2000.

BAUMOL, W. - "The Transactions Demand for Cash: an Inventory Theoretical Approach", *Quarterly Journal of Economics*, November de 1952.

BOUGHTON, J.M. - "Forecasting Money Demand with Jorgenson's lag and Tobin's q", *Journal of Macroeconomics*, Fall 1982, pp. 405-418.

DAVIDSON, PAUL (1996) – "Economic Theory and Reality", *Journal of Post Keynesian Economics*, 1996, 18, 479-508.

DEQUECH, D. (2000) – "Some Foundational Issues for Post Keynesian Economic in the 21st Century", paper presented at the 6th International Post Keynesian Workshop, Knoxville, June 2000

FELLNER, W. - *Monetary Policies and Full Employment*, Berkeley & Los Angeles, University of California Press, 1946.

FISHER, Douglas - "Money Demand Variability: a Demand System Approach", *Journal of Business and Economic Statistics*, Apr.1992, pp.143-151.

FORD, J. (1983) - *Choice, Expectation and Uncertainty*, Oxford: Basil Blackwell.

FORD, J. (1990) – "Shackle's Theory of Decision-Making Under Uncertainty: A Brief Exposition and Critical Assessment", in Frowen, S. (ed.) *Unknowledge and Choice in Economics*, London: MacMillan.

FRIEDMAN, M. - "The Quantity Theory of Money - a Restatement", reprinted from *Studies of the Quantity Theory of Money*, University of Chicago Press, 1956 by THORN - Monetary Theory and Policy, Praeger Publ., N. York, 1976.

GERRARD, B. (1995) - Probability, Uncertainty and Behaviour: a Keynesian Perspective, in Dow, S. and J. Hillard (eds.) Keynes, Knowledge and Uncertainty, Hants: Elgar.

GOLDFELD, S.M. - "The Demand for Money Revisited", 1973, reprinted by THORN(1976), pp.189-247.

HUNT, B.F. & VOLKER, P.A. - "A Simplified Portfolio Analysis of the Long-run Demand for Money in Australia", *Journal of Monetary Economics*, 8 (1981), pp. 395-404.

JUDD, J.P. e SCADDING, J.L. - "The Search for a Stable Money Demand Function: a Survey of the Post-1973 Literature", *Journal of Economic Literature*, Set.1982, pp. 993-1023

KEYNES, John Maynard – *The General Theory on Employment, Interest and Money*, 1936.

LAIDLER, David E.W. - *The Demand for Money - Theories, Evidence and Problems*, HarperCollins College Publishers, N. York, 1993

LEWIS, Paul & RUNDE, Jochen – "A Critical Realist Perspective on Paul Davidson's Methodological Writngs on-and Rhetorical Strategy nfor-Post Keynesian Economics", *Journal of Post Keynesian Economics*, 1999, 18, 35-56.

PATINKIN, D. - *Money, Interest and Prices*, Harp & Row, N. York, 1956.

SOSIN, K. & TURNER, K. - "Monetary Policy, Uncertainty, Money Elasticities and Interest Rates", *Akron Business and Economic Review*, Summer 1990, pp.78-93.

THORN, R. - *Monetary Theory and Policy*, Praeger Publ., N.Y.,1976

TOBIN, J. - "Liquidity preference as behavior towards risk", *Review of Economic Studies*, vol. 25, Feb.1958, pp.65-86.

-"A Dynamic Aggregative Model", *Journal of Political Economy*, Apr.1955, reprinted by THORN(1976) pp. 172-188.

ROGOFF, K. - "The Optimal Degree of Commitment to an Intermediate Monetary Target", *Quarterly Journal of Economics*, vol. C, November 1985, pp. 1169-1189.

SRIRAM, Subramanian S., "Survey of literature on Demand for Money: Theoretical and Empirical Work with special Reference to Error-Correction Models", *IMF Working Paper*, IMF, 1999.

WINSLOW (1995) – "Uncertainty and Liquidity-preference", in Dow, S. and J. Hillard (eds.) *Keynes, Knowledge and Uncertainty*, Hants: Elgar.