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Demand for money as a financial asset – theory and evidence*

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Abstract

In this paper, the demand for money is generalised on the basis of the theory of financial investments and the problem of its elasticity is examined, so as to re-discuss the issue of monetary policy effectiveness in areas of low interest rates. We are concerned to know whether these areas exhibit the high elasticities which some economists assume. The analysis is carried out in the light of both theoretical contributions and acknowledged econometric estimation. In this context the algebraic proof of the money demand function presented in Tobin (1958) is resumed, since this author only presented a graphical solution. This article presents the theoretical background for econometric research as well as a review of the most recent literature on demand for money. First results of econometric estimates with Portuguese data are provided. A Cointegration method was used to estimate long and short-run regressions.

Key words: money, demand, cointegration.

1. Introduction

This paper generalises money demand on the basis of the theory of financial investments, and examines the problem of its elasticity, so as to re-discuss the issue of monetary policy effectiveness in areas of low interest rates. We are concerned to know whether these areas exhibit the high elasticities which some

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economists assume. The analysis is carried out in the light of both theoretical contributions and acknowledged econometric estimation.

In this context, we resume the algebraic proof of the money demand function presented in Tobin (1958), since this author only presented the graphical solution. This methodology had been approached previously in other works (see Barata, 1998, pp.179-198; Barata and Variz, 2000). This paper presents the theoretical background for econometric research as well as a review of the most recent literature on demand for money.

In his formulation of the theory of preference for liquidity, Keynes took the risk problem into account in its twofold form - market risk (i.e., the possibility of incurring capital losses due to reductions in bond prices) and credit or insolvency risk (translated into the risk premium that is added to pure interest). However, only the former is relevant in the context of the optimal portfolio choice theory.

In the framework of traditional economic analysis, risk has no effect on the rational individual's behaviour. Neo-classical theory assumes that the behaviour of individuals is limited to the maximisation of utility associated to his consumption over distinct periods of time, subject to the market interest rate, constraints imposed by the initial wealth level and future income balances, which were taken as known. As a result, the financial market would play no role in determining the risk of financial assets, though in practice future income flows are indeed uncertain.

Breaking with the preceding theory, Keynes pioneered the distinction between saving formation and its application (i.e., financial investment): while funds saved are determined as a function of income, the investment of these funds depends on the interest rate and risk levels. As he wrote, "it should be evident that the interest rate cannot provide a reward for saving or abstinence as such. When an individual accumulates savings in the form of cash balances no interest earnings are raised. On the other hand, the interest definition per se indicates shortly that it is a reward for the renunciation to liquidity for a given period" (c.f. The General Theory..., Chapter 13, Sec. II).

Thus, the financial market takes the important role of allocating a certain part of total savings to investment, alternatively to holding them in the form of money. The fact that this allocation is made in this way exposes financial investments to the consequences of securities price changes, that is, the investment portfolios are affected by market risk and this influences the behaviour of investors, forcing them to forecast securities price changes in order to buy when prices are low and to sell later when they are high, as Keynes supposed.

Fellner (1946, pp. 145-151), criticised that view, arguing that it would not be reasonable to assume that the financial agents had expectations of interest rate rises just when they fell. If such expectations did exist, he argued, then other variables related to economic recovery - namely investment - would act on the interest rate time structure in such a manner that they would not fall.

Leontief, quoted by Tobin (1958), also argued also that the speculative motive for liquidity preference is necessarily null in equilibrium situations, whatever the interest rate be (one must observe that Keynes has said the same, provided that the assumption of a world without uncertainty concerning interest rates holds). As a matter of fact, Leontief noted that the existing deviation between the observed and the expected interest rates would be eliminated, since experience shows investors how to build correct forecasts; even if an interest rate is persistently very low, one may accept it as "normal", if it remains with such values for a long time.

Later, the monetarists also contested the keynesian theory, denying the relevance of the speculative motive for liquidity preference as an explanation for money demand, hence merging the real and the monetary spheres of the economy.

As a reply to all this criticism, Tobin wrote the article quoted above, in which he derived the behaviour of money demand due to the speculative motive for liquidity preference, pioneering the idea of an existing financial investment motive behind holding money, so as to cover market risk. This supposes that money is considered as one of the financial assets included in the investors' portfolios. The investor is supposed to be rational, in the sense that he aims to obtain an optimal combination of investments, in such a manner that the portfolio yield is maximised and its risk is minimised. However, he did not derive a formula for money demand and did not find its elasticity. Moreover, he did not take into account that money is an asset yielding some income, since it includes time deposits earning interests. This paper will restate this theory considering the missing issues just mentioned.

Another aspect in Tobin's model deals with the alleged disregard for uncertainty and its faithfulness to Keynes's interpretation of uncertainty. This point also deserves to be studied and we shall do so in this paper.

One may say that after this summit of monetary theory, we are unable to observe equally important theoretical developments, unless the inclusion of rational expectations in monetary theory is seen as a theoretical improvement. However, it seems that such a step was taken in order to get "stable" results for money demand, that is, it responds to an empirical need to suit econometric estimates, instead of providing a solid theoretical framework in the field of monetary theory.

After reading the book written by Laidler (1993) on demand for money, we think that this idea may be confirmed. A more recent work, whose author is S. Sriram (1999), allows us to maintain that statement. As a matter of fact, the core of research on money demand has to do with empirical analysis, especially against the background of monetarism. Moreover, over the course of the last two decades - especially after the 1985 article by Rogoff - leading monetary literature was mainly concerned with the issue of credibility and the independence of central banks.

One may put the question whether the latter literature, including also the developments on monetary rules, is not the unique way for pertinent research, within the framework of the Stability Treaty of the European Monetary Union, which practically abolished the economic policy in the countries involved. We do not think so, because most of that literature has to do with ergodic models rather than with economic theory. As a matter of fact, a macroeconomic theoretical background is indispensable to understand correctly and evaluate the results that occur in the economies.

For these reasons, we think it is worth outlining the theoretical work on money demand within the framework of the Keynesian theory and to develop empirical analysis on it, considering recent econometric techniques, like co-integration. In doing so, we do not intend to produce ergodic models. We simply wish to check how the theory fits to reality, considering past evidence. This paper is organized as follows: in section 2 and 3 we present the theoretical framework, in section 4 we present the econometric findings and in section 5 we conclude.

2. The preference for liquidity in the presence of risk

2.1 - The theory of Tobin

As a starting point, Tobin (1958) built a formalisation for the reasoning of Keynes as regards the assumptions on money demand for speculation reasons. Take investment in consolidated bonds or perpetual rents, at an annual interest rate r (which is fixed at the issuing date). The interest rate that will be recorded in the market in a certain future moment is unknown, but can be estimated or anticipated by economic agents. We denote this rate by re.

At the date of issuing, the interest rate on bonds equals the market interest rate. Thus, if its interest takes the value of one currency unit, its current price shall be 1/r.

If the market interest rate always equalled r, the value of the capital generating one currency unit at the end of the following year would also equal 1/r and the investor would incur in no losses. Obviously, if the interest rate decreases, bond prices rise (hence the investor raises gains), falling otherwise. Nevertheless, these developments are unknown at present, hence the different existing expectations regarding re. The rate of capital gains or losses, g, can therefore be written as follows:

$$g = \frac{1/re - 1/r}{1/r} = r/re - 1.$$

and the anticipated total rate of return is

$$(1) \qquad r + g = r + \frac{r}{re} - 1.$$

Tobin considers two assets: asset 1 is money, x_1 representing its share in total portfolio value. This asset yields no return, hence bearing no risk[1]. Asset 2 stands for the risky asset, which yields a positive income and accounts for x_2 of the portfolio. Thus, we have $x_1+x_2=1$, each share falling between zero and one. Introducing the concept of riskless asset, that is money, with the portfolio theory he concluded that, in such situations, the optimal portfolio of each investor always encompasses part of the riskless asset and part of the risky asset.

The starting point for Tobin is the Keynesian principle that asset 2 comprises consolidated bonds (i.e., bonds giving rise to a perpetual rent), which by assumption are issued by the government (so the insolvency risk is null). The market risk exists because bond prices vary conversely to the market interest rate, as seen above. Therefore, g (the rate of profits/losses) is a random variable.

With Rp standing for the average rate of return of portfolio p and R2 denoting that of asset 2 in the portfolio, σ p as the standard deviation of the rate of return of the portfolio and $\sigma 2$ the corresponding for asset 2, we have:

I) $E(R_p) = x_2 E(R_2)$

II) $\sigma_p = x_2 \sigma_2$

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[1] Eventual risk associated with changes in money valuation other than those due to return fluctuations are disregarded in the analysis.

Solving this equations system, the mathematical expression of the efficient frontier is found to be

$$E(R_p) = \frac{r}{\sigma_g} \sigma_p.$$

The following step consists of determining the portfolio chosen by the investor, which shall fall over this straight line. Such choice is determined by the tangency point of an indifference curve derived from the utility function the investor exhibits towards the return of the portfolio, which is denoted U(R).

Tobin assumes that function U(R) is quadratic, given that return is measured by the expected value of R and risk is measured by a second order moment (the variance, which in square roots gives the standard deviation). Hence, with the decision principle defined according to these two parameters, the acceptance is implicit of the existence of indifference curves based upon the same elements. Indeed, if U(R)=R stood for the mathematical hope that utility were equal to the expected value of R (which is a constant μ) and from the viewpoint of economic behaviour, this would mean that maximising the first would be the same as maximising the return in an environment of certainty.

Considering the methodological statement adopted here, this function must be viewed as a game rule, accepted by those who act in financial markets following "rational expectations". As a matter of fact, no one can prove that they actually have a quadratic utility function or any other formula. Therefore, admitting the following

$$U(R) = (1+b) R + b R^2,$$

yields

$$E[U(R)] = (1+b)\ \mu + b\ (\sigma^2 + \mu^2),$$

where $-1 < b < 0$ for a risk-averse investor (i.e., an individual for whom marginal utility of return is always equal to, or greater than, zero at any pint, and is also a decreasing function in R).

Every time E[U(R)] is held constant an indifference curve is obtained. Plotting the family of indifference curves in the axis Es, one of these will be tangential to the efficient frontier, determining the point the investor chooses.

Therefore, every rational individual who is risk-averse and optimises his expected utility of R, invests a part of his wealth in money and another part in bonds, even if he is sure that his interest rate forecast will not fail, as Leontief suggested. This is the well-known separation theorem. This holds, provided that the existence of risk is recognised (i.e., even such sureness is always subject to the possibility of some error) and the behaviour of individuals is risk-averse.

At the tangency point, the angle coefficient of the straight line equals the derivative of the indifference curve[2],

$$\frac{r}{\sigma_g} = \frac{x_2\,\sigma_g}{-(1+b)/2b - x_2\, r}$$

thus

$$(2) \qquad x_2 = \frac{r}{r^2 + \sigma_g^2.}\,[-(1+b)/2b].$$

It is concluded that, ceteris paribus, the proportion invested in bonds rises with the interest rate (since $dx_2/dr > 0$). If x_2 rises, then the share of money in the portfolio ($1-x_2$) decreases – i.e., money demand for speculative motives decreases.

Tobin analyses the effects of the reduction of risk, everything else beingconstant, which meets the argument of Leontief with regard to investors' learning process. If the behaviour of monetary authorities – through declarations and market practices – reduces uncertainty as regards interest rates, then σ_g decreases, which has the same effect as an interest rate increase, that is, money demand decreases (see the expression of x_2).

The aggregation of individual demand functions gives rise to overall money demand. Though this function was not explicitly indicated in the article, it can be shown here based upon the elements contained in Tobin's article and taking into account his concept of wealth as a sum of monetary assets with other assets, subject to the so-called portfolio equilibrium principle (Tobin, 1955). This principle states that the community breaks down its wealth into real capital (possibly financed through bonds) and monetary assets in a way that this composition satisfies the community and, in a context of stable prices, no demand excess exists in any of these assets.

Let W stand for total wealth, i.e. the overall portfolio comprising immediate means of settlement (i.e., assets in the form of liquidity, denoted L_2) and bonds (B):

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2 For more details, see Barata and Variz , "Demand for Money and Risk", (2000), http://www.iseg.utl.pt/cief/papers

$W = L_2 + B$.

Then $W = L_2 + x_2 W$, and

(3) $L_2 = (1 - x_2) W$.

Replacing the share of securities in the portfolio with the respective value at the point where the efficient frontier is tangent to an indifference curve yields:

$$(4) \quad L_2 = \{1 - \frac{r}{r^2 + \sigma_g^2} [-(1+b)/2b]\} W.$$

This function can be proven to be downward sloped and concave. This curve almost crosses the interest rate axis at $r = \sigma_g$ and crosses the money axis at $L_2 = W$ (when r equals zero). Therefore, this function differs from the Keynesian function L(i), which at no point coincides with the money axis (instead, it tends asymptotically to the line of the minimum interest rate).

It should be noted also that the theory of Tobin was criticised, as it assumed a concept of money as a non-income-raising asset, hence excluding quasi-money.

2.2 - The missing liquidity trap principle

As a foundation for the keynesian money demand for speculative motives, based on the portfolio choice principles, the theory of Tobin was incomplete, since it did not allow the correct study of the elasticities. He calculated and analysed the elasticity of the demand for securities, with the following expression

$$([\sigma_g^2 - r^2]/[r^2 + \sigma_g^2]).$$

The latter form is simple and does not depend on parameters or variables other than r and σ_g. If one assumes that $r < \sigma_g$, which helps to determine the sign of elasticities. This assumption is quite realistic, as the volatility of nominal interest rates is generally high.

From here Tobin concluded that the demand for bonds is less elastic for higher interest rates than for lower ones. Indeed, this elasticity is always smaller than one and tends to 1 when the interest rate becomes close to zero. However, this does not seem to allow for conclusions on the validity of the keynesian principle that the preference for liquidity presents high elasticities in the low interest zone, which is contrary to Tobin's remarks at the end of his article.

The elasticity of Tobin's money demand function with respect to the interest rate (recall that $L_2=(1-x_2)W$) is

$$E_{L_2,r} = \partial L_2/\partial r\, r/L_2 = -\frac{\sigma_g^2 - r^2}{(r^2+\sigma_g^2)^2}\left(-\frac{1+b}{2b}\right)W\frac{r}{(1-x_2)W}$$

from which results, given $r = x_2(r^2 + \sigma_g^2)[-2b/(1+b)]$ from equation (2),

$$\text{(5)}\qquad E_{L_2,r} = -\frac{\sigma_g^2 - r^2}{(r^2+\sigma_g^2)}\,\frac{x_2}{1-x_2}.$$

Replacing x_2 by its value gives rise to the alternative expression

$$\text{(6)}\qquad E_{L_2,r} = -\frac{(\sigma_g^2 - r^2)\,a\,r}{(r^2+\sigma_g^2)(r^2+\sigma_g^2 - a\,r)},$$

with $a = -(1+b)/2b > 0$.

Indeed, the analysis of the elasticity function, in its alternative forms ((5) or (6)) leads to the conclusion that when r tends to zero this elasticity also becomes close to zero. This is more easily conveyed by (6).

Therefore it becomes apparent that this reasoning will not provide a conclusive appraisal of the validity of the keynesian principle, according to which money demand elasticity in the zone of lower interest rates is very high.

When r tends to a given intermediate value, midway to its maximum value σ_g (or in its neighbourhood), elasticity converges to -1: this is shown by calculating the limit of (5) when r tends to $\sigma_g\sqrt{2x_2-1}$. Finally, when r tends to σ_g, elasticity tends again to zero.

In effect, the analysis of function (5) and its derivative (which requires some burdensome algebra, avoided here) shows that the absolute value of this elasticity, when r ranges between zero and σ_g, starts at zero, peaks for intermediate levels of interest rates (this peak may exceed 1, depending on the value of a), decreasing afterwards until zero, the latter being its trend value at the point r reaches the neighbourhood of σ_g. Its image resembles an inverted U over the r axis.

Therefore, this is not the correct approach to elasticities growing ad infinitum in the zone of low interest rates to which the traditional keynesian theory refers and the liquidity trap principle is missing in Tobin's theory.

3. The demand for money as financial asset

3.1 - The liquidity preference above a minimal interest rate

First of all, one must observe that the concept of money strictu sensu (aggregate M_1, that is, cash plus demand deposits) is the most obvious, but not necessarily the most adequate nor is it an imposition resulting from any sort of irrevocable theory. However, the same applies much more to the enlarged concepts of money, since they include assets not immediately liquid and were introduced to meet monetarists' need to find variables that lend support to the postulate/hypothesis of "monetary variables stability" in the context of econometric researches (apparently, this is why aggregate M_1 was replaced by M_2, since the latter delivered the results predicted by their theory).

Nevertheless, it could be argued that nowadays, it is broader monetary aggregates that are most widely adopted by central banks. The crucial aspect is that assets included in the money concept are risk-free, that is to say, they are subject to no capital losses due to interest rate changes. This is what currently occurs, even when adopting aggregate M_3 and the broader aggregates as used a decade ago.

So, to meet the present statistical practices of central banks, one must use a broader monetary aggregate and this implies reformulating Tobin's theory.

One must consider that in a modern economy, there is an amount of financial funds coming from savings and credit that are applied to finance physical investment and financial investment. Financial investment is composed of several securities, bank deposits and cash. Some of these assets produce income (e.g. dividends, interest) and may have market risk (as a consequence of the volatility of their quotes) and others, such as cash, and most demand deposits have no income and no risk.

Let us take M_2 (cash plus total bank deposits) as the concept of money aggregate. Thus, we are dealing with a financial asset yielding some income, since time deposits raise interests, and there is no market risk. One part of this aggregate is destined to purchase goods and services (transactions in the economy) and the other part is a financial investment, since its destination is to buy securities when this operation is considered worthy by investors, or it is kept liquid until they judge the right moment has come. They take this decision considering the level of interest rates in the market and their evaluation of the prices of securities and expectations of changes. The last part of the aggregate is the demand for money as financial asset. It is equivalent to Keynes' speculative motive for liquidity preference.

We shall derive hereafter the function of the demand for money as* financial asset. We consider all the financial assets in the economy grouped in three sets: money (M), equities (E) and bonds (B). Money has no market risk and yields some income, which is the interest of deposits. Equities and bonds have market risk. In the set of bonds, those issued by enterprises and those issued by the Government, as well as Treasury Bills, are included. Usually bonds have a smaller market risk (denoted σ_B) than equities and also a lower rate of return (denoted R_B). Some have their interests indexed to a market interest rate and Treasury Bills have no risk. So, in the plan (E(R), σ) – where E(R) or $\overline{R}$ denotes the rate of return expected values – the locus of the investment opportunities in E and B is a hyperbole, going from B to E (see Fig. 1), as it is known in the portfolio theory. As investors also have the possibility of investing their funds in time deposits, which compose M_2, there is an asset without risk. In this case, the portfolio theory also shows that the new efficient frontier is the segment of the straight line tangent to the hyperbole, from R_M until the efficient portfolio e. We assume a quadratic utility function, like that of Tobin, with the restriction that $R>i_m$, where i_m is a very low interest rate (to be discussed in the next section). As is known from the literature concerning Tobin's separation theorem, the investors equilibrium portfolio (p) is given by the tangency of an iso-utility curve and the straight line efficient frontier in the point ($\overline{R}_p$, σ_p).

Let us study the problem algebraically. The equations that will determine the efficient frontier are:

I') $\overline{R}_p = x_M R_M + (1-x_M)\ \overline{R}_e$

II') $\sigma_p = (1-x_M)\,\sigma_e$,

where $\overline{R}_e$ and σ_e denote, respectively, the expected rate of return and the risk measure of the efficient portfolio e; $x_M+x_e=1$, with x_M the fraction of the total financial investment placed in money and x_e the fraction invested in the efficient portfolio e (composed of equities and bonds).

Taking equation II'), we derive

$$x_M = 1 - \frac{\sigma_p}{\sigma_e};$$

replacing this expression in equation I'), we obtain the straight line (see Fig.1):

$$\overline{R}_p = R_M + [(\overline{R}_e - R_M) / \sigma_e] \sigma_p.$$

Considering that the derivatives of the iso-utility curve and the efficient frontier are equal in the tangency point, we have

$$\frac{(1-x_M)\sigma_e}{-(1+b)/2b - x_M R_M - (1-x_M)\overline{R}_e} = \frac{\overline{R}_e - R_M}{\sigma_e};$$

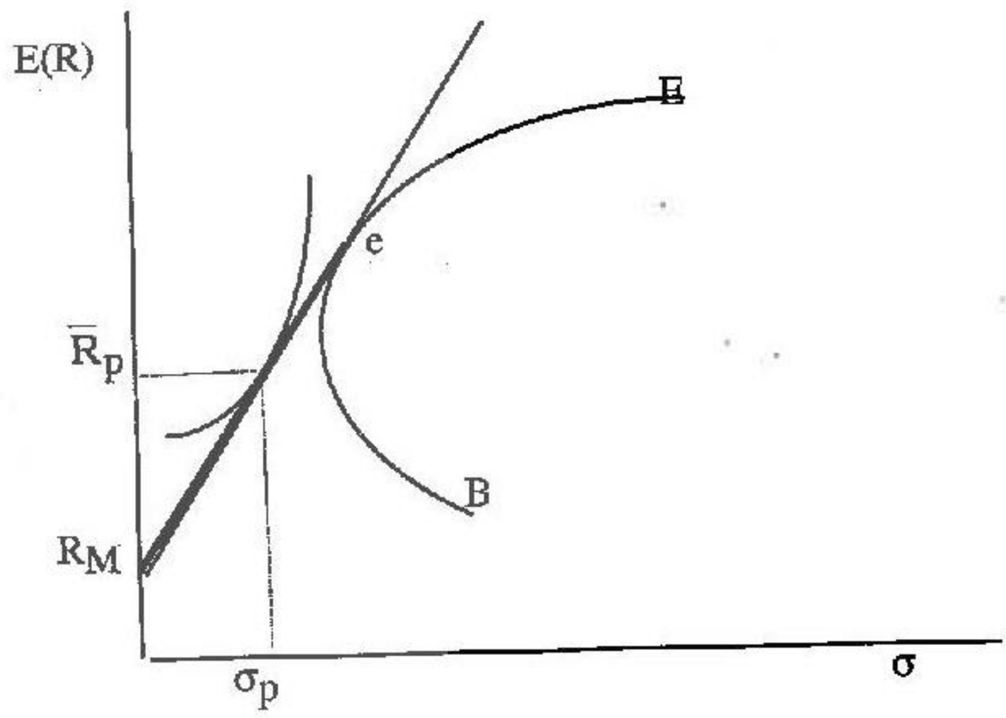

Fig. 1: Efficient frontier when money yields income

Let us denote with i the interest rate in the market, corresponding, for example, to the market money rate for a period of 90 days, as this term may be considered reasonable to forecast changes in expected interest rates. Considering that $\overline{R}e$ is higher than i, because it incorporates a risk premium (denoted rp), we may write $\overline{R}e = i + rp$. On the other hand, as most assets included in M2 do not yield interest and that the structure of this aggregate changes when interest rates rise, we consider that the relationship between its interest rate RM and i is not linear and respects proportional relationship. Let us pose $RM=\alpha i$, with $\alpha<1$. To simplify the resolution formula, let us denote $\overline{R}e - RM = u$ and $(1+b)/2b=a$ (remember that $a<0$) . Note that $u = (1-\alpha)i + rp$ and $du/di=1-\alpha$. .

Thus, resolving the last equation, we obtain

$$x_M = \frac{\sigma_e^2 + a\,u + \overline{R}_e\,u}{\sigma_e^2 - R_M\,u + \overline{R}_e\,u}$$

hence

$$x_M = \frac{\sigma_c^2 + u\,(a + i + r_p)}{\sigma_e^2 + u^2}\,.$$

The derivative dx_M/di (with a too long formula) is negative as i increases. Let us examine the following derivative:

$$\frac{d\,x_M}{d\,i} = \frac{[(1-\alpha)(a+i+r_p)+u](\sigma_e^2+u^2) - 2u(1-\alpha)[\sigma_e^2+u(a+i+r_p)]}{(\sigma_e^2+u^2)^2}$$

This formula is negative if, and only if

$$[(1-\alpha)(a+i+r_p)+u](\sigma_e^2+u^2) < 2u(1-\alpha)[\sigma_e^2+u(a+i+r_p)].$$

After a burdensome algebra, it is concluded that this inequality is true, since it is equivalent to

$$|a| > \frac{\overline{R}c[(2-\alpha)-2u(1-\alpha)] - R_M - 2u(1-\alpha)\sigma_e^2}{(1-\alpha)(1-2u)}$$

and this is true because the denominator is positive (α and 2u are less than 1) and the numerator is negative – because, among other obvious reasons, $[_{(2-\alpha)-2u(1-\alpha)}] < 0$ since $\alpha-2<0$, which guarantees $u>(\alpha-2)/2(1-\alpha)$.

With that derivative negative, the fraction of M_2, x_M, in the global portfolio of financial investors will decrease as i rises.

Let us examine graphically the effect of a rise in the interest rate of the market, i.

In Fig.2, it is supposed that an economic growth and productivity increase had caused company profits to rise and this, in turn, caused $\overline{R}_E$ to jump. In order to slow this potentially inflationary situation, the central bank decided an interest rate increase, in order to "cool" the economy, which provoked a rise of $\overline{R}_B$. So, the frontier hyperbole moved upwards (to the thick shape). The interest rate of money increased also. The tangency with a higher iso-utility curve of investors determined a portfolio with a higher expected rate of return ($\overline{R}'_p$) and a higher risk too (σ'_p), as a consequence of effects of income and substitution.

So, considering that

$$x_M = 1 - \frac{\sigma_p}{\sigma_e},$$

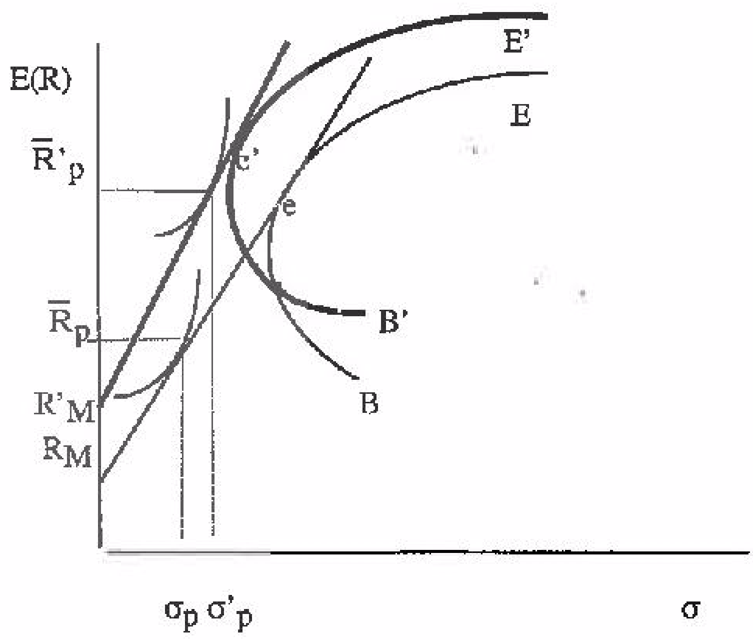

Fig. 2: The effects of a rise of the interest rates R_B and R_M

in such a situation, with an increase of σ_p and a decrease of σ_e , x_M decreases, and the same happens with the demand for money as financial investment, denoted L_2.

Let us consider the national wealth, noted W, computed with the formula Y/i, where Y is the national income. Assuming that the investors place in financial investments a proportion ν of W, their global portfolio values ν W. Consequently, the demand for money as financial investment amounts to

$$L_2 = x_M \nu W.$$

Replacing x_M by its value derived above,

$$L_2 = \frac{\sigma_e^2 + u(a + i + r_p)}{\sigma_e^2 + u^2} \nu W.$$

Replacing u and W by the variables they express, we obtain

$$(7) \quad L_2 = \frac{\sigma_e^2 + [(1-\alpha)i + r_p](a + i + r_p)}{\sigma_e^2 + [(1-\alpha)i + r_p]^2} \nu \frac{Y}{i}$$

As $d L_2/d i = \nu (d x_M /d i \ . Y/i + d(Y/i)/di \ x_M)$, this derivative is negative, which means that L_2 is decreasing. The second derivative is positive because the opposite hypothesis leads to a false conclusion, as we shall show hereafter. So the function is concave upward (see demonstration in Appendix).

3.2 - The zone of the liquidity trap principle

Until now, we have not put any restriction on the value of the interest rate and have not questioned the possibility of its reaching zero. The portfolio theory of Tobin ignores this point. So, uniquely based on this theory, the L_2 curve may cross the abscissa axis. This means to ignore the keynesian theoretical problem of the zone where elasticities of L_2 are infinite, that is, the liquidity trap zone.

Keynes argued that a minimal interest rate exists, because when one lends money he has, at least, to be paid for the risk of default. Otherwise, nobody would lend money. Beyond this argument, one must consider that it makes no financial sense to have interest rates decreasing until zero. Considering that a fixed-rate bond's price equals, ceteris paribus, its interest divided by the market interest rate, if the interest rate tends to zero, bond prices in the stock market are boosted immeasurably, tending to infinitum. No investor in such a situation would refrain from selling his securities to make fabulous profits just before the interest rate arrived at zero. The only remaining issue would be to identify the purchaser, since hardly anyone would continue to forecast further interest rate reductions. This purchaser could be the banking system, if it were devoted to lowering the rates by purchasing bonds in open market operations. However, in this case, from a given interest rate level i_m, all investors would already have sold their bonds to the system.

This means that, as Keynes admitted, a minimum interest rate does in fact exist (noted i_m). It is that which speculators already consider being so low that that it becomes worthwhile to sell all securities in their portfolio and collect all capital gains. This is when x_e becomes zero and all financial wealth is kept in the form of money (equities would no longer be a financial investment, they would be kept merely as representing the property of enterprises).

This issue implies that investors' utility function has to be re-formulated so as to take into account the above-mentioned behaviour of selling all bonds when i_m is reached.

Indeed, equation (2) shows that, given a significant interest rate, when it equals a very low level i_m, x_e only becomes zero with b=-1, which is not possible, because b was assumed to be smaller than 1. In this situation, the expression of the quadratic utility function U(R) should equal zero and not only when R=0. Therefore, its form should be:

$$U(R) = \begin{cases} (1+b)\,R + b\,R^2, \text{ for } -1 < b < 0 \text{ and } R > r_m; \\ 0\,, \text{ for } R \leq r_m \end{cases}$$

As stated before, concerning Tobin's U(R), this function may be viewed as a game rule, accepted by those who behave in financial markets following "rational expectations", since no one can prove that they actually have a quadratic utility function.

As for the demand for money for financial investment reasons (the speculation motive), taking income as the scale factor yields:

$$(12) L_2 = \begin{cases} \dfrac{\sigma_e^{\,2} + [(1-\alpha)i + r_p](a + i + r_p)}{\sigma_e^{\,2} + [(1-\alpha)i + r_p]^{2}} \; v\,\dfrac{Y}{i} & \text{for } I > i_m \\ vW & \text{for } i \leq i_m \end{cases}$$

This function does not tend asymptotically to the horizontal line starting at i_m: instead, it merges with this line straight from the point where the initial curve crosses it up to the vertical line, beginning at vW. When the interest rate i_m is reached, all financial wealth is in the form of money and for any value of W it is = vW. Therefore, the demand curve has a section that is perfectly horizontal for $i = i_m$, as shown in Fig. 3.

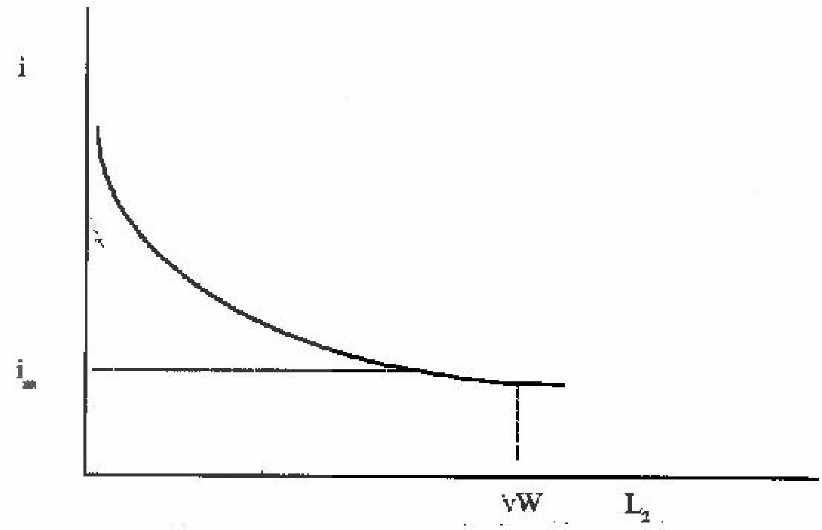

Fig. 3 : The demand for money –financial investment

This curve leads to a typically keynesian demand for money, because if money supply intersects this horizontal section, the liquidity trap will indeed occur. However, we must remember that Keynes wrote, in The General Theory, chapter 15,III, about the second reason for the existence of i_m (liquidity preference may become virtually as a consequence of too low interest rates)[3], that "whilst this

[3] If the real interest rate is used, it may be negative sometimes, when there is inflation and monetary illusion. In that case, a positive i_m near zero only exists without monetary illusion.

limiting case might become practically important in future, I know no example of it hitherto".

4. Econometric findings and elasticities

Finally, the last issue deals with knowing the effect of elasticities on overall money demand, $M_d = L_1 + L_2$, where L_1 is the transactions motive demand for money and L_2 is the financial investment motive demand for money.

It should be noted that, according to mathematics textbooks, the elasticity of M_d is

$$E_{Md,i} = E_{L1,i} \frac{L_1}{Md} + E_{L2,i} \frac{L_2}{Md}.$$

Bearing in mind the theory of money demand for transaction purposes of Baumol (1952) and the money demand function for speculation reasons given by $L_2 = x_M \nu Y/i$, the Keynesian money demand in terms of real assets gains the following form:

$$(13) \qquad \frac{Md}{P} = k P^{\gamma-1} X i^{\gamma} + x_M \frac{\nu}{i} X,$$

where X=Y/P and γ generalises the Baumol hypothesis of the coefficient ½, being $0 < \gamma \leq ½$.

The elasticity of the function M_d/P is

$$\frac{d\,Md/p}{d\,i} \frac{i}{Md/p} = -\gamma \frac{L_1/P}{Md/P} + \nu \left(\frac{d\,x_M}{di} - x_M \right) \frac{X}{Md/P}$$

Examining this formula, we conclude that its absolute value may not usually be very high. As a matter of fact, γ is smaller than ½ and L_1/M_d is lower than 1; the term X/Md/P is the velocity of the circulation of money; this may be high, but it comes multiplied by values with an absolute value which is usually low ($\nu < 1$; $0 \leq x_M \leq 1$; $d\,x_M/di < 0$). The absolute value of the derivative becomes high only when i is high and in this case x_M approaches 0.

What kind of behaviour does this elasticity have when i becomes very low? As x_M approaches 1 in this situation and the absolute value of the derivative of x_M

becomes very low (it is concave), the elasticity becomes lower. So, this is the opposite of what Tobin expected and stated in his article (1958). These are theoretical results. Let us see what happens with econometric estimations.

Instead of the model $M_d=L_1+L_2$ suggested by Keynes as a simple approach, we decided to use $\ln M_d=\ln L_1+\ln L_2$ because it is more feasible to estimate a log-linear model. For L_1, the Cambridge equation was used, since the Baumol model would imply co-linearity of the interest rate used with the same variable used in L_2. The first step was to transform equation (7) in order to obtain a multiplicative model. Posing

$$A = a(1-\alpha) + (2-\alpha)\, r_p$$
$$B = (1-\alpha)$$
$$C = \sigma_e^2 + a\, r_p + r_p^2$$
$$D = 2(1-\alpha)\, r_p$$
$$E = \sigma_e^2 + r_p^2$$

Equation (7) becomes

$$(7') \qquad L_2 = \frac{C(\,A/C\ i + B/C\ i^2 + 1)}{E(B^2/E\ i^2 + D/E\ i + 1)\ i}\ \nu\, P X$$

After the following variable transformation, we obtain:

$$(\,A/C\ i + B/C\ i^2 + 1) = (i^2 + i + 1)^{\lambda_1}$$
$$(B^2/E\ i^2 + D/E\ i + 1)\ i = (i^2 + i + 1)^{\lambda_2}$$

and, with $\lambda = \lambda_1 + \lambda_2$,

$$(8) \qquad L_2 = C/E\ (i^2 + i + 1)^{\lambda}\ \nu\ P X$$

Considering $L_1/P= kX$ and multiplying it by equation (8) divided by P the log-linear model is

$$(9) \qquad \ln \frac{Md}{P} = \ln(k\ C/E\ \nu) + \beta \ln X + \lambda \ln R$$

with $R=(i^2 + i + 1)$.

If the Cambridge equation holds, the β value will be 2, otherwise the generalised model of Baumol will prevail.

Using Portuguese data collected from long series provided by the Bank of Portugal (1954 to 1993), the model was estimated through the cointegration method of TSP. First, 2SLS was run, using $\ln M_1$ as a dependent variable and the constant term, "time", velocity of circulation of M_1, investment, logarithm of nominal GDP and inflation rate as instrumental variables. The interest rate data concern "total public debt", since it is the only data available for the whole period. Two dummies were included: D1 for the colonial war (1962 until 1974) and D2 for the 1974 Revolution (effects from 1974 until 1986, the year of Portugal's entrance to the EU). The first step with 2SLS was justified in order to avoid problems concerning seemingly unrelated regressions. The best result for the long run estimation with OLS was:

$$\ln \frac{M1}{P} = -5{,}036 + 1{,}385\ln X - 0{,}653\ln R - 1.375\ln P - 0{,}165D1 - 0{,}162D2$$

t-statistic -4.38 13.4860 -2.04 -27.89 -3.21 -4.42

Adjusted R-squared = 99.55%

Durbin-Watson = 2.328 [.553,.979]

The best result with 2SLS was:

$$\ln \frac{M1}{P} = 0{,}937\ln X - 2.853\ln R - 1.147\ln P - 0{,}162D2$$

t-statistic 389,01 -4.49 -65,66 -3,05

Adjusted R-squared = 98,9%

Durbin-Watson = 2.31299 [.437,.987]

Tests[4] have shown that real money and real product have unit roots and that R (computed with real interest rate) is stationary[5]. Real M_1, real M_2 and real GDP are cointegrated.

A third step of the estimations was to compute long run and short run estimator values in the same regression. The best results were[6]:

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4 The Author is grateful to **Ana Cecília Campos** for cointegration tests, made in the framework of a research financed with a scholarship granted by FCT through CIEF.

5 See outputs in "output_Md.doc", http://www.iseg.utl.pt/cief/papers

6 "*" means a significant acceptance level at 5% or less.

Method of estimation = Ordinary Least Squares
Dependent variable: ΔlnM1/P
R-squared = .744296
Adjusted R-squared = .694805
Durbin-Watson = 2.41552 [.556,.995]
Durbin's h = -3.94826 [.000]
Durbin's h alt. = -2.01480 [.044]
Breusch/Godfrey LM: AR/MA1 = 4.05940 [.044]
Breusch/Godfrey LM: AR/MA2 = 2.93969 [.230]
Breusch/Godfrey LM: AR/MA3 = 14.1189 [.003]
Breusch/Godfrey LM: AR/MA4 = 16.0641 [.003]

Variable	Estimated Coefficient	Standard Error	t-statistic	P-value
lnX	.661137E-02	.306392E-02	2.15781	* [.039]
lnR	.122411	.321753	.380451	[.706]
lnP	-.051622	.015093	-3.42027	* [.002]
D1	-.034597	.039138	-.883959	[.384]
D2	-.279520	.048202	-5.79891	* [.000]
Δ lnM1/P$_{(-1)}$	-.461211	.149044	-3.09446	* [.004]
U$_{(-1)}$	-.221885	.148089	-1.49832	[.144]

As $U_{(-1)}$ = residual of the SLS equation, its value is minus the second member of that regression $\ln \frac{M1}{P}$. Computing its expression and replacing it in the last regression, we obtained:

$$\ln \frac{M1}{P}_t = -0{,}2758\ln X - 0{,}1573\ln R - 0{,}3935\ln P - 0{,}5927\ln(M\text{-}1/M\text{-}2) - 0{,}0444D1 - 0{,}3592D2$$

1,7536
(p=0,0206)

The statistic was computed manually, taking the variances of the involved parameters and shows that the elasticity of real demand for money is significantly lower than unity.

These results show that the model fits well to the collected data. The elasticity of real demand for money with respect to the real interest rate is $-0{,}157\times(2i+1)(i/R)$. For low interest rates, let us say 1%, this elasticity has the value of -0,0016. This is a confirmation of the previous theoretical analyses, in which we concluded that for low interest rates the demand for money is rigid. These results are like those obtained by Goldfeld.

Only at the level of the minimal interest rate does the elasticity suddenly become infinite (the theoretical liquidity trap principle).

Goldfeld (1973) used multiplicative models to specify these functions and presented a remarkable study containing several experiences of econometric estimations based on quarterly data for the USA from 1952 to 1972. The estimations of Goldfeld made use of monetary aggregate M_1, while taking income as the scale variable. For the Keynesian-type function, Goldfeld reached an elasticity of -0,019 with respect to the interest rate on commercial paper, and -0,045 for the interest rate on time deposits (cf. op. cit. p.193). These results were -0,015 and -0,038, respectively, in the case of the monetarist specification.

Judd and Scadding (1982), through a comprehensive revision of literature on this matter from 1973 onwards, give information on the results of numerous published works. Results are similar to those of Goldfeld. For instance, in the most recent research among those cited – the regression in the work of Kimball (1980) – the elasticity of M_1 with respect to the interest rate on Treasury bills is -0,019, being -0,038 for the interest rate on time deposits (Judd and Scadding, pp.1016-1017).

Boughton (1982) estimated for the USA a multiplicative model similar to that of Goldfeld, adding to its arguments Tobin's "q " (i.e., the relative price of capital goods), with quarterly data running from 1952 up to 1978, and found an elasticity of -0,012 for commercial paper and -0,013 for time deposits.

Mehra (1992) estimated a post-keynesian model like that of Goldfeld, using cointegrating regressions, with USA data; his results show that the elasticity of M_1 with respect to interest rate varies from -0.05 to -0.29 (cf. p.15).

Given this wide range of results, we may conclude that real money demand elasticity with respect to interest rates is substantially smaller than one, the function being particularly inelastic. As a consequence, the central banker knows that it is efficient to control the interest rate through manipulations of high-powered money in the open market, even for the lowest interest rate levels.

5. Conclusion

In the previous sections, a money-demand function was derived in the light of the optimal portfolio choice theory. Results which confirm the shape of Keynes' L_2 were found, but not the high elasticities in the zone of low interest rates. Instead of a curve of L_2 tending asymptotically to the straight line departing from

i_m, the curve merges with this line at a certain point. The liquidity trap case would occur only on this straight line. Thus, monetary policy on the basis of open market operations is always efficient for interest rate manipulations.

The problem of knowing whether the risk measure used in this approach is a good interpretation of the keynesian uncertainty concept was considered. We think that a theory based on Tobin's principles appears to be at the same time compatible with Keynes' analysis of speculative behaviour, in a more operational context than that which fundamental uncertainty could offer, and most likely to provide a more realistic theoretical framework, although his analysis was incomplete.

Econometric estimations of the demand for money function, with the derived post-keynesian model, were made with Portuguese data, using the method of 2SLS (instrumental variables), followed by cointegration. The results confirmed the theoretical analyses and has provided a rigid function with respect to the interest rate.

APPENDIX

Demonstration that $d^2L_2/d\,i^2$ is negative.

As a matter of fact,

$d^2L_2/d\,i^2 = (d^2x_M/d\,i^2.Y/i + d(Y/i)/di.d\,x_M/d\,i + d^2(Y/i)/di^2.x_M + d\,x_M/d\,i.\,d(Y/i)/di)$

$= Y\,(d^2x_M/d\,i^2.1/i - 2/i^2.d\,x_M/d\,i + 2/i^3.x_M)$, noting that $d(Y/i)/di = -Y/i^2$ and $d^2(Y/i)/di^2 = 2Y/i^3$.

Let us suppose that $d^2L_2/d\,i^2<0$, that is, L_2 is convex upward. As the second and third terms within the brackets are positive, d^2x_M/di^2 has to be negative and with an appropriate absolute value, in order for this hypothesis to be respected. Then, as $0 \leq x_M \leq 1$, resolving the inequality with respect to x_M,

$$d^2L_2/d\,i^2<0 \Rightarrow x_M < i/2(-\,d^2x_M/d\,i^2\,.\,i + 2\,d\,x_M/d\,i),$$

with the first term within the brackets supposedly positive and the second negative.

When i becomes very low, approaching zero, we obtain $x_M = 1$.

Hence, $1 < i/2(-\,d^2x_M/d\,i^2\,.\,i + 2\,d\,x_M/d\,i)$. In this situation, as L_2 was supposed convex upward, the absolute value of the negative dx_M/di becomes extremely high, the supposedly positive first term within the brackets becomes very low, with i approaching zero. The value within the brackets approaches zero. Multiplied by i/2, the whole approaches zero and it cannot be higher than 1. So the *conclusion* is false and the departing hypothesis must be the opposite, that is, $d^2L_2/d\,i^2>0$, which implies that L_2 is concave upward.

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