UNIVERSIDADE TÉCNICA DE LISBOA

INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

MESTRADO EM: Economia

WELFARE AND SUSTAINABILITY MEASURES IN DYNAMIC ECONOMIES:
GREEN ACCOUNTING FOR PORTUGAL, 1992 - 2004

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Abstract

The focus of this dissertation is on the theory and practice of what can be called the economic theory of sustainability. Our argument is that traditional discussions and analysis of savings and investment at the macroeconomic level can be greatly enriched by integrating the environment into the macroeconomic picture. Here we take sustainability to mean, generically, non-decreasing welfare for the economy as a whole. The critical concepts are the green net national product (NNP), and the genuine savings. Green net national product is a welfare measure that proposes corrections to the usual national account's aggregates in order to account for environmental and well-being concerns. The genuine savings indicator is a notion of net savings that nets out the depreciation of all forms of capital including natural capital. Regarding a measure of sustainability, decreasing green NNP is equivalent to negative genuine savings and indicates unsustainability. We devise a model to estimate these aggregates incorporating the costs of a vector of air pollutants to households, and the depreciation of commercial forests in Portugal for the years, 1992 – 2004. The pollution disamenity term is around 6 – 8% of NNP, and the depreciation of commercial forests ranges from -0.7% in 1991 to 0.4% in 1996. So, the total environmental adjustments are of the magnitude of 6 – 9% of NNP. This may seem small compared to Portuguese NNP; however we did not include some relevant stocks of natural capital such as fish, mineral, water and soil. Regarding genuine savings, we find consistent evidence of unsustainable development for Portugal after 2003. There is also a clear tendency towards unsustainability throughout the period.

Keywords: sustainable development, indicators, welfare, optimal growth, green accounting, genuine savings.

Resumo

Esta dissertação aborda a teoria e a prática, do que se pode chamar, teoria económica da sustentabilidade. O nosso argumento é que as discussões e análises tradicionais da poupança ou investimento ao nível macroeconómico podem ser grandemente enriquecidas tomando o ambiente como parte integrante da macroeconomia. Define-se desenvolvimento sustentável em termos de bem-estar não decrescente para a economia como um todo. Os conceitos centrais utilizados, são o Produto Interno Líquido (PIL) verde e a poupança genuína. O PIL verde é uma medida de bem-estar que propõe correcções aos agregados usais da contabilidade nacional, para ter em conta preocupações ambientais. O indicador de poupança genuina é uma noção de poupança que incorpora a depreciação do capital natural. Em termos de indicadores de sustentabilidade, a mensagem é: PIL verde decrescente equivale a obter poupança genuína negativa, o que indica desenvolvimento não sustentável. Para estimar estes indicadores, apresentamos um modelo de uma economia dinâmica que tem em conta o custo das emissões de um vector de poluentes atmosféricos para as famílias, e o valor da depreciação das florestas comercias Portuguesas em 1992 – 2004. O custo, em desutilidade, da poluição atmosférica é aproximadamente 6 – 8% do PIL, e o termo da depreciação das florestas comerciais varia entre -0.7% do PIL em 1991 a 0.4% do PIL em 1996. Assim, os ajustes ambientais são da ordem dos 6 – 9% do PIL. Embora se argumente que o valor é baixo, é de notar que os temas e stocks ambientais considerados deixaram de fora outros relevantes: pescas, minérios, recursos hídricos, e solos. Em relação à poupança genuína, os resultados evidenciam desenvolvimento insustentável para Portugal depois de 2003. É também clara ao longo de todo o período uma tendência para a insustentabilidade (poupança genuína decrescente).

Palavras Chave: desenvolvimento sustentável, indicadores, bem-estar, crescimento óptimo, contabilidade verde, poupança genuína.

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Preface

After 20 years of the publication of the Brundtland report, sustainable development has been adopted as an overarching goal of economic and social development by United Nations agencies, by *Agenda 21*, and by many individual nations, local goverments and even corporations. It has generated a huge literature, although much of what has been written about sustainable development is more heat than light. It seem politicians prefer it to be vaguely defined. Economists, however, have addressed a particular interpretation which others find monistic because it fits sustainable development into a fairly mainstream approach to economic development.

Those who prefer pluralist approaches see sustainable development as serving many different goals - rising standard of living, concern for the poor, sound environmental services, and so on. Although pluralistic views are important, they tend to obscure many insights from a more direct focus. To this end, we present the concept of sustainable development as it has developed in the economic literature. We believe that economics has gone further than any other discipline in developing a consistent story about sustainable development: what it is, what are the conditions to achieve it and how it can be measured.

The focus of this dissertation is on the theory and practice of what can be called the economic theory of sustainability. Our argument is that traditional discussions and analysis of savings and investment at the macroeconomic level can be greatly enriched by integrating the environment into the macroeconomic picture. So, we consider the environment as capital, with the consequence that running it down is a recipe for unsustainable development. We therefore, advocate the idea of measuring sustainability in a macroeconomic context so that decision-makers can be more alert to the underlying 'true' trends in the economy and to the way in which their policies may affect those trends.

The critical concepts here, are the green Net National Product, and the Genuine Savings. Green Net National Product is a welfare measure, that proposes corrections to the usual national account's aggregates in order to account for environmental concerns. The genuine savings indicator, is a notion of savings in the economy that nets out the depreciation of all forms of capital including natural capital. We are more than conscious that the actual indicators presented here require improvement.

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This dissertation would not have been possible without the relevant contributions of Prof. Victor Martins from ISEG and Prof. Tiago Domingos fom IST. Through several discussions they particularly showed how to be critical about economic data and results, sometimes ranging a scope wider than that of this dissertation. I also would like to thank the firms that gave attention to my requests about (marginal) costs of harvesting, particularly, Carlos Ribeiro from Globulus. The financial support through the grant PTDC/AMB/64762/2006 is gratefully acknowledged.

During the conferences I attended, conversations with John Pezzey, John Hartwick, Thomas Aronsson and Geir Asheim were important to understand the theory, its drawbacks and the way to proceed to empirical estimations. Throughout this period, I worked in the Environment and Energy Section in IST, and a special thanks for interesting conversations and comments goes to Tânia Sousa, João Rodrigues, Ana Simões, Rosa Trancoso, Ricardo Teixeira, Tatiana Valadas and particularly Cristina Marta Pedroso.

I thank my family, particularly my mother, brother and father for supporting me in more anxious times. Special thanks to Rosa Trancoso for her love and patient support.

1 Introduction

Sustainable development is a widely spread and accepted concept for the development of an economy, expressing concerns with the very long run. Particularly, with the possibility of an economy to continue to grow or at least to have a steady state in the far future. The popularized notion of sustainable development is one that aims to achieve a development path that ensures the satisfaction of needs of current generation, without, however, compromising the possibilities of satisfaction of the needs of future generations. It offers a vision of progress that integrates immediate and longer-term needs, local and global needs, and regards social, economic and environmental needs as inseparable and interdependent components of human progress.

This definition and the concerns in its base, were built in several policy statements, e.g., Agenda 21 or the Millennium goals at the global level, in the Community Action Programmes on the Environment or the Lisbon Strategy at the European Union level, and at the national level, e.g., National strategy of sustainable development (Resolução do Conselho de Ministros nº 109/2007). Considering this last document, it lists a series of indicators that are supposed to be capturing the three pillars of sustainable development - economic, social and environmental.

The social indicators concern a characterization of the employment and unemployment, level of education, the inequality in distribution of income and a poverty rate indicator. The environmental indicators include greenhouse gases emission rates and energy intensity, i.e., the ratio of total energy consumption by gross domestic product (GDP). Finally, the economic indicators include, GDP in purchasing power parities, productivity per worker, price levels and private investment as % of GDP.

Looking at the chosen indicators, what is striking is that, in effect, there is no integration of the three pillars. Each indicator, gives a separate message of the society. In fact, only the energy intensity relates physical measures of energy use with the wealth generated in each year. So, in order to evaluate the ongoing policies and to estimate the impact of proposed policies to implement a sustainable development there is a need to complement the list of indicators with indicators that truly integrate the different pillars of this concept of development. This is the motivation for this dissertation. That is, to present a theory that integrates social, environmental and economic concerns, from which indicators can be derived, and to estimate indicators of sustainable development for Portugal.

Although, the notion of sustainable development proposed by the Brundtland report, and discussed in sections 2.1 and 2.2, retains the basic rationale for integration; in terms of real politics, there is little evidence. It is progress within the economic dimension made so far to obtain the desired integration that we are interested. Particularly relevant, is the recent development of formal approaches to green national accounting.

Green national accounting refers to the application of a theoretical framework (namely, optimal growth models), which presents an appealing way to incorporate concerns with the environment or other, that in the end is able to propose adjustments to usual measures of national accounting. Typically, these include pollution effects in production and welfare, depletion of natural renewable and exhaustible resources and environmental amenities.

A considerable amount of research has been devoted to issues surrounding the use and design of the national accounts. One of the basic ideas behind this research has been to provide a coherent framework for measuring national well-being, and thus, also the name, welfare accounting. We will use green and welfare accounting interchangeably.

But why the need for adjustments? There is now a widespread agreement that the conventional system of national accounts, in most countries based upon the System of National Accounts (SNA) designed by the United Nations Statistical Office, is not adequate as a mean of measuring or monitoring the impact of environmental changes in welfare or income, which we are interested in. The conceptual basis and scope of the national accounts were governed by definitions of income and wealth, in the 1940s and 1950s, which did not make any allowance for the depletion of natural capital or the costs of environmental damages such as pollution. Specifically, it has been argued that it is not possible from the national accounts to determine whether an economy is genuinely growing , or merely living off its capital (Atkinson et al., 1997).

GDP is a measure of production, that is to say, it was designed primarily, among other national accounts' aggregates, as a planning tool to guide the huge production effort of the second world war. Movements of such aggregates, and their associated price and volume measures, are used to evaluate the overall performance of the economy and hence to judge the relative success or failure of economic policies pursued by governments.

Two sources of difficulty in the national accounts are pointed out. One is the non-imputation of the value of environmental goods and services, and the other is the absence of any allowance for the depreciation, depletion or degradation of environmental assets (Perrings and Vincent, 2003). Whereas produced assets are positively valued (at market prices), and written-off against the value of current output as they depreciate, environmental assets are valued at zero, and not written-off against the value of current output they depreciate. Consequently, while the sale of environmental assets augments current income, there is no indication that it also involves costs in terms of the future capacity of the economy. This is only defensible under the assumption that environmental assets are both costless and in limitless supply.

There is another reason for using data from the system of national accounts. Economic literature has provided a rationale linking GDP changes and changes in some measures of welfare. In fact, economic theory shows that there is a link between welfare measures, sustainable development and national accounting. This framework provides a theoretically sound (although very demanding empirically) way to propose corrections to the national accounts, either in the way actual data is being organized, or in identifying what data is relevant for what situations and further insights about what types of prices and valuation techniques should be used.

The UN's Handbook of National Accounting notes that there is no consensus on how "green GDP" can be calculated, and, in fact, still less consensus on whether it should be attempted at all. It should be stressed that the main drawbacks identified that lead to the statement above are concerned, not about the general usefulness of such an indicator, but rather, about the uncertainty that calculating prices for non-market benefits of natural resources, brings to the SNA. Still, traditional discussions and analysis of savings and investments at the macroeconomic level can be greatly enriched by integrating the environment into the macroeconomic picture.

What are the uses for green accounting? Dasgupta and Mäler (2000) list three potential uses. The first has to do with the fact that there is a need for an aggregate index of economic activity, of a kind that would help one to summarize a macroeconomy. The second reason arises due to the need of a quantitative measure of social welfare, not only for making welfare comparisons across space and time, but also for evaluating alternative economic policies with cost-benefit analysis. The third reason is academic. It stems from a desire to estimate the levels of aggregate consumption an economy is capable of sustaining along alternative economic programmes.

1.1 Reading guide

In a nutshell, first we define the object of the study - sustainable development - in the context of an economic framework, then we present the relevant economic techniques to characterize and indicate sustainable development and finally we estimate the proposed measures by the theory.

The task of presenting sustainable development is carried out in chapter 2, which is divided in three parts. First, we present a brief history of the appearance of sustainability concerns, particularly, in politics and economic theory. Then in section 2.2 we interpret the commonly taken definition of sustainability and argue for the need for a more specific definition, which is provided using an economic approach in section 2.3. In this section, first we try to state clearly the focus of sustainability, then we give particular emphasis to the ethical questions for choosing sustainability and their implications on the modelling choices and finalize the section by presenting common economic definitions of sustainable development and our particular choice.

In chapter 3 we present the general theory and results in the basis of the sustainability indicators to estimate subsequently. First we review the literature on sustainability of dynamic economies and welfare or green accounting on section 3.1. A review of the empirical green accounting works is also provided. We note that, as it will become clear, some of the results on welfare measurement are very recent which means that the theory is still in development and it is impossible to give a comprehensive review of the bibliography at this point. So, we present a complete though selective review of the literature.

Following, in section 3.2 we present the formal theory using a multisector optimal growth model to show the results relevant to characterize and indicate sustainable development. In this chapter, green net national product (gNNP) and genuine savings (GS) are defined.

In chapter 4 we explore the possibilities of analyzing different sustainability concerns as particular cases of the multisector optimal growth model with special emphasis on using the national account's data set. So, first, in section 4.1, we present the general view and scope of the UN's System National Accounts concerning the economic-environmental and its "green" upgrade, the System of integrated Environmental and Economic Accounting (SEEA). Then we proceed with addressing specific concerns that have been raised in the literature of green accounting, specifically, renewable resources in section 4.3, environmental amenities in section 4.4, pollution emissions in section 4.5, the model the World Bank to estimate its measure of GS in section 4.6, and finally a model for a small open economy that is used to estimate welfare and sustainability measures for Portugal in section 4.7.

The calculation of the Portuguese gNNP and GS for the years 1992 -2004 is conducted is chapter 5. First we present the data necessary and estimate the disutility costs of air pollution emissions in section 5.1. Then, the data used and the value of the depletion of commercial forests in Portugal is addressed in section 5.2. Section 5.3 concludes the chapter by presenting and analyzing the results concerning the gNNP and GS.

Chapter 6 concludes and presents possible ways for further development of the methods used here.

2 Sustainability of what?

2.1 A Brief History of Sustainable Development

The notion of sustainable development was popularized twenty years ago with the publication of "Our common future", (WECD, 1987), also named the Brundtland report. The decade of the 1980s has witnessed a fundamental change in the way governments and development agencies think about environment and development. The two are no longer regarded as mutually exclusive. The concepts grew out of the "Limits to Growth" debate of the early 1970s (Meadows et al., 1972), which discussed whether or not continuing economic growth would inevitably lead to severe environmental degradation and societal collapse on a global scale. By the late 1970s and after much further debate, an apparent resolution of the problem was reached: economic development could be sustained indefinitely, it was held, but only if development is modified to take into account its ultimate dependence on the natural environment (Pezzey, 1992).

Concerns about the role of the environment and the scarce resources to economic development, can be traced back to classical economists, particularly, Malthus, Ricardo and Mill. The conjunction of fixedness of land resources, population growth and diminishing returns in production, led to the conclusion that the economy had a tendency to converge in the long run to a stationary state. Later classical writers, more optimistically, pointed out ways in which new resource discoveries and technical progress could offset diminishing returns (Perman et al., 1996).

Neoclassical growth theory developed in the twentieth century, rejected the classical hypothesis of a long run tendency towards steady state. The assumption of continuing technical progress was sufficient to conclude, in this framework, that economic growth (rather then some level of output) could be sustained perpetually.

Classical economists view value as arising from the labour power embodied in output, whereas neoclassical economics envisaged value as being determined in exchange, so reflecting preferences and costs of production. That is to say, values could be measured in terms of consumer preferences (Perman et al., 1996). This change in emphasis paved the way for the development of welfare economics. The subsequent theoretical development has little of intrinsic interest to environmental and resource economics *per se*. Suffice to say that, when Keynesian economics stimulated a resurgence of interest in growth theory, in the middle of the twentieth century (Harrod, Domar and Kaldor), the development of a neoclassical theory of growth (Solow) provided economic growth models which had underlying the absence of land, and any wider category of natural resources from the production function. So, classical limits to growth arguments do not have any place in these models.

The introduction of natural resources into neoclassical models of economic growth followed some pathbreaking work completed during the 1960s and 1970s, in which economists systematically investigated the efficient and optimal depletion of resources. The original investigation of optimal depletion dates back to the seminal paper of Hotelling (1931), "The economics of exhaustible resources". Incidentally, the original work that laid the ground for optimal accumulation theory in economics was done by Frank Ramsey in its seminal paper "A mathematical theory of saving" (Ramsey, 1928)[1].

The main contributions of the model were firstly the initial question Ramsey posed and secondly the method of analysis, the intertemporal maximization (optimization) of collective or individual utility by applying techniques of dynamic optimization. The Ramsey model is today acknowledged as the starting point for optimal accumulation theory although its importance was not recognized until many years after its first publication.

The models of Ramsey (1928) and Hotelling (1931) provided a foundation upon which a more general and extended structure was built later by Dasgupta, Heal, Solow, Stiglitz and Hartwick among others. These writers developed models of efficient and optimal growth for economies whose production function included as factors of production exhaustible and renewable resources as well as capital and labour inputs. The *Review of Economic Studies* published in 1974 a special issue dedicated to the analysis of these models: The Review of Economic Studies Symposium on the Economics of Exhaustible Resources. One important and related issue addressed in this literature and examined below, concerns the characterization of sustainable development, that is to ask, if sustainable development is implemented, what does it look like? The seminal works are of Solow (1974) and Hartwick (1977) and further developments are Dixit et al., (1980) and Asheim et al. (2003).

In 1972, the meeting of the UN Conference on the Human Environment, held in Stockholm, led to the formation of the United Nations Environmental Programme (UNEP).

[1] Ramsey used calculus of variations to determine the optimal amount an economy should invest (save) rather than consume so as to maximize future utility, or in Ramsey's words how much of its income should a nation save? The prevailing mathematical technique for these kind of problems, nowadays stated as optimal control problems, was develop by Pontryagin (1962), generalizing the calculus of variations.

The broad concept of sustainable development was first widely publicized by the 1980 World Conservation Strategy of the International Union for the Conservation of Nature and has since become central to thinking on environment and development. Quoting the Brundtland report, "sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (WECD, 1987, p.43).

2.2 Interpreting Sustainable Development

What is being referred to when talking about sustainable development? The term 'sustainable' is not open to much dispute: it means 'enduring' or 'lasting'. So, sustainable development is development that lasts. Economic development is broadly defined traditionally in terms of real Gross Domestic Product (GDP) *per capita*, or real consumption *per capita*. Alternatively it could be broadened to include other indicators of development such as education, health, inequality or some measure of the 'quality of life', such as the Human Development Index (HDI) created for the United Nations Development Programme. This combines measures of social goals to provide an index of relative achievement, that is, a score which is defined in terms of a country's position relative to other countries (Atkinson et al., 1997).

Sustainable development now includes economic, social and environmental requirements. Also, the Brundtland report looks at sustainability both as a requirement for intragenerational justice and as a requirement for *inter*generational justice. Embracing sustainable development, then, seems to be a recognition that too many things have gone wrong, and that past development efforts have achieved part of what should truly comprise human progress.

The above definition contains within it two key concepts:

- the concept of needs, particularly the essential needs of the world's poor; and
- the concept of limitations imposed by the state of the technology and social organization on the environment's ability to meet present and future needs (WECD, 1987, p. 43).

Sustainable development is interpreted anthropocentrically, that is to say, in terms of the extent to which economic activities can meet human needs, rather than in terms of protection of the biosphere or specific natural systems. Secondly, no presumption was made against the desirability and feasibility of economic growth itself (Perman et al., 1996). The Brundtland report states that, "Far from requiring the cessation of economic growth, [sustainable development] recognizes that the problems of poverty and underdevelopment cannot be solved unless we have a new era of growth in which developing countries play a large role and reap large benefits" (WECD, 1987, p. 40). The emphasis is placed on growth alleviating poverty rather than on poverty being eliminated through redistribution. This concept of sustainable development seems to hold out the promise of economic development, coupled with no further degradation of natural environments and a significant improvement in the absolute and relative lot of the poor (Atkinson et al., 1997). Furthermore, it seems to forget the trade-offs between the goals of economic development, equity and sound environments. Moreover, this definition of sustainable development requires that we can satisfactorily define a reasonable standard for human needs.

But there are prior steps to be taken before policies for sustainable development can be formulated. A fundamental step is finding indicators to measure sustainable development for otherwise it would not be possible to say if an economy is on, or not, in a sustainable path of economic development. So, in terms of usage, it becomes quite clear that the Brundtland report does not give a precise definition. Sustainable development has been applied to a vast array of situations, and clearly an appropriate criterion will depend very much upon the operational context.

We will now proceed to informally lay down the basis for a formal theory of sustainable development or sustainability, which we will use interchangeably throughout the text.

2.3 Economics of Sustainable Development

This approach to sustainability is also termed the neoclassical ("weak") economics of sustainability[2]. Following Pezzey and Toman (2002), the "economics of sustainability" is taken to include any work with some concern for intergenerational equity or fairness in the decision-making of a whole society over many generations; some recognition of the role in this of finite environmental resources; and some recognizable use of economic concepts

[2] Weak sustainability corresponds to a view of sustainability that allows unlimited substitutability between productive inputs. This is in contrast to strong sustainability where the focus is preserving certain fundamental (irreplaceable) types of natural capital. For more on weak versus strong sustainabilty see Neumayer (2003).

such as cost, production, instantaneous utility, or some kind of aggregating of utility over time into (intertemporal) welfare.

First of all, to keep the scope manageable, as Pezzey and Toman (2002), we make three key omissions. We have excluded:

- the now common, everyday use of sustainable to mean just environmentally desirable, focusing instead on a meaning of equitable across several generations; so we give no attention to 'sectorial sustainability';
- intragenerational equity from our definition of sustainability; and
- any discussion of population growth.

Since we are using the formal structure and techniques of the science of economics to address the issue of characterization and measurement of sustainable development, then there are a number of assumptions about human behavior tagged along with the models presented in the next chapters. Economists assume that people rationally try to maximize their welfare, which in turn reflects some well-defined, insatiable, individualistic, though unexplained, set of preferences for absolute levels of marketed and unmarketed goods and services (Pezzey and Toman, 2002).

Welfare is usually taken to be the present value, or discounted sum, of utility from some time onwards, using a constant utility discount rate. People or households are also often assumed to be perfectly informed, including having perfect foresight, and to act competitively, that is, taking everyone else's actions as independent of their own. And it is usually assumed that smooth trade-offs (substitutions) are always possible among different inputs used to produce goods and services, and among goods and services that yield wellbeing or utility. Finally, economists typically assume that an economy can in principle attain an equilibrium where all supplies and demands of inputs and outputs are in balance, even though in practice the economy is always changing because of changing circumstances.

2.3.1 Time horizon and geographical scale

Concerning the choice of time horizon to address the issues of sustainability it is usually chosen to be infinite, even though it is impossible for this planet's civilization to be sustained forever, given the finite life of our sun. Pezzey and Toman (2002) defined the time horizon as several generations (at least 100 years, which takes us into the "far future") that reaches a time when there will be many descendants of an average person now, but when the vast majority of each descendant's genes will come from many other, unknown people alive now. It also reaches well beyond the most far-seeing futures and insurance markets in the world. Notwithstanding, the theoretical inconsistency of using an infinite time in models of economies is greatly reduced by the general use of a constant discount rate in the definition of present value (see next section). Pezzey and Toman (2002) note that when it comes to empirical measurement, economists use current market or non-market valuations and discount rates which in practice ascribe no value to anything beyond about a generation or two hence, because of the great uncertainty and absence of futures markets then.

The geographical scale often adopted is that of a nation, not global or regional (subnational) level. This is mainly because most of the relevant data are collected and processed at the national level, and decision-making power does rest largely with nation states.

2.3.2 Why commit to sustainable development?

After deciding what economic techniques are appropriate to tackle the problems of sustainable development, it becomes important to address the question of why is it desirable for our generation to contribute to the implementation of sustainable development. This concerns the normative approach of the ethics of sustainability, that is to say, philosophical rationales for intergenerational concern. The ethics of intergenerational equity criteria can then be debated in terms of the appeal of the underlying axioms.

Obviously, sustainability is a goal that in principle very few people would oppose to. Nonetheless, whatever one might think about intergenerational responsibility in concept, the issue is questionable in practice because there may be good reason to expect technical and economic progress to continue, leaving future generations better off than today. Why then should we refrain to consume now so that the future generations have a better quality of life?

Another point of view, is that economic progress and ecological protection over several centuries are not so automatically assured, and humankind and its economic activity has reached a scale that is potentially big enough to threaten the welfare prospects of future generations. The problem of intergenerational equity steams from the fundamental asymmetry between the present and the future, i.e., harm that is undertaken now cannot as such be undone in the future, but, equally, present sacrifices for the benefit of the future cannot be compensated for by the future (Neumayer, 2003).

Economic theories of natural and environmental resources usually seek to answer the following question: How can an efficient management of natural and environmental resources be achieved? The objective is to get the real economy to imitate a perfect market economy through internalizing external effects and to promote economic efficiency through regulating the use of natural and environmental resources when such internalization is not feasible (Asheim, 2007). At any time the present generation determines how the resource base is being managed. We will not go into detail here about the axioms for a normative foundation for sustainability that rule out present behavior that leads to inequitable consequences for future generations. We refer the reader to the works of Asheim (1994), Chichilnisky (1996), Asheim et al. (2001), Heal (2005) in Mäler and Vincent's (2005) "Handbook of Environmental Economics, Volume 3" and the work's they cite. Suffice to say that economic efficiency does not necessarily lead to intergenerational fairness.

There are, very broadly, three approaches in the literature. First, the classical utilitarianism, which is concerned with ends - individual welfare - but rejects the notion that individuals should be regarded differently just because of the time when they are alive. This is termed non-discounted utilitarianism. The objective of an economy following non-discounted or classical utilitarianism, is to maximize the sum of utilities for all generations in the planning horizon. This is easy when we are dealing with a finite number of generations, but when dealing with infinite time horizons this approach is not feasible simply because the objective function would be infinite and it would be impossible to rank and choose the best consumption path as maximizing the objective function subject to the set of feasible consumption sequences. The only way of making sure that the sum converges is to treat generations unequally, and in particular to give little weight to "most" of them, which is what discounting does.

So, there is apparently a practical reason for discounting. As Heal (2005) points out, it is a way of ensuring that we have a well-defined preference order over the set of alternative consumption sequences between which we must choose, and that this ordering can be represented by a real-valued function[3]. So, it is not in general possible to evaluate

[3]Ramsey (1928) stated that 'discounting of future utilities is ethically indefensible and arises purely

consumption sequences over time in a manner that gives equal weight to all generations if at the same time we insist on working with finite horizons and a classical utilitarian framework.

There are two other problems besides the weighting of generations which Heal (2005) addresses. Those are the dynamic consistency , and the intertemporal Pareto efficiency of a consumption path. A choice of a consumption path is dynamically efficient if at some date during the execution of the chosen path we stop and ask what path we would now chose, given what we have done to date, then (provided no parameters have changed) the answer is that we continue with the original choice. A consumption path that is intertemporally Pareto efficient means that it is a path with the property that no variation about it will make some generation better off and none worse off.

The second approach, and the most widely used framework in dynamic welfare economics, is discounted utilitarianism. Each generation as a utility weight so that, in the present value welfare there is a discount term. Usually the discount term is taken as exponential with a constant discount rate. Koopmans (1960) provided the axiomatic justifications for this approach. In a nutshell, if we rank utility sequences in a fashion that satisfies two key axioms (stationarity and independence) plus three other more technical conditions (continuity, sensitivity and the existence of a best and worst path), then it follows that we can represent our ranking by the sum,

$$\sum_{t=1}^{t=\infty} u(c_t)\rho^{t-1}$$

where c_t is the consumption rate at date t, $u(c_t)$ is the utility function and ρ is the utility discount rate or the rate of time preference. Or in continuous time,

$$\int_0^\infty u(c)e^{-\rho t}dt.$$

The development that results from maximizing the present value of utility is called PV-optimal or just optimal. Thanks to Koopmans' axiomatic foundation, many economists defend a (utility) discount factor $e^{-\rho t}$ as a perfectly good reflection of "intergenerational

from a weakness of imagination'. Moreover, Ramsey works with an infinite time horizon. Ramsey was clearly aware of the difficulty and had an ingenious alternative. Along with rather special assumptions, he assumed that utility levels are bounded above and then sought to minimize the total shortfall over time of actual utility levels from its maximum level, termed 'bliss'.

equity" (Pezzey, 2002). Many people criticize constant discounting as very inequitable for what is called the far future. Such critics must then, implicitly or explicitly, reject at least one of Koopmans' underlying axioms.

The most common form of non-constant discounting that deals with those critics is hyperbolic discounting. There is also the Chichilnisky criterion where she proposes that the discounted integral of utilities be replaced by one where it includes a term that is a function which depends only on the limiting behavior of utility over time, such as the long run average. Chichilnisky recommends, thus, a mixture of two approaches: the discounted utilitarianism mixed with the approach which ranks paths according to their very long run sustainable utility levels (Heal, 2005).

The third approach is based on a more explicitly rights-based or entitlements-based view of intergenerational obligation. Rawls' (1971) now-classical construction of a rationale for maximizing the well-being of the least well-off (across space, and at least to some extent across time) is a relatively familiar example of this approach in the sustainability literature (see Solow, 1974; Dixit et al., 1980 or Asheim, 2000, 2003). Solow (1974) adapted Rawls' arguments that originally did not concern the welfare of an entire generation, by stating that maximizing social welfare amounts to maximizing the utility of the generation with the smallest utility. This approach towards the paradigm of sustainable development is termed the maximin approach.

Here we adopt discounted utilitarianism as a modelling choice for dynamic economies.

2.3.3 Economic definitions of sustainable development

The definitions we will mention all use the concept of a 'representative agent' of the entire generation of people alive at a particular time, and so automatically ignore any age, geographical and especially economic inequities within a society.

Pezzey (1992) lists several definitions of sustainable development. To present but some definitions, a sustainable state is one in which:

- aggregate output or consumption is non-declining;
- utility is non-declining;
- non-declining renewable resources;
- non-declining aggregate resource stock;

- non-declining resources and non-increasing pollution stock; and
- non-declining discounted present value of whatever one thinks should be maintained.

Perman et al. (1996) distinguished six alternative conceptualizations. A sustainable state is one in which:

- utility is non-declining through time;
- consumption is non-declining through time;
- resources are managed so as to maintain production opportunities for the future;
- natural capital stock is non-declining through time;
- resources are managed so as to maintain sustainable yield of resource services; and
- satisfies minimum conditions of ecosystem stability and resilience.

It is now clear that the Brundtland definition of sustainable development is not precise, and that in precise terms we need to make a choice in order to proceed to the formal theory of sustainable development. Here we adopt Pezzey and Toman's (2002) sets of sustainable definitions. They devised four groups of ways of approaching sustainable development.

- The best known criterion is to keep utility constant from some time onwards at its maximum sustainable level $U_m(t)$, then (first explored by Solow 1974, after Rawls' 1971 'maximin' criterion).
- Sustainability can be intuited from political rhetoric as a constraint on utility over time. Possible variants are forever constant utility ($\dot{U}(t) = 0$ for all t (as in Solow, 1974)), non-declining utility ($\dot{U}(t) \geq 0$ for all t, as is (Pezzey, 2004)), forever non-declining welfare (Dasgupta and Mäler, 2000), and stopping current utility exceeding the current maximum sustainable level ($U(t) \leq U_m(t)$ for all t)[4]. If it does, then actual utility must eventually fall below its current level, so the current level is unsustainable. Note that this last criterion is generally implied by the non-declining utility criterion.

[4] A notation comment. For a continuous variable dependent on time, $X(t)$, we have the following definition: $dX(t)/dt := \dot{X}(t)$.

- Sustainability can instead be defined as a constraint on changes in opportunities, rather than changes in outcomes. This represents quite a shift in political philosophy to concern over what future generations inherit from us, rather than concern over what they enjoy (they can get a good inheritance, but still spend it thoughtlessly if they choose). The constraint most frequently suggested is non-declining wealth or aggregate capital, instead of non-declining utility (see for instance, Arrow et al., 2004).

- A concern for sustainability could be included as a change in the definition of intertemporal welfare, which would then be maximized without constraint, rather than as an overriding constraint on maximizing welfare. The utility discount rate could decline over time, rather than stay constant (hyperbolic discounting); one could maximize some weighted sum of PV and the long-run limit of utility (Chichilnisky criterion); or one could include a preference for the growth, as well as the level, of consumption in the definition of utility (Pezzey, 1997).

We adopt as a definition of sustainable development a development in which current utility does not exceed the current maximum sustainable level of utility, $U_m(t)$. This definition is presented formally in section 3.2.2 after we presented the general model of multisector optimal growth. This maximum sustainable level is the economy wide analogue to the notion of income suggested by Hicks (1946), (p. 174): an individual's income "must be defined as the maximum amount of money which the individual can spend this week, and still expect to be able to spend the same amount in real terms in each ensuing week". So, extending this concept to an economy as a whole, national income would represent the maximum well-being that can be enjoyed in a given period, leaving the economy with the capacity to generate the same well-being in each ensuing period.

3 Sustainability in dynamic economies

Here we review very briefly the relevant literature for the analysis of sustainable development in dynamic economies. After that, we present the formal theory of sustainability. This review is divided into two parts. The first deals with the question of characterizing a sustainable development path, where the Hartwick rule and the work of Dasgupta and Heal are the keystone topics. The second deals with the literature on green or welfare accounting and also presents some empirical works on sustainability.

3.1 Review of results

3.1.1 The 1970's results

During the 1970's, there existed a growing concern about whether or not an economy could grow forever based on exhaustible resources. Concerns were raised about the possibility that in the long run, the limited availability of production commodities would begin to act as a constraint on the economics growth potential.

On this topic, three papers became an essential foundation for later work on sustainability. These were: "The Optimal Depletion of Exhaustible Resources" by Dasgupta and Heal (1974), "Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths" by Stiglitz (1974), and "Intergenerational Equity and Exhaustible Resources" by Solow (1974). These study different aspects of sustainability using models of a very simple economy in which a non-renewable resource is extracted and combined with the services of built capital goods, human labor, to produce output (with a Cobb-Douglas production function) that can be consumed or re-invested to expand built capital.

In Dasgupta and Heal (1974) and in subsequent refinements by Pezzey and Withagen (1998), the utility discount rate is constant. Hence, society's objective is what we have referred to here as PV optimality in section 2.3.2. As Pezzey and Toman (2002) put it, 'a key finding from Dasgupta and Heal's (1974) analysis was that the PV-optimal outcome is grim for far-distant generations'. Dasgupta and Heal (1974) found that PV-optimal consumption, and utility, typically rises to a single peak, and then declines forever towards zero consumption asymptotically. This result is the direct consequence of a positive utility discount rate, combined with the inherent scarcity of the nonrenewable resource. Under these circumstances, consumption is concentrated in earlier years of relative resource plenty, and capital investment is not adequate to offset the effects of resource depletion on output (Pezzey and Toman, 2002).

Stiglitz (1974) points out that one way to avoid this undesirable outcome is ongoing technical progress. In his model, the rate of exogenous technical progress is assumed to be large enough to offset the effects of resource depletion. This assumption implies that the PV-optimal path can have sustained increases in per capita consumption (even with a growing population, which is omitted from Dasgupta and Heal's model). Stiglitz, hence, made sustainability appear less challenging by showing that if technical progress endlessly augments the productivity of the non-renewable resource at a rate fast enough compared to the discount rate, than forever rising consumption is not just feasible but also PV-optimal.

Solow (1974) paper was the first widely read paper to suggest, in the context of formal economic growth theory, a sustainability-like objective for society quite different from PV-optimality. Solow justifies his focus by referring to Rawls' (1971) principle of maximizing the minimum realized consumption level as stated in section 2.3.2. Solow's direct focus is on conditions under which a constant consumption path is feasible. In the challenging case when technical progress is absent, Solow shows that with Cobb–Douglas production (and a constant population), constant consumption could be sustained despite declining resource flow by a suitable path of capital accumulation. Solow shows that to achieve constant consumption, it is necessary that the resource flow accounts for less than half the value of production (Pezzey and Toman, 2002). Basically, this condition amounts to built capital being sufficiently substitutable for the resource that the entire economy can be run by accumulating enough built capital, even if the inflow of natural materials is negligible. This latter assumption obviously is questionable given the materials conservation law. But it could be side-stepped in a more complex model that allowed for a long term steady-state with a renewable energy substitute (Pezzey and Toman, 2005).

For our purposes, the sequel to Solow's (1974) paper was from Hartwick (1977): "Intergenerational Equity and the Investing of Rents from Exhaustible Resources." This paper, presents what came to be known as the Hartwick's rule: Under many circumstances in an economy with depletable resources, the rent derived from resource depletion is exactly the level of capital investment that is always needed to achieve constant consumption over time.

To put it more clearly, in resource economics two intertemporal allocation rules have attracted particular attention: the Hotelling rule and the Hartwick rule. The Hotelling rule is the fundamental no-arbitrage condition that every efficient resource utilization path has to meet. In its basic form it implies that the net price of an exhaustible resource must grow at a rate that equals the interest rate. The Hartwick rule, in contrast, was formulated for a production economy where consumption at any point of time depends not only on the resource extraction but also on the stock of man-made capital available at that point in time.

In such a Dasgupta-Heal-Solow model, Hartwick (1977) showed that a zero value of net investments entails constant consumption over time, provided the Hotelling rule holds as a condition for local efficiency (Asheim et al., 2003). It is clarifying, to differentiate between the Hartwick rule - a prescription to hold the value of net investments (also know as genuine savings or investment) constant and equal to zero - and the Hartwick result - following such a prescription leads to constant utility.

In a one consumption good economy endowed with two stocks - a stock of an exhaustible non-renewable resource and a stock of man-made capital - Hartwick's rule means that if the accumulation of man-made capital always exactly compensates in value for the resource depletion, then consumption remains constant at the maximum sustainable level (Withagen and Asheim, 1998). This relates to the Hicksian definition of income presented in section 2.3.3. Dixit et al., (1980), in a very elegant paper, made the observation that the Hartwick's rule can be generalized. Following the generalized Hartwick's rule, which is the prescription of holding the present value of net investments constant, but not necessarily equal to zero, then in an economy with constant population and constant technology, utility is constant.

The converse of the generalized Hartwick's rule - an efficient constant utility path is characterized by the value of net investments being zero at each point in time - was proved in an economy with stationary instantaneous preferences and a stationary technology by (Whitagen and Asheim, 1998).

For constant consumption to be feasible when the resource is nonrenewable, some kind of unlimited capital resource substitutability is needed (as with the Cobb–Douglas production function), so in recent times, Hartwick's rule has come to be known as a weak sustainability approach. And because capital investment minus resource rents is the net investment in all the economy's productive stocks, the rule also reads as, "zero net investment forever results in constant consumption forever" (Pezzey and Toman, 2002). For a thorough review of the assumptions and results related to the Hartwick rule see Asheim et al. (2003).

3.1.2 Welfare accounting and sustainability results

The seminal paper that paved the way for all the subsequent developments allowing the construction of a theory of welfare accounting was Weitzman's (1976) "On the welfare significance of National Product in a Dynamic Economy". Generally speaking, this paper provided a relationship between sustainability and the national accounts. More precisely, in the optimal path of a dynamic economy with a stationary technology and perfect competition, where the social objective is to maximize a discounted utilitarian welfare function, Weitzman was able to show that the NNP (measured as the sum of consumption and investment) in the current period is proportional to the maximum welfare attainable along the optimal path.

One important aspect of Weitzman's result is, therefore, the interpretation of NNP as a static equivalent to future utility. Following Asheim (2007) and entailing that dynamic welfare corresponds to a discounted utilitarian welfare function, this result is presented with a clearer relation to welfare accounting as, dynamic welfare is improving if and only if NNP is improving, along the optimal path. NNP improving means that net investments are positive. The concept of NNP should be interpreted in a broader sense to include, in addition to physical capital, natural resources and stocks of knowledge resulting from learning and research activities (Weitzman, 1976). Net investment interpreted in this manner has been termed genuine savings by Hamilton and Clemens (1999).

However, Weitzman (1976) assumes that the utility function is a linear function of a vector of consumption goods and that the objective of the economy is to follow a discounted utilitarian path. Asheim and Weitzman (2001) prove that the welfare interpretation of NNP holds if the utility function is not a linear function of a vector of consumption goods, under the provision that NNP is deflated by a consumer price index. The interpretation still holds even if dynamic welfare does not correspond to a discounted utilitarian welfare function (Asheim and Buchholz, 2004). The debate on how to deflate green NNP to obtain a measure of welfare in a comprehensive setting is still unresolved (see for instance Sefton and Weale, 2006; Li and Löfgren, 2006; and Asheim, 2007).

There is perhaps a simpler interpretation of NNP in the context of this framework: NNP is what a social planner would choose to maximize, subject to certain efficiency conditions, at each point in time in order to maximize the present value of utility. This view of NNP has inspired much of the subsequent research on social accounting and welfare measurement, where different aspects of capital formation have been addressed (Aronsson et al., 2004). Some of the prime examples of this approach include Hartwick (1990), Mäler (1991), who both look at resource depletion and environmental damage from pollution. Hartwick follows a presentation of incorporating adjustments models by model, whereas Mäler considers a model with several adjustments.

Concerning the empirical application of the green accounting theory, Repetto et al., (1989) first calculated green NNP to include subsoil assets (petroleum) and agricultural soils, in addition to forest resources in a study conducted by the World Resources Institute in Indonesia for the period 1970-84. This paper, which popularized the net price approach for estimating net investment and valuing the timber stock, found that net investment in timber was substantially smaller than for petroleum and substantially larger than for agricultural soils, and was equivalent to approximately 5 percent of GDP and 25 percent of gross domestic investment.

After this study many came. Vincent and Hartwick (1998) obtained and reviewed more than 30 studies for incorporating environmental resources into the national accounts, of more than 20 countries since the late 1980s. Concerning the depletion of natural resources, such as oil and other exhaustible assets, forest and agricultural assets, the adjustment would be equivalent to only 0.2 - 4 percent of GDP for all the studies.

Measuring sustainability has often been done using just a measure of green net investment, for example by Pearce and Atkinson (1993), Atkinson et al. (1997) or Hamilton and Clemens (1999). Pearce and Atkinson (1993) used data for 18 countries, from the USA to Burkina Faso, in which they relied on savings instead of investment data to calculate the net increase in built capital. The value of changes in natural capital was calculated using data on net changes in resource stocks valued at current market prices. Rough adjustments also were made for the flow of different environmental disamenities.

Hamilton and Atkinson (1996) devised a simple model treating air emissions as cumulative pollutants is used to derive measures of 'green net national product'. They found that genuine savings ranged from -4 - 14 % of GDP in Europe and that, air pollution damage as percent of GDP in Europe is about 1 - 8 % of GDP.

Concerning sustainability, the only formal proof available, so far, to test if a development path is sustainable was presented in Pezzey (2004). He proves that, if an economy with multiple consumption goods (including environmental amenities) uniquely maximizes present value with constant discounting, it is unsustainable at some time if either of two measures—augmented net investment, or the change in augmented green net national product—are zero or negative then. "Augmented" denotes that time is treated as a productive stock, which includes in each measure the value of future, exogenous changes in technology or terms of trade.

Pezzey et al., (2005) estimate and compare two empirical measures of the weak sustainability of an economy for the first time: the change in augmented Green Net National Product (gNNP), and the interest on augmented Genuine Savings (GS). Yearly calculations are given for each measure for Scotland during 1992–1999. They found that the change in augmented gNNP greatly exceeds the interest on augmented GS even when macroeconomic fluctuations are taken into account. This is a mismatch which poses an unresolved problem with the theory.

3.2 Welfare, sustainability and national income accounts - Green accounting

In this chapter we present formally the fundamental results establishing a theory of welfare or green accounting. We will follow closely the works of Weitzman, compiled in his 2003 book "Income, Wealth and Maximum Principle", Asheim compiled in his 2007 book "Justifying, Characterizing and Indicating Sustainability" and Pezzey and Toman's (2002) "Progress and Problems in the Economics of Sustainability" chapter of Tietenberg and Folmer (2002).

Therefore, this section presents the theoretical welfare significance of net national income or product. Recently there as arisen a branch or application of economic analysis that Weitzman (2003) calls "pure theory of comprehensive national income accounting". Through the core of this theory runs a common strand attempting to connect a currently observable index of comprehensive net national income or product with some appropriate but not currently observable welfare measure of future power to consume, which typically has a "sustainability-like" flavour. This is related to the well know Hicksian concept of income paraphrased as saying that income is the maximum amount that can presently be consumed without compromising the future ability to consume at the same level. This is a fair paraphrasing of the widespread notion of income that finds implicit expression the Brundtland report's definition of sustainability, as well as the mutual strand of the conceptual apparatus used by the three great economists who did fundamental theoretical work on the concept of income: Fisher, Lindahl and Hicks (Weitzman, 2003).

About the notion of sustainable development what is important to note is the sustainable-equivalent gain or utility that the development path generates. In the words of Weitzman (2003), the sustainability of a development program is to be identified with the hypothetical level of constant utility over time that would yield the same degree of overall well-being as what the actual development program may yield. Hence, the word sustainability applied to economic development stands for, and is measured by, the corresponding level of sustainable-equivalent utility (Weitzman, 2003).

3.2.1 Multisector optimal growth and welfare

Consider a continuous-time, representative agent, competitive, time autonomous, deterministic, constant population economy. The m-dimensional consumption bundle $\mathbf{C}(t)$ contains everything that influences well-being $U(\mathbf{C}(t))$, where $U(\cdot)$ is concave, non-decreasing and as smooth as required[5]. More specifically, component i of $\mathbf{C}(t)$ measures the instantaneous flow of consumption services from consuming at the rate of $C_i(t)$ units of commodity i per unit time at the instant t, for $i = 1, 2, ..., m$. The consumption vector is conceptualized as a complete list containing everything that influences current well-being, including environmental amenities and other externalities. Consumption here would ideally include all components that determine the true "standard of living". As stated by Weitzman (2003), 'not just goods we buy in stores and the governments services "purchased" with our taxes, but all non-market commodities, such as those produced at home, and environmental services, such as those rendered by natural capital like forest and clean air.' We assume that income accounting is complete in the sense that comprehensive consumption is presumed to be fully observable, along with its associated m-vector of competitive efficient prices.

Suppose that there are n capital goods that include natural resources, man-made cap-

[5] The convention throughout the text is that vectors are represented in bold.

ital, human capital (education and knowledge accumulated in R&D) and foreign capital, forming a vector $\mathbf{K}(t)$. The stock of capital of type i $(1 \leq i \leq n)$ existent at time t is denoted by $K_i(t)$, and its corresponding net (of depreciation) investment flow is $I_i(t) = \dot{K}_i(t)$. Hence the n-vector of net investments is $\mathbf{I}(t) := \dot{\mathbf{K}}(t)$. Thinking of a natural capital asset like a commercial forest, the net investment flow of the resource would be negative whenever the overall harvest rate exceeds its natural regeneration rate. So, in general, net investment of a natural asset is positive (negative) whenever the assets is being build up (depleted).

Assume that the attainable possibilities of the underlying production is time autonomous[6]. In Weitzman's (2003) words, ' the coverage of capital goods is so comprehensive, and the national accounting system so complete, that there remain no unaccounted-for residual "atmospheric" growth factors'. National income is perfectly complete because all sources of future growth have been attributed as proper investments with proper investment prices. For the case of the welfare significance of national income in imperfect economies see Aronsson et al., (2004).

It is required that the attainable possibilities of consumption and investment at any time can be described as a function only of the capital stocks existing at that time. Therefore, a consumption-investment pair $(\mathbf{C}(t), \mathbf{I}(t))$ is attainable at time t from the capital stock $\mathbf{K}(t)$ if and only if $\{\mathbf{C}(t), \mathbf{I}(t), \mathbf{K}(t)\} \in A$, where A is a convex attainable production possibility set with free disposal (Weitzman, 2003).

Having this stated, we assume that, seeking to maximize intertemporal welfare with a utility constant discount rate $\rho > 0$, the representative agent (or central planner, or benevolent dictator) chooses paths of consumption $\mathbf{C}(t)$ and net (of depreciation) investment subject to a smooth and convex production possibility set A, with initially given capital stocks $\mathbf{K}(0) = \mathbf{K}_0$, i.e., the *multisector optimal growth* model is of the form

$$\max_{\mathbf{C},\mathbf{I}} \int_0^\infty U(\mathbf{C}(t))e^{-\rho t}dt, \tag{3.1}$$

subject to the constraints

$$\{\mathbf{C}(t), \mathbf{I}(t), \mathbf{K}(t)\} \in A, \tag{3.2}$$

[6] For the non-autonomous case see, Aronsson et al., (2004), Pezzey and Toman (2002) and Asheim (2007).

or equivalently, the s production possibilities constraints

$$\hat{F}_k\left(\mathbf{C}(t), \mathbf{I}(t), \mathbf{K}(t)\right) \geq 0,\ \ k = 1, ..., s$$

the differential equations

$$\dot{\mathbf{K}}(t) = \mathbf{I}(t), \tag{3.3}$$

and obeying the initial conditions

$$\mathbf{K}(0) = \mathbf{K}_0.$$

It is of paramount importance to stress that, for better or for worse, the results presented here depend critically on the assumption of discounted utilitarianism in the functional in equation 3.1. As an assumption, this model does not allow technological changes in the production possibilities set. Note also, that $U(\mathbf{C}(t))$ is the rate of utility at time t, and has units of utils/time.

Assume that a solution to the problem exists and is unique for all $\mathbf{K}_0 > 0$. Define the current-value Hamiltonian, $H^c(\mathbf{C}, \mathbf{I}, \mathbf{\Psi}) := U(\mathbf{C}) + \mathbf{\Psi}^c\mathbf{I}$ for each t, where $\mathbf{\Psi}^c$ is the vector of current shadow investment prices (co-state variables) in utility numeraire[7]. $\mathbf{\Psi}$ represents the extra utility value of the objective function that could be obtained upon reoptimization after relaxing the corresponding constraint equation by one unit (Weitzman, 2003). Assume that there exists a unique trajectory $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t))$ that solves the multisector optimal growth problem. Hence, by Pontryagin's Maximum Principle, $(\mathbf{C}^*(t), \mathbf{I}^*(t))$ maximizes $H(\mathbf{C}(t), \mathbf{I}(t); \mathbf{\Psi}(t))$ at each t, subject to $\hat{F}_k\left(\mathbf{C}(t), \mathbf{I}(t), \mathbf{K}(t)\right) \geq 0$. Assume there exists a vector of piecewise continuous functions $\boldsymbol{\lambda} = (\lambda_1, ..., \lambda_s)$. Define the Lagrangean,

$$L(\mathbf{C}, \mathbf{I}, \mathbf{K}, \mathbf{\Psi}, t) = H(\mathbf{C}, \mathbf{I}, \mathbf{\Psi}, t) + \boldsymbol{\lambda} \cdot \hat{\mathbf{F}}\left(\mathbf{C}(t), \mathbf{I}(t), \mathbf{K}(t)\right)$$

The solution $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t))$ maximizes $L(\mathbf{C}, \mathbf{I}, \mathbf{K}, \mathbf{\Psi}, t)$ with respect to (w.r.t) the controls $\mathbf{C}^*(t)$ and $\mathbf{I}^*(t)$. This provides the first order conditions.

[7]The current value Hamiltonian relates to the present value Hamiltonian as $H^c(\cdot) := e^{\rho t}H^p(\cdot)$, such as the current and present shadow prices obey $\mathbf{\Psi}^c(t) := e^{\rho t}\mathbf{\Psi}^p(t)$. We drop the superscrit c in the text, so $\mathbf{\Psi}(t)$ and $H(\cdot)$ are the vector of current value co-state variables and the hamiltonian respectively.

Additionally, $\Psi(t)$ obeys the Euler equations

$$\dot{\Psi}(t) = \rho\Psi - \nabla_{\mathbf{K}} L(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t), \Psi(t), t); \tag{3.4}$$

and the transversality conditions $\lim_{t\to\infty} \Psi(t)\mathbf{K}^*(t)e^{-\rho t} = 0$ are verified[8]. Also, on the optimal path, $\lambda_k(t) = 0$ if $\hat{F}_k(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)) > 0$, otherwise, $\lambda_k(t) \geq 0$. This in particular implies that $\lambda_k \hat{F}_k^*(\cdot) = 0$.

Before going on it is of critical importance to interpret the maximum principle in economic terms. This has been done elsewhere and in many ways (see for instance Seierstad and Sydsaeter, 1987), but it lays at the heart of the interpretations of the following results.

As Weitzman (2003) stresses, probably the single most important idea in all of economics is Adam Smith's famous insight that ferocious competition in the marketplace, far from being the formula for chaos and decay that it seems at first glance to be, actually induces an allocation so orderly that the result is as if guided by an "invisible hand". The rigorous mathematical essence of the modern version of the invisible hand principle is that there exists a fundamental isomorphism between "resource allocation as a constrained optimization problem" and "resource allocation as a competitive equilibrium". Loosely speaking, every allocation of resources that can be described as a solution of a constrained optimization problem can also be described or interpreted as being the outcome of a competitive equilibrium - and vice-versa. The maximum principle of optimal control theory fits this paradigm exactly. For more on this see Weitzman (2003, p.99) or Asheim (2000, p.28).

Define the maximized intertemporal welfare at time t as

$$W^*(t) := \int_t^\infty U(\mathbf{C}^*(s))e^{-\rho(s-t)}ds. \tag{3.5}$$

This quantity is also termed dynamic welfare in Asheim (2007).

Proposition 1 (Hamilton-Jacobi-Bellman Equation) *For the multisector optimal growth model, if $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t))$ is an optimal solution starting at time t, then*

$$\rho W^*(t) = U(\mathbf{C}^*(t)) + \Psi(t)\mathbf{I}^*(t) \tag{3.6}$$

[8] $\nabla_{\mathbf{K}} L(\cdot)$ represents a vector of partial derivatives w.r.t. each component of the vector $\mathbf{K}$.

is true for any t.

Proof. Take the total derivative of the maximized Lagrangean

$$L^p(\mathbf{C}^*, \mathbf{I}^*, \mathbf{K}^*, \Psi^p, \boldsymbol{\lambda}, t) = U(\mathbf{C}^*)e^{-\rho t} + \Psi^p \mathbf{I}^* + \boldsymbol{\lambda} \cdot \hat{\mathbf{F}}(\mathbf{C}^*, \mathbf{I}^*, \mathbf{K}^*)$$

with respect to (w.r.t.) time,

$$\frac{d}{dt}L^{p*}(\cdot) = \dot{\mathbf{C}}^* \nabla_{\mathbf{C}} L^p(\cdot) + \dot{\mathbf{I}}^* \nabla_{\mathbf{I}} L^p(\cdot) + \dot{\mathbf{K}}^* \nabla_{\mathbf{K}} L^p(\cdot) + \dot{\Psi}^{p*} \nabla_{\Psi} L^p(\cdot) + \dot{\boldsymbol{\lambda}}^* \nabla_{\boldsymbol{\lambda}} L^p(\cdot) + \frac{\partial L^p(\cdot)}{\partial t}.$$

Since we are evaluating welfare changes on the optimal path, $\nabla_{\mathbf{C}} L^p(\cdot) = 0 = \nabla_{\mathbf{I}} L^p(\cdot)$, the third and forth terms cancel since $\dot{\Psi}^{p*} = -\nabla_{\mathbf{K}} L^p(\cdot)$ and $\nabla_{\Psi} L^{p*}(\cdot) = \mathbf{I}^* = \dot{\mathbf{K}}^*$. From the first order conditions for the controls, $\nabla_{\mathbf{C}} L^{p*}(\cdot) = 0 \Leftrightarrow \nabla_{\mathbf{C}} U(\mathbf{C}^*)e^{-\rho t} = -\boldsymbol{\lambda} \cdot \nabla_{\mathbf{C}} \hat{\mathbf{F}}(\mathbf{C}^*, \mathbf{I}^*, \mathbf{K}^*)$ since $\nabla_{\mathbf{C}} \hat{\mathbf{F}}(\mathbf{C}^*, \mathbf{I}^*, \mathbf{K}^*) < 0$ and $\nabla_{\mathbf{C}} U(\mathbf{C}^*) > 0$ then $\lambda_k(t) > 0$. This implies that $\nabla_{\boldsymbol{\lambda}} L^{p*}(\cdot) = \hat{F}_k^*(\cdot) = 0$ and so we obtain

$$\frac{dL^{p*}(t)}{dt} = \frac{\partial L^{p*}(t)}{\partial t} = -\rho U(\mathbf{C}^*(t))e^{-\rho t}.$$

Integrating in $[t, \infty[$ and using the result, $\lim_{t \to \infty} H^{p*}(t) = 0$ (Seierstad and Sydsaeter, 1987) we obtain $L^{p*}(t) = \rho \int_t^{\infty} U(\mathbf{C}^*(s))e^{-\rho s} ds$. Now, transforming this expression into current value variables we obtain expression 3.6. ■

This result is interpreted in Weitzman (2003) as the Wealth and Income version of the Maximum Principle. Now, think of a simple model with one composite consumption good consumed at rate, $c(t)$, and define $c^*(t) + \Psi(t)\dot{K}(t) := Y^*(t)$ as NNP as conventionally measured (in monetary units) by the sum of consumption expenditures and net investments. Now, it is possible to rewrite the Hamilton-Jacobi-Bellman (HJB) equation as

$$\int_t^{\infty} \mathbf{C}^*(s)e^{-\rho(s-t)} ds = \int_t^{\infty} Y^*(t)e^{-\rho(s-t)} ds. \tag{3.7}$$

This interpretation is the fundamental result of Weitzman (1976) and can be stated as ' the maximum welfare actually attainable from time t on along a competitive trajectory (right hand side (RHS) of 3.7) is exactly the same as what would be obtained from the hypothetical consumption level $Y^*(t)$ in each ensuing instant. Hence, in this simplified setting, NNP is what can be called a stationary equivalent of future consumption.

The HJB equation is the key relation that will allow us to go from observations about an economy dynamic competitive path to inferences about future welfare. Note that proposition 1 is a remarkable result in the sense that it means that changes in the stock of forward looking welfare can be picked up by changes in the flow of the value of current net product. More specifically, this proposition relates current intertemporal welfare, which needs information from the future to be calculated, to a quantity that only uses information from current time t, namely the stocks' initial conditions $\mathbf{K}_t$, and the initial conditions for the controls that put the economy on the optimal path, $\mathbf{C}_t^*$ and $\mathbf{I}_t^*$.

Due to the HJB equation the Hamiltonian is sometimes termed 'utility-NNP', here $Y_u^*(t)$, since it is the sum of the utility value of consumption and the utility value of net investments in the various capital stocks, and corresponds to the constant-equivalent of future utilities, or in other words, utility-NNP is the 'interest on maximized intertemporal welfare' (Asheim, 2007).

So, according to proposition 1 the welfare significance of real utility-NNP is the following:

- along the optimal path, changes in real utility-NNP are proportional to changes in dynamic welfare.

Proposition 1 allows to prove the fundamental result about the welfare significance of net investments.

Proposition 2 (Utility-GS) *Under the assumptions of proposition 1,*

$$\dot{W}^*(t) = \mathbf{\Psi}(t)\mathbf{I}^*(t) \tag{3.8}$$

holds for any t.

Proof. Using the Leibniz rule, $\dot{W}^*(t) = \rho\int_t^\infty U(\mathbf{C}^*(s))e^{-\rho(s-t)}ds - U(\mathbf{C}^*(t)) = \rho W^*(t) - U(\mathbf{C}^*(t))$, which is the difference between interest on the value of total discounted future utility $\rho W^*(t)$ and current utility $U(\mathbf{C}^*(t))$. For the optimal trajectory, the HJB equation can be used on the above expression to obtain expression 3.8. ■

The RHS of expression 3.8 represents the genuine investment with utility as numeraire, and it is this term, but in monetary units, that is called adjusted net savings or genuine savings (GS) (Hamilton, 2000). The welfare significance of the genuine investment is the following: along the optimal path, genuine investment with utility as numeraire measures changes in dynamic welfare, i.e., having instantaneous positive net investment is equivalent to having increasing instantaneous dynamic welfare.

Proposition 2 also allows for welfare interpretations of utility NNP in the optimal path. Using the Hamiltonian defined above, expression 3.8 rewrites as $\dot{W}^*(t) = Y_U^*(t) - U(\mathbf{C}^*(t))$ which means that, on the optimal path, increasing instantaneous welfare is equal to having instantaneous utility-NNP higher than the utility value of instantaneous consumption.

Furthermore, taking the time derivative of the HJB equation and using proposition 2 we obtain a relation between utility-NNP and utility-GS, stating that infinitesimal time changes in the utility NNP equal the interest on genuine investment, i.e.

$$\frac{d}{dt}Y_U^*(t) = \rho\mathbf{\Psi}(t)\mathbf{I}^*(t). \tag{3.9}$$

In other words, increasing (decreasing) real utility-NNP correspond to positive (negative) utility-GS.

Our aim is to infer future welfare from present observations of the current competitive economy. At first glance, it seems that proposition 1 provides a way to do just this. There are, however, two fundamental problems. First, all consumption goods are aggregated in the utlity function that is not observable to the national income accountant. Second, as stated above, prices of investment goods are expressed in utils (utility as numeraire) and the national accountant does not know how to deflate the money prices into utils. No quantity can be measured in utility units from a real economy. Moreover, NNP as usually defined and calculated in the national accounts, is linear in quantities, with the weights being at least in part revealed by observable market prices (Dasgupta and Mäller, 2000). Propositions 1 and 2 will not be useful in the real competitive economy where competitive money prices are observable.

In the green accounting literature, measurable NNP is frequently defined making use of a linear approximation of the maximized hamiltonian using a first order approximation of $U(\mathbf{C}) \approx \nabla_{\mathbf{C}}U(\mathbf{C})\mathbf{C}$ (Hartwick, 1990; Hamilton, 2001). Dasgupta and Mäller (2000) agree with the welfare interpretation of the Hamiltonian as utility NNP when they state that "the Hamiltonian equals constant-equivalent utility". However, since "both theory and empirics imply that the Hamiltonian is a non-linear function of consumption and leisure", they do not agree that a linearized version of the utility NNP could have any welfare significance. This result has been proved wrong by Asheim and Weitzman (2001), showing that deflated by an appropriate Divisia consumer price index (transforms utility metrics into real Divisia prices) growth in real NNP can indicate welfare improvement.

In order to obtain welfare measures, since what can be observed are nominal, and indirectly, real prices, we define NNP in real Divisia prices as

$$Y^*(t) := \mathbf{P}(t)\mathbf{C}^*(t) + \mathbf{Q}(t)\mathbf{I}^*(t), \tag{3.10}$$

where $\mathbf{P}(t)$ and $\mathbf{Q}(t)$ are the vectors of real prices for consumption and net investment. $\mathbf{PC}^*$ represent the consumption expenditures and $\mathbf{QI}^*$ the real net investment (genuine savings). What is the welfare significance of a NNP in monetary units (real prices), i.e. in what conditions do the previous results follow to the linear index of national production in 3.10.

The vectors of real Divisia prices for consumption and net investment are defined, respectively, as, $\mathbf{P}(t) := \nabla_{\mathbf{C}} U(\mathbf{C}^*(t))/\Lambda(t)$ and $\mathbf{Q}(t) := \mathbf{\Psi}(t)/\Lambda(t)$, where $\Lambda(t) > 0$ is an extended price index verifying $\dot{\mathbf{P}}(t)\mathbf{C}^*(t) = 0$. This is the Divisia property of price indices but in continuous time. For a discussion on (Divisia) price indices in continuous time see the appendix in Asheim (2007).

Asheim and Weitzman (2001) use $\Lambda(t) = \lambda(t)\pi(t)$, where $\lambda(t) > 0$ is the not directly-observable marginal utility of current expenditures and $\pi(t) > 0$ is a Divisia consumer price index satisfying

$$\frac{\dot{\pi}(t)}{\pi(t)} = \frac{\dot{\mathbf{p}}(t)\mathbf{C}^*(t)}{\mathbf{p}(t)\mathbf{C}^*(t)},$$

which implies that, $\dot{\mathbf{P}}(t)\mathbf{C}^*(t) = 0$. For a thorough discussion on the interpretations of $\lambda(t)$ and $\pi(t)$ see Weitzman (2003).

So, Asheim and Weitzman (2001) define nominal consumption and investment prices as $\mathbf{p}(t) := \nabla_{\mathbf{C}} U(\mathbf{C}^*(t))/\lambda(t)$ and $\mathbf{q}(t) := \mathbf{\Psi}(t)/\lambda(t)$, and a nominal interest rate at time t, $r(t)$, given by $r(t) := \rho - \dot{\lambda}(t)/\lambda(t)$. The corresponding real prices are defined as $\mathbf{P}(t) := \mathbf{p}(t)/\pi(t)$ and $\mathbf{Q}(t) := \mathbf{q}(t)/\pi(t)$.

It is then possible to show that,

Proposition 3 *Under the given assumptions,*

$$\dot{Y}^*(t) = R(t)\mathbf{Q}(t)\mathbf{I}^*(t) = \frac{R(t)}{\Lambda(t)}\dot{W}(t) \tag{3.11}$$

where $R(t) := \rho(t) - \dot{\Lambda}/\Lambda$ *holds for any* t.

Proof. For simplicity drop the time argument in all the functions. Since $\dot{\mathbf{P}}\mathbf{C}^* = 0$, then $\dot{Y}^* = \mathbf{P}\cdot\dot{\mathbf{C}}^* + d(\mathbf{Q}\cdot\mathbf{I}^*)/dt$. Taking the time derivative of the HJB equation, $\rho\dot{W}^* = \nabla_{\mathbf{C}}U(\mathbf{C}^*)\dot{\mathbf{C}}^* + d(\mathbf{\Psi}\mathbf{I}^*)/dt$. Making use of the definitions of real prices we obtain $\rho\dot{W}^* = \Lambda\mathbf{P}\dot{\mathbf{C}}^* + d(\Lambda\mathbf{Q}\mathbf{I}^*)/dt$. Proposition 2 in real prices is $\dot{W}^* = \Lambda\mathbf{Q}\mathbf{I}^*$. Substituting this in the last expression obtained and rearranging, we have $\rho\Lambda\mathbf{Q}\mathbf{I}^* = \Lambda\left(\mathbf{P}\dot{\mathbf{C}}^* + d(\mathbf{Q}\mathbf{I}^*)/dt\right) + \dot{\Lambda}\mathbf{Q}\mathbf{I}^*$. Rearranging and using again proposition 2 in real prices we obtain the desired result. ∎

The second equality comes from rewriting proposition (2) in real prices, and allows to conclude that, provided that $R(t) > 0$, instantaneous changes in real NNP deflated by a consumer price index have the same sign as changes in welfare at that same instant. Asheim (2007) states that "*provided that real consumption interest rate is positive, growth in real NNP (deflated by a consumer price index) in fixed net investment prices can be used to measure welfare improvement along the optimal path*". Note also that with $R(t) > 0$, and using the definition of NNP in real Divisia prices, $\dot{W}(t) > 0 \Leftrightarrow Y^*(t) > \mathbf{P}(t)\mathbf{C}^*(t)$, which amounts to state that maximized welfare is instantaneously increasing if and only if real price NNP exceeds the real value of consumption at the same instant.

3.2.2 Multisector optimal growth and sustainable development

Concerning the results for sustainability lets first state the Hartwick rule and result mentioned in section 3.1.1, using the setting of the multisector optimal growth model.

Definition 1 (Hartwick rule) *Given* $T > 0$, *the Hartwick rule of investment is followed along a path* $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t))_{t=0}^{\infty}$ *for* $t \in (0, T)$ *if the path is competitive with prices* $(\mathbf{P}(t), \mathbf{Q}(t))_{t=0}^{T}$ *and* $\mathbf{Q}(t)\mathbf{I}^*(t) = 0$ *for all* $t \in (0, T)$.

Proposition 4 (Hartwick result) *Given* $T > 0$, *if the Hartwick investment rule is followed for* $t \in (0, T)$, *then utility is constant for all* $t \in (0, T)$.

As mentioned in section 3.1.1, Dixit et al., (1980) made the observation that the Hartwick result can be generalized.

Proposition 5 (Generalized Hartwick result) *For the conditions of definition 4, if the generalized Hartwick investment rule, of having* $\mathbf{Q}(t)\mathbf{I}^*(t)$ *constant is followed for* $t \in (0, T)$*, then utility is constant for all* $t \in (0, T)$.

For a proof of these statements see Asheim et al., (2003). Regarding the converse of Hartwick rule the following proposition is demonstrated in Withagen and Asheim (1998).

Proposition 6 (The converse of Hartwick rule) *If the utility path is egalitarian (constant) along a competitive path then the generalized Hartwick investment rule is followed for all t .*

In the context of discussing the myths and facts about the sustainability implications of the Hartwick rule of investment, Asheim et al., (2003) provided examples that show that a non-negative value of net investments (genuine savings) on an open interval is not a sufficient condition for having consumption sustainable. However, it has been an open question whether it is a necessary condition: Does a negative value of net investments during a time interval imply that consumption exceeds the sustainable level?

Asheim et al. (2003) state that it is true that the value of net investments measures the present value of all future changes in utility. This allows them to conclude that if along an efficient path utility is monotonically decreasing/increasing indefinitely, then the value of net investments will be negative/positive, while utility will exceed/fall short of the sustainable level. The value of net investments thus indicates sustainability correctly along such monotone utility paths. Finally, Asheim et al. (2003), state clearly the relation of the Hartwick result to sustainability, by saying that 'if the value of net investments $\mathbf{Q}(t)\mathbf{I}^*(t)$ is constant for all $t \in (0, \infty)$, then the rate of utility realized at any time t can be sustained forever'. They conclude the paper by stating that it is their opinion that the Hartwick investment rule has little prescriptive value for decision-makers trying to ensure that development is sustainable. The Hartwick investment rule is, however, of interest when it comes to describing an efficient path with constant utility.

Following Pezzey (2004) let us define a sustainable economic path.

Definition 2 *A sustainable economic path at time t is one that obeys* $U(\mathbf{C}(t)) \leq U^m(t)$*, where* $U^m(t)$ *is the maximum sustainable utility, defined as* $U^m(t) := \max U$ *subject to* $U(\mathbf{C}(s)) \geq U$ *for all* $s \geq t$.

This definition is generally implied by the definition of sustainability as forever non-declining utility. With this definition of a sustainable path, Pezzey (2004) showed the following result, which he called the 'one-sided unsustainability test':

Proposition 7 (One-sided sustainability test) *Under the given assumptions (with a unique non-constant utility path), at t,*

$$\mathbf{Q}(t)\mathbf{I}^*(t) \leq 0 \text{ or } \dot{Y}^*(t) \leq 0 \Rightarrow U(\mathbf{C}^*(t)) > U^m(t).$$

Proposition 7 is equivalent to state that measuring non-positive real net investment (genuine savings) or non-rising real NP at instant t means that the optimal economy is in an unsustainable development path. At time t, current utility (or consumption, if $U(\mathbf{C})$ is monotonically increased with $\mathbf{C}$) is higher than the maximum sustainable utility, implying that utility will decrease at some future time. However, positive net investment or genuine savings do not entail that the economy is sustainable. In fact, Pezzey and Toman (2002) stress that so far no general test for sustainability is known. Moreover, in a small open economy, where all prices are exogenous world prices, and with just one consumption good, the sustainability test is then two-sided.

Incidentally, proposition 3 provides a way to test empirically the underlying theory, by estimating both the change in green NNP (here Y^*) and the interest on genuine savings (here $R\mathbf{QI}^*$), on the first equality,

$$Change\ in\ green\ NNP = interest\ on\ GS$$

for both changes in real NNP and real genuine savings can be measured (approximately) in real economies. Nonetheless, Vincent (1997) for Malasya, Hanley et al. (1999) and Pezzey et al. (2005) for Scotland, seem to be the only works with estimates for both real NNP and real genuine savings.

4 Welfare accounting with the SNA

In the last chapter we presented and discussed the most important results for welfare measurement and sustainability in the context of a very general model - the multisector optimal growth model. However, in order to get an empirical measure of welfare or sustainability we need to define a disaggregate vector of capital stocks and of consumption goods that is specific to a given economy. Moreover, since there are many links between the generation of welfare, sustainability and the functioning of the economy and they are of very different impacts, it becomes necessary that we focus on a few.

The purpose of this section is, then, to illustrate the construction of a measure of welfare based on several models of dynamic optimizing economies. As stated above, different motivations to welfare measurement or sustainability lead to different indicators. So, this section will illustrate this point further by adding terms to a benchmark measure of welfare, which steams from specific concerns that have been considered in the published literature.

Before we present corrections to the national accounts we must briefly describe the way nature is already accounted for in the SNA. This allows the identifications of SNA aggregates' in the models of the rest of the chapter.

4.1 SNA93 and SEEA

The measures developed in the last chapter used optimal control techniques and in the end, intend to use the data obtained from the System of National Accounts (SNA93). Hence, together with understanding the models, their assumptions, results and applicability, it is necessary to understand the view behind the SNA and the way the data is obtained and organized, in order to propose corrections to the standard measures of economic activity - e.g., GDP, NNP, total savings and whatnot - to measure welfare changes and the degree of sustainability of the economy.

The SNA, is an internationally agreed framework for the systematic compilation and presentation of economic data for purposes of economic analysis, decision-taking and policy-making, and includes two main categories: flows of goods and services and stocks of assets used in the production of goods and services. The primary objective of the national accounting system is the compilation of flow accounts, producing principally income and product and the various magnitudes that flow around and within these accounts. These magnitudes include GDP and national income, value added, consumption, savings and investment, exports and imports, the fiscal balance and the balance of payments. These indicators are vital inputs for macroeconomic analysis and policy.

Production — "a physical process, carried out under the responsibility, control and management of an institutional unit, in which labour and assets are used to transform inputs of goods and services into outputs of other goods and services" (SNA93, par. 6.15) — is the fundamental concept determining which economic activities the SNA is aimed to include.

The revised SNA explicitly included, natural resources in its balance sheets and accumulation accounts, and introduced environmental accounting in a satellite accounting framework (1993 SNA, chaps. XII and XXI). Naturally occurring assets such as land, subsoil assets and uncultivated forests are included in the balance sheets provided that institutional units (households, government units, corporations and non-profit organizations) exercise effective ownership over these assets and draw economic benefits from them. The two criteria of enforced ownership and actual and potential benefits make them economic assets (1993 SNA, par. 10.2), qualifying these assets for inclusion in the balance sheets and asset accounts (UN, 2000).

The SNA93 defines assets as an entity functioning as a store of value (SNA93, par. 10.2):

(a) over which ownership rights are enforced by institutional units, individually or collectively; and

(b) from which economic benefits may be derived by its owner by holding it, or using it, over a period of time[9].

Thus, in the SNA93, environmental assets must be owned and capable of bringing economic benefits to their owners, given the technology, scientific knowledge, economic infrastructure, available resources and set of relative prices prevailing on the dates to which the balance sheet relates or expected in the near future (1993 SNA, par. 10.10 and 10.11). The types of environmental assets considered in the SNA93 are presented in figure 1.

[9]The economic benefits consist of primary incomes derived from the use of the asset and the value, including possible holding gains/losses, that could be realized by disposing of the asset or terminating it.

Figure 1: Environmental assets within the SNA93 (UN, 2003; Table 7.1).

- AN.1 Produced assets
  - AN.11 Fixed assets
    - AN.111 Tangible fixed assets
      - **AN.1114 Cultivated assets**
        - **AN.11141 Livestock for breeding, dairy, draught, etc.**
        - **AN.11142 Vineyards, orchards and other plantations**
    - AN.112 Intangible fixed assets
      - ***AN.1121 Mineral exploration***
  - AN.12 Inventories
    - AN.122 Work in progress
      - **AN.1221 Work in progress on cultivated assets**
- AN.2 Non-produced assets
  - AN.21 Tangible non-produced assets
    - **AN.211 Land**
      - **AN.2111 Land underlying buildings and structures**
      - **AN.2112 Land under cultivation**
      - **AN.2113 Recreational land and associated surface water**
      - **AN.2119 Other land and associated surface water**
    - **AN.212 Subsoil assets**
      - **AN.2121 Coal, oil and natural gas reserves**
      - **AN.2122 Metallic mineral reserves**
      - **AN.2123 Non-metallic mineral reserves**
    - **AN.213 Non-cultivated biological resources**
    - **AN.214 Water resources**
  - AN.22 Intangible non-produced assets
    - ***AN.222 Leases and other transferable contracts***

Environmental assets that fall outside the SNA93 boundaries include, environmental assets over which ownership rights cannot be established and resources that do not bring any economic benefit either because there are no market prices attributed to the asset products, or because it is not profitable to exploit the resource with the current resource prices and technology. Land under cultivation in the SNA covers all agricultural land and some wooded land. All agricultural land appears in the SNA category of land under cultivation. However, some wooded land may lie outside the SNA boundary; for example, virgin forests too remote for economic use or not subject to ownership. Some wooded land may not appear in land under cultivation simply because it is impossible to separate it from vineyards, orchards, cultivated forests, etc. or from the non-cultivated forest resources which occur on it; therefore, it will appear grouped with those assets.

Regarding the valuation of assets, the SNA93 does not attempt to determine the utility of the flows and stocks which come within its scope. Rather, it measures the current exchange value[10] of the entries in the accounts in money terms, i.e., the values at which goods and other assets, services, labour or the provision of capital are in fact exchanged

[10] Value v is defined in the SNA93 as $v = p(q)q$, where q is the quantity of the good or service, and p is the price per unit of quantity which varies with q. Unlike prices and quantities, values are expressed in terms of a common unit of currency and are commensurate and additive across different products (SNA93, par. 16.9).

or else could be exchanged for cash (SNA93, par. 3.70). If the exchange values of a transaction of goods or services between institutional units is known, the data required by the system is directly available. If the exchange value is unknown (non-monetary transactions), values should be taken from markets of similar goods (traded against cash). If there are no appropriate markets, flows and stocks are to be recorded at the discounted present value of expected future returns, which may be difficult. If there are no markets and it is not possible to calculate the net present value, then the cost of producing the asset should be used as a lower bound on the asset's value.

The production oriented view of the SNA implies that, by definition, it focuses on goods and services that are produced using human labor and other factors of production and that are bought and sold in markets. Hence, any forest products or services outside of this production boundary are not accounted for in the SNA. Natural resources are often sold in markets, and so, to some extent, are reflected in the conventional national accounts. However, the prices of resources may not always reflect the cost of renewing renewable resources, nor the true (full) costs of depletion of non-renewable resources. Natural assets and their services of resource supply, waste absorption and other amenities of the environment often have no price at all, being treated as "free" goods, so that their use is not fully reflected in the national accounts. The result is that, in presenting the value of the actual monetary transactions in the economy, the national accounts systematically understate or omit the environmental costs incurred by those transactions, in terms of environmental depletion and degradation. GDP and related indicators thus contain a substantial element of consumption of natural capital, which is unaccounted for as a significant cost of production (UN, 2000 par. 17).

In 1993, the United Nations Statistics Division (UNSD) elaborated a satellite System of integrated Environmental and Economic Accounting (SEEA) in a handbook of national accounting, setting out a framework to systematically account for the stocks and flows of environmental resources in a way that was consistent with the SNA. Such compatibility refers, in particular, to the measurement of the production and consumption of goods and services in market prices or production costs rather than the measurement of their utility or human welfare through contingent and related valuations. The environmental cost adjustments proposed in the SEEA are only to be carried out in satellite accounts (UN, 2000).

The emphasis under the SNA on extending asset boundaries to environmental resources reflects concern about the conservation of resource stocks and related national wealth (UN, 2000; par. 21). However, the SEEA maintains the production focus of the SNA, in the sense that it is interested in the environmental impacts of economic activity and not the other way around (Hartwick and Vincent, 1997).

In the SNA, an asset, even an environmental asset, is defined in terms of the "benefit" limited to the provision of income or a stock of wealth which can be converted to monetary terms. For the SEEA, the concept of an environmental asset is linked to the provision of environmental "functions". Natural resource assets are defined in the SEEA as those elements of the environment that provide use benefits through the provision of raw materials and energy used in economic activity (or that may provide such benefits one day) and that are subject primarily to quantitative depletion through human use. Figure 2 shows the SEEA asset classification.

Figure 2: SEEA asset classification (UN, 2003; Table 7.2).

EA.1 Natural Resources
- EA.11 Mineral and energy resources (cubic metres, tonnes, tonnes of oil equivalents, joules)
- EA.12 Soil resources (cubic metres, tonnes)
- EA.13 Water resources (cubic metres)
- EA.14 Biological resources
  - *EA.141 Timber resources (cubic metres)*
  - *EA.142 Crop and plant resources, other than timber (cubic metres, tonnes, number)*
  - *EA.143 Aquatic resources (tonnes, number)*
  - *EA.144 Animal resources, other than aquatic (number)*

EA.2 Land and surface water (hectares)
- EA.21 Land underlying buildings and structures
- EA.22 Agricultural land and associated surface water
- EA.23 Wooded land and associated surface water
- EA.24 Major water bodies
- EA.25 Other land

EA.3 Ecosystems
- EA.31 Terrestrial ecosystems
- EA.32 Aquatic ecosystems
- EA.33 Atmospheric systems

Memorandum items – Intangible assets related to environmental issues (extended SNA codes)
- AN.1121 Mineral exploration
- AN.2221 Transferable licenses and concessions for the exploitation of natural resources
- AN.2222 Tradable permits allowing the emission of residuals
- AN.2223 Other intangible non-produced environmental assets

Concerning forests, the SEEA handbook of national accounting (UN, 2003) argues that it is more informative, than constructing accounts for timber, to look at the total value of forested land, paying attention to the timber, the land on which it grows and other forms of ecosystems and biological assets supported by the forests. The SEEA accounts for forest land and related ecosystems, biological assets (plants, animals and so forth) in the forest and other assets related to forests.

Cultivated economic forest land corresponds to land over which ownership rights are enforced, and for which natural growth and/or regeneration of timber and other biological assets is under the direct control, management and responsibility of institutional units and is likely to produce economic benefits to the owner of the land. SEEA alerts that in developed countries, typically all exploitable forest land are generally classified as cultivated economic forest land.

However, in Portugal there are considerable areas of forest land devoted to ecosystem or habitat protection. These protected areas by definition are not considered in the usual national accounts. In the SEEA, protected areas are classified as non-economic environmental forest land - land of both protected and non-exploitable forest for economic reasons. The SEEA also defines uncultivated economic forest land as land with ownership rights but where the owner accrues economic benefits with no direct control of the regeneration rates, e.g. paid parks. Also contributing to the total value of forests in the SEEA are biological assets - flora and fauna living in forests - distinguished between produced and non-produced economic biological resources and non-economic (environmental) biological resources.

Concerning valuation, the SEEA suggests several methods for non-market valuations of stocks for forests and other types of natural capital. We do not review the methods here. For more on the SEEA it is instructive to browse the special issue of March 2007 of the Journal of Ecological Economics - Environmental Accounting: Introducing the System of Integrated Environmental and Economic Accounting 2003.

4.2 National accounting aggregates in a dynamic model

Consider as a benchmark model the following version of the Frank Ramsey's (1928) model, derived to answer the question of "what is the optimal savings rate for an economy?"[11]. Starting from the multisector optimal growth of section 3.2.1, the Ramsey model assumes that the economy's objective is to maximize a discounted utilitarian function, as in expression 3.1. All functions are assumed to be functions of time unless stated otherwise.

[11] Rigorously, the model presented below is usually termed Ramsey-Cass-Koopmans model, due to the works of Tjalling C. Koopmans (1961, 1965, 1967) and David Cass (1965) reasoning in favor of a time preference in the objective functional.

The accumulation of physical capital in Ramsey's model follows

$$I = f(K) - C, \tag{4.1}$$

or in other words, the economy produces a homogenous consumption/investment good C, using physical capital K as a factor of production. The population is taken to be constant. Assume also that the gross investment is given by $I = \dot{K} + \delta K$, where δ is the rate of depreciation of physical capital (consumption of fixed capital), and hence, $\dot{K}$ is net investment. These two equations state that a change in physical capital is the part of gross output ($f(K)$) that is not consumed minus the depreciation of capital. The solution of this model is given by the optimal trajectories of the consumption rate, physical capital (and incidentally the shadow prices for capital) and it is widely studied. Using the results of section 3.2.1, and the Euler equations 3.4 for the Ramsey model, it is possible to define the NNP in real Divisia prices as,

$$Y_{Ramsey} = P^C \left\{ C + \dot{K} \right\}. \tag{4.2}$$

In this case, the Divisia price index property writes as $\dot{P}_C C = 0$ meaning that without loss of generality we can assume $P_C = 1$. This formula corresponds to the definition of the expenditure approach of NNP used in the SNA93 for a closed economy with no government sector, namely, the sum of all expenditures on goods and services, including net investment, provided by the economy. The formula for the GNP is then $C + I$. In accounting terms it states that aggregate supply is equal to aggregate demand. Note that since the economy is closed, NNP and NDP are the same.

The output approach to NNP can be seen by using equation (4.1) to write $Y_{Ramsey} = f(K) - \delta K$. NNP is the sum of all net value added of all productive agents. Finally, the income approach measures NNP as sum of compensation to the factors used in the production process (compensation to the employees, rents on capital, etc) plus operating surplus. An expression for the income approach in the context of the Ramsey model is obtained by assuming a production function homogenous of degree one and then using the fact that, since we are dealing with optimal trajectories of a centralized economy, the marginal productivities of capital are the interests on the capital considered, i.e.,

$Y_{Ramsey} = rK - \delta K$, where r is the interest rate $(f_K(K))$[12]. The operating surplus is zero in the optimal trajectories, since all production is allocated to compensate the factors of production. Each 'factor of production' is paid according to its marginal participation in the productive process. This illustrates how the three approaches to national income acounting can be obtained in the setting of a dynamic economy.

4.3 Natural commercial (non-)renewable resources

As stressed in section 3.1.1, in the 1970s, seminal contributions were made for the analysis of sustainability of economies depending solely on non-renewable resources. Initially, the focus was on the effects on production and economic growth, of natural limits of exhaustible resources. But it was only in the beginning of the 90's that a concern for practical corrections of the standard national accounts aggregates' began to appear. The first proposals to correct national accounting aggregates into measures of welfare and sustainability dealt with non-renewable resources, with a particular focus on energy resources.

Consider the following model of an economy harvesting renewable resources. This is a generalization of the Dasgupta and Heal (1974) model. All functions are assumed to be functions of time unless stated otherwise. Assume a closed economy produces a composite consumption good using physical capital, K, and extracted resources, R, according to the net production function $F(K, R)$ with the usual neoclassical assumptions. The only source of utility for the households is the rate, C, at which they consume the homogeneous good. Unperturbed by human action, the renewable resource follows a density dependent natural growth, $G(S)$. Note that the Dasgupta-Heal model is for exhaustible resources as mentioned in section 3.1.1 which is obtained as a particular case of the following model by considering $G(S) = 0$. Consider all the functions used to be as smooth as needed in order to ensure the existence of solutions. Formally, we have the optimal control problem,

$$\max_{C,R} \int_0^\infty U(C)e^{-\rho t}dt, \tag{4.3}$$

[12]For instance, in the case of an economy with a constant inelastic supply of labor l and with wage rate w, then $Y_{Ramsey} = wl + rK - \delta K$, where, as expected, $w = f_l(\cdot)$.

subject to

$$\dot{K} = F(K,R) - C, \tag{4.4}$$

$$\dot{S} = G(S) - R, \tag{4.5}$$

with given initial endowments of physical capital $K(0) = K_0$ and natural resource $S(0) = S_0$. The central planner is assumed to control the rate of consumption and extraction of the renewable resource. The current value Hamiltonian for this problem is $H(C, K, S, \Psi^K, \Psi^S) := U(C) + \Psi^K \dot{K} + \Psi^S \dot{S}$.

Before going on to present the green NNP and genuine savings for this model let us illustrate the maximum principle stated in the beginning of section 3.2.1, by deriving the famous Ramsey rule for the consumption path and Hotelling rule for the prices of resources. From the maximum principle, we know that the optimal path $(C^*(t), K^*(t), S^*(t))$ maximizes the Hamiltonian w.r.t. the controls providing the conditions[13]

$$U_C^* = \Psi^K \tag{4.6}$$

$$F_R^* \Psi^K = \Psi^S \tag{4.7}$$

The Euler conditions given by equation 3.4 are

$$\frac{\dot{\Psi}^K}{\Psi^K} = \rho - F_R^* \tag{4.8}$$

$$\frac{\dot{\Psi}^S}{\Psi^S} = \rho - G_S^* \tag{4.9}$$

Taking the time derivatives of the first order conditions and using the Euler conditions we obtain

$$\frac{\dot{C}^*}{C^*} = \frac{F_K^* - \rho}{\eta(C^*)}, \tag{4.10}$$

$$\frac{\dot{F}_R^*}{F_R^*} = F_K^* - G_S^*, \tag{4.11}$$

where $\eta(C^*) := -U_{CC}^* C^* / U_C^*$ is the elasticity of the marginal utility of consumption.

[13] Here we abuse the usual notation to ease the exposition. For a function of one or more variable, $f(x_1, ..., x_n)$, the symbol f_{x_i}, where $i = 1, ..., n$, represents the partial derivative w.r.t. to x_i keeping all $x_{j \neq i}$ constant, i.e., $f_{x_i} := \frac{\partial f(\cdot)}{\partial x_i}$.

Hereafter, all the derivations are done on the optimal path, so we drop the star superscript on the equations.

Condition 4.10 is the Ramsey rule for the optimal consumption path and condition 4.11 is the Hotelling rule for the behavior of the resource prices along the optimal path. To put it simply, the fundamental result of Dasgupta and Heal (1974) is that the interest rate (price of man-made capital), F_K, eventually falls below the discount rate ρ, so that the long run consumption tends asymptotically towards zero, implying unsustainable development. Concerning condition 4.11, $\dot{F}_R/F_R$ is the proportional change of the price or rent of the resource. So, whenever G_S is (negative) positive (assuming logistic behavior, this is for stock levels (above) below the maximum sustainable yield) the resource price is rising at (more) less than the interest rate.

Returning to green accounting, the model above is of the form of the multisector growth model considered in section 3.2.1, so the propositions derived there apply here. However, this is not a very interesting case, since the model is too simple. However, it is a good starting point to introduce corrections to the usual national accounts NNP.

The most common adjustment to NNP is derived by considering that the resource is not costless to extract or harvest. So, assume that the harvesting of the resource stock, S at a rate R bears a cost at rate $f(R, S)$. That is to say $f(\cdot)$ represents firm's extraction costs. In addition, we assume $f_R > 0$ and $f_S < 0$, i.e., the harvest costs increases with increasing resource harvest and decreasing resource stock. Therefore,

$$\dot{K} = F(K, R) - C - f(R, S). \tag{4.12}$$

This, in turn, implies that the first order condition 4.7 is now, $\Psi^K (F_R - f_R) = \Psi^S$.

Define the real value prices for consumption P^C, investment in physical capital Q^K and in the renewable resource, Q^S, the same way as defined for equation 3.10. Therefore, from proposition 3 it is known that, either the quantity $R\left(Q^K\dot{K} + Q^S\dot{S}\right)$, or $P^C C + Q^K\dot{K} + Q^S\dot{S}$, can be used to reflect changes in welfare. Now, using $P^C = U_C/\Lambda$ and $Q^i = \Psi^i/\Lambda$, $i = K, S$, the first order conditions become $P^C = Q^K$, and $Q^S/P^C = F_R - f_R$. These conditions can now be used to write the NNP in real money terms as $Y = P^C\left\{C + \dot{K} + (F_R - f_R)\dot{S}\right\}$.

Now, from section, 3.2.1, the Divisia price index property for this model is $\dot{P}^C C = 0$.

Since optimal consumption is positive, then P^C is constant and can be set to 1 without loss of generality. In this case the expression for genuine savings or net investments is simply the real money value of the net investments. Summing up, and identifying the national accounts NNP as consumption plus net investment we have,

$$Y = NNP + (F_R - f_R)\,\dot{S}, \tag{4.13}$$

$$GS = \dot{K} + (F_R - f_R)\,\dot{S}. \tag{4.14}$$

This expression for green NNP indicates, in particular, that the resource stock should be priced using what is usually called the unit resource rent or marginal net price, $(F_R - f_R)$, price minus marginal cost of extraction/harvest. Moreover, in competitive equilibrium it is known that F_R should be the real price of renewable resource changes, and the quantity $(F_R - f_R)\,\dot{S}$ is generally called Hotelling or scarcity rent, marginal rent or even economic depreciation (Hartwick, 1990; Atkinson et al., 1997). This indicates that current Hotelling rents, should be netted out of usual NNP to arrive at green NNP. Or, using equation (4.5) to write,

$$Y = NNP - (F_R - f_R)\,R + (F_R - f_R)\,G \tag{4.15}$$

we can interpret this expression as: net product when there is a living commercial resource is measured as traditional NNP (consumption plus net investment), less the value of current resource rents, plus the net growth of the resource valued at its rental rate (Atkinson et al., 1997).

So, just as the national accountant nets out depreciation of physical capital (the usual 'wear and tear') from GDP to obtain NNP, by the same token depreciation of commercial natural resources should also be netted out from GDP to obtain the environmentally adjusted NNP, namely, green NNP.

A note of clarification is required. In this section we have treated commercial renewable and exhaustible resources altogether. This did not pose any problem, since concerning the production uses of the resources (and the level of abstraction considered in the production function, and the functioning of the economy for that matter) both resources are indistinguishable. However, concerning other types of links and uses of resources in the economic functioning it becomes apparent the need to distinguish between renewable and exhaustible resources.

Nonetheless, there is an interesting case in which one could approximate the behavior of an economic system exploring forests as if they were exhaustible resources. This case is when one believes that the forest is being harvested in a non-renewable fashion. This is the circumstance analyzed in Motta and Amaral (2000) for the Brazilian Amazon. They assumed that timber is a non-renewable resource, based on the fact that 'timber exploitation derives from forestland conversion devoted to agricultural production and cattle-raising'. Therefore, in this case, land is usually fully converted and the possibility of second growth forest is almost nil.

4.3.1 Estimating Hotelling rents for exhaustible resources

The biggest source of difficulty to include natural capital depreciation in the context of the national accounts or welfare measures is to estimate the marginal costs of extraction/harvest. At this point, it is important to briefly review the main approaches to estimating Hotelling rents. We shall briefly present the two most used methods to calculate Hotelling rents for a non-renewable resource - net price and El Serafy methods. The results are also hold when considering renewable resources, but the formulas are a bit messier. So, we defer the interested reader to Appendix 2 of Vincent and Hartwick (1998).

The question of estimating Hotelling rents is controversially debated in the relevant literature (see for instance Vincent and Hartwick, 1998, Neumayer, 2003, or Davis and Moore, 2000). We have showed that in the context of a competitive inter-temporally efficient closed economy, the Hotelling rent term is of the form $(P - MC)\, R$, where P is the average market price of the resource, MC is the marginal cost of extraction and R is the resource extracted in the accounting period. Calculating directly the term $(P - MC)\, R$ is usually called the net price method.

Since the data on marginal costs are frequently unavailable, the obvious solution is to consider the average cost of extraction instead. This is for instance what the World Bank does to estimate its genuine savings or more recently 'adjusted net savings' (Bolt et al., 2002). However, as Hartwick (1990) stated, as long as marginal extraction costs are increasing, using average extraction costs in place of marginal in the formula $(F_R^* - f_R^*)\, R$ will overestimate true economic depreciation.

Let us present the different methods to estimate Hotelling rents in a more specific context. The value of a natural resource asset $V(t)$ is defined as the discounted sum of the net returns (total resource rent, $PR(t) - f(R(t))$) it generates over time[14]. Net accumulation is defined as $D(t) = \frac{dV(t)}{dt}$ or its discrete-time counterpart $D(t) = V(t+1) - V(t)$. According to the net price method net accumulation in non-renewable asset values is calculated by

$$D = -\left[P - f_R(R)\right] R. \tag{4.16}$$

Assuming a special cost function (marginal cost function elastic w.r.t. quantity extracted, with elasticity β: $f_R(R) = \alpha R^{\beta}$), i.e., $f(R) = \frac{\alpha}{1+\beta} R^{1+\beta}$ then, according to Vincent and Hartwick (1998) and Vincent (1997), expression 4.16 can be written, either, as

$$D = \left[P - (1+\beta)\, AC\right] R, \tag{4.17}$$

or

$$D = TR \frac{(1+\beta)}{\left(1 + \beta e^{r(T-t)}\right)}, \tag{4.18}$$

where AC is the average cost $f(R)/R$, TR is the total resource rent $PR - f(R)$, whether data is available on β and AC, or β and TR, respectively.

The El Serafy method was developed to calculate depreciation of non-renewable resource stocks based on the concept of user cost. Net rents derived from an asset are split into two components: those that accrue at the expense of asset degradation (the 'user cost'), and those earned independently of asset degradation ('true' income). The method equates the discounted finite stream of annual rents (TR_t) earned from exploitation of the resource for a finite period of n years until the reserves are fully depleted, to a stream of discounted annual interest (X) to infinity resulting from selling the exhaustible resource and associated physical capital and investing the revenue in financial assets[15]. The difference between TR and X is the user cost measure of depreciation (Vincent and Hartwick, 1998). By further assuming that annual rent TR_t is constant over time it is possible to

[14] $V(t) = \int_t^T \left[pR(s) - f(R(s))\right] e^{-r(s-t)} ds$ where r is the continous discount rate and $C(q(t))$ is the total extraction costs; or its discrete-time counterpart $V(t) = \sum_{s=t}^{T} \left[pR(s) - f(q(s))\right] \frac{1}{(1+i)^{s-t}}$, where i is the discrete discount rate.

[15] $\sum_{j=0}^{n} \frac{R_{t+j}}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{X}{(1+r)^j}$

write,

$$D(t) = TR - X = TR\frac{1}{(1+i)^{n+1}}. \quad (4.19)$$

El Serafy proposes estimating n annually, by dividing the remaining stock by the quantity extracted, S_t/q_t. The reasoning for the El Serafy method is: receipts from non-renewable resource extraction should not fully count as what El Serafy calls 'sustainable income' in a Hicksian interpretation because resource extraction leads to a lowering of the resource stock and thus brings with it an element of depreciation of the resource capital stock (Neumayer, 2003). While the receipts from the resource stock will end up at some finite time, 'sustainable income' must last forever. Hence, as Neumayer (2003, p.149) puts it 'sustainable income' is that part of resource receipts which if received infinitely would have a present-value just equal to the present-value of the finite stream of resource receipts.

Vincent and Hartwick (1998) present a generalization both of the El Serafy and the net price methods by using the aforementioned cost function, optimal exploitation and the transversality condition ($MC(T) = AC(T)$) that enables to write the following expression for the net accumulation of an exhaustible resource,

$$D(t) = -TR(t)\frac{1+\beta}{1+\beta(1+i)^{T-t}}. \quad (4.20)$$

If $\beta \to 0$ the net price method is obtained and if $\beta \to \infty$, the El Serafy method is obtained.

Both these methods have been widely put in practice. For a review see Vincent and Hartwick (1998) and Neumayer's (2003) section 5.1.4.2 (p. 146). For an application of all these methods, consider Motta and Amaral (2000).

4.4 Environmental amenities

The distinction between commercial and non-commercial resources is important in terms of valuation of the resource in question. As seen from the results of the previous section, if the resource is only indirectly a source of utility as a mean of producing the consumption good, then changes in the resource stock should be valued at the Hotelling rent - market price minus extraction/harvest cost. But, on the other hand if we are considering forests not simply as a source of timber or non-timber products, or even other nonmarket forest products, but instead, also as a source of less tangible forest amenities consumed by households it becomes evident that the unit resource rent is not capturing these effects.

The approach we follow here is to assume that the stock of a natural resource (or a part of it) has some direct utility value for the households. Formally, this amounts to assuming two composite goods in the utility function.

What changes if there are two composite goods that are sources of utility, for instance, the consumption rate and the stock of renewable resource? In the above setting, the utility function is now $U(C, S)$ and the real price used to value the stock of resource is then, $P^S = U_S/\Lambda$. Given that most frequently there are no markets for amenity services, the most widely used approach to estimate P^S is through contingent valuation, namely, willingness-to-pay (or accept), travel cost methods and hedonic pricing. The appearance of new markets may also be of interest for valuing natural capital services. For instance, the Katoomba group's Ecosystem Marketplace seeks to provide information on markets and payment schemes for ecosystem services, such as water quality, carbon sequestration and biodiversity.

The first order conditions are the same, $P^C = Q^K$ and $Q^S/P^C = (F_R - f_R)$ and using $NNP = C + \dot{K}$, imply that green NNP, Y, in real terms is now given by

$$Y = P^C \left\{ NNP + \frac{P^S}{P^C} S + (F_R - f_R)\, \dot{S} \right\} \tag{4.21}$$

$$GS = P^C \left\{ NNP - C + (F_R - f_R)\, \dot{S} \right\} \tag{4.22}$$

So, this suggests that in order to arrive at measure of welfare that takes into account natural assets that are both a source of inputs into the production process and also a direct source of utility for the households, then, starting from NNP we have to add the value of the natural asset valued by willingness-to-pay techniques and subtract (if $\dot{S} < 0$) the value of the depreciation of the asset due to harvest, or add (if $\dot{S} > 0$) the value of the appreciation of the asset.

Moreover, applying the Divisia price property does not yield a satisfactory way to deflate the utility-NNP and utility-GS, which is problematic. However, as it shall be clear in the next section, there are other ways of including the environmental amenities in optimal control models. Nonetheless, the above results suggest that the environmental amenities should be valued as the marginal utility in monetary units of a change in the environmental asset providing the amenity. This information can be obtain by contingent valuation methods, including the willingness-to-pay for a extra unit of the environmental service. Note that the introduction of the environmental amenity did not entail any change in the genuine savings. Only the introduction of capital stocks in the model will alter the genuine saving formula.

4.5 Pollution emissions

Pollution is a by-product of the production and consumption processes. Depending on the pollutant, it is absorbed and accumulated at some rate on the natural environment. It may have direct (dis)utility effects on the health of the population and indirect as degrading the environmental quality. Accepting that, generally, the national accounts measure the goods but not the "bads" resulting from economic activities (Hamilton, 1996), in what way should the usual national accounting aggregates be corrected to account for the "bad" effects of pollution in a measure of welfare?

In the literature of green accounting several dynamic models have been considered to analyze combinations of such different topics as, flow and stock pollutants, the role of abatement and how should it be accounted for, disamenity, production and environmental effects and finally defensive expenditures. Basically, there are two approaches to analyze these effects, either with a big model including all the effects, or by using small models, where terms are analyzed one by one comparing the differences to simpler models. The first, is the approach followed by Mäller (1991), and the second approach is followed by Hamilton (1996) or Hartwick (1990). In regard to dynamic models that include some effect of pollutants, these three papers are the basis of the correction terms proposed to adjust the usual national accounts aggregates.

Starting with a flow pollutant, assume that only production is responsible for the emission rate, so that we have the pollution emission rate $E(F(\cdot))$. A simple model of a flow pollutant would be to consider a central planner that chooses consumption rates so that the a discounted utilitarian function is maximized as in 3.1 with $\mathbf{C} = (C, E(F))$, subject to the dynamics of accumulation of man-made capital, $\dot{K} = F(K) - C$. Following the optimal path, this economy yields, in real monetary units, a green NNP given by,

$$Y = P^C \left\{ C + \dot{K} + \frac{P^E}{P^C} E \right\}. \tag{4.23}$$

What is the meaning of P^E/P^C? It represents the change in utility in result of changing emission rates by one unit, i.e., the marginal social cost of emissions.

If we introduce firm's abatement expenditures, a, then capital accumulates as $\dot{K} = F(K) - C - a$, and the emission rate is now $E(F, a)$. In this case it is easy to show that

$$Y = P^C \left\{ C + \dot{K} - eE \right\}, \tag{4.24}$$

where $e := (E_a)^{-1}$, is the marginal cost of abatement, i.e., how much is the cost of reducing one unit of pollutant? Comparing these two expressions for green NNP, since these are optimizing models, we know that the marginal cost of abatement must be equal to the marginal social cost along the optimal path (Atkinson et al., 1997). What happens differently is that the central planner in the second model has control over the emission rates through the abatement expenditures, and while seeking to maximize the welfare of the economy, he/she sets price of the emissions as the abatement cost of the last unit of emission removed. Moreover, according to the Divisia price index formula $\dot{\mathbf{P}}(t)\mathbf{C}^*(t) = 0$ and the first order conditions, we obtain expression 4.42, which allows to set P^C equal to unity. In the first model, however, this deflator does not apply, and it is not possible in general to set $P^C = 1$.

Noting that, in the last model, the GDP is given by, $GDP = F(K) = C + \dot{K} + a$, equation 4.24 yields a different interpretation. The green NNP is now, $Y = GDP - a - eE$, meaning that starting from GDP, to arrive at green NNP we have to deduct the abatement expenditures from GDP and then subtract the value of emissions valued at the marginal abatement cost. Abatement expenditures are, hence, considered, in effect, intermediate consumption (Hamilton, 1996).

This setting also allows for the inclusion of environmental amenities. Assume that environmental services, B are negatively affected by the pollution emissions flow as, $B(E) = B_0 - \alpha(E)$, where, B_0 represents the level of environmental services that flow from a pristine environment, and $\alpha(E)$ is the amount by which ecosystem functions decline due to increases in emission flow. Now, in the green NNP in 4.24, a new term appears, namely, $P^B/P^C B_0$, which represents the value of the flow of environmental services provided. These services should be valued, as stated in the last section, at the price that utility-maximizing consumers would be willing to pay for a marginal unit of environmental service. Every time a variable is considered directly in the utility function, this is the way it appears in the green NNP, i.e., summing and valued at its marginal utility in monetary units. Moreover, the introduction of new commodities do not affect the formula for the genuine savings.

Now, considering stock effects, this is generally the case where we have the accumulation of pollution X as in, $\dot{X} = E(F, a) - d(X)$, where $d(X)$ is the natural rate of dissipation of the pollutant. What changes in formulas for green NNP and genuine savings. Every time a stock is introduced, a new term appears both in the green NNP and genuine savings formulas. However, there is a slight detail. This term is measured, no longer in terms of marginal utility, as in the previous model, but in terms of the shadow price for the stock in question, that is to say, priced at the changes in the welfare due to changes the stock. Nonetheless, with appropriate controls, and in the optimal path, the valuation of changes in pollution stock can be done using marginal costs of abatement. It can be shown that in a model with a stock pollutant with abatement and disutility effects, the green NNP formula for green NNP is equal to $Y = C + \dot{K} - eE$, provided that $d(X) = 0$. However, the genuine saving is now $GS = \dot{K} + Q^X/P^c\dot{X}$, where Q^X is the shadow price of the stock of pollution.

What is important to note from this section, is that despite the several ways the interactions between pollution emissions and the economy and the environment, it is widely accepted that the damage from pollution should be valued at the marginal social cost (or marginal cost of abatement) and this value should be deducted from the conventional NNP to get a measure of welfare. However, as a critique to Hamilton (1996) and Hartwick (1990), it is worth mentioning that comparing just the formulas for the green NNP is not sufficient for two reasons. Firstly, the genuine savings formula changes when stocks are introduced, and secondly the price deflator (which is not considered in these two papers) becomes important and it can also change according to the controls utilized by the central planner.

More interactions between pollutants and the economic activity are considered in the literature, namely the pollutants associated to renewable and exhaustible resources, which next section presents as an example of a more complete model that is used in practice.

4.6 World Bank's Genuine Savings Model

This section follows closely Hamilton (2000). All functions are assumed to be functions of time unless stated otherwise. Consider a closed economy with a single resource used as an input to the production of a composite good that may be consumed, invested in produced assets or human capital, or used to abate pollution, so that $F(K, R, N) = C + \dot{K} + a + m$, where R is resource use, a is pollution abatement expenditures, N is human capital, and m is investment in human capital (current education expenditures). Function $q(m)$ transforms education expenditures into human capital that does not depreciate (it can be considered to be a form of disembodied knowledge), so that $\dot{N} = q(m)$. Labour is fixed and is therefore factored out of the production function.

Pollution emissions are a function of production and abatement, $E = E(F, a)$, and pollutants accumulate in a stock X such that $\dot{X} = E - d(X)$, where d is the quantity of natural dissipation of the pollution stock. The flow of environmental services B is negatively related to the size of the pollution stock, so that $B = \alpha(X)$, $\alpha_X < 0$. Resource stocks S grow by an amount G and are depleted by extraction R, so that $\dot{S} = -R + G(S)$, and resources are assumed to be costless to produce and harvest. The utility of consumers is assumed to be a function of consumption and environmental services, $U = U(C, B)$. There is a fixed pure rate of time preference ρ. It is assumed that the social planner chooses consumption, pollution abatement expenditures, investment in human capital and resource extraction rates in order to maximize welfare as follows,

$$\max_{C,a,m,R} W(C) = \int_0^\infty U(C, \alpha(X))e^{-\rho t}dt \tag{4.25}$$

$$\dot{K} = F(K, R, N) - C - a - m - \delta K \tag{4.26}$$

$$\dot{X} = E(F, a) - d(X) \tag{4.27}$$

$$\dot{S} = -R + G(S) \tag{4.28}$$

$$\dot{N} = q(m) \tag{4.29}$$

This formal model may be criticized, in particular relative to the relations between the environment and the production process. Note that it is assumed that the environment's rate of pollution dissipation is independent of the stock of natural capital and that the pollution stock has no effect on the natural capital regeneration. Also, this model assumes that there are no expenditures for exploring and harvesting renewable resources. Nonetheless, this can be understood due to the difficulty in finding data on these topics.

Hamilton (2000) obtained the following expression for the genuine savings,

$$GS \equiv \dot{K} - \delta K - (1 - eE_F)F_R\dot{S} - e\dot{X} + 1/q_m\dot{N}, \tag{4.30}$$

where $e := -1/E_a$ is the marginal cost of pollution abatement which is equal to the marginal social cost of pollution emissions (emission Pigouvian tax). Using the Hamilton-Jacobi-Bellman equation we get $U_C GS = rW - U = \dot{W}$ which means that measuring negative genuine saving at a point in time implies that future utility is less than current utility over some period of time on the optimal path. This expression also implies that Hicksian income - the maximum level of consumption that leaves total welfare instantaneously constant - is given by,

$$gNNP = C + GS. \tag{4.31}$$

The standard measure of NNP (in a closed economy) is $C + \dot{K} - \delta K$. So, expanding the asset base implies that standard NNP should be adjusted by deducting net depletion of natural resources (valued at the net price) and the marginal damages from net pollution accumulation, and by adding investments in human capital. Hamilton (2000) argues that current education expenditures are not consumption and therefore should be included in saving, just like other defensive expenditures.

4.6.1 The World Bank computations

For real data, with n being the net resource rental rate and σ being the marginal social cost of pollution Hamilton (2000) arrives at the following equation,

$$GS = GNP - C - \delta K - n\dot{S} - \sigma\dot{X} + m. \tag{4.32}$$

This formula for genuine savings is a simplification of equation 4.30, and shows the steps to obtain genuine savings from GDP in practice. Starting from GNP, subtract consumption expenditures and the value of consumption of fixed capital to obtain net saving. Then, follow equation 4.30 with the approximation that marginal social costs are close to the marginal costs of abatement, and the accumulation of human capital is equal to the value of investments in human capital (current education expenditures).

The definition of the published indicator is as follows (Bolt et al., 2002):

$$GS = \left(GNS - CFC + CSE - \sum_i R_i - CD \right) / GNI, \tag{4.33}$$

where GNS is the Gross National Saving, CFC is the depreciation of produced capital, CSE is the current (non-fixed capital) expenditure on education, R_i is the rent from depletion of natural capital i, CD is the damage from carbon dioxide emissions and GNI is Gross National Income at market prices.

An annual average genuine savings (or adjusted net savings) was calculated from 1970 onwards for all countries for which the above data were available. Bolt et al. (2002) discusses the technical information required for the calculation of each element of GS. Here we shall, for now, just present the main drawbacks of the World Bank's computations.

The World Bank assigns all damage from CO_2 to the emitting country. It counts natural capital depreciation due to resource depletion to the country of resource extraction, not the country of resource consumption. It has been shown (Neumayer, 2003; p.145) that if natural capital depreciation is attributed to the country of resource consumption, then the GS position of resource-exporting developing countries improves, whereas that of resource-importing developed countries is not so positive any longer. The main critique of the WB's computations refers to the method used for computing of resource rents. Neumayer (2003) argues that changing the method changes substantially the results.

4.7 Small open economy

All the results presented in this chapter were for closed economies: in which prices of resources and other economic variables are generated endogenously. In practice, when one attempts to measure the sustainability of a particular country's economic course, one inevitably is dealing with an economy open to some amount of international trade and exhibiting some degree of price-taking behavior. The basic net investment rule requires reformulation in an open-economy context (Vincent et al., 1997).

We will consider, then, a model of a small open economy which is adapted from Pezzey et al. (2005). This model is itself a particular case of the multisector optimal growth model of section 3.2.1. Concerning the capital stocks of this economy we have the vector $\mathbf{K} := (K, K^f, \mathbf{S})$. Let $\mathbf{S}$ represent the vector of stocks of commercial renewable natural resources, K be the stock of domestic man-made capital, which grows at the rate of gross investment (Domestic Fixed Capital Formation) I minus depreciation δK, as in

$$\dot{K} = I - \delta K, \tag{4.34}$$

and K^f represent the stock of net foreign capital held privately or by the government, which earns a return at the exogenous, constant world interest rate r. Let K^f grow as a result of interest on the capital plus exports K^X minus imports K^M according to

$$\dot{K}^f = rK^f + K^X - K^M. \tag{4.35}$$

The stock of commercial renewable natural resources is harvested for domestic use in the production process, $\mathbf{R}^d$, and for export, $\mathbf{R}^X$, and regenerates at the natural rate, $\mathbf{G}(\mathbf{S})$. Therefore, $\mathbf{S}$ changes according to,

$$\dot{\mathbf{S}} = \mathbf{G}(\mathbf{S}) - (\mathbf{R}^d + \mathbf{R}^X). \tag{4.36}$$

Production, using fixed technology (does not depend explicitly on time), uses the stock of man-made capital along with the domestic commercial resources harvested and imported to produce a consumption/investment good, as in $F(K, \mathbf{R}^d + \mathbf{R}^M)$. Part of the natural resources is exported at world market prices $\mathbf{Q}^{\mathbf{R}}$. The stock of natural assets imported is valued also at world market prices. So, the net value of resource transactions is $\mathbf{Q}^{\mathbf{R}}\left(\mathbf{R}^X - \mathbf{R}^M\right)$, and adds to the production of the consumption good and to net imports $K^M - K^X$, to be used for material consumption, gross investment I, firms' pollution abatement current expenditure, a and harvesting, with the firm's harvesting cost function, $f(\mathbf{R}^d + \mathbf{R}^X, \mathbf{S})$ with the properties stated in section 4.3. Formally,

$$F(K, \mathbf{R}^d + \mathbf{R}^M) + \mathbf{Q}^{\mathbf{R}}\left(\mathbf{R}^X - \mathbf{R}^M\right) + K^M - K^X = I + C + a + f(\mathbf{R}^d + \mathbf{R}^X, \mathbf{S}).$$

Rewriting this expression in a more familiar manner, we have,

$$\dot{K} = F(K, \mathbf{R}^d + \mathbf{R}^M) + K^M - K^X + \mathbf{Q}^{\mathbf{R}}\left(\mathbf{R}^X - \mathbf{R}^M\right) - C - a - f(\mathbf{R}^d + \mathbf{R}^X, \mathbf{S}) - \delta K. \tag{4.37}$$

Regarding the household's utility function, we assume that $U(\mathbf{C}) = U(C, \mathbf{E})$, where C is material consumption, and $\mathbf{E}$ is a vector of emission flows, dependent on resource use and abatement expenditure, $\mathbf{E}\left(\mathbf{R}^d, a\right)$. In fact, we could have the vector of emissions dependent on production and abatement that it wouldn't change the formulas for the genuine saving and green NNP derived below. Each emission level $E^i(\cdot)$, depends on domestic resource use $\mathbf{R}^d$ and abatement expenditure a^i for each pollutant $i = 1, \ldots$ The marginal cost of abating pollutant i, is denoted by

$$e^i := -E^i_{a^i}. \tag{4.38}$$

Consider that the vector of marginal costs of abating pollution emission is, $\mathbf{e} = \left(e^1, \ldots, e^i, \ldots\right)$.

In the context of discounted utilitarianism, we assume that the central planner for this economy acts to maximize the PV-utility as in 3.1 subject to the economic dynamics given by 4.37, 4.35 and 4.36. In order to achieve this objective, the central planner controls C, R^d,R^X, R^M, a, and $K^M - K^X$.

In accordance with simple national accounts procedures we identify NNP in this setting as

$$NNP := C + \dot{K} + \dot{K}^f. \tag{4.39}$$

Pezzey et al., (2005) comment that environmental and resource spending by firms (here a on pollution abatement, and $f(\cdot)$ on resource extraction) is not part of conventional NNP, as mentioned in section 4.5. This is because by national accounting conventions, firm (as opposed to governmental or household) expenditures are treated as intermediate, and thus already excluded from all calculations of national product (whether gross or net, domestic or national) in order to avoid double counting.

All functions are assumed to be as smooth and convex as necessary for the propositions of section 3.2.1 to apply. Accordingly, we have

Proposition 8 (GNNP and GS) *For the economy described above, the green NNP is given by*

$$gNNP: \; Y = NNP + (\mathbf{Q^R} - f_{\mathbf{R}})\dot{\mathbf{S}} - \mathbf{eE}, \tag{4.40}$$

and the genuine saving by

$$GS : Q\dot{K} = NNP - C + (\mathbf{Q^R} - f_\mathbf{R})\dot{\mathbf{S}}. \qquad (4.41)$$

Proof. According to the maximum principle in 3.2.1 the Hamiltonian for this problem is $H(C, I, \Psi)$, where the arguments are the vectors of consumption rates $C = (C, \mathbf{E})$, gross investment $I = \left(\dot{K}, \dot{K}^f, \dot{\mathbf{S}}\right)$ and investment prices $\Psi = \left(\Psi^K, \Psi^{K^f}, \mathbf{\Psi^S}\right)$. The first order conditions, i.e., $H_i(\cdot) = 0$ with i representing the control variables C, R^d,R^X, R^M, a, and $K^M - K^X$, respectively imply that $\Psi^K = U_C$, $\Psi^K(F_{\mathbf{R}^d} - f_{\mathbf{R}^d}) = \mathbf{\Psi^S}$, $\Psi^K\left(\mathbf{Q}^R - f_{\mathbf{R}^X}\right) = \mathbf{\Psi^S}$, $\Psi^K\left(F_{\mathbf{R}^M} - \mathbf{Q}^R\right) = 0$, $U_\mathbf{E} = -\mathbf{e}U_C$ and $\Psi^K = \Psi^{K^f}$. Noting that $f_{\mathbf{R}^X} = f_{\mathbf{R}^d} := f_\mathbf{R}$ and $F_{\mathbf{R}^d} = F_{\mathbf{R}^M}$ implies that $Q^R = F_{\mathbf{R}^d}$. Using the real Divisia prices $P = \left(P^C, \mathbf{P^E}\right)$ and $Q = \left(Q^K, Q^{K^f}, \mathbf{Q^S}\right)$, these conditions imply that $P^C = Q^K$, $Q^K\left(\mathbf{Q^R} - f_\mathbf{R}\right) = \mathbf{Q^S}$, $Q^K = Q^{K^f}$ and $\mathbf{P^E} = -\mathbf{e}P^C$. These conditions allow to rewrite equation 3.10 and real genuine savings $\mathbf{QI}$, respectively as,

$$\begin{aligned} Y &= P^C\left\{C + \dot{K} + \dot{K}^f + \left(\mathbf{Q^R} - f_\mathbf{R}\right)\dot{\mathbf{S}} - \mathbf{eE}\right\}, \\ \mathbf{QI} &= P^C\left\{\dot{K} + \dot{K}^f + \left(\mathbf{Q^R} - f_\mathbf{R}\right)\dot{\mathbf{S}}\right\}. \end{aligned}$$

Now, rewriting the Divisia price index property, $\dot{\mathbf{P}}\mathbf{C} = 0$, using $\mathbf{P^E} = -\mathbf{e}P^C$, we obtain,

$$\frac{\dot{P}^C}{P^C} = \frac{\dot{\mathbf{e}}\mathbf{E}}{(C - \mathbf{eE})}. \qquad (4.42)$$

We assume that $\dot{\mathbf{e}} = 0$, due to lack of data on the evolution of marginal damage costs. This way, P^C can be set to unity, yielding expressions 4.40 and 4.41. ■

Equations 4.40 and 4.41 show the adjustments necessary to reach to gNNP from the usual NNP:

- deduct the amenity cost of emission $\mathbf{eE}$;
- deduct the value of rents from resource stock depletion $(\mathbf{Q^R} - f_\mathbf{R})\dot{\mathbf{S}}$.

Basically, these are the expressions we will estimate for Portugal (1990 - 2004) in chapter 5. As it could be perceived throughout this section there is an endless number of different ways that one could use to integrate different concerns about the environment.

For instance, can the stock (environmental concentration) of a cumulative pollutant be abated by human action, or only the flow (emissions)? Is abatement effort the result of current spending, or of capital equipment which is the result of past spending? Are extraction costs affected by the extraction rate, the remaining stock, or both? Indeed, Pezzey and Toman (2002) conclude, the results found show that well-known methods of accounting for pollution or resource depletion in gNNP measurement are often far from general.

5 Green accounting in Portugal

This chapter uses the formal theory of green (or welfare) accounting presented is section 3.2 and further specified in section 4.7 to estimate a measure of welfare and a measure of the extent to which Portugal is on a path of sustainable development. Namely, the green net national product and the genuine savings formulas given in 4.40 and 4.41. We include the damages from air pollution flows as a disamenity, and the depletion of commercial forests in Portugal for the years 1992 - 2004. First, we describe the data and then we present the calculations and results. The discussion and interpretation of the results will be made while presenting the results.

The major problem with performing green or welfare accounting is data (un)availability. So, the main purpose of this section is not to specify thoroughly the way to proceed to derive measures of welfare and sustainability from the national accounts, but instead, and as the first approximation of these measures for Portugal, to indicate the difficulties and drawbacks behind the calculations of a green accounting aggregate. In this context, we propose to adjust the usual national accounts aggregates to include a valuation of resource depletion and the disutility due to air pollution emissions. Here, we work with € of 2000.

5.1 Pollution emissions and valuation

For each pollutant considered, the term to be included in the calculations of the green NNP and genuine savings is the product of the vector of emission flows multiplied by an estimate of the either the marginal benefit of abatement (also termed the marginal social costs) or marginal costs of abatement. As seen in section 4.5, the basic question in valuing air pollution emissions is whether to use marginal social costs (also known as the marginal damage cost (MDC) of pollution) or marginal abatement costs. Note, however, that this distinction makes sense only away for the optimum, since in the optimal path both these costs are equal. In this case, according to Atkinson et al., (1997, p. 87) if we assume that the current state of the economy is one of overpolluting, then the marginal social costs provide an upper bound on the value of optimal pollution emissions. If underpollution is the case, then marginal social costs provide a lower bound on the optimal emission value. Therefore, using MDC should be viewed as an upper limit estimate and interpreted accordingly. This implies, in addition that the deduction for pollution emissions in the welfare measure will decrease as the optimum is approached, which is a desirable property. On the other hand, using marginal abatement costs to value emissions will not lead to an unequivocal direction of bias in the estimates of the value of pollution (Atkinson et al., 1997). If the economy is overpolluting, then marginal abatement costs will be below the optimal and emissions above.

We followed the practical convention proposed by Pezzey et al., (2005) where:

- if there are data on only marginal benefits or marginal costs of environmental improvement, use whichever is available;
- if there are data on both benefits and costs, but of very different reliability, use the generally more reliable data;
- if there are data on both marginal benefits and costs, of broadly similar reliability, use the bigger figure. This will be the marginal benefit if, as one often expects, pollution is excessive;
- in any case, be explicit about what choices were made and why, and about how much difference they make to the final results.

Following this convention, in our case, and for Pezzey et al. (2005) for that matter, for all pollutants considered, we used data for marginal damage costs (MDC) rather than for marginal abatement costs, because the former were the only available.

Using the preceding model of cumulative pollutants, we wish to estimate the value of air pollution in Portugal caused by carbon dioxide (CO_2), sulphur dioxide (SO_2), nitrogen oxides (NO_x), particulate matter (PM10), volatile organic compounds (VOC), methane (CH_4), nitrous oxide (N_2O) and carbon monoxide (CO). Concerning the greenhouse effects of the pollutants, Pezzey et al., (2005) warns that the marginal damage cost of each of these takes into account its atmospheric lifetime effect or global warming potential.

The emission data for the air pollutants considered, except PM10, was taken from the Environment Institute's submissions in the context of the United Nations Framework Convention on Climate Change and from the National Inventory Report (2007) referring to the period 1990 - 2005. The emission data on PM10 was taken from the Environment Institute's submissions in the context of the Convention on Long-range Transboundary Air Pollution (UNECE) relative to the period 1990 - 2005.

In terms of the mass of pollutant emitted, the CO_2 is the most emitted pollutant accounting for 94% of total emissions 1990 and steadily rising to 96% in 2005. Methane, carbon monoxide, and volatile organic compounds together amount to 3 - 7% of total emissions during the period. Finally, SO_2, N_2O, $NO_x$and PM10 together amount to 0.7 - 1.5% of total emissions during the period. Most of the pollutants' emissions are not decreasing in 1990 - 2005. For instance, the CO_2 emissions have a clear tendency to rise. Also, the ratio of total emissions to GDP is almost constant for the relevant period.

As stated above the value attributed to each emission of pollutant is the marginal damage cost, rather than the marginal abatement cost, due to data availability. The marginal damage costs were obtained from a literature review. For most of the pollutants considered, various estimations of marginal damage costs were available in the literature, and a few estimations concerned Portugal. For others, such as the N_2O, few or only one value is obtained from the literature. Whenever possible values for Portugal were used. All the prices estimated were considered constant throughout the period of accounting due to lack of data on the evolution of these estimates.

The studies used were considered relevant for Europe: COWI (2000), ExternE project for Portugal (Martins et al., 1998), and BeTa database (Holland and Watkiss, 2002).

The ExternE project was the first comprehensive attempt to use a consistent 'bottom-up' methodology to evaluate the external costs associated with a range of different fuel cycles. It developed and demonstrated a unified methodology for the quantification of the externalities of different power generation technologies. The Portuguese national implementation covered four fuel cycles: biomass, hydropower, coal, and natural gas. Biomass, coal and hydropower (Martins et al., 1998). The methodology adopted by the ExternE project is, a linear dose-response function used to quantify physical effects, and a valuation of years of life lost. Morbidity costs were based on the cost of hospital stays, emergency visits, restricted activity days, symptom days, asthma attacks and bronchitis attacks. Pezzey et al.,(2005) argue that 'this technique is considered to be highly relevant for the analysis of these pollutants'.

The ExternE project for Portugal yielded estimations of marginal damage costs for the following pollutants: CO_2,SO_2,NO_x and PM10. The marginal damage costs were for 1995 € and were, then, converted to 2000 prices. All price conversions were conducted using the UNSTAT's consumer price index (series n^o 4620) for Portugal.

The BeTa project (Holland and Watkiss, 2002), was intended to provide a simple ready reckoner for estimation of the external costs of air pollution. The original methods for calculating the estimates have been adapted using the ExternE methodology. This follows the 'impact pathway approach' tracing emissions through dispersion and environmental chemistry, to exposure of sensitive receptors, impacts (calculated using exposure-response functions) and finally economic valuation using the willingness to pay approach. The main effects considered are effects on mortality and morbidity. Overall, however, it is considered that the externalities taken into account in the database are likely to dominate the full external costs. Many of those not quantified are likely to be small, as shown in past analyses.

From the BeTa database, we used the estimation of marginal damage costs for the pollutants, SO_2, NO_x and VOC. These are values specific to Portugal, specifically, they estimated MDC for rural areas in Portugal. Urban externalities of SO_2 for cities of different sizes are calculated by multiplying results for a city of 100,000 people by the multiplying factors (Holland and Watkiss, 2002). These results are independent of the country in which the city is located. We estimated the data for Portugal by using a weighted average of the rural estimations and the estimations for the cities (the factors mentioned above), where the weights reflect the approximate distribution of population in Portugal through rural and cities of different sizes.

COWI (2000) estimated MDC for the eight pollutants considered, aiming to present "an overview of the environmental externalities that need to be taken into account when evaluating different waste management policies and how they can be integrated into cost-benefit analysis." They considered an overview of the types of externalities arising from landfill disposal and incineration of waste, and also a quantification of the main externalities according to typical scenarios for landfill disposal and incineration of waste both in physical and economic terms. They provided MDC estimations specific for Portugal.

In table 1 we present the values we used to calculate the term eE in the green NNP expression 4.40.

The best estimation refers to values of the above studies that were calculated specifically for Portugal. Averages were taken when more than one value existed for Portugal. The low and high estimations in table 1 were calculated from all the values above (for Portugal or not) to give an idea of the different values existing in the literature. For the

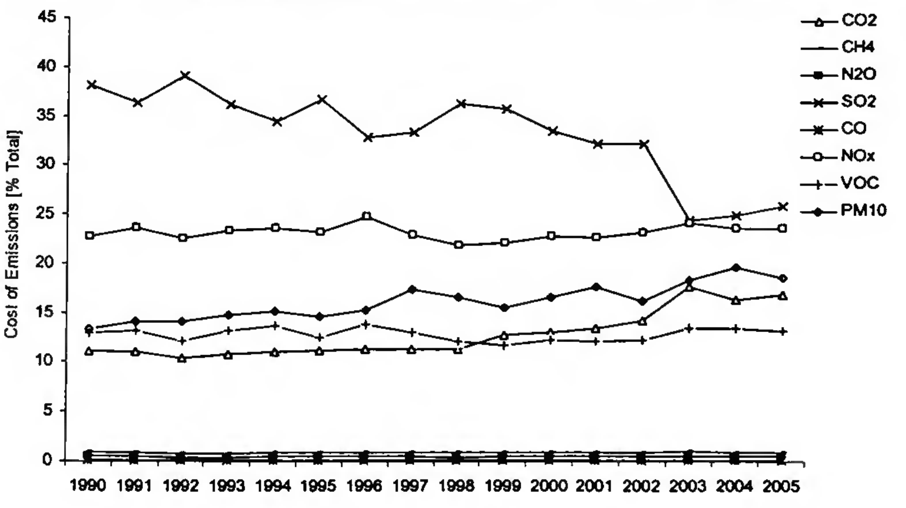
Table 1: Estimates of marginal damage cost by air pollutant in Portugal [€2000/ton].

	Best	Low	High
CO_2	16	3	52
CH_4	108	67	300
N_2O	1836	1836	1836
SO_2	7898	169	14126
CO	9	2	9
NO_x	5985	286	20778
VOC	1201	113	5111
$PM10$	10005	1056	23620

CO_2 we have also have considered the famous estimation by Frankhauser (1994), of the marginal global damage per ton of carbon emitted of \$20[16]. This valued is used by the World Bank to estimate genuine savings (Bolt et al., 2002). To bound the range of values found in the literature we also used benefit transfer techniques, in particular we have adjusted values estimated from other countries to Portugal by using purchasing power parities. The values in table 1 compare to the values used by Pezzey et al., (2005) for Scotland.

Figure 3 illustrates the costs of emissions as percentage of the total cost for the best estimation.

Figure 3: Cost of air emissions per pollutant as percent of total. Best estimate for Portugal.



Note that, surprisingly, comparing to the physical quantities of emissions, the SO_2 is the pollutant that bears the higher costs in the order of 25 - 40% of the total. Then

[16] Since the data is for CO_2 and the damage estimate is per ton of carbon the estimated marginal damage for CO_2 emissions is $20 \times 12/44 = 5,4545$ \$/ton CO_2.

follows, NO_x about 25% and then a group of three pollutants between 10 - 15% of total cost composed of emissions from PM10, VOC, and CO_2. The costs corresponding to CO, N_2O and CH_4 are almost negligible as a percentage of total costs. This result deserves more attention, since the global warming potentials (GWP) of N_2O and CH_4 that are, respectively, of the order of 310 and 21 (compared to the GWP of CO_2 which is 1) suggest that looking only at the greenhouse effect costs, they should be higher than the costs of emitting CO_2. This serves to make a remark about the way that prices were estimated with the ExternE methodology. The main focus is on disutility effects related to health problems. This is in accordance with modelling the pollution flow as a 'bad' in the utility function.

As mentioned above, and in accordance with the practical convention we have considered an upper and lower bound for the marginal cost of emissions of the pollutants in question. The figure 4 illustrates the range of MDC estimates encountered in the literature. Note that these are values estimated for Portugal and in the worst case scenario, transferred to Portugal using adjusted benefit (value) transfer techniques.

Figure 4: Total cost of emissions [% GDP]

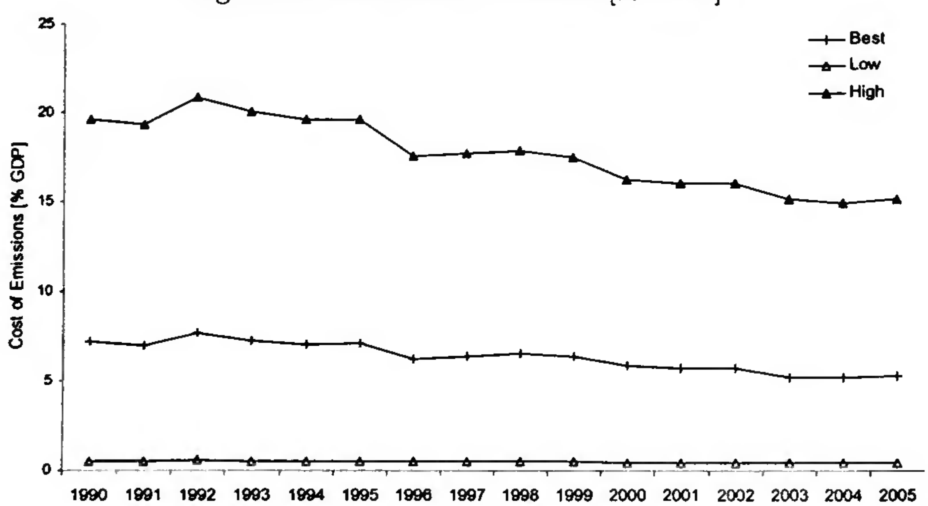


Figure 4 presents the cost of emissions as % of GDP. The costs are very high but declining, because of decreasing emissions of air pollutants during 1990 - 2005. The lower bound is also decreasing but it is not evident from the figure. Atkinson et al., (1997) estimated the damages of PM10, SO_2, NO_x, and CO_2. They estimated a value of 8.7% of GDP in 1980 for Portugal, and noted that 'this seems high, with the probable cause lying in the adoption of the estimates of marginal social costs based on UK emissions (weighted for differences in per capita income)'. For the pollutants they consider we have that in 1990 the total cost is about 5.0% of GDP and this value steadily declines until it reaches 3.5% of GDP in 2005. Our results, then, suggest that differences in income do not explain the high value of costs of emissions because we have considered values estimated for Portugal specifically. Nonetheless, the values estimated for the ExternE project for Portugal are believed to be a bit high, but in the absence of better estimates, and keeping in mind that the prices obtained should be understood as upper bounds to the true costs of air emissions, those were the adopted prices.

5.2 Depreciation in commercial Portuguese forests

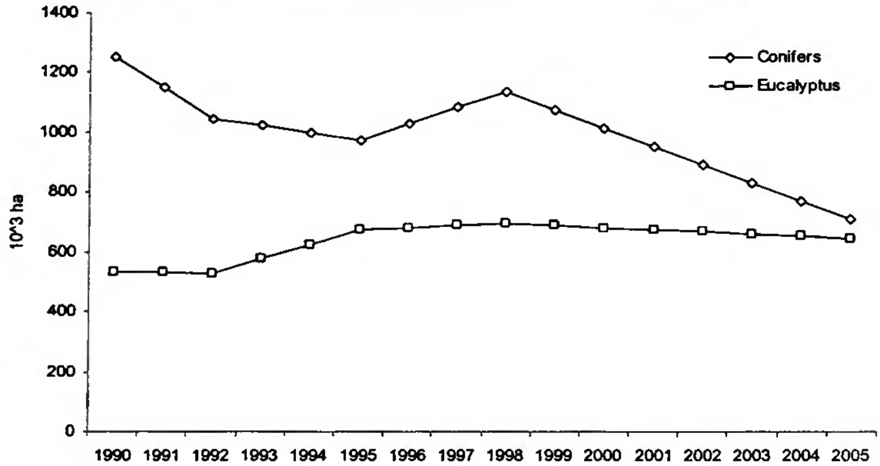
In this section we present the values used to estimate $(\mathbf{Q}^R - f_\mathbf{R})\dot{\mathbf{S}}$ in expressions 4.40 and 4.41. This term is the most hard to obtain data to estimate. Both data on stocks, prices and especially marginal harvest costs. Concerning commercial forests we have considered to study two of the most important commercial sources of wood in Portugal, that is, conifers and eucalyptus. Mendes (2005) divides Portuguese forestland into two main functions: in 1995, the main function of 51.8% (24.4% of conifers, 17.7% of broad-leaves and 11.6% of mixed stands) of forestland was wood supply, and the second function, corresponding to 48.2% of the forestland, was for non-wood forest products (NWFPs), mostly cork production in the southern regions.

In 1998, the forest sector represented 2.93% of the GDP, which places the country in a top position within the EU 15, in terms of this indicator, being surpassed only by Finland and Sweden (Mendes, 2005). Most of this value added was due to cork products. Corks exports are the most important part of total forest exports. However, since Mendes (2005) argues that 'it is believed that the industrial demand for cork induces harvesting of all sustainable production' but not more, we have considered the net growth of "cork forests" equal to zero, and so we did not consider it in estimating the depreciation of commercial forest use in Portugal. This may be a question to be addressed elsewhere.

We estimate $\dot{\mathbf{S}}$ directly, i.e., we obtained data on the stocks for some years, estimated the gaps, and then used the approximation $\dot{\mathbf{S}} \simeq \mathbf{S}(t+1) - \mathbf{S}(t)$. The data was obtained from the National Forest Inventory 2005/06 (IFN) of the DGRF (Direcção-Geral dos Recursos

Florestais[17]) for the years 1990, 1992, 1995 and 2005 in terms of area. This is depicted in figure 5.

Figure 5: Estimated evolution of land use in Portugal



The data on volumes of standing stock (m^3/ha) was also obtained from the IFN 2005/06 and was considered to be 85.5 m^3/ha and 55 m^3/ha throughout the period 1990 - 2005, for conifers and eucalyptus, respectively. This allowed us to calculate the stocks $S(t)$. The area figures presented do not include burnt areas. However, since the data is being collected with a 5 year interval this information is difficult to interpret. Also, in order to use these data in our very simplified model of section 4.7, we have to assume that there is a myriad of decisions concerning the optimal harvest time that overlap each other, so that we approximate the manage of a forest with a continuous harvest rate. This might just be the Portuguese case since most of our forest is privately managed and not subject to aggregate interests concerning the ideal age to cut the trees. Vincent and Hartwick (1998) present some formulas to calculate the depreciation of forest resources when there is a waiting period and a certain optimal age to cut.

Information on relative to prices was obtained through the SICOP system (Sistema de Informação de Cotações de Produto Florestais na Produção)[18], for the period 2000 - 2005, and directly from DGRF for the period 1990 - 1995 based on roadside prices. The gaps were estimated using a linear approximation. Also, the World Bank provided data on timber prices (1970 - 2004), in their calculations of the adjusted net savings, these

[17] www.dgrf.min-agricultura.pt/

[18] http://cryptomeria.dgrf.min-agricultura.pt/

prices were estimated as a weighted average of the fuelwood price and industrial wood price (Bolt et al., 2002).

Figure 6: Estimated evolution of timber prices

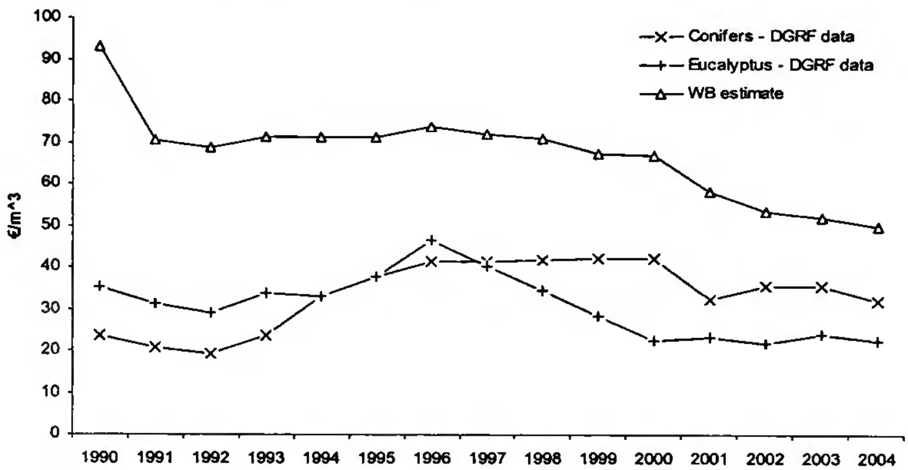
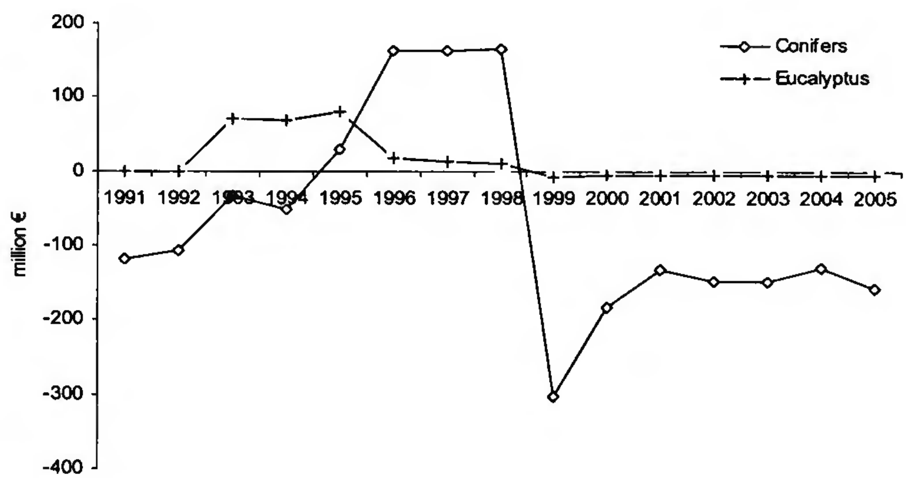


Figure 6 depicts the estimation of the evolution of the timber prices.

Figure 7: Forest depreciation by species in Portugal.



The data on marginal cost of harvesting was impossible to obtain. An average value for the marginal cost of harvesting 7 €/m^3 was obtained through inquiries with several firms that provide forest services. Figure 7 presents the estimated forest depreciation by species in Portugal. Figure 8 serves to show the sensitivity test of using both data sources on prices. When calculating the gNNP and GS we choose to work with the World Bank figures since they yield the higher depreciation. This is consistent with using an upper bound for the air pollution term in the previous section.

Figure 8: Forest depreciation by price data source.

We see that the commercial forests considered here have been depleted from 1990 - 1993 and from 1999 - 2004, 1999 being the year of the highest depletion. Between this period, the value of net depletion $(\mathbf{Q}^R - f_{\mathbf{R}})\dot{\mathbf{S}}$ was positive, meaning that the forest appreciated then. Rising the marginal costs of extraction would lower the value of the depreciation of commercial forests throughout. Since the value added of the forest sector is around 3 %, this implies that the value of the change of commercial forests in Portugal is of the order of 10 % of the value added of the forest sector in the years of 1998 an 1999 (the highest depreciation values). This has the interpretation that in 1999, the forest value added should have been 10 % less, to account for the loss of 'potential' timber in the future due to harvest in the current period.

5.3 Green Net National Product and Genuine Savings results

We now proceed to the calculation of green NNP and genuine savings according to expressions 4.40 and 4.41. The main results are shown in table 2 (1990 - 1997) and table 3 (1998 - 2004). The values of GDP and consumption of fixed capital (CFC - UNSTAT's series n^o 30227) were taken from UNSTAT for Portugal[19]. Consumption of fixed capital is subtracted from GDP to obtain net national product (NNP). We then subtracted the total cost of emissions of air pollutants and added the term related to depletion of commercial

[19] http://unstats.un.org/unsd/snaama/dnllist.asp

forests, to obtain the an estimation of the green net national product for Portugal.

Table 2: Green NNP, Genuine savings and their components for Portugal 1990 - 1997 [10^6 €2000].

	1990	1991	1992	1993	1994	1995	1996	1997
GDP	91960	95977	97023	95040	95957	100066	103693	108037
CFC	13695	13880	13542	13572	13813	15814	16373	16914
NNP	78265	82097	83481	81468	82144	84253	87320	91124
eE	6578	6694	7443	6889	6755	7132	6475	6878
Forest Depletion		−562	−545	39	39	227	345	336
gNNP		74841	75492	74618	75427	77348	81190	84582
gNNP/NNP		91%	90%	92%	92%	92%	93%	93%
GS		8761	8842	7357	6218	4301	3423	3894
Change gNNP			651	−875	810	1921	3842	3392
Interest on GS (2%)		175	177	147	124	86	68	78

Table 3: Green NNP, Genuine savings and their components for Portugal 1998 - 2004 [10^6 €2000].

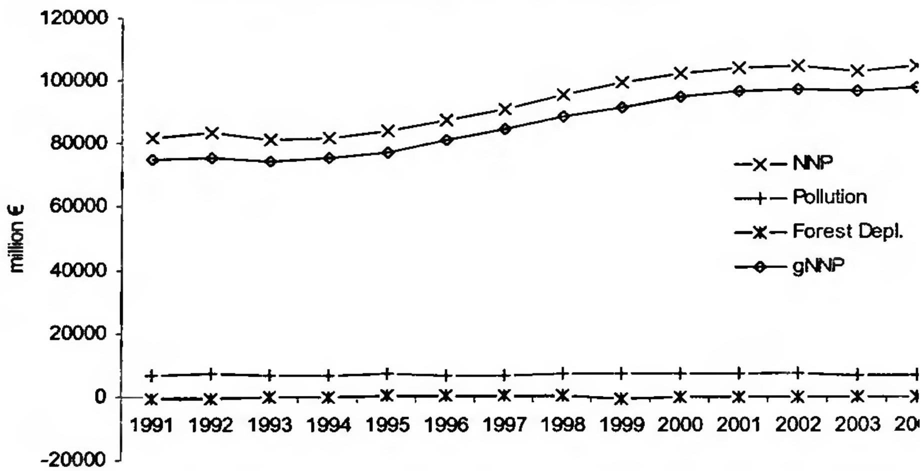
	1998	1999	2000	2001	2002	2003	2004
GDP	113180	117636	122252	124717	125669	124261	125738
CFC	17555	18301	20091	20731	20902	21050	21188
NNP	95625	99335	102161	103986	104767	103211	104550
eE	7396	7499	7147	7172	7180	6478	6512
Forest Depletion	331	−541	−336	−286	−259	−251	−240
gNNP	88560	91294	94679	96528	97328	96481	97797
gNNP/NNP	93%	92%	93%	93%	93%	93%	94%
GS	4247	2208	303	254	363	−823	−2256
Change gNNP	3978	2734	3384	1849	800	−847	1316
Interest on GS (2%)	85	44	6	5	7	−16	−45

Green NNP is about 10 – 6% less than NNP, with the gap falling steadily over 1991–2004. This means that the green terms do not make a big difference to the results. We underline that we have considered few environmental corrections, and many more can be made in this context, namely, concerning other resources in Portugal like, fish stocks, mineral, water, and also important soil (erosion). Due to data availability we did not estimate these here.

Nonetheless, compared to Scotland's results, this value is somewhat larger. This is so for two reasons. First, the air pollution emissions costs estimated for Portugal were higher, and second, in Portugal we had a higher depreciation term due to commercial forests harvesting than Pezzey et al., (2005) found.

Figure 9 shows the green NNP and the factors that compose it. Note that the pollution term is depicted as positive in the figure but it is subtracted in the calculations. It is apparent from this figure that these green terms are in fact small compared to NNP. In fact the cost of emissions decreases from more or less 9% in the beginning of the period to 6% of NNP in 2004. The term correspondent to commercial forest depletion ranges between - 0.7% in 1991 and 0.4% in 1996.

Figure 9: Green NNP and its components



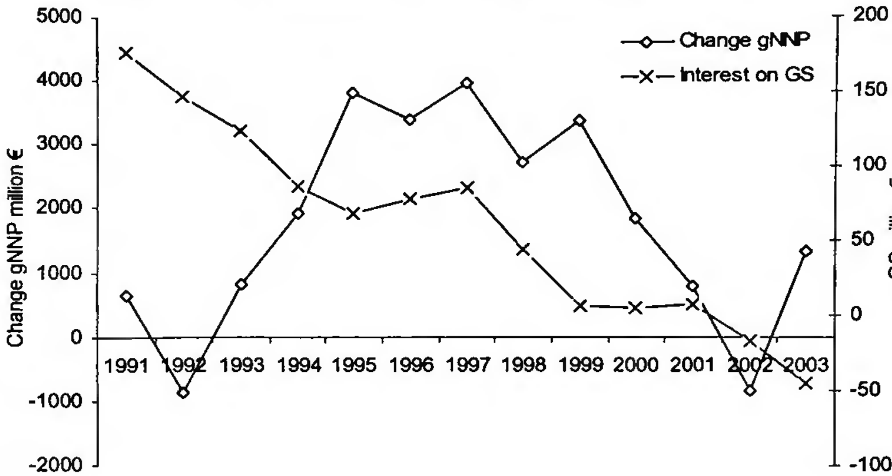
So, this suggests a reduction in the need for the various sensitivity analyses which clearly could be undertaken of any of the above assumptions.

From the evolution of the green NNP it is possible to estimate the genuine savings for Portugal. This can be done also from expression 4.41. Looking at the change in green NNP for Portugal in figure 10 we find evidence of non-sustainable development in 1992 and 2002, that is to say, green NNP decreased in those years. Elsewhere, the green NNP is positive.

In order to calculate the genuine savings for Portugal we first estimated the net national saving for Portugal with no environmental terms. We did this in four ways, common in the national accounts' literature, to test the sensitivity of the results. Then, and according to the expression for genuine saving in expression 4.41 we have added the value of depreciation of commercial forests.

In the first approach, we used the common national accounts formula, that states that net savings is equal to gross national savings minus consumption of fixed capital, $NS = GNS - CFC$, as in Bolt et al. (2002). The data was taken from UNSTAT's series

Figure 10: Green NNP and interest on genuine savings for Portugal



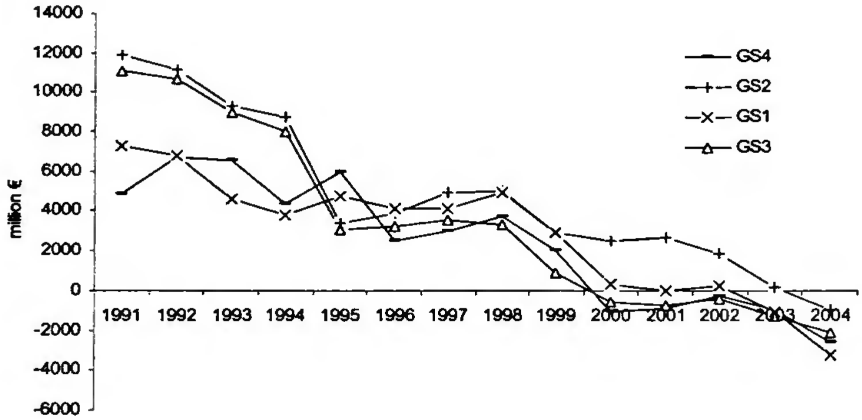
n^o 30243 and series n^o 30227, respectively. Genuine savings calculated in this manner is termed GS1. From figure 11 we conclude that GS1 becomes negative in 2003 and stays thereafter, suggesting unsustainable behavior.

The second way we have calculated genuine savings is by the simple formula that states that net savings equals NNP minus total consumption expenditures (here given by the sum of household and government consumption expenditures), i.e., $NS = NNP - C$. The data was obtained from UNSTAT, series n^o 30235 and series n^o 30230, for households and government consumption expenditures, respectively. We called the genuine savings calculated using this net savings as GS2. From figure, 11 we conclude that, once again, there is evidence of unsustainable development in 2004.

The third way to calculate net savings for Portugal uses the formula $NS = NNI - C$, where net national income $(NNI) = GNI - CFC$, and , gross national income (GNI) at market prices is the sum of gross primary incomes receivable by resident institutional units/sectors. It is commonly denominated GNP. In contrast with GDP, GNI is not a concept of value added but a concept of income (UN, 1993, par. 7.16). UNSTAT provided the data for gross national income (series n^o 326). Using this we have calculated genuine savings for Portugal, GS3. Figure 11, shows that this measure becomes negative in 2000 and stays negative thereafter. This suggests unsustainable development from 2000 on, with a greater tendency towards unsustainability from 2002 on.

The fourth way we have calculated net savings follows Pezzey et al. (2005). We

Figure 11: Genuine savings for Portugal.



know that net savings is equal to NNP minus total consumption expenditures. Now using expression 4.39 for the NNP we obtain that $NS = I - CFC + \dot{K}^f$. That is to say, net savings equals net investment added by net foreign capital. Gross investment was taken from UNSTAT series n^o 30232, and net foreign capital from series n^o 6705. The measure of genuine savings is termed GS4 in figure 11. It can be seen that again there is evidence of non-sustainable development from 2000 on. In fact, the first two net saving estimates for Portugal are negative after 2003 (inclusive for net saving number 2), and the last two are negative after 2000. The average of this measures is taken to be a proxy for the genuine savings for Portugal.

These four measures of net savings for Portugal all depicted a clear tendency towards diminishing net savings until it reached a negative value in the end of the accounting period. This unsustainable pattern arises from the economic data, i.e., when no environmental correction have been made. We note that during the period GDP was increasing (though slowly) almost everywhere. In spite of this, there is no signs of alteration of the decreasing tendency of net savings. It would be interesting to do these calculations for previous years (the 1980s) to account for a higher GDP growth.

From the theory presented in section 3.1, we would expect that changes in green NNP should be equal to interest on genuine savings as in proposition 3. We used the interest rate of 2% following Pezzey et al. (2005) and the comparison of both aggregates is depicted in figure 10. It is evident that the value of changes in green NNP is much higher, for all years, than the value of genuine savings. The ratio of the two measures, i.e., $rGS/\dot{Y}$, is quite variable and ranges between about 0.18 and 27%. The ideal value for this ratio would be 100% according to proposition 3. Just to compare, Pezzey et al. (2005) found ratios that ranged between 6 and 70%. So our estimations have a higher mismatch.

This suggests one of three things, the rejection of the underlying theory presented in section 3.2, the rejection of the underlying model developed in section 4.7 or evidence showing that the data provided by national accountants is not adequate for these models.

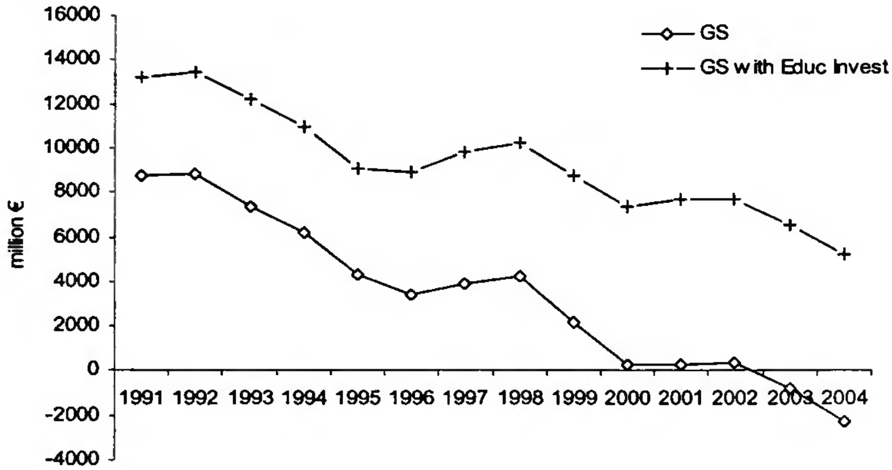
There are some other ideas to explain the mismatch. Let us see what is the effect of changing the marginal damage cost of pollution emission according to the range in table 1. Considering the low (high) estimates of marginal damage cost of pollution emissions, the ratio $rGS/\dot{Y}$ ranges between 0.2 and 13% (0.2 and 63%). As expected the mismatch is somewhat reduced when using the high estimates. However, this does not amount to much in explaining the mismatch and moreover, it is believed that the estimations of marginal damage costs are an upper bound. Changing the interest rate to be 6% we see that the mismatch ranges between 0.5 and 82%, which shows the great variability of the mismatch, and still does not explain the mismatch convincingly.

The third sensitivity test, which also increases the interest on GS relative to the change in green NNP, is to follow Hamilton and Clemens (1999, p. 346). They argue that current, ultimately arbitrary conventions in national accounting practice treat the vast majority of educational expenditure as consumption, which is better reclassified as investment in human capital. Doing this in our theoretical model is so simple that a formal treatment is unnecessary. Reclassifying items from consumption to investment increases GS, but leaves gNNP unchanged which may explain part of the mismatch.

Figure 12 shows the result of adjusting the education expenditures as investment in human capital in the calculations of genuine savings. We see that in terms of sustainability, there is no longer evidence of unsustainable behavior, though there is a clear tendency towards unsustainable development. Regarding the mismatch problem, we found that the mismatch ranges between 5 and 41%. Meaning that it increased a bit the explanation of the mismatch, but again the variability is big. We note that, particularly in Portugal, the problems of efficiency of the education expenditures may difficult the interpretation in terms of investment in human capital.

There are other explanations of the mismatch proposed by Pezzey et al. (2005) like

Figure 12: Comparison of GS with and without considering education expenditures as investment.



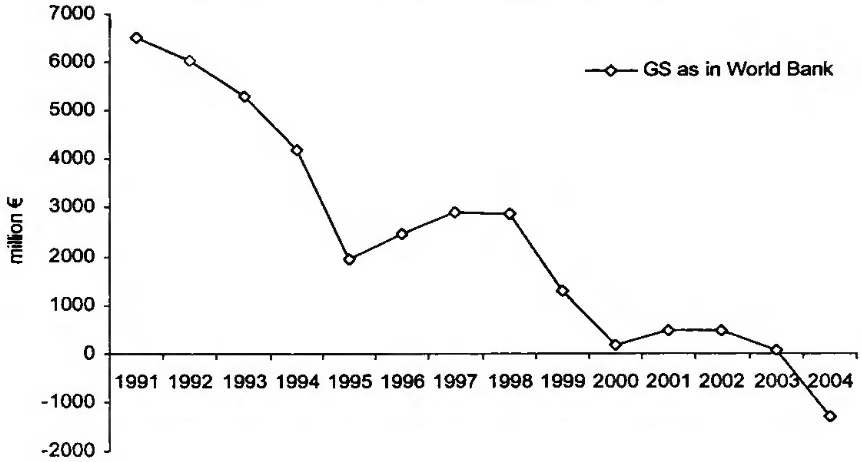
noting that the theory of section 3.2 assumes full capacity utilization at all times and thus excludes business cycles and as Asheim commented we should use the potential GDP instead of the actual GDP in these calculations. However, though the inclusions of business cycles made a change in the green NNP, to Pezzey et al. (2005), this did not convincingly explain the mismatch. The value of technical progress and changing terms of trade, using projections for total factor productivity and world prices for resources, was incorporated in Pezzey et al. (2005) analysis, but again the mismatch is not convincingly explained.

Other more fundamental questions should then be asked concerning the basic assumptions of optimal growth theory. These questions are addressed in the conclusion and future work. For now, we do not have clear information to resolve this problem, but it appears that we left out some important environmental corrections, so, the model of section 4.7 is misspecified; the data obtained is not yet adequate and consistent with the assumptions made, and finally there may be some explanation of the mismatch related to misspecifications of the utility function in section 3.2.

It is illustrative of the implications of different choices for the models in chapter 4 to estimate the genuine saving for Portugal using the approach of the World Bank in section 4.6. This amounts to estimate the following expression,

$$GS = NNP + (\mathbf{Q}^R - f_{\mathbf{R}})\dot{\mathbf{S}} - \mathbf{eE}. \tag{5.1}$$

Figure 13: GS 'as if' World Bank's calculations



We have included education expenditures as investments in human capital. From figure 13, it is clear that the GS is almost negative in 2003 and negative in 2004, providing evidence of unsustainable development then. Again there is a tendency towards unsustainability throughout the period. This is also consistent with the World Bank's calculations of adjusted net savings for Portugal.

6 Conclusions and future work

We have presented the commonly stated notion of sustainable development as a departure point. For empirical uses the Brundtland definition of sustainable development is not very precise. So, we followed the economic theory of welfare accounting, which provides powerful techniques to address the paradigms of sustainable development. A clear definition of sustainability is provided in terms of a country's aggregate welfare and a formal theory is presented to address measuring the degree of sustainability. The definition of sustainability is given in the end of section 2.3.3 and the formal definition and a test of (un)sustainable development is presented in section 3.2.2. Basically we define sustainability in terms of non-decreasing welfare. This is a weak definition of sustainability and we do not address here other approaches to sustainability, as strong sustainability. These are more related to the preservation of physical stocks of irreplaceable natural resources and the main critics to the weak economic approach is the degree of substitution between sources of welfare, namely man-made and natural capital. Neumayer (2003) and Pezzey and Toman (2002, 2005) present a thorough discussion on the assumptions behind weak economic and strong physical measures of sustainable development.

We have analyzed the SNA93 and the SEEA to sketch the way natural resources are accounted in national accounts' schemes, and found that there are considerable environmental effects and resources that are not accounted for in the National accounts. We then presented, through a formal approach, ways to propose adjustments in the national accounts' aggregates. The most widely accepted adjustment terms are the deduction of pollution costs and the depreciation (or appreciation) of natural resources (used for production).

We note that the power of the optimal growth framework is to provide a consistent way of analyzing and including the different aspects one wishes to incorporate in the national accounts. On the other hand, the generality of the theory provided here has the drawback that the way one includes terms in the models is somewhat arbitrary and dependent on the person who is developing a particular model. This means that different models provide different measures of sustainability. Notwithstanding, we believe it is always more instructive to compare these different measures based on different particular models of the multisector optimal growth theory than to compare two different 'ad-hoc' measures. This is because in discussing ad-hoc measures it becomes very difficult to discuss the inclusion of some terms in detriment of others, and we prefer to discuss this inclusion of terms in a more fundamental theoretical level. If one does not agree with one term, (s)he has to disagree with at least one fundamental assumption of the framework in question.

It should be noticed that the fundamental results from the theory of welfare or green accounting are rather recent, much theoretical work is in development. So, though it is not clear from the text, we have put a considerable effort in finding a clear way to present the fundamental theoretical results of welfare accounting in section 3.2. Usually, on the relevant literature the results and proofs are disperse, or different models are used, so slightly different results are obtained.

In a nutshell, from section 3.2 we know that there is a link between welfare and a current measure of production and savings - green NNP and genuine savings. Also, there is a connection between green NNP and genuine savings (changes in green NNP are reflected as interest on genuine savings). Further, these results are established in real prices. Finally, a definition of sustainable development is given in terms of non-decreasing welfare, which allows to obtain a one-sided sustainability test. A general test for sustainability is still yet to be found.

Concerning our definition of sustainable development we presented a framework that allowed us to estimate green NNP and genuine savings for Portugal 1992 - 2004. The terms we included in the calculations of green NNP and GS are, the disamenity of air pollution emissions to households and the depreciation of commercial forests - pine and eucalyptus. As shown in section 4.5 disamenities from air pollution emissions should deducted from usual NNP, and they should be valued at marginal abatement costs or marginal damage costs. Regarding commercial forest depreciation, section 4.3 shows that the depreciation of natural resources should be deducted from usual NNP, much in the same way as consumption of fixed capital is deducted from GDP to obtain NNP. The depreciation of commercial forests should be valued at the net price, i.e. resource price minus marginal extraction/harvest cost.

The pollution disamenity term is around 6 - 8% of NNP, and the depreciation of commercial forests of the magnitude of - 0.7% in 1991 and 0.4% in 1996. So, the environmental adjustments are of the magnitude of 6 -9% of NNP. This may seem small compared to Portuguese NNP, however, as stated before, we did not include such relevant stocks of natural capital as fish, mineral, water and soil. We believe a substantial change in the results should be evident when including those terms.

Analyzing the evolution of green NNP suggests unsustainable development for Portugal in 1992 and 2003. Moreover, analyzing the evolution of the genuine savings indicator suggests a tendency towards unsustainable development throughout the period with actual indications of unsustainability after 2002. When is, according to the theory, Portugal's welfare going to decrease is not clear, although it is clear that consistent negative genuine savings is unsustainable (Atkinson et al., 1997).

Concerning the magnitudes of the environmental corrections, the best estimation of the total cost of pollution emissions is considerably high for Portugal, mainly due to the high marginal damage costs obtained from the literature specific to Portugal's case. These marginal prices, as stated in section 5.1, should be interpreted as an upper bound or the true costs. Since the costs are taken to be constant throughout the period, in terms of dynamics, the effect captured in the data is only the quantity effect and not the price effect. It remains to be shown the effect of changing marginal damage costs. For this matter, proposition 8, shows the need to use a price deflator that incorporates these changes in prices, namely expression 4.42. Ideally we would like to have annual estimates of marginal abatement costs and marginal damage costs for each pollutant considered here. Or, at least only for, SO_2, NO_x, PM10, CO_2 and VOC, which were those with the highest costs. Note that, it is also important to provide a detailed analysis of the way the estimations are made, since the assumptions made to estimate those prices should not conflict with the assumptions of the dynamic models used.

In regard to the depletion of commercial forests, the highest value of depreciation term is estimated to be around 10% of the total value added in the forest sector in the year 1999, which is considered high in the context of the specific sector. The obtained data for marginal costs of extraction is at best a crude approximation of the true costs (or even the average cost), and this is a subject that deserves further attention in future green accounting studies in Portugal. When using average costs, with increasing marginal costs, one is obtaining a measure that is overvaluing commercial forest depreciation. So, this is consistent with the use of an upper value in the pollution correction term.

Also, using a more detailed model for the ideal age to cut, that is to say, to use a model that integrates the age differences in the different forests in Portugal could be of significative importance. Nonetheless, compared to green NNP, it is expected that this value will continue to be small. Note, however, that only commercial forests are concerned. There are other benefits from forest resources that are not considered here and that could have an importance in sectorial terms.

Calculating the genuine savings for our model in four different ways we have found consistent evidence of unsustainable development for Portugal after 2003, even when calculating genuine savings in a different setting, namely, that of the World Bank model in section 4.6. It should be stressed that for all four measures of net savings calculated, there is a clear tendency throughout the period of accounting, towards unsustainable development. The net savings are always decreasing, even when Portugal experienced (slight) GDP growth, and eventually becomes negative around year 2003. This is also visible, from the data from the World Bank estimations of the Adjusted Net Savings for Portugal. This is stating that, clearly, Portugal has economic problems in terms of maintaining its welfare at a non-decreasing level, and that the reason for this is not environmental[20]. So, as an important demonstration of the power of green accounting in providing a framework that really integrates the three dimensions of sustainability, is that the economic problems in Portugal are seriously pulling towards unsustainable development.

As showed in proposition 3 there is a result stating that changes in green (comprehensive) NNP should be equal to the interest on genuine savings. We have estimated both these aggregates and found a considerable mismatch. As shown from the sensitivity analysis, this mismatch problem is very little affected by any green adjustments to our results. Pezzey et al., (2005) suggest that including the effect of retirement on net human capital formation, or by reclassifying some parts of health spending as investment, and introduce business cycles might explain the mismatch.

We note that, after correcting for investments in human capital, in the model of section 4.7, this conclusion changes which opens the discussion of what kind of expenditures could be considered as investments and thus are giving the wrong picture in usual national accounts. This is still an open question in the literature, nonetheless it is a good example of the integration of social concerns. Further detailed analysis of what composes investment and expenditures in the Portuguese system of national accounts should be carried out.

Moreover, it should be included the value of the time change (technological progress

[20] At least considering air pollution emissions and commercial forests only.

and changing terms of trade) using the total factory productivity and projections of world prices for the resources.

However, the most interesting comment made is that of misspecified utility function. This suggests the discussion of the inclusion of status effects of the utility function and other parametrizations in terms of happiness, for instance. However, one must be cautious because looking at the theory of section 3.2, it becomes important the form of the consumption bundle since it is critical to obtain the price deflator to get real money measures with welfare significance. Thus it is also pertinent to question the role of the price deflator in different models as discussed in section 4.2.

This work should be seen as a beginning step towards a sound theoretically discussion about welfare measures in Portugal and as a continuation of the discussion of the mismatch in the relevant literature.

Even though the interest in finding an aggregate measure of the degree of sustainability of an economy may be criticized (van der Bergh, 2007), the use of the gNNP and GS should be made alongside with other indicators, such as the analysis in Hanley et al. (1999). Moreover, although not presented here, welfare accounting theory provides means and formulas to address the question of Cost Benefit Analysis. For instance, comparing two alternative policies should be done by comparing the different welfare measures the policies yield along the accounting period. For more on this, see Aronsson et al., (2004).

Two important messages from this work are:

- the message that all indicators considered in the last section showed that in fact, after 2003 there is consistent evidence of unsustainable development in Portugal.
- the consistent non verification of the first equality in proposition 3.

In any case, it remains true that if politicians and other institutional agents want to use aggregate indicators of welfare or sustainability that are soundly grounded on economic theory, then the green accounting theory presented and used here is still the best way to proceed.

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