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EQUILIBRIUM AND GROWTH REVISITED: AN EXTENSION OF THE DOSSO MODEL OF CAPITAL ACCUMULATION (*)

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1 — The original formulation of the model

The DOSSO linear programing model of capital accumulation may be stated in the format:

$$Max\ Z = k\ s(T) \tag{1.1}$$

$$s.t\, B^* \Delta s(t) \leqq s(t-1) - B^* c(t)$$

$$\Delta s(t) \geqq 0 \text{ and } t = 1, 2, \ldots, T$$

Where $s(0)$ and $c(t)$ are given, $\Delta s_i(t)$ is the output of goods from sector i used for investiment, i. e., increase the stock of capital good of type i, k is a nonnegative constant $(1 \times n)$ vector that characterizes the preference over capital goods. $s(t)$ is the $(n \times 1)$ stock vector of capital, $c(t)$ is the $(n \times 1)$ consumption sector, $B^* = B\ (I-A)^{-1}$ is the $(n \times n)$ matrix of gross capital requirements where B is the $(n \times n)$ capital coefficient matrix and A is the $(n \times n)$ domestic input-output coefficient matrix. It is assumed that A satisfies the Hawkins-Simon (1949) conditions.

The model (1.1) is a «Block triangular» programme and the dual variables in this model are interpreted as «Discounted shadow values» on the capital stocks at different moments of time (1). It should be emphasized that while the model captures some important features of the interrelationship among sectors of the economies, there are many features which have not been taken into account. For example, it does not consider depreciation of capital goods and the model is built up in terms of a closed economy. Without depreciation

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(1) In fact DOSSO drop the condition $s(t) \geqq 0$ for $t = 1, 2, \ldots, T$ in their formulation of the model in chapter 12. Once this assumption it dropped the model is isomorphic to McKensie (1963) or Tsukut's (1966) models as a special case.

we do not have deccumulation of capital and if we do not consider explicity the international trade bottlenecks would soon appear because most of the countries need to import machinery, essencial for growth. In this paper we tackle both the depreciation and international trade problems. At the same time we search for analytical results in terms of sustained balanced growth.

Although our extended DOSSO model may be considered a natural extension of a model that produces a turnpike, it seems that a turnpike convergence theorem cannot be sustained for our extension without strong assumptions that would virtually convert it into a closed model (2). We discuss the problem of steady-growth and the main difficulties we may find in similar analysis.

2 — New features of the extended DOSSO model

The rationale of the forward-looking measure of capital is provided by the notion that a larger stock of capital goods ought to make a larger contribution to future output than a smaller stock. As we know, the treatment of depreciation in input-output dynamic models is necessary in order to make them more realistic, but this brings with it some new problems. A proper treatment of fixed capital would inevitably require the assumption of joint production. Although the DOSSO model involves fixed capital, it ruled out joint production (3). So, in order to eliminate logical contradiction, we should assume that depreciation is fixed time-invariant fraction of current capital stock. As this seems to be intractable, as a proxy, we assume the time pattern of decay of each capital good takes place at constant rate in each period, whether or not the total capital is used (4).

Let R be a exogenously specified $(n \times n)$ non-negative diagonal matrix of capital depreciation rates. We also assume thate the same rates are valid for the capital accumulated during the plan horizon, if the measurement of capital stock $s(t)$ is made at the end of each period, and if $q(t)$ is a n-dimensional non-negative vector of gross investment (which cannot be used during the period of production), the availability of capital stock is given by:

$$s(t) = (I - R)\, s(t-1) + q(t) \tag{2.1}$$

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(2) The turnpike concept asserts that given a constant technology, specified consumption paths and the objective to maximize the size of a capital stock having a specified terminal structure, then the efficient growth path will run close to the von Neumann facet for most of the planning period, and the proportion of time spent in the neighbourhood of the von Neumann faced increases with the length of the plan. Morishima (1964) pointed out that this linear programming approach is not free of exceptions. These exceptions are known as «cyclic cases».

(3) If we want to introduce joint production, we need to generalize the model into the direction first investigated by von Neumman.

(4) We have to express uneasines on this matter. In fact, this approach to the depreciation may be criticized on a variety of grounds. It sidesteps some rather fundamental points in the economic theory.

Now we consider, explicitly, the foreing trade. It is clear that changes in volume of trade should be considered along with price variations. When both the imports and exports of goods are made responsive to relative prices we obtain, in general, more realistic models [5]. Nevertheless, an adequate formulation and estimation of a model involving prices mechanism is a difficult task. It involves a careful analysis of complex pattern of lags and non-linearities. Here we use a simpler approach to facilitate a balanced growth result.

We assume that $m'(t)$ is the vector of imported capital goods and $m''(t)$ imported consumption goods. Both, as well as $e(t)$, the vector of export possibilities, are $(n \times 1)$. In this version of the DOSSO model there is no import of raw material (we assume the country is self-sufficient on this matter), and we consider $m''(t)$ in each period as determined outside the model. The critical point is that now we consider the import of capital goods as essential for growth. So, we relate $m'(t)$ to the change in domestic output. This is given by a first order difference equation:

$$m'(t) = M[x(t) - x(t-1)] \qquad \textbf{(2.2)}$$

where M is a non-negative matrix of fixed coefficients that represents the amount of imported capital to support a unit of output.

It is required that the total demand for each period, in each sector, is attended by the availability of each commodity. So:

$$x(t) = (I-A)^{-1}[c(t) + e(t) - m''(t) + q(t)] \qquad \textbf{(2.3)}$$

and for simplification we call:

$$c(t) + e(t) - m''(t) = z(t) \qquad \textbf{(2.4)}$$

As in the DOSSO model:

$$Bx(t) \leqq s(t-1) \qquad \textbf{(2.5)}$$

Substituting (2.3) and (2.4) into (2.5), we have:

$$B^{*}q(t) \leqq s(t-1) - B^{*}z(t) \quad \text{for } t = 1, 2, \ldots, T \qquad \textbf{(2.6)}$$

We assume that the measurement of capital stock $s(t)$ is made at the end of each period and that imported capital goods, $m'(t)$, and domestic capital

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(5) We need to be careful on this matter. The dynamic Leontief model has some interesting properties amongst which, under certain assumptions the most relevant is the nonsubstitution theorem. Given this assumption and a perfect world market for all goods, there exists a unique comparative advantage and foreign trade leads to a dramatic simplification — only the advantageous goods will be produced and all others will be imported.

formation, $q(t)$, are not used during the period of import and production, respectively. So, the available capital stock at the end of each period is given by:

$$s(t) = (I - R)\, s(t-1) + q(t) + m'(t) \qquad \textbf{(2.7)}$$

Assuming that the initial stock of capital $s(0)$ is given and $M^* = M(I - A)^{-1}$, by substitution we obtain the objective function. The balance of payments is given by:

$$\mu e(t) - \mu m'(t) - \mu m''(t) \geqq 0 \qquad \textbf{(2.8)}$$

Where μ is a n-dimensional raw unit vector and

$$m'(t) = M^*[z(t) - z(t-1) + q(t) - q(t-1)] \qquad \textbf{(2.9)}$$

For completeness we write the new version of the open model of capital accumulation in the linear programing form:

$$\left.\begin{array}{l} Max\ Z = k \sum\limits_{1}^{T} (I-R)^{T-t}[(I + M^*)q(t) - M^* q(t-1)] \\ \text{subject to:} \\ B^* q(t) - \sum\limits_{\tau=1}^{t-1} (I-R)^{t-\tau-1}[(I - M^*)q(\tau) - M^* q(\tau-1)] \leqq \\ (I-R)^{t-1}s(0) + \sum\limits_{\tau=1}^{t-1} (I-R)^{t-\tau} M^*[z(\tau-1)] - B^* z(t) \\ \mu M^*[q(t) - q(t-1)] \leqq -\mu e(t) - \mu M^*[z(t) - z(t-1)] - \mu m''(t) \end{array}\right\} \textbf{(2.10)}$$

where $q(t) \geqq 0$ and $t = 1, 2, \ldots, T$.

3 — Balanced growth

We think the model presented has some value in giving insights into the dependence on imported capital, but this requires numerical computation and in few countries it is possible to find the matrix M of fixed imported capital per unit of output. So, we do not think that the approach is of much practical use, but this is an empirical problem and has nothing to do with the rather theoretical points that we would like to stress. In fact we intend to show that a balanced growth path exists on certain assumptions, but the assumptions are such as virtually to eliminate all the important features of an open economy [6].

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(6) The search for a turnpike is of relevance for if the optimal path of capital accumulation is of the turnpike form, we can be reasonably sure that some strategy of balanced growth will be satisfactory in a variety of situations. Nonetheless, this extended DOSSO model does not belong to the class of closed linear sistems, so it is rather doubtful that a turnpike, in the exact sence of the word, may be defined.

In order to carry out the study of balanced growth, we need to drop the balance of payments constraint and this, no doubt, is a fundamental simplification that seems to be disastrous. The implication is that the economy is able to import all the necessary capital goods without any concern with export earnings (7).

It is also necessary to treat all the relevant constraints as equalities and to make the assumption that all relevant matrices are indecomposable.

So, if equalities are to hold instead of (2.6) we have:

$$B^* [q(t) + z(t)] = s(t+1) \quad \textbf{(3.1)}$$

Taking the equation (2.9) and assuming that steady-state at the rate α is sustainable we have:

$$m'(t) = \frac{\alpha}{1+\alpha} M^* [q(t) + z(t)] \quad \textbf{(3.2)}$$

Now we assume that $z(t)$ is a linear function of outputs. Thus, we define N to be a matrix of propensities to consume (including exports). So:

$$z(t) = N[q(t) + z(t)] \quad \textbf{(3.3)}$$

$$\therefore\ z(t) = \bar{N} q(t) \quad \textbf{(3.4)}$$

where:

$$\bar{N} = (I - N)^{-1} N \quad \textbf{(3.5)}$$

Now let us forward-lag (3.1) and to substitute in (3.6), then:

$$B^* [q(t+1) + z(t+1)] = (I - R) s(t-1) + q(t) + m'(t) \quad \textbf{(3.6)}$$

Substituting (3.1) and (3.2) into (3.6), we have:

$$B^* [q(t+1) + z(t+1)] = (I - R) B^* [q(t) + z(t)] + + q(t) + \frac{\alpha}{1+\alpha} M^* [q(t) + z(t)] \quad \textbf{(3.7)}$$

In steady-state at the rate α,

$$B^* [q(t+1) + z(t+1)] = (1 + \alpha) B^* [q(t) + z(t)] \quad \textbf{(3.8)}$$

(7) We could consider the possibility to obtain the required imported capital goods financed by aid, loans, direct foreign investment, etc. Here we do not discuss such possibility that would lead us far from the theorectical results we deal with.

and substituting (3.8) into (3.7):

$$(1+\alpha)B^*[q(t)+z(t)] = (I-R)B^*[q(t)+z(t)] + \\ + q(t) + \frac{\alpha}{1+\alpha}[q(t)+z(t)] \qquad \textbf{(3.9)}$$

Substituting (3.4) and (3.5) into (3.9), after the necessary transformation we have:

$$[I - RB^*(I+\bar{N}) - (B^* + \bar{N} - \frac{1}{1+\alpha}M^* - \frac{1}{1+\alpha}M^*\bar{N})]q(t) = 0 \qquad \textbf{(3.10)}$$

but

$$(B^* + B^*\bar{N} - \frac{1}{1+\alpha}M^* - \frac{1}{1+\alpha}M^*\bar{N}) = (B^* - \frac{1}{1+\alpha}M^*)(1+\bar{N}) \qquad \textbf{(3.11)}$$

therefore

$$\{[I - RB^*(I+N)] - \alpha(B^* - \frac{1}{1+\alpha}M^*)(I+\bar{N})\}q(t) = 0 \qquad \textbf{(3.12)}$$

Now we show that steady growth at the rate is possible. Theorem: $\exists\ \alpha > 0$, $q > 0$ such that (3.12) is true.

Proof:

a) We assume that some growth is possible so that when $\alpha = 0\ \exists\ q > 0$ such that $[I - RB^*(I+\bar{N})]\ q > 0$, so that $RB^*(I+\bar{N})$ satisfies the Hawkins-Simon condition, and therefore:

$$[I - RB^*(I+\bar{N})]^{-1} > 0 \qquad \textbf{(3.13)}$$

b) Premultiplying (4.12) by $[I - RB^*(I+\bar{N})]^{-1}$, we have:

$$\{I + \alpha[I - RB^*(I+\bar{N})]^{-1}(B^* - \frac{1}{1+\alpha}M^*)(I+\bar{N})\}q = 0 \qquad \textbf{(3.14)}$$

Let $[I - RB^*(I+N)]^{-1}(B^* - \frac{1}{1+\alpha}M^*)(I+\bar{N}) = H(\alpha)$ **(3.15)**

so that we seek $\alpha > 0$, $q > 0$ such that:

$$[I - \alpha H(\alpha)]q = 0 \qquad \textbf{(3.16)}$$

We shall find $\alpha_1 > 0$, $\alpha_2 > \theta$, $q > 0$ such that:

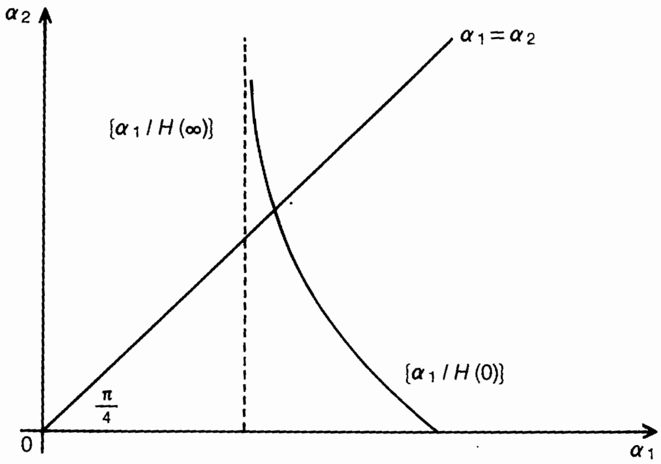
$$[I - \alpha_1 H(\alpha_2)]\ q = 0 \quad \text{and } \alpha_1 = \alpha_2 \qquad \textbf{(3.17)}$$

which clearly satisfies (3.16). It happens that (3.17) can be written as:

$$[(1/\alpha_1)I - H(\alpha_2)]\ q = 0 \qquad \textbf{(3.18)}$$

Where $H(\alpha_2)$ is an increasing function of α_2 and $(1/\alpha_1)$ is the Frobenius root of $H(\alpha_2)$. Therefore, by the Frobenius theorem $(1/\alpha_1)$ increases as α_2 increases (8). Then, we are able to draw figure 1 as follows:

FIGURE 1



Formally, we have, by the Frobenius theorem:

$$\exists\, \alpha_1 > 0 \text{ and } q > 0 \text{ such that } [(1/\alpha_1) I - H(\alpha_2)] = 0 \qquad \textbf{(3.19)}$$

Now, when $\alpha_2 = 0$, we have $\alpha_1 > 0$; when $\alpha_2 = \infty$, we have α_1 finite and positive, and as α_2 changes, α_1 varies continously. Thus, $\exists\, \alpha_1 = \alpha_2 = \alpha > 0$, $q > 0$, as in figure 1, such that $[I - \alpha_1 H(\alpha_2)]\, q = 0$. The value α that satisfies the equation (3.16) is unique since as α_2 increases α_1 decreases. Q. E. D.

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(8) The relevant part of the Frobenius theorem states: if P and Q are both semipositive indecomposable and of the same order then $P >> Q$ implies $\lambda^* P > \lambda^* Q$. This theorem is relaxed in Takayama (1974): if $P \geqq Q \geqq 0$, in fact it sufficies to assume that P is indecomposable.