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MASTER´S FINAL WORK
INTERNSHIP REPORT

CALCULATIONS OF THE VALUE AT RISK FOR A DIVERSIFIED PORTFOLIO

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RESUMO

O presente relatório apresenta o trabalho realizado num estágio curricular com duração de cinco meses e meio na seguradora Fidelidade, na equipa de *Asset and Liability Management*, pertencente à Direção de Gestão de Risco. O principal objetivo do estágio foi desenvolver um modelo em *R* que permitisse calcular o Value-at-Risk (VaR) da carteira da Fidelidade de forma eficiente, dada a complexidade de cálculo necessária para uma carteira diversificada.

Para melhorar as metodologias de gestão de risco da empresa, o processo de desenvolvimento do modelo em *R* iniciou-se com a pesquisa das principais metodologias de cálculo do VaR. O método escolhido, dadas as características da carteira da Fidelidade, foi o método da simulação de Monte Carlo. Uma vez que os instrumentos de renda fixa têm características específicas, utilizamos uma metodologia diferente das metodologias de "VaR mapping" (também analisadas), chamada de "New approach", para que o cálculo do VaR pudesse ser similar ao realizado noutras classes de ativos. Devido a restrições de tempo, as classes de ativos como derivados, participações e investimentos estratégicos não foram incluídos na análise da carteira.

Utilizando dados históricos de um ano, com data de referência de 31 de dezembro de 2021, desenvolvemos o modelo em *RStudio*. O modelo foi testado inicialmente por classe de ativo e, posteriormente, aplicado à totalidade da carteira. Para validar os resultados utilizamos a ferramenta "PORT" da plataforma *Bloomberg* que produziu resultados próximos aos nossos, validando até certo ponto a *performance* do modelo criado.

Palavras-chave: Value-at-Risk; R; Simulação de Monte Carlo; Gestão de Risco.

ABSTRACT

This report presents the work produced during a five-and-a-half-months curricular internship at the insurance company Fidelidade, in the Asset and Liability Management team, which is part of the Risk Management Department. The main goal of the internship was to develop an *R* model that would allow the Value-at-Risk (VaR) of Fidelidade's portfolio to be calculated efficiently, given the complexity of the calculation required for a diversified portfolio.

In order to improve the company's risk management methodologies, the process of developing the *R* model began with research into the main methodologies for calculating the VaR. Given the characteristics of Fidelidade's portfolio, the Monte Carlo simulation method was chosen. Since fixed income instruments have specific characteristics, we used a methodology different from the "VaR mapping" approaches (also analysed), called the "New approach", so that the VaR calculation could be similar to that applied to other asset classes. Due to time constraints, asset classes such as derivatives, participations and strategic investments were not included in the portfolio analysis.

We developed the model in *RStudio*, using one year of historical data with a reference date of 31st of December 2021. The model was first tested by asset class and then applied to the entire portfolio. To validate the results, we used the "PORT" tool on the *Bloomberg* platform, which produced results close to our own, thus validating the performance of the model created to a certain extent.

Key Words: Value-at-Risk; R; Monte Carlo Simulation; Risk Management.

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1. INTRODUCTION

The master's program in Mathematical Finance considers several topics that can be applied in many areas of financial institutions, one of them being risk management, which was my main interest since the beginning of my academic journey. Courses such as Financial Markets and Investments, Foundations of Financial Theory, Interest Rate and Credit Risk Models, as well as Programming Techniques, were important to prepare me to the project related to the internship that I proposed myself to in Fidelidade – Companhia de Seguros, S.A., in the Department of Risk Management (DGR) – Asset and Liability Management (ALM) team. The curricular internship had the duration of five and a half months.

1.1.Description of the organization

Fidelidade is a Portuguese insurance company, founded in Lisbon in 1808, being the third oldest insurance company in the world, and the oldest in Portugal. Being part of Fosun International Limited, this insurance company leads the insurance market in Portugal in life and non-life, having as values the experience, protection, credibility, innovation, competitivity, efficiency, leadership, loyalty, trust and stability. The company is also present in other countries such as Spain, France, Luxembourg, Cape Verde, Angola, Mozambique, Macao, Liechtenstein, Peru, Bolivia, Paraguay and Chile.

The team that I was part of for the internship was based at the working office located in the head office of the company, at Largo do Calhariz, 30, P-1249-001 Lisbon, Portugal.

The actions of the DGR department must comply with the Solvency II regime, using the supplementing Directive 2009/138/EC of the European Parliament and the Council, applied since 1st of January 2016. An example of the DGR responsibilities is the computation of the Solvency Capital Requirement (SCR), Pillar 1 of Solvency II, a challenging task requiring sometimes nontrivial mathematics, applied in all the teams, because of all the risks that are assessed (e.g., market risks, default risk, life and non-life business risks, operational risk).

In the team of ALM, I could learn from all its members, and could meet the supervisors of the project – Tiago Boura and Daniela Matos – everyday, contacting them every time I needed help or to clarify any doubts, even though some days of the week were remote work model, as the company adopted an hybrid model. The responsible for the team – Hugo Duarte – was also available every time I needed and would be aware of every part of the process and all the updates, every week, at the team's weekly meetings.

1.2.Description of the project

The present report is based on the study and application of the Value-at-Risk (VaR) to a diversified portfolio. The main goal of the project was to improve the risk methodologies used in the company. Since VaR was not being applied to the whole portfolio of Fidelidade, it would be important to develop a system of using it in a more widespread way.

Combining the knowledge acquired in Programming Techniques with the rest of the courses and using information tools such as Danielsson, J. (2011) and Tsay, R. S. (2013), it would be possible to create a model in *RStudio*, adapted to Fidelidade's portfolio, in order to calculate VaR in an easier and quicker way, since there are several methods that can be applied, but some of them are quite expensive and difficult to put into practice. Therefore, with the intention of applying VaR to the whole portfolio, some of the methodologies, such as the ones enumerated below, were studied and tested – first by asset class, and at the end on the whole portfolio, following the internship plan and its timeline made by the supervisors of the project (see the Appendix).

Primarily, the three main approaches usually used to apply VaR – Variance-Covariance method; Historical method and Monte Carlo simulation method – were considered alongside with their pros and cons. After all the research, especially from the explanations of the various VaR methods in Jorion (2006), the conclusion that was made - considering the capacity of incorporating more complex assets or instruments, which is important in the design of a model for a diversified portfolio, and the capacity of simulating extreme and unexpected events - was that the Monte Carlo simulation seems to be the most appropriate model, although it had the disadvantage of being more complex and more expensive to apply to large portfolios than the other methods. Having that into account, without other main disadvantages, this was the method chosen to apply to the Fidelidade's portfolio. Since the main goal was to calculate VaR for the whole portfolio, it is difficult to apply it to fixed income instruments with traditional approaches, because, as explained in Section 2.2.4, fixed income instruments have some characteristics such as, for instance, coupon rates or maturity dates that make the construction of a model harder than using, for example, just historical prices. VaR mapping techniques (Jorion, 2006), in its three approaches – Maturity Mapping, Cashflow Mapping and Duration Mapping – were considered as one option to overcoming this difficulty, but it would be interesting and easier to use one approach that would simplify the calculation of VaR and would be similar to the ones used for the other asset classes. The methodology used was called as the "New Approach" in which the historical prices and returns of bonds are adjusted, to be used directly in the VaR computation.

The project was in its entire time supported by *Bloomberg*, whether for collecting data or to compare the results of the methodology used to compute the VaR with the ones obtained by this platform.

1.3.Description of the activities

After defining the topic in which I would be working on at Fidelidade, my supervisors at the company worked on an internship plan with the timeline (see the Appendix), in which the main activities would be the following:

1. Research about VaR: Analyze the different kinds of methods, parametric and non-parametric:

    a. Variance-covariance method;

    b. Monte Carlo method;

    c. Historical method.

   Definition of the methodology to be applied;

2. Computer implementation of the methodology – Application of the methodology by asset class;

3. Study of the correlations and definition of correlation and covariances matrices;

4. VaR application to the whole portfolio;

5. Analysis of the results and its impacts;

6. Review of the project:

    - Review of the programming code;

- Review of the results.

Following the guidelines, as referred previously, the starting point was the research to select the methodology of VaR that would be applied, and then the process to apply it, finishing in the analysis of the results. Subsequently, the next chapters of the report are an image of the process that was designed in the plan.

2. THEORY AND METHODOLOGY

2.1. Value-at-Risk

In order to apply the Value-at-Risk, it is important to define it, as well as describing the methodologies that can be used in its implementation.

VaR[1] is a risk measure which outputs the expected loss, over a given time horizon (denoted as h), with a given level of confidence (defined by α), that a portfolio or an investment can have. By what is written on page 17 of Jorion (2006) "VaR describes the quantile of the projected distribution of gains and losses over the target horizon". Defining α as the confidence level, VaR is the 1- α lower tail level.

Let L be the loss (measured as a positive number, by convention); then, VaR can be defined as the smallest loss, in absolute value (as it is a measure of risk, which is always positive), such that

$$P(L > VaR) \leq 1 - \alpha. \qquad (1)$$

The confidence level or the significance level is usually defined by an entity. The choice and determination of the confidence level, according to Dowd (2005), is related to the purpose of the risk measure, being that the reason why it is different, depending on the entity. The same happens with the holding period, it depends on the way each institution operates. Dowd (2005) also says that the usual holding periods are one-day or one-month, but there are institutions that operate in periods of a quarter. It depends on some factors as, for instance, the liquidity of the markets where the institution is inserted in – it would be "the length of time it takes to ensure orderly liquidation of positions in that market". In what concerns the confidence level, Dowd (2005) defends that it would be preferable to use low confidence levels, to obtain a better excess loss proportion. Other purposes, like comparison with other companies, can make the choice of confidence level change.

VaR is used more often by banks, and the choice of the parameters above is defined in banking regulation, based on Basel IV. In the case of this project, since it is an insurance company – that follows Solvency II - a longer time horizon is considered because of the type of business and products; therefore, we assumed a time horizon of one-year, at a 99.5% level of confidence, like the one used in the standard formula of Solvency II

[1] Popularized in the early days of RiskMetrics – see Morgan, J., Reuters, & Limited, R. (1996).

implemented by European supervision – European Insurance and Occupational Pensions Authority (EIOPA). In the Directive 138/2009/EC, it is said that the Solvency Capital Requirement should be calculated having into account the obligations of the subsequent 12 months, to accomplish them with a probability of 99.5%, as can be seen in point 3 of article 101 of Section 4, Chapter VI: "It shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5 % over a one-year period.".

2.2.Value-at-Risk Methods

VaR has many different applications in order to face the different characteristics of portfolios or financial instruments. It can be computed analytically by making assumptions about return distributions for market risks, and by using the variances and covariances across these risks, or it can also be estimated by running hypothetical portfolios through historical data or from Monte Carlo simulations - for example, see Alexander C. (2008) and Manganelli and Engle (2001) for additional details on VaR models.

With that being said, there are three main approaches that have to be considered: (i) the variance-covariance approach, also called delta-normal or linear VaR, that is a parametric method; (ii) the Historical method, that is a non-parametric method; (iii) and the Monte Carlo simulation method, that is a semi-parametric approach (Abad et al., 2014).

2.2.1. Linear/Variance-Covariance Method

The parametric method of VaR (naturally) requires parameters, such as the standard deviation and the mean of the returns of assets. It assumes that those returns of assets are normally distributed, the correlations between risk factors are constant and the delta (or price sensitivity to changes in a risk factor) of each portfolio is also constant. This method involves using the variance-covariance matrix of asset returns to calculate VaR through matrix multiplication, making it computationally efficient and straightforward to implement (Hull, 2012).

To apply the parametric VaR method, the volatility of each risk factor is extracted from the historical observation period. Estimating the volatilities of the risk factors requires using historical data of investment returns. The requirement of collecting historical data to calculate VaR, as referred, applies not only to this method but also to other VaR methods, and this is the reason why some authors do not agree with the term "historical method" for the method presented in the next section, since historical data is also required for other methods (Jorion, 2006). However, extracting volatilities from historical data is an important part of the parametric VaR calculation. Worked out from the component's delta, with respect to a particular risk factor and that risk factor's volatility, is then determined the potential effect of each component of the portfolio on the overall portfolio value. This assessment allows to understand the contribution of each component to the overall risk profile of the portfolio.

Besides the already referred advantage of the straightforward nature of the method (because of the simple computation using matrices multiplication), which is useful in situations where quick risk assessments are required, see Dowd, K. (2002), there is also another important benefit from this approach, as it is suitable for diversified portfolios, with different asset classes, consisting of positions in different investment options.

Despite its extensive use, this method has some limitations such as the assumptions of a normal distribution and constant correlations. Many variables in financial markets have distributions that are different from the normal distribution. As a result, this assumption may lead to inaccurate risk estimates, especially during extreme market conditions or events characterized by non-normal returns, including heavy tails and skewness. Another characteristic of this method is that, in fact, although we can apply the linear approach to a portfolio with no derivatives, consisting of positions in stocks, bonds, foreign exchange, and commodities - in which case the change in the value of the portfolio is linearly dependent on the percentage changes in the prices of the assets comprising the portfolio - in the case of nonlinear instruments, such as options, this method would be inappropriate.

2.2.2. Historical Method

The Historical Simulation method uses historical data of portfolio returns assuming that all possible future variations on data have been experienced in the past and the historically simulated distribution is identical to the returns distribution over the forward looking risk horizon.

The process of calculating VaR with the Historical Method is described in Choudhry, M. and Ketul, T. (2006, pp.36-37) as initially collecting data from the past, “for instance, if a 1-day VaR is required using the past 100 trading days, each of the market factors will have a vector of observed changes that will be made up of the 99 changes in value of the market factor. A vector of alternative values is created for each of the market factors by adding the current value of the market factor to each of the values in the vector of observed changes. The portfolio value is calculated using the current and alternative values for the market factors. The changes in portfolio value between the current value and the alternative values are then calculated. The final step is to sort the changes in the portfolio value from the lowest to the highest value and determine VaR based on the desired confidence interval. For a 1-day 95% confidence level VaR using the past 100 trading days, the VaR would be the 95th most adverse change in the portfolio value.”

Being nonparametric, it is a method that makes no assumptions about the distribution of the risk factors and does not model volatility. With this method, the idea is to use the same distribution observed in the past, since it uses a sample historical data to estimate that distribution.

The problem behind using past information, and the main disadvantage of this approach, is that it may not be true that the future will resemble the past. There are events that occurred in the past that will not be considered, and if there are rare events that can happen in the future, it will not take them into account as well. If there are changes in the market structure or shifts in regimes, for instance, trusting merely on historical data for risk estimation may introduce potential imprecisions and fail to capture emerging risks and unforeseen events that may impact the portfolio we are dealing with.

The simplicity of this method is its main advantage, since it uses directly the observed historical data, making no assumptions on distributions or correlations between assets. As said by Jorion (2006), the method is intuitive, as, to explain the backgrounds of the VaR measure, it is just required to go back in time and understand what occurred.

Being aware of the limitations, a third way to calculating the portfolio risk is to use historical simulation VaR alongside other risk measurement techniques - for instance, see C. Hull (2002) for further details.

2.2.3. *Monte Carlo Simulation Method*

The Monte Carlo simulation in the context of VaR estimation generates a large number of simulated future scenarios based on probability distributions of key risk factors and then it involves evaluating the resulting distribution of the portfolio values to determine VaR. By simulating a large number of scenarios, VaR can be estimated as the appropriate quantile of the distribution of the simulated portfolio values.

The process of calculating VaR with Monte Carlo Method is described in Choudhry, M. and Ketul, T. (2006) as, firstly, the creation of a vector with alternative values for each market factor, by adding its current value to each of the values in the vector of simulated changes, which represent the expected fluctuations or random variations of that market factor. After that, the portfolio value is calculated using both the current values and the alternative values of the market factors for a better understanding of how the portfolio could change under different market conditions: "Once this vector of alternative values of the market factors is obtained, the current and alternative values for the portfolio, the changes in portfolio value and the VaR are calculated exactly as in the historical method" (see Choudhry, M. and Ketul, T. (2006), page 37).

This method, also known as stochastic simulation, is said by Jorion (2006) to be a parametric method, but it is different from the well-known "parametric method" which is the variance-covariance. If we want to compare the Monte Carlo with another method, we can do it by comparing it with the Historical approach, except that, in the Monte Carlo method, the hypothetical changes in prices are created by random draws from a prespecified stochastic process instead of sampled from historical data. The main difference between these two approaches is that Monte Carlo allows to choose a statistical distribution that is believed to approximate the changes; it allows the risk manager to use the historical distributions if it is not an option to assume normal returns. This method is more likely to estimate VaR accurately because it is more realistic.

As shown in Glasserman (2004), one of the advantages of this approach is the capacity of incorporating complex risk factors and dependencies as well as being able to include portfolios with nonlinear instruments, complex derivatives or non-traditional assets. Another advantage is related to the fact that Monte Carlo simulation is more realistic, since it can incorporate future market scenarios or capture extreme events, considering the impact of changings in market conditions.

The main limitation of this approach is the computational complexity and resource requirements, which make the method time consuming and expensive, especially when calculating the VaR for large portfolios with a large number of risk factors, since it requires generating a large number of scenarios. Other disadvantage of the Monte Carlo simulation is the necessity to estimate probability distributions for the risk factors, which may be complex in some cases and lead to inaccurate risk estimates.

2.2.4. Approach for Fixed Income Instruments

Fixed Income instruments are really significant in investment portfolios, particularly insurance's investment portfolios, so it is important to estimate their risk in an effective way. The computation of the VaR for fixed income using the traditional approaches presented above, is particularly challenging due to the high level of complexity related to the diversity of risk factors that affect the price of these instruments. Consequently, calculating returns in this case requires considerably more effort compared to equity. For instance, the historical returns of bonds cannot be applied to compute VaR, as they are for stocks, because in equity the observed prices can be directly used to calculate those returns, but bonds characteristics make it difficult to implement the three VaR methods presented previously. In fact, bonds usually pay coupons (except if they are zero-coupon bonds), have a time to maturity that changes every day, and cash flows until the maturity date (Vlaar, 2000). As mentioned in Darbha (2001), the estimation of VaR for these instruments is complicated by two reasons: (i) the changes in market values of the securities are non-linearly related to changes in spot interest rates, which complicates the making of assumptions about the distribution of the returns; (ii) and the cash-flow interpolations may also cause imprecise approximations.

There are three VaR mapping approaches that can be used as an alternative solution to simplify the computation of VaR for fixed income. These are the Maturity mapping, the Duration mapping and the Cash-Flow mapping. The process is made by the replacement of the current values of the portfolio by exposures on the risk factors (Jorion, 2006).

Maturity mapping considers time as the risk of the bond, i.e., it associates the risk of the bonds only with the maturity of the principal payment. This method is quite straightforward because it assumes that the closer the maturity date, the lower the risk will be, as the probability of receiving the principal payment increases. The main issue with this method is that, by focusing on the time dimension, it does not consider other risks that may affect the value of the security, such as changes in interest rates, for example.

Duration mapping connects this risk of a bond portfolio with the risk of a zero-coupon bond. To compute VaR with this methodology, the risk of the portfolio is associated to the risk of a theoretical zero-cupon bond that has a maturity equal to the duration of the portfolio. Therefore, the zero-coupon bond will be impacted by the possible changes in interest rates, as they impact the portfolio. The estimation of VaR is then made by calculating the modified duration of the portfolio and making an assumption on the distribution of interest rate variations. Limitations of this approach are associated to the possibility of working with non-linear relationships between bond prices and interest rates or, in the case of dealing with bonds with embedded options, because it may lead to the need of using alternative or additional methods to compute risk for fixed income with a better technique.

The Cash-Flow mapping is the more complex of these three approaches because it decomposes the bonds into the risk of each cash-flow. Each cash-flow "is represented by the present value of the cash payment, discounted at the appropriate zero-coupon rate" (see Jorion, (2006), page 284). By valuing each cash-flow separately, this method of VaR mapping captures the specific risk associated with each payment in the portfolio, which makes the method the most precise one. However, it is computationally complex because instead of considering only one risk factor as the other mapping methodologies – the principal in the case of the Maturity mapping or the duration in the case of the Duration mapping – it considers several risk factors, being the risk of a bond calculated by the risk of each of its cash-flows.

Merging various mapping procedures would be a good approach to apply to fixed income instruments. The problem with this, when we are trying to calculate the VaR for a diversified portfolio (which is the case of this project), is that it requires the fixed income instruments to be treated in a very different way, when comparing with other asset classes. By this, the best way to calculate VaR of a diversified portfolio would be treating this kind of securities in a way similar to equity.

A methodology that would simplify the calculation of VaR when considered alongside with other asset classes, would be the "New Approach" proposed in Esqu and Gaspar (2014). It is a methodology that adjusts bonds historical prices and extracts from them an adjusted history of returns and this makes it possible to calculate VaR of bonds without using yields. With this method the historical prices of bonds can be used directly in the VaR computation. The returns are adjusted (Adjusted Historical Returns – AHR) as follows

$$AHR(n, N, n_{VaR}) = \frac{\left(\frac{P}{p(n-N,T)}\right)^{\frac{T-n_{VaR}}{T-(n-N)}}}{\left(\frac{P}{p(n,T)}\right)^{\frac{T-(n_{VaR}+N)}{T-n}}}, n = N+1, \dots, n_{VaR}, \qquad (2)$$

where n is the day of the historical return, N the time horizon, n_{VaR} the day of the VaR computation, P the principal, $p(n,T)$ the historical prices, and T the maturity.

The proposal in this new approach is, as referred, to calculate VaR for each bond as it would be calculated in the traditional methods, using the historical returns. To obtain these returns, the past prices are adjusted for the "pull-to-par" effect, calculating pulled prices. The "pull-to-par" effect is the tendency of bonds to move or "to be pulled" to their par (face) value as getting nearer to their maturity date. The pulled prices calculated in the referred article tend to correct that "pull-to-par" effect of bonds and pull the historical prices to the maturities which are significant for the computation of the VaR. By doing so, the bond's price trajectory is modified incorporating the "pull-to-par" effect early on the historical prices. The pulled prices are compared with historical prices in the graph below, presented in Esqu and Gaspar (2014):

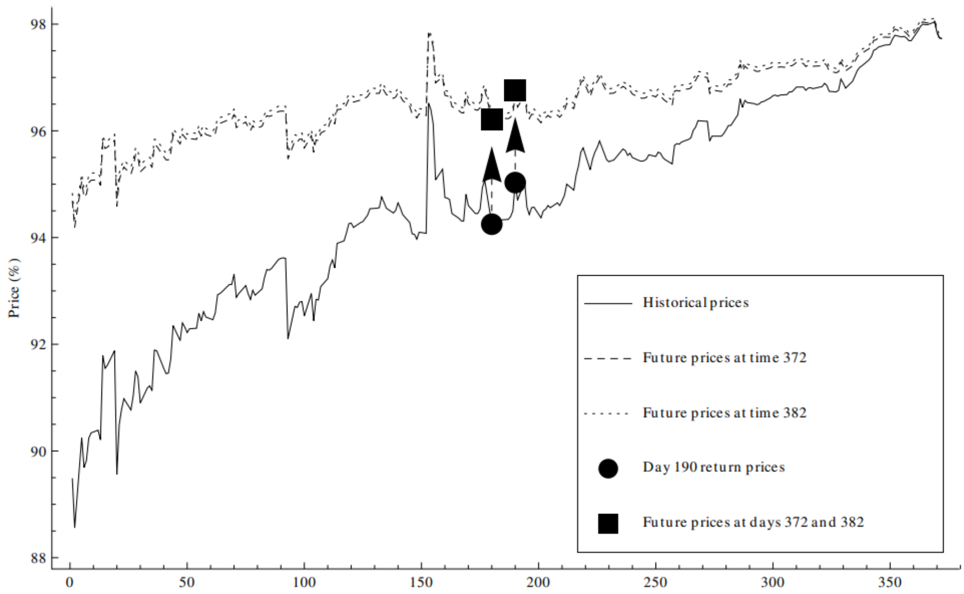

Figure 1 – Representation of pulled prices compared to historical prices

Source: Esqu and Gaspar (2014)

In this way, the returns that are going to be obtained (AHR) will incorporate the adjustment mechanism of the prices considering the bond's characteristics.

2.3.Comparing the Value-at-Risk Methods

After the investigation of the referred methodologies, the choice of the methodology to use was made considering the characteristics of the diversified portfolio in analysis.

In view of all the methodologies presented before, the one which appeared to be the most suitable model to apply in the case of this project was Monte Carlo Simulation method. Despite its questionable efficiency when applied to large portfolios, because the computation is slower and more expensive than for the other methods, this is the most realistic methodology, comparing to the remaining options. Monte Carlo method allows the incorporation of various risk factors, as well as extreme events that cannot be predicted when using other methods based only on historical data. Consequently, this was the chosen method to implement on the calculation of the VaR of Fidelidade's portfolio.

For the Fixed Income instruments, the mapping approaches were tested alongside with the "New approach", which became the selected method as the results were identical to the results on mapping approaches and, moreover, it had the advantages presented previously, namely its practicality among other asset classes, since it can be worked in a similar way of equity.

Subsequently, after the choice of the methodology was made, the data was collected so that the model of VaR could be created and applied.

3. Implementation and Results

The ALM team is responsible for managing the risks that can arise from mismatches between the Fidelidade Group's assets and liabilities, in particular the interest rate, currency and liquidity risks. The ALM team is also responsible for ensuring the alignment between the duration and liquidity of assets and liabilities, optimizing the allocation of assets and liabilities, and ensuring the company's long-term financial stability and profitability. These strategies are based on different risk metrics.

The VaR is one tool which supports the risk management, helping to define strategies related to Assets and Liabilities to better manage the risks. This is important on a team that is part of the decision-making process of the Company and performs mitigation exercises.

This chapter explains and details the application of VaR within the scope of the ALM team. The implementation of Monte Carlo VaR was performed in *RStudio* and the distribution used in the code was the multivariate normal distribution with *mvrnorm()* function with 10000 simulations. As already said, 99.5% is the confidence level appropriate to insurance companies, so it was the one applied. The calculation of VaR was made for a one-day period and then multiplied by the square root of the days to obtain the 99.5% VaR for other periods, as one-month (20 days) or one-year (252 days[2]). First, to accommodate the specifications of each one, the implementation was made by asset class. After having the VaR calculated for every asset class, the code was extended and adjusted to include the entire portfolio.

At the end of this chapter the results obtained by the application of the model are presented and analyzed.

3.1. Data

The first step of the creation and implementation of the model was the analysis of the portfolio on the date of 31st of December of 2021, since the risk appetite of the company is reflected in the portfolio's composition. The Fidelidade's portfolio was composed of assets which covered a set of liabilities. In other words, the assets in the portfolio are

[2] This conversion to days refers to trading days since some stock markets are closed on weekends and holidays.

chosen with the goal of meeting up the expenses and obligations of an insurance company of this size. This composition was complex and related to the Lines of Business (LoB) of the company, such as Life and Non-life.

31st of December 2021[3] was the reference date, not only for the VaR calculation, but also as the reference date of historical data. This means that all the historical prices that were used to compute VaR, were prices from one-year interval (31st December 2020 until 31st December 2021). The historical data was mainly gathered from *Bloomberg* but there were some prices which were not available at *Bloomberg*, such as the non-listed assets, that were collected from *Binfólio*, Fidelidade's financial asset management platform.

To collect data from *Bloomberg* we used the field "PX_LAST", which is the closing price of a security requiring the International Securities Identification Number (ISIN) of the security and the particular day on which the price is required. All of this information was gathered using the *Bloomberg* add-in *@bdh()* formula in *Excel*.

Fidelidade's portfolio on 31st of December 2021 had a composition of Equity, Real Estate, Fixed Income, Cash, Strategic Investments, Derivatives and Participations[4]. As an assumption - mainly due to lack of time to analyze these asset classes – the Participations were excluded from the analysis, as well as the Strategic Investments and the Derivatives, because of their complexity, as shown for instance in Duffie and Pan (1997). Therefore, having a portfolio with this composition to work with, it was necessary to address the various particularities that some of these asset classes had, as explained next.

In what concerns the Equity asset class, some of the equities of the portfolio are unlisted, which means that it was not possible to find the corresponding data. In such cases, as an alternative, we used adequate benchmarks, like indexes. For some of them, the indexes were applied by country or, in the cases of Venture Capital Funds, Private Equity, Venture Capitals and Infrastructure Funds, a different index was used – the one that would be more appropriate for the portfolio. The indexes used as benchmarks, in this and other asset classes, which are being referred in this report, are due to an internal study that was made at Fidelidade, which allowed using benchmarks for all asset classes.

[3] It may seem like an old date, given the date of this report, but since the internship occurred in 2022, this is the date that corresponds to the closing of the previous year at the time.

[4] For confidentiality reasons, it is not possible to reveal the specific assets and the respective market values.

Therefore, it is important to clarify that in Section 3.2.2. the results of VaR of the benchmarks will be shown, meaning that all the asset classes in analysis have a benchmark, as referred previously. Having access to the use of benchmarks for all asset classes from the internal study does not mean that results of the VaR of benchmarks were used for all asset classes; in some cases, those results were used just in terms of comparison, as was the case of Equity and Fixed Income. Since this internal study was available, we could compute the VaR from benchmarks simultaneously with the calculation of the VaR with real data from the actual assets. Consequently, we had a way to compare our results.

On the Fixed Income asset class, there were some bonds with missing data, and alternative bonds had to be chosen as benchmarks. The choices were made by assuming the same country risk or similar characteristics such as same or alike rating, type of coupon, with similar industry and similar average maturity and higher outstanding amount. To support this choice, the “SEC” tool on *Bloomberg*, which is the “Security Finder” was used, choosing the category of Fixed Income.

Real Estate and Cash asset classes cannot be analyzed like the other classes, since the data in the market is of a different kind, which means their valuation is different from the other types of assets. While Equity or Fixed Income are tangible assets, as funded by ownership or contracts, the valuation of Real Estate or Cash depends on wider market factors as, for instance, location or geopolitics in the case of Real Estate. Then, also for these two asset classes, indexes were used as benchmarks. Real Estate was divided by geographic areas in which the company has investments of that kind, for each area there is an index. The (five) areas are Portugal, Europe, USA, UK and Global.

With all the assumptions and exclusions, the number of assets to be considered decreased: the 90 assets in Equity decreased to 72 assets; the 135 assets on Cash have been replaced with only one index; for the 1065 assets on Fixed Income, only 864 assets were included, accounting for all the benchmarks. Concerning Real Estate, there was a particular treatment. There were 18 Real Estate assets, but the benchmark was not considered for the total of market values of those assets. The Real Estate asset class was composed of participations in different companies including direct property, investment funds and cash. We used a benchmark resulting from the five indexes, for 89% of the

Real Estate in the portfolio, and the remaining market value was considered in the Cash asset class.

For the final analyses, the total number of assets used was 942: 11% of Equity, 16% of Real Estate, 69% of Fixed Income and 4% of Cash, making a total of 13,198,644,846.21 euros.

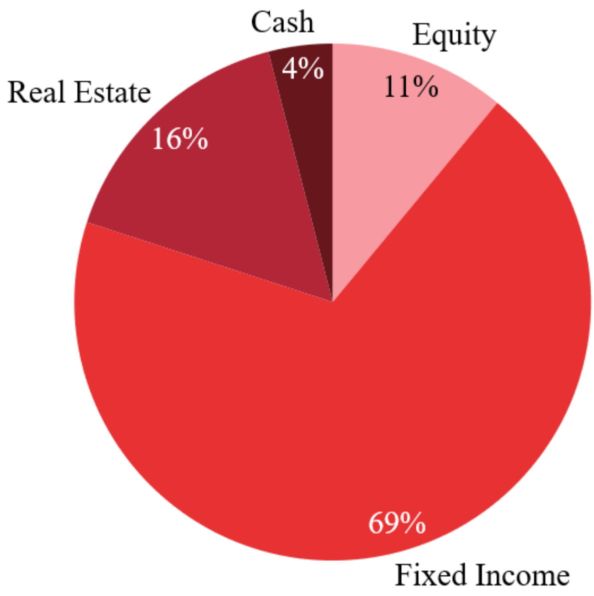

Figure 2 – Portfolio's composition.

Source: Author's creation

3.2. Value-at-Risk Calculation

This section will be divided in three subsections: the first shows the process of the conception of the model, with tests of some methodologies; the second one contains the results by parts (by asset classes) with comparisons with *Bloomberg* and benchmarks; in the last subsection the final results for the whole portfolio will be presented. In each of these sections there are tables with the VaR value in percentage – this means the percentage of the market value corresponding to the asset class or, if not explicit, the market value of the whole portfolio.

3.2.1. Value-at-Risk Calculation Process

After selecting the data to be used, it had to be collected and prepared in *Excel* files as an input to *RStudio*. In the case of Equity, only the prices of one year were needed to calculate the VaR, implementing the Monte Carlo Simulation method. In the case of the benchmark of Real Estate and Fixed Income, the process was the same as in Equity. In the case of the benchmark for Cash (with only one index), the Monte Carlo Simulation did not work, so the Historical Value-at-Risk method was applied[5].

For Fixed Income a different approach was used. In the first place, we tested the three Mapping approaches - Maturity mapping, Duration mapping and Cash-flow mapping - and also the "New Approach", on a portfolio of three bonds. The values of the four approaches were all very similar:

Approaches	VaR (%)
"New approach"	0.13
Cash-flow mapping	0.14
Principal mapping	0.12
Duration mapping	0.11

Table 1 – VaR of Fixed Income by approach

Source: Author's creation

Since the results were very close to each other, and because the "New Approach" had many advantages for the calculation, it was the one chosen.

Concerning the *Excel* file to be read in *RStudio*, for Equity, Real Estate and Cash, it contained the prices of one year of observations. Each column displays the prices for each asset. In the case of Fixed Income, the adjusted historical returns (AHRs) had to be calculated in *Excel* before, being provided as an input to the code. To calculate these returns the necessary data was also collected. This data was more than the prices; the issue date was needed, as was the maturity date (for perpetual bonds the benchmark was 2025), the redemption value and the dirty prices. The prices used were the dirty prices since these include the accrual corresponding to the coupon. This assumption makes it possible to adjust the prices and returns in a way that the coupon is also considered. After taking this

[5] The implementation of Historical VaR method was made just as a reference for the calculations by asset class, since for the entire portfolio, VaR calculation was made with Monte Carlo Simulation method.

data, the AHRs were obtained in *Excel* following the “new approach” explained previously. These AHRs were inserted into another *Excel* file as an input to *RStudio*.

3.2.1.1. Building the R code

To build the code[6], the objective was to enable a one-time execution to obtain the Value at Risk result for the complete portfolio, by having data collected in an *Excel* file.

The initial task involved preparing the *Excel* files: *Prices.xlsx* with prices of each asset in the portfolio (excluding Fixed Income, as specified below), and *Weights.xlsx* with the respective weight of each asset in the portfolio.

Since the "New Approach" was used for Fixed Income, we modelled returns in *Excel* as adjusted historical returns, rather than prices, like in the other asset classes. We labelled the input for the code with these returns as *FixedIncome.xlsx*.

Once we had the inputs, we needed to prepare the files used as input for the code, to ensure there were no errors. It could not have missing data, which proved to be very difficult, since the data was collected from different types of assets and the frequency of data is different from asset to asset. Therefore, the initial stage of the R code would involve data adjustments. Although the data could be worked in *Excel*, this would lose the purpose of maximizing efficiency. So, initially, the code must eliminate all the empty cells (#N/A) from the *Excel*. To achieve this, we adjusted the code to examine all columns and assume the number (in this scenario, the price) of the last cell of the column wherever missing values are present. The value is assumed to be maintained because the price of each asset remains the same until it is replaced by a new one, so it is reasonable to assume that the price is equal to the one in the previous day.

After handling the missing values, we proceeded to calculate the returns of each asset using the data set provided in the *Prices.xlsx Excel* file. The price returns were computed using the *diff()* function, which provides – in values - the difference between consecutive prices and then it was divided by the last element of prices.

Subsequently, the returns from *FixedIncome.xlsx* were appended to the previously computed returns in a unified *array*.

[6] The code in *R* was not shared in this file due to confidentiality reasons.

After operating the necessary calculations to collect all returns, we applied Monte Carlo simulations. We started by simulating 1 000 000 runs of the process, but as the database was too big there were no response in a properly time, or without errors or bugs, derived from the complexity. Then we were forced to decrease the number of simulations until we reached reasonable results. We ended up with 10 000 simulations. We believe that with other computer skills we could target a higher number of simulations, but with the available resources this was the possible value. As referred before this was one of the issues of using the Monte Carlo method, since it is known that the computers and its performances are limited; as a consequence of this disadvantage we were unable to use as many simulations as we would have liked.

Having specified the number of simulations, we used the multivariate normal distribution function *mvrnorm()* to generate 10 000 scenarios for all assets in order to model potential future outcomes. Then, simulated portfolio returns were calculated for each scenario by multiplying the generated asset returns by the portfolio weights provided in the *Weights.xlsx Excel* file.

Using the Monte Carlo simulation results, the *quantile()* function was used to calculate the value below which a specified proportion of data resides, which means this function measures the potential loss that a portfolio might experience at a certain level of confidence. In this case, with the defined confidence level of 99.5%, the percentile for the simulated portfolio returns to be found was 0.5%, so the proportion *p* of the function *quantile()* was defined as 0.005. To obtain the VaR result, the *quantile()* function was multiplied by the total Market Value (MV) of the portfolio. As previously stated, in order to calculate monthly and yearly VaR, the *quantile()* function was also multiplied by the square root of 20 and 252 days, correspondingly. The results were then divided by MV to express the daily, monthly and yearly results as a percentage of the total portfolio. Figure 3 shows the code flowchart.

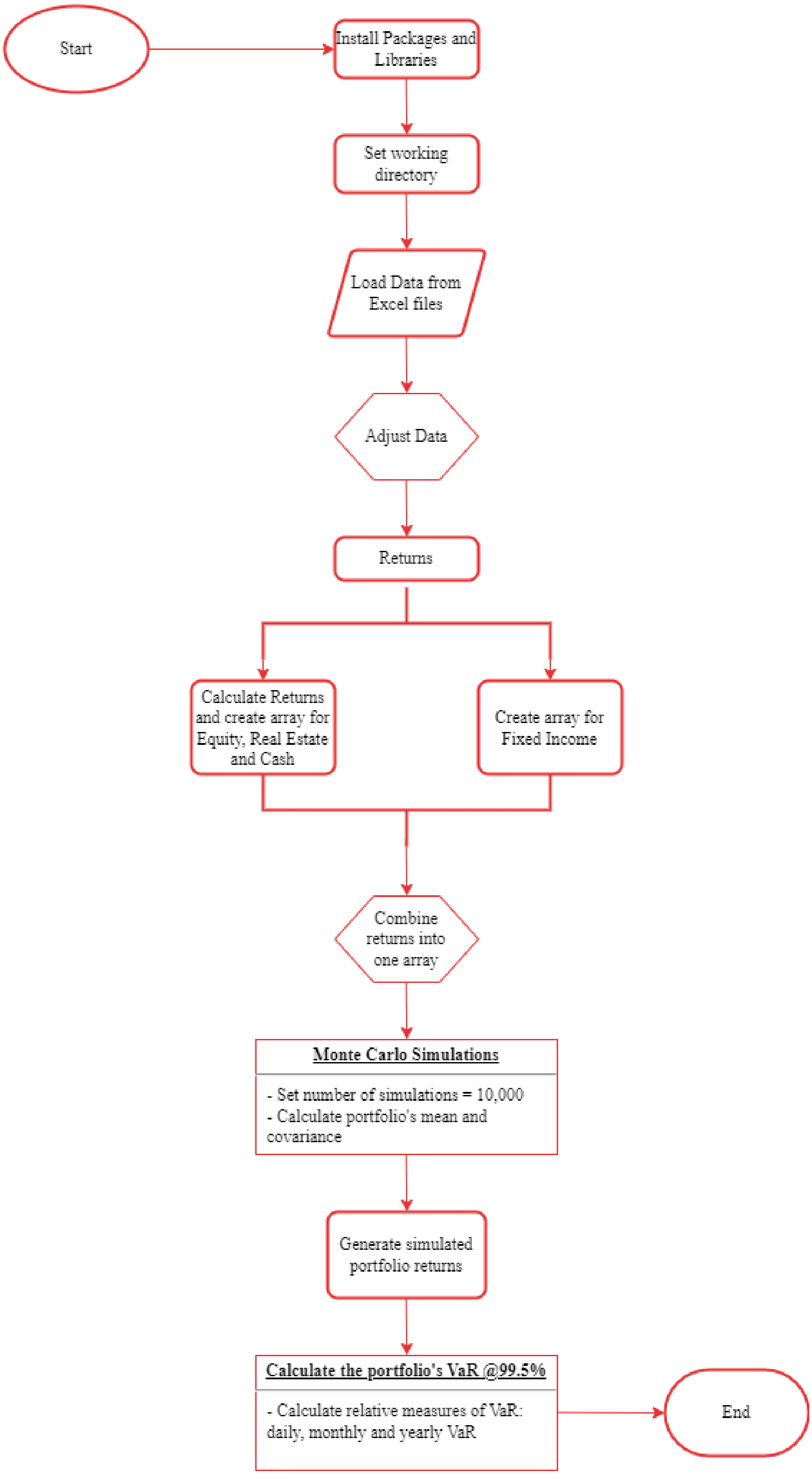

Figure 3 – Code Flowchart

Source: Author's creation

3.2.2. Value-at-Risk by asset class - Results

Following the process described in *3.2.1.*, the results by asset class were:

Asset Class	1 day VaR (%)	1 month VaR (%)	1 year VaR (%)
Equity	1.042	4.659	16.538
Fixed Income	0.215	0.960	3.408
Real Estate	1.748	7.815	27.741
Cash	0.004	0.020	0.069

Table 2 – VaR results by asset class

Source: Author's creation

If we observe the individual results, Equity and Real Estate are the asset classes with the higher potential losses according to the VaR calculation. For one-year VaR calculation the result was 16.54%, which was the second highest value in this analysis. That is because equity investments lead to potentially higher returns but have also a higher volatility and, consequently, risk. With Real Estate, the data collected from some of the indexes could be an issue since the frequency of market data is lower than, for instance, equity. This could affect the accuracy of the estimation, but other characteristics of real estate market do impact the results and make them higher in risk level: Real Estate is a less liquid asset class, which increases its volatility. The direct changes in interest rates can also contribute to a higher VaR, as it can be seen by the 27.74% one-year VaR, although this result is similar to the 25% shock that should be applied to Real Estate in the capital requirement for property risk formula on Solvency II regulation. This result of the Real Estate VaR could also be partially impacted by the geographical areas on which this asset class have exposure. On the other hand, Cash and Fixed Income are much more stable asset classes, with less risk. Cash, or cash equivalents, are extremely liquid assets, which results in the least risky asset class in this portfolio with only 0.07% one-year VaR. Fixed Income, which is the major asset class of the portfolio, is generally an asset class with lower risk and have historical lower volatility, resulting in a 3.41% one-year VaR. Although the VaR of bonds can vary because of characteristics such as duration, type of bond or credit quality, this type of asset has regular interest payments and defined maturity dates, which make the cash flows more predictable.

To confirm if all the results calculated on *RStudio* were consistent results, they were compared in two ways: a comparison with the VaR of the benchmarks and a comparison with the VaR calculations on *Bloomberg*, by using the function “PORT”.

The results of the benchmarks that were obtained with *RStudio*, were the following:

Asset Class	1 day VaR (%)	1 month VaR (%)	1 year VaR (%)
Equity	1.630	7.290	25.875
Fixed Income	0.172	0.768	2.725
Real Estate	1.748	7.815	27.741
Cash	0.004	0.020	0.069

Table 3 – VaR results of benchmarks by asset class

Source: Author’s creation

As explained previously, for Real Estate and Cash, we used a benchmark, so this is the reason why the results are the same for these asset classes, if we compare Table 2 and Table 3.

For the comparison with *Bloomberg*, some assets that were considered in *RStudio* were not compatible with *Bloomberg* (for example the ones that were unquoted and did not have data on *Bloomberg* - the data was collected on Binfólio). This means that, to be able to compare, some adjustments had to be made in the portfolio, excluding some assets, and recalculating the VaR for the adjusted portfolio with *RStudio*, making all the necessary modifications. In this way, the results of Tables 4 and 5 will not be consistent with the results on the previous tables. The results of the comparison were the following:

	1 day VaR (%)		1 month VaR (%)		1 year VaR (%)	
Asset Class	R	Bloomberg	R	Bloomberg	R	Bloomberg
Equity	1.19	1.52	5.33	7.11	18.93	24.06
Fixed Income	0.18	0.26	0.82	1.21	2.92	4.08

Table 4 – Comparison of VaR results (Equity and Fixed Income): RStudio vs.Bloomberg

Source: Author’s creation

For the case of Real Estate, since *Bloomberg* was not able to calculate VaR with all the indexes, a real estate investment fund – Fundimo PL – was considered and compared with the calculation using *RStudio* and with the benchmarks used for Real Estate. The results were the following:

	1 day VaR (%)		1 month VaR (%)		1 year VaR (%)	
Asset Class	R	Bloomberg (Fundimo)	R	Bloomberg (Fundimo)	R	Bloomberg (Fundimo)
Real Estate (Benchmark)	1.8	0.6	7.8	3.0	27.7	10.1
Real Estate (Fundimo)	0.9	0.6	4.0	3.0	14.3	10.1

Table 5 - Comparison of VaR results (Real Estate): RStudio vs. Bloomberg

Source: Author's creation

Having in mind that *Bloomberg* makes the calculation of VaR in a very different way compared to our model, using factor models – which weren't used in the approaches made in this project – it was predictable that there would be some differences between the results extracted from *RStudio* and from the platform of *Bloomberg.* In addition to that, the benchmarks are not the proper assets, so there would naturally be a difference between the results of benchmarks contrasted with the results of the assets. Hence, interpreting the results obtained and the comparisons made, in our team we found these results acceptable, although Real Estate was not a fair comparison.

As a point of clarification, VaR can be computed in many ways and using various methods, but it remains VaR. For this reason, to validate the results of our model and the *R* code, we needed to compare it with reliable platforms, such as *Bloomberg*. Since they use factor models to calculate VaR, we were aware that there would be some margin for discrepancies, but if our results were too far from the ones obtained in *Bloomberg* it would be an indication of a problem in the model. In this way, these comparisons could be an aid to the validation of our process.

3.2.3. Value-at-Risk of portfolio - Results

After having all the results for each asset class, the VaR of the total portfolio was calculated, using the Monte Carlo VaR Method. The results were:

	1 day VaR (%)	1 month VaR (%)	1 year VaR (%)
Total Portfolio	0.39	1.74	6.19

Table 6 – VaR final results of the portfolio

Source: Author's creation

To obtain these results, as explained before, the AHRs were added to the returns calculated in the *R* code for the other asset classes and 10 000 scenarios of other returns were generated by the Monte Carlo simulation. Calculating the VaR of the whole portfolio, after obtaining a code for each asset class individually, was a fairly simple computation, as the covariances and correlations were automated in the code, as all the returns were in just one *array*. The problem with this calculation, which differed from the individual computation, was the limited capacity of the computer to run more random scenarios using Monte Carlo, since the returns used as input were too many as they related to the entire portfolio, so the code had to be adjusted.

According to the results by asset class, since the portfolio is mostly composed of fixed income (69%), it was already expected that the VaR of the whole portfolio would be closer to the results of Fixed Income individually. This means a much lower risk compared to the values obtained in Equity or Real Estate that represented only 27% of the portfolio.

For a one-year time horizon with a 99.5% confidence level, the portfolio's VaR was 6.19%. It is worth noting that although this percentage is not that high, it is in line with the diversified composition of the portfolio in analysis, which is predominantly fixed income - a less risky asset class.

4. Conclusion

To measure risk in companies like Fidelidade, especially the risk of their own portfolio, it is important to use a reliable model, known and used in most of the companies, and that is the case of VaR. The purpose of this internship was to create a model in which the majority of the assets of the portfolio would be included, and which could calculate, all at once, the VaR of the portfolio.

By the theory presented in this report, we can conclude that there are several methods to calculate VaR, with different degrees of complexity and versatility, but some of them cannot be applied to entire diversified portfolios, only to specific asset classes. Given the composition of the portfolio in analysis, the options under consideration had to be complete and versatile; therefore, some of the existing methods were not even considered in this case.

The Monte Carlo method was recognized as the most realistic and the most complete, being the chosen one to apply in our project. It was also understood that, for some specific asset classes, the approaches to consider could not be the same, as was the case of Fixed Income investments. For Fixed Income the research focused on finding a way to work these assets not very different from the rest, which lead to the implementation of a "New approach", that could adjust prices of bonds using the "pull-to-par" effect. Using this technique, it was possible to correlate all the assets of the portfolio using the same model, Monte Carlo VaR, without analysing bonds in a separate way.

Even though it provided successful results, this model could be improved in the future, in order to be able to consider other asset classes, since some of them were excluded as an assumption, for instance, the Derivatives.

One of the major difficulties throughout the process resulted from the fact that some equities were unlisted; consequently, to be able to calculate VaR, we had to use benchmarks, which are approximations and will affect the accuracy of the results. Other than that, the results were quite satisfactory and were in line with those obtained from *Bloomberg*, which is a very reliable platform.

This model, being improved in the ways referred previously, can be a simple tool used for internal analysis of the company.

About the internship as an experience, it was a great opportunity for personal and professional growth, since it was a learning immersion not only about the topics I worked on, but also about the tools, as *RStudio*, which I had never used before, or as *Bloomberg*, for example. The complexity of working with real data from a company of the dimension of Fidelidade, the major insurance company in Portugal, in the area I wanted to, applying some of the knowledge from the Master's program, was a really enjoyable experience.

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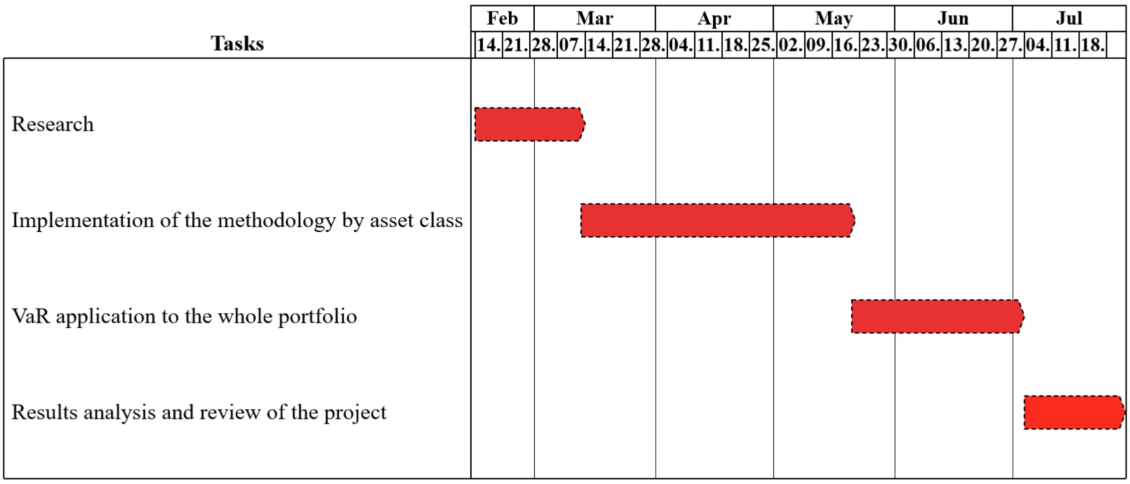
Appendix – Internship timeline



Source: Author’s creation