UNIVERSIDADE DE LISBOA
INSTITUTO SUPERIOR DE ECONOMIA E DE GESTÃO

DOUTORAMENTO EM GESTÃO

# Three Essays on Liquidity

José Eduardo Boto Correia

Professor Doutor João Luis Correia Duque, Orientador

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Lisboa, Janeiro de 2015

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# RESUMO

Esta dissertação consiste em três estudos. O primeiro refere-se ao problema do *dealer* face aos negociadores informados e os não informados. Em equilíbrio, o dealer coloca um bid-ask spread, de modo a maximizar o seu lucro esperado. Ele reduz o spread até um mínimo de modo a promover negócios dos negociadores de liquidez, mas aumenta o spread até um ponto onde a probabilidade de aparecerem negociadores informados seja reduzida. Este spread pode ser visto como uma strangle no preço do active subjacente. Os preços bid podem ser vistos com o preço a que o dealer é obrigado a comprar (preço de exercício de uma opção de compra) e o preço ask pode ser visto como o preço a que o dealer é obrigado a vender (preço de exercício de uma opção de venda). Desenvolve-se uma condição de equilíbrio para o valor teórico do spread, pelo qual calcula-se as volatilidades implícitas para os activos subjacentes. Depois, testa-se e conclui-se que existe uma relação linear entre as volatilidades implícitas, calculadas pelos preços bid e ask, e as volatilidades históricas, observadas no mercado durante o dia de negociação.

No segundo estudo, avalia-se o impacto da crise financeira de 2008 na (i)liquidez das mercados accionistas da Eurozone. Analisa-se alguns indicadores de liquidez, como a taxa turnover, o bid-ask spread e a medida de Iliquidez definida por Amihud (2002). A crise financeira de 2007/2008 foi uma das maiores crises financeiras desde a Grande Depressão e reestruturou o mundo das finanças. Houve vários eventos durante a crise financeira, mas foi em Setembro de 2008, que a crise atingiu o seu período mais crítico. Definiu-se o 15 de Setembro de 2008, o dia em que o Lehman Brothers entrou falência, para separar dois períodos de crise financeira. Após a falência do Lehman Brothers, os mercados financeiros caíram dramaticamente. Analisamos os países que tiveram mais

problemas (Portugal, Irlanda, Itália, Grécia e Espanha - PIIGS) versus os outros países da Eurozone.

No terceiro, pretende-se analisar o impacto da diluição e dos dividendos na adequação de alguns modelos de avaliação de warrants, a um produto financeiro pouco liquido num mercado com pouca iliquidez - o mercado de warrants Português. Para evitar o enviesamento da pesquisa, e para testar os efeitos dos dividendos e da diluição, decidiu-se utilizar apenas o modelo Black-Scholes original e três derivações. Utilizou-se os dados dos warrants da Euronext Lisbon entre 1998 e 2000. O estudo empírico a elaborar, consiste em obter valores teóricos para os quatro modelos de avaliação utilizados e calcular um erro percentual médio para cada um deles em relação aos preços de mercado dos warrants. Conluiu-se que o modelo de Black-Scholes ajustado à diluição e aos dividendos funciona melhor no mercado Português.

# ABSTRACT

This dissertation consists of three essays. The first essay is about the dealer problem facing informed and uninformed traders. In equilibrium the dealer sets the bid ask spread in order to maximize his expected profit. This is obtained by reducing the spread to a minimum in order to promote trading from liquidity traders, but increasing the spread to a point where the probability of facing informed traders is reduced.

The spread can also be viewed as a written strangle on the underlying stock price. Bid prices can be seen as the price by which dealers are obliged to buy (exercise price of a written call option) and the ask prices can be seen as the prices by which they are obliged to sell (exercise price of a written put option) whenever traders arrive and close the deal (exercise the option). We developed an equilibrium condition for the theoretical value of the spread from which we derived implied volatilities for the underlying stocks. Then we tested and concluded that it seems to exist a linear relation between implied volatilities, extracted from closing bid ask spreads, and historical volatilities, observed in the market during the trading day. This relation holds for every stock in the sample but seems to be dependent on liquidity: more liquid stocks present lower residuals.

In the second essay, we evaluate the impact of the financial crisis 2008, on (il)liquidity of Eurozone stock markets. We analyse some indicators of liquidity, like the bid-ask spread, turnover rate and Amihud Illiquidity measure.

The financial crisis of 2007/2008 was one of the greatest financial crises since Great Depression and restructured the world of finance. There were many events during financial crisis, but was in September 2008, that crisis hit its most critical period. We use the 15th of September of 2008, the day that Lehman Brothers filed for Chapter 11 bankruptcy protection, to separate two periods of the financial crisis. His assumed that

was after the bankrupting of the Lehman Brothers, that the stock markets fall deeper. We examine countries in deeper trouble (Portugal, Ireland, Italy, Greece and Spain - PIIGS) versus the other countries in the Eurozone.

In the third, we analyse the impact of dilution and dividends on the goodness of fit of warrant pricing valuation models, to a illiquidity financial product in a illiquidity market - the Portuguese warrants market. In order to avoid modelling bias over the research design, and to test dividend and dilution effects we decided to keep this empirical research under the Black-Scholes framework. Therefore, four pricing models were used: the original Black-Scholes model and three derivations. Using these four models we empirically estimate values for actual warrant prices, computing the mean percentage error for each (the difference between model prices and market prices). We found that the original Black-Scholes model when adjusted to account for dilution as well as for dividends works best in the Portuguese market. The analysis uses data collected from the Euronext - Lisbon, between 1998 and 2000.

*In memory of my parents, Fortunata and Eduardo*
*To my son, Eduardo*

# ACKNOWLEDGEMENTS

My heartfelt appreciation goes out to João Duque, my advisor, who supported me in all the moments.

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### *Abstract*


The paper addresses the dealer problem facing informed and uninformed traders. In equilibrium the dealer sets the bid ask spread in order to maximize his expected profit. This is obtained by reducing the spread to a minimum in order to promote trading from liquidity traders, but increasing the spread to a point where the probability of facing informed traders is reduced.

The spread can also be viewed as a reverse strangle on the underlying stock price. Bid prices can be seen as the price by which dealers are obliged to buy (exercise price of a written call option) and the ask prices can be seen as the prices by which they are obliged to sell (exercise price of a written put option) whenever traders arrive and close the deal (exercise the option).

We develop an equilibrium condition for the theoretical value of the spread from which we derived implied volatilities for the underlying stocks.

Then we tested and concluded that a linear relation seems to exist between implied volatilities, extracted from closing bid ask spreads, and historical volatilities, observed in the market during the trading day. This relation holds for every stock in the sample but seems to be dependent on liquidity: more liquid stocks present lower residuals.

Then implied volatilities were used as a forecasting tool of future volatility. However, the results failed to support such a relation.

# *1. Introduction*

One of the critical aspects of a developing stock exchange is that dealers create sufficient liquidity so that investors can buy and sell significant volumes of shares at reasonable transaction costs without significantly affecting market prices.

There is a large literature (Amihud and Mendelson (1986), Glosten and Milgrom (1985), Ho and Stoll (1980), Huang and Stoll (1997), Stoll (1978)), primarily regarding developed exchanges, on market microstructure, which relates the actual or effective bid-asked spread to the dealer inventories, persistence of buy and/or sell orders, share price volatility, and the presumption (and prominence) of informed versus noise traders.

Using a database of the spreads and prices of the stocks of four national indices of the Euronext (AEX, BEL-30, CAC-40 and PSI-20) between 3rd January of 2005 and 13th June of 2008, we examine whether spreads appear to be reasonable, given the liquidity and volatility conditions on four developing exchanges (Amsterdam, Brussels, Paris and Lisbon), and whether those spreads are related to estimates of transaction frequency and volatility.

The next section reviews some of the relevant literature on market microstructure and on volatility estimates and presents the determinants of the spreads. Section three presents the theoretical model and section four describes the database and characteristics of the four stock exchanges that may be relevant for understanding the apparent empirical results. Then three hypotheses relating spreads to theoretically related variables are tested. Section five presents the empirical results and section six concludes with a review of the findings.

## *2. Literature Review*

The dealer's problem was formulated by Stoll (1978), who defined three costs the dealer faces in providing dealer services: holding costs of less than optimal inventories; order-processing costs; and the costs of trading with informed traders, an asymmetric information cost. Easley et al. (1996) added a new source of explanation relating market structure to the spread.

One plausible explanation for the spread is inventory costs. When market making the dealer buys and sells stocks that change their efficient portfolio composition. Therefore in order to balance this inconvenient inventory spreads can lead traders to avoid deal with dealers. Moreover, after a dealer buy or sell, prices should react accordingly in order to establish the desired efficient portfolio, as explained by Stoll (1978).

Order processing costs are the costs incurred by dealers to execute orders and keep running the office activity. This spread component has typically a fixed behaviour at least for a group of homogeneous stocks, although fixed administrative costs can be decreasingly significant as volume increases.

Market structure can also partly explain the spread justifying two different phenomena: spread differences between markets and the U-shaped intraday pattern of the spreads. Markets with monopoly dealers should present higher spreads than markets with competitive bidding and asking. But markets with monopoly dealers tend to present a U-shaped intraday pattern of the spreads, as opposed to competitive markets, since monopoly dealers may be forced to accumulate unwanted inventories during peak trading volumes. Therefore opening and closing high activity may well reflect on higher spreads for monopoly dealer markets.

Finally, spreads can be justified by asymmetric information. In a semi-strong efficient market, market agents may have asymmetric information, creating possible abnormal returns whenever trading with dealers that stand firm bid and offer quotes. After big blocks of stocks were being traded, dealers are supposed to readjust quotes in order to accommodate the new information (Glosten and Milgrom (1985)). These quotes can be viewed as options expecting to be exercised, as explained by Copeland and Galai (1983).

Copeland and Galai (1983) assume that the dealer faces two possible types of traders: informed traders (with some non-public information) and liquidity traders (who pay a "fee" for the immediacy of the trade). When a liquidity trader faces the dealer, the trader has some probability of trading and a complementary probability of not trading. When quoting a bid and an ask price the dealer is writing a put and a call options on the stock facing a potential conflict of motivations: first the dealer needs to set small spreads since he is attracting liquidity traders from whom he profits when trading; but, second, he needs to increase the spread in order to reduce the probability of facing an informed trader from whom he loses when trading. Copeland and Galai (1983) show that the dealer is writing strangles on the stock and, consequently, the bid and ask spread is a positive function of the price level and the variance of the stock, and a negative function of market activity, depth and continuity and is negatively correlated with the degree of competition.

Glosten and Milgrom (1985) have also analysed the dealers' problem when facing possible informed traders, extending the adverse-selection-portfolio spread to the information-based spread. These two explainable theories were empirically supported by empirical research presented in Glosten and Harris (1988) using the first 800 successive prices beginning on December 1, 1981 for a sample of 250 stocks listed in NYSE.

Admati and Pfleider (1988) studied the spread information-based explanation and concluded that spreads should narrow when volume is high and prices are more volatile. However, Foster and Viswannathan (1990) doing similar research predicted narrow spreads when volume is high and prices are less volatile.

Stoll (1989), examining the quotations for the National Market System securities on the NASDAQ system during the last quarter of 1984, concluded that the spread components appear to be proportionally invariant and the most important factor, apart the order processing cost, seems to be the adverse information cost. However, other studies summarised in Brooks and Masson (1995), show that the percentage of the spread attributable to the adverse information cost may be dependent on the market studied and the method used, ranging from 4% to 80% of the total spread. However, in all studies reviewed by Brooks and Masson (1995), the adverse information cost percentage is greater than the adverse-selection-portfolio cost. Nevertheless, Huang and Stoll (1997), show that by on the contrary, with a sample of the 20 largest and most actively traded stocks composing the Major Market Index and listed in NYSE, inventory costs account for a major slice of the spread when compared with the information cost[1].

Manaster and Mann (1996), observing the spreads in futures transactions data, reject the neutrality of the adverse-selection-portfolio cost and suggest new models absorbing these with the information component cost. However, it is highlighted that the speed of inventory adjustments in future markets seems to be higher than for equity markets.

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[1] Excluding the order processing costs that keep on being the most significant part of the spread

Easley et al. (1996) studied the probability of information based trades and found that the risk of adverse information trades is lower for liquid stock than for infrequently traded securities. This effect results on higher spreads for illiquid stocks although the phenomenon is not perceptible when low and high volume stocks are compared. The authors suggest that these results cannot be attributable to the fact that market makers for infrequent stocks have a monopoly for quotation on these securities, nor to risk bearing undesirable inventories for longer time periods: "Less active stocks are riskier because they are subject to more information-based trading" leading to the observation that "risk-adjusted returns increased significantly with bid-ask spreads" (pp. 1428).

Spread analysis can also be used to identify changes in price behaviour or information contents. Brooks, Johnson and Su (1997) used spread analysis to identify a possible fading of information asymmetry between informed and uniformed traders. When studying the effects of CEO presentations to financial analysts' societies (as an investor-relation programme) on trading activity, the authors used spreads as a way to capture effective changes on information asymmetry. However, the absence of any significant difference leads them to conclude that these programs are much like maintenance programmes for keeping the investor base.

Gwilym, Clare and Thomas (1998) apply the referred spread decomposition to the LIFFE index options contracts and find the spread to seem to be driven by the underlying volatility instead of being the volatility of the options market itself. This is opposed to previous empirical findings when the equity market was under scope. The reason for such behaviour is presented by the authors as a result of the importance of the underlying volatility on option pricing.

Barclay *et al* (2003) demonstrated that informed traders prefer negotiate in Electronic Communication Networks, because the anonymity. Gramming *et al* (2001) shows that the Probability of Informed Trading (PIN) is higher in a electronic trading compared to the floor of Frankfurt Stock Exchange.

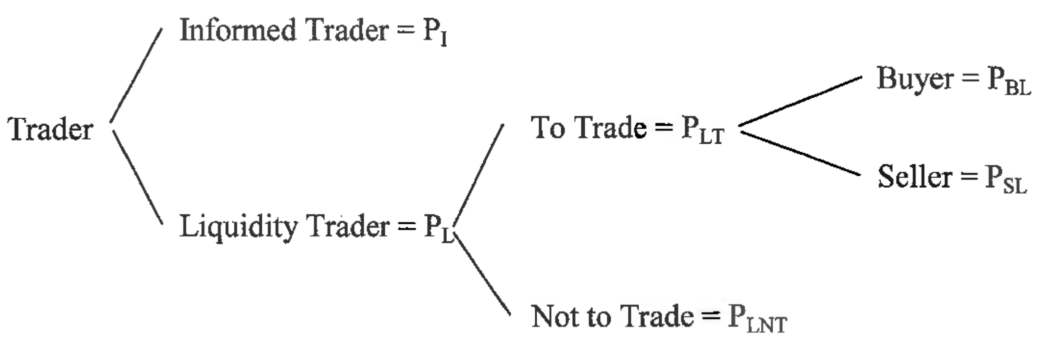
Giouvris and Phillips (2008) who studied the bid-ask spread components and determinants for the FTSE100 and FTSE250 stocks under different trading regimes. He concluded that in a quote driven market, the asymmetric component of the spreads is higher and the stock volatility affects more this component. Another conclusion is that change of the market (from quote driven to hybrid) does not reduce the asymmetric information component of the spread, because there are obligations by the market makers to provide liquidity.

Buti and Rindi (2013) defined a theory to determine an optimal submission strategy in a limit order book, in which traders choose among limit, market, and reserve orders and simultaneously set price, quantity, and exposure.

# *3. The Theoretical Model*

Assuming that order processing costs are constant and that monopoly dealers on specific stocks do not exist in Euronext Stock Exchange trading system, spread changes along time can only be the result of either information or adverse-selection inventory costs. As according the survey of Brooks and Masson (1995) information costs are far more significant than inventory costs, we based our research on the assumptions Copeland and Galai (1983) extending the way of facing the information asymmetry dealer problem.

We will assume, as in Copeland and Galai (1983), that a liquidity trader is possibly a buyer or a seller with the corresponding probability $P_{BL}$ and $P_{SL}$. Therefore, the dealer faces the following traders with the corresponding associated probabilities:




In this model, prices exhibit a semi-strong form of efficiency allowing abnormal returns for traders with superior information. So, the dealer is assumed to profit from liquidity traders but to lose when trading with informed traders. We will also assume:

1. $P_I + P_L = 1$ **(1.1)**

with $0 < P_I < 1$;

2. $P_{LT} + P_{LNT} = 1$; **(1.2)**

3. $P_{BL} + P_{SL} = P_{LT}$; **(1.3)**

Therefore, a liquidity trader can only become a buyer, a seller or a non-trader.

4. $P_{BL} + P_{SL} + P_{LNT} = 1$; **(1.4)**

Additionally we will assume:

5. No tax effects on trading activity;
6. Short sales are unconstrained;
7. The instantaneous risk free rate for borrowing and lending is constant and equal to $r_f$;
8. The "true" underlying asset price S follows a stochastic process f(S) known ex-ante by the entire market;
9. The information arrives to the market through an exogenous process by informed traders. Dealers and liquidity traders are assumed to be uninformed.
10. The dealer price bids ($K_B$) and price asks ($K_A$) for a limited volume of shares until the next trade happens;
11. $\frac{\partial P_{BL}}{\partial K_A} < 0$ and $\frac{\partial P_{SL}}{\partial K_B} > 0$

As the dealer increases the ask ($K_A$), for a given "true" unobservable price of the underlying asset, the probability for a liquidity trader to buy decreases, meaning that the "fee" paid for immediacy becomes too "heavy". Similarly, when the bid ($K_B$) moves too low for that "true" unobservable price of the underlying asset the same restrictive movement from liquidity traders (sellers) occur, decreasing the trading volume.

Then, the dealer sets his objective as the maximisation of his net profit, taking into consideration that he profits from trades with liquidity traders and loses from informed traders. If he widens the spread too much, he risks losing liquidity traders but reduces the probability of loss from informed traders. If he narrows the spread, he increases the probability of dealing with liquidity traders but increases the risk of losing with informed traders.

Not knowing whether the bid or ask price will be hit the bid and ask prices are viewed as call ($c(K_A)$) and put ($p(K_B)$) options provided by the dealer to the traders. Bid prices can be seen as the price by which dealers, who introduced the order in the trading system, are obliged to buy (exercise price of a written put option), and ask prices as the prices by which they are obliged to sell (exercise price of a written call option), whenever traders arrive and execute the deal (exercise the option).

Both options are clearly out-of-the-money, but while informed traders deal whenever they believe that $S<K_B$ or $S>K_A$, liquidity traders exercise the out-of-the-money option if they are willing to lose, paying for immediate liquidity. In such a case if the trade is a result of a liquidity trader when the "true" unobservable stock price stands at S, the dealer profits $S-K_B$ from liquidity sellers and $K_A-S$ from liquidity buyers.

As the dealer offers free call and put options to the market (assuming a profile of a reverse strangle) he is exposed to the risk of being exercised by informed traders, with a probability of $P_I$. In such a case, the dealer expected cost equals the integral of the loss function for all scenarios where the "true" unobservable price is greater than the ask price or less than the bid price. If $f(S)$ stands for the probability distribution function of $S$, in a continuous framework:

$$E[\text{Cost}] = P_I \left[ \int_{K_A}^{\infty} (S - K_A) f(S) dS + \int_0^{K_B} (K_B - S) f(S) dS \right] \quad \textbf{(1.5)}$$

or:

$$E[\text{Cost}] = P_I \left[ c(K_A) + p(K_B) \right] \quad \textbf{(1.6)}$$

It is interesting to note that the specific risk of the underlying stock ($\sigma$) influences the spread. As the volatility increases the call and put values increase. In order to keep unchanged the expected loss (E[Cost]), the dealer must change the exercise prices of the strangle, decreasing $K_B$ and/or increasing $K_A$ as the underlying asset price or its volatility change.

Alternatively, the dealer faces uninformed traders who create a profit for the dealer whenever a trade occurs. As previously explained, the liquidity trader can become a buyer, a seller, or he can just opt for avoiding to trade. Therefore, the expected profit for the dealer becomes:

$$E[\text{Revenue}] = P_L \left[ P_{BL}(K_A - S) + P_{SL}(S - K_B) + P_{LNT}.0 \right] \quad \textbf{(1.7)}$$

This is a non-linear function, since it was assumed, $\frac{\partial P_{BL}}{\partial K_A} < 0$ and $\frac{\partial P_{SL}}{\partial K_B} > 0$.

The dealer dilemma is now an optimization problem, where he tries to fix the best bid and ask prices (the strangle exercise prices) in order to maximize his profit:

$$Max_{K_A, K_B} : \; P_L \left[ P_{BL}(K_A - S) + P_{SL}(S - K_B) \right] - P_I \left[ c(K_A) + p(K_B) \right] \quad \textbf{(1.8)}$$

or in a different way,

$$Max_{K_A, K_B} : \; P_L P_{BL}(K_A - S) + P_L P_{SL}(S - K_B) - P_I c(K_A) - P_I p(K_B) \quad \textbf{(1.9)}$$

According to Copeland and Galai (1983), this can be optimized if:

$$P_L P_{BL}(K_A - S) = P_I c(K_A) \tag{1.10}$$

and

$$P_L P_{SL}(S - K_B) = P_I p(K_B) \tag{1.11}$$

Replacing $P_L$ in these previous equations by equation (1.3) results into the following system of two simultaneous equations:

$$(1 - P_I) P_{BL}(K_A - S) = P_I c(K_A) \tag{1.12}$$

$$(1 - P_I) P_{SL}(S - K_B) = P_I p(K_B) \tag{1.13}$$

Assuming the probability of being a liquidity buyer equals the probability of being a liquidity seller ($P_{BL} = P_{SL}$) and adding up both sides of equations (1.12) and (13):

$$(1 - P_I) P_{BL}(K_A - S) + (1 - P_I) P_{BL}(S - K_B) = P_I c(K_A) + P_I p(K_B) \tag{1.14}$$

With some extra algebra, we conclude that:

$$\frac{1 - P_I}{P_I} P_{BL}(K_A - K_B) = c(K_A) + p(K_B) \tag{1.15}$$

Using the Black-Scholes (1973) option pricing model[2], the call and put values are:

$$c(K_A) = SN(d_1) - K_A e^{-r\tau} N(d_2) \tag{1.16}$$

$$d_1 = \frac{\ln\left(\frac{S}{K_A}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

---

[2] This model is for European options, while the dealer's exposure is for equivalent to American options.

$$p(K_B) = -SN(-d_3) + K_B e^{-r\tau} N(-d_4) \quad (1.17)$$

$$d_3 = \frac{\ln\left(\frac{S}{K_B}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_4 = d_3 - \sigma\sqrt{\tau}$$

Several parameter estimates are required for the solution of equation (1.15). Therefore, different alternatives are available:

1. We can estimate the theoretical spread, setting $K_A$ (or $K_B$) and calculating $K_B$ (or $K_A$) by using a numerical approximation that solves equation (1.15) for a given pair of $P_I$ and $P_{BL}$ and some assumed volatility;
2. We can estimate the $P_I$ (the implied probability of a trader being an informed trader) or $P_{BL}$ (the implied probability of a trader being a liquidity buyer) by solving equation (1.15) for a given pair of $K_A$ and $K_B$ and for some assumed volatility;
3. We can estimate the implied volatility by a numerical approximation that solves equation (1.15) for a given pair of $K_A$ and $K_B$.

In this paper we relax the fixed parameters assumed by Copeland and Galai (1983) for $P_{BL}$ and $P_{SL}$, to satisfy assumption 11 ($\frac{\partial P_{BL}}{\partial K_A} < 0$ and $\frac{\partial P_{SL}}{\partial K_B} > 0$). Easley et al. (1996) found empirically that the probability of an information based trade stands between 16% and 22%. We will assume this probability constant:

$$P_L = 0.8 \quad \text{and} \quad P_I = 0.2$$

Additionally will take:

$$P_{BL} = P_{SL} = 0.3\frac{K_B}{K_A}$$

As a consequence:

$$P_T = P_{BL} + P_{SL} = 2\left(0.3\frac{K_B}{K_A}\right)$$

$$P_{NT} = 1 - 2\left(0.3\frac{K_B}{K_A}\right)$$

Then assumption 11 is fulfilled since:

$$\frac{\partial P_{BL}}{\partial K_A} = -0.3\frac{K_B}{K_A^2} < 0 \quad \text{and} \quad \frac{\partial P_{SL}}{\partial K_B} = 0.3\frac{1}{K_A} > 0$$

Replacing the above for $P_{BL}$ and leaving $P_I$ as a free variable, equation (1.15) becomes:

$$\frac{1-P_I}{P_I}0.3\frac{K_B}{K_A}(K_A - K_B) = c(K_A) + p(K_B) \qquad \textbf{(1.18)}$$

or, replacing also $P_I$:

$$1.2\frac{K_B}{K_A}(K_A - K_B) = c(K_A) + p(K_B) \qquad \textbf{(1.19)}$$

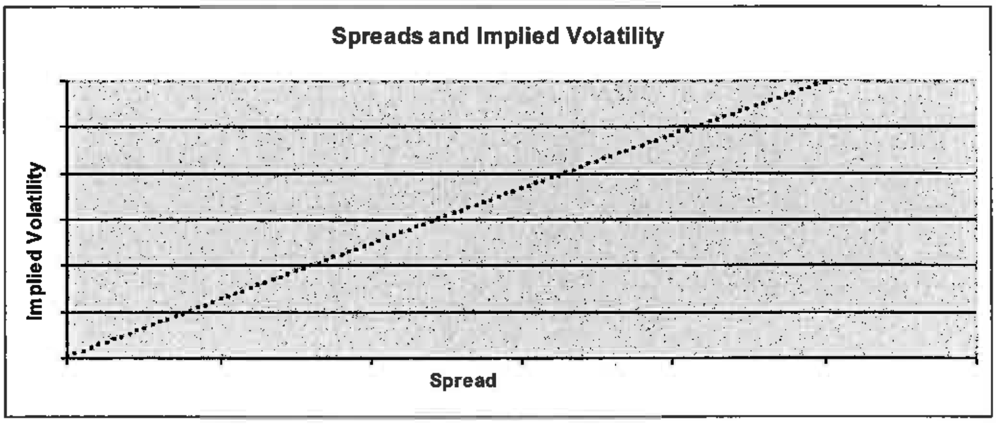
Equation (1.18) presents some interesting properties. First, when $K_A$=$K_B$ (both exercise prices become equal) the left side of equation (1.18) equals zero. In this case, both options are at-the-money and worthless[3]. This only happens at maturity and therefore, we would conclude that, if there is no spread, then both options have just expired. In contrast, whenever there is a dealer offering a quotation, unless a trade is occurring he has to quote a spread. Otherwise he is offering two valuable options with absolutely no return for the writer.

Other observations regarding equation (1.18) concern the impact of some of the variables affecting option prices. Equation (1.18) states an equilibrium framework. Therefore, when one of the variables affecting the option price changes, either equation (1.18) is invalid or another variable has to change in order to keep the equilibrium.

When volatility changes, it implies that the right hand side of the equation changes. As the strangle vega is positive, the right side of equation (1.18) increases with volatility, which, in equilibrium, forces the left hand side to increase. This increment results into an increase of the spread which means an ask price increase and a bid price decrease. Such widening of the spread reduces the price of the options, since they become more out-of-the-money. In conclusion, in equilibrium, a change in volatility does not have the same impact on the spread, as it would have if written on a straightforward plain vanilla strangle. Figure 1 shows a plot of different pairs of implied volatilities and spreads that keep the equilibrium in equation (1.18).

[3] As the sum of the call with the put sum zero and being each greater then zero, each have to equal zero.

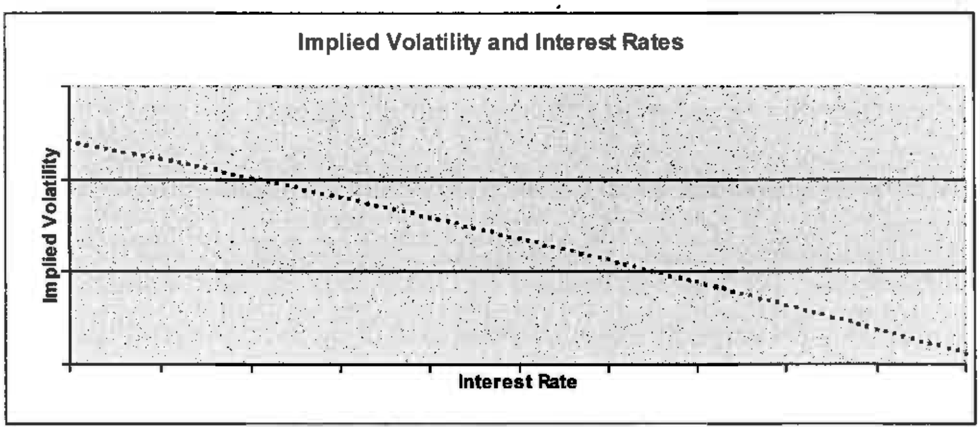
**Figure 1.1. Spreads and Implied Volatility**




Source: author's own estimation based on Euronext data.

If, instead, we keep the spread constant and allow changes of interest rates, volatility has to change in order to satisfy equation (1.18). Figure 1.2 show that when interest rates increase and the spread is kept constant, then volatility implied in the options decreases.

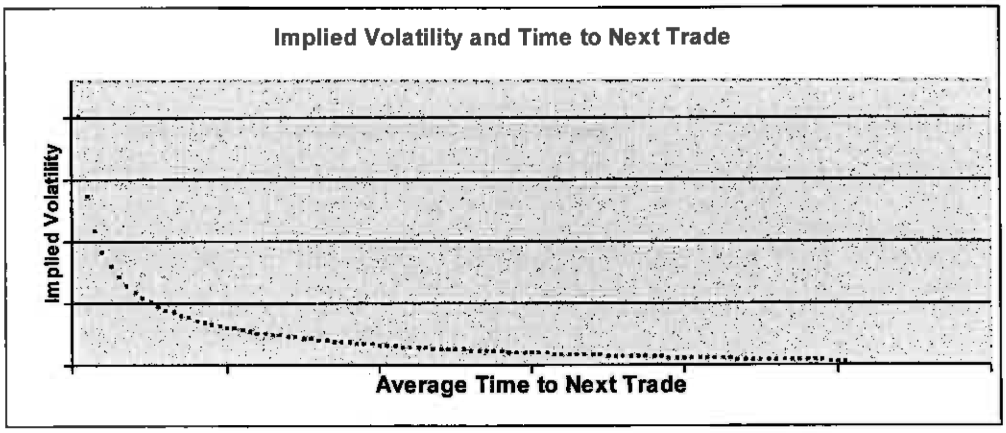
**Figure 1.2. Implied Volatility and Interest Rates**




Source: author's own estimation based on Euronext data.

If the spread is constant but the time to the next trade varies, then the volatility has to adjust to satisfy equation (1.18). Figure 1.3 illustrates this adjustment. Actually, volatility is negatively related to the time to the next trade, since this usually increases the value of both options (even if the put option is out of the money for liquidity traders). As a consequence, to preserve equilibrium in equation (1.18) the implied volatility has to decrease.

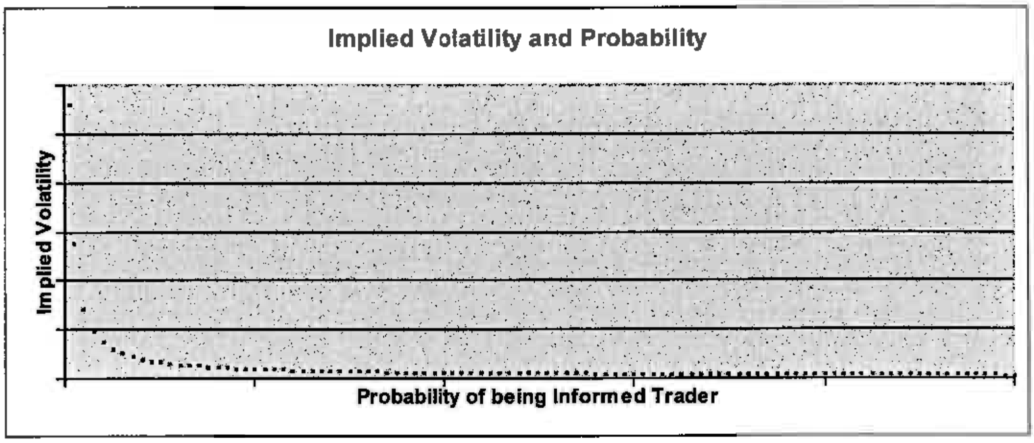
**Figure 1.3. Implied Volatility and Time to Next Trade**




Source: author's own estimation based on Euronext data.

When the spread is kept constant and volatility changes, in order to satisfy equation (1.18), it is possible to adjust the probability of any trader being an informed trader ($P_I$). In such a case, the increase on the probability of being an informed trader results on a decrease of the volatility implied by equation (1.18), as shown by Figure 1.4.

**Figure 1.4. Implied Volatility and Probability of being an Informed Trader**

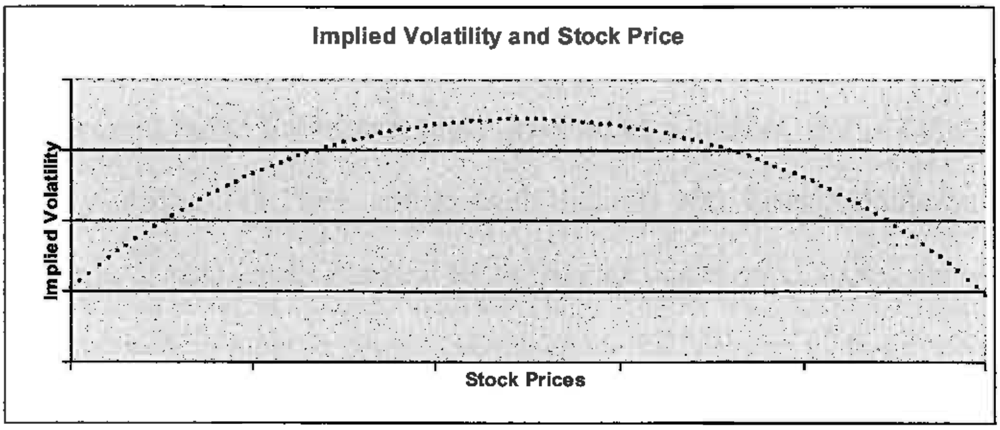



Source: author's own estimation based on Euronext data.

Lastly, we discuss the relation between the underlying stock price and the implied volatility extracted from equation (1.18).

When the stock price varies, the values of both the call option and the put option vary. In order to satisfy equation (1.18), either the spread varies, the implied volatilities vary or both. Assuming a constant spread, a change on the stock price would force a change on the implied volatility. However, this relation, as Figure 1.5 shows, is non-linear. If the price is at the bid price, meaning that the put option is at-the-money and the call "deep" out-of-the-money, the implied volatility consistent with equation (18) is at the minimum. A raise in the stock price, the spread does not necessarily change. If it is kept constant, and volatility increases, a new equilibrium can be set. Implied volatility is at a maximum when the stock prices stands in the middle of the spread, declining then for new increases of the underlying stock price.

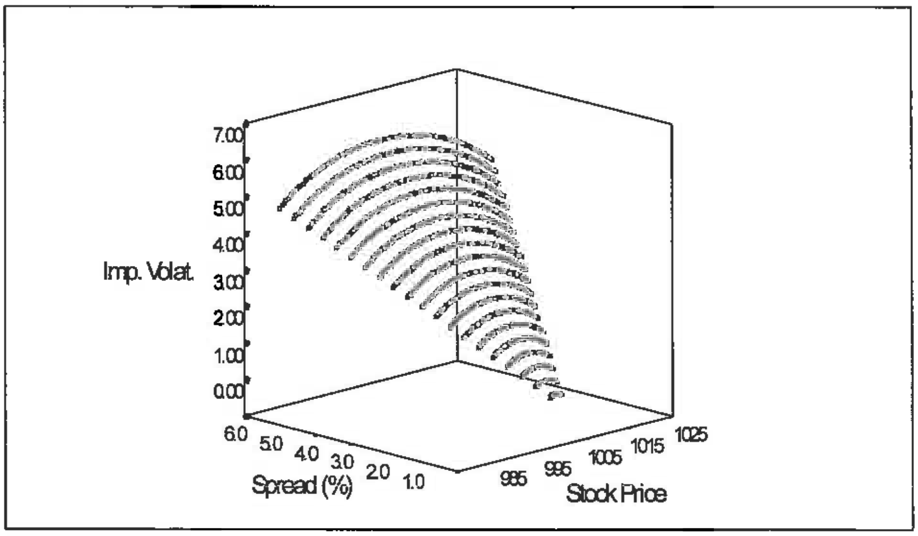
**Figure 1.5. Implied Volatility and Stock Price**




Source: author's own estimation based on Euronext data.

The relation in Figure 1.5 is observed for different spreads. As is shown in Figure 1.6, as the spread increases we observe the same pattern for the implied volatility change according to changes in stock price.

**Figure 1.6. Implied Volatility, Spread and Stock Price**




Source: author's own estimation based on Euronext data.

In summary, changes in volatility do not necessarily imply changes in spreads if we let the underlying stock price move in between the bid and the ask prices.

## *4. Data and Empirical Research Design*

Euronext is the Eurozone's largest cash equities market, and is composed by the stock exchanges of Paris, Amsterdam, Brussels and Lisbon (Euronext Securities Market). According with Euronext Rules "Market orders are unlimited bid or ask orders to be executed at the next prices determined by the Euronext trading system, with any remaining unexecuted portion being added to the Central Order Book for execution as soon as possible at the next prices."

Before the beginning of each Trading Day, an opening auction is held. The main trading session, takes place on a continuous basis and in the end of the trading session, a short period is provided at the close of a Trading Day during which orders can be entered for execution at the last traded price. During the main trading session, a market-to-limit order is converted into a limit order at the best bid price (for sell orders) or best ask price (for buy orders). In the Euronext Cash market model's order-driven system, there is one figure that acts as market maker – the Liquidity Providers (LPs).

The four national indices, represents the largest outstanding market capitalisation of the companies listed in each stock exchange country. In Paris, the index represents 40 firms, in Amsterdam, 25 and in Lisbon and Brussels, 20. As the numbers of companies composing the indices differ, we chose the stocks belonging to the 5 biggest and the 5 smallest of each index.

We collected closing prices as well as the best bid and ask prices at the close of the market between January, 3, 2005 and June, 13, 2008. We choose this period to avoid the financial crisis. Statistics on the average number of trades per month and the average time between trades are shown in Table 1.1.

**Table 1.1 Average Time to Maturity**

| | Average Number Trades/Day | Average Time to Maturity (minutes) |
|---|---|---|
| **AMSTERDAM** | | |
| 5 Biggest | 6,736 | 0.08 |
| 5 Smallest | 1,911 | 0.36 |
| **All** | **4,324** | **0.22** |
| **BRUSSELS** | | |
| 5 Biggest | 2,550 | 0.28 |
| 5 Smallest | 687 | 1.00 |
| **All** | **1,618** | **0.64** |
| **LISBON** | | |
| 5 Biggest | 1,305 | 0.64 |
| 5 Smallest | 301 | 6.08 |
| **All** | **803** | **3.36** |
| **PARIS** | | |
| 5 Biggest | 8,604 | 0.06 |
| 5 Smallest | 3,563 | 0.16 |
| **All** | **6,083** | **0.11** |
| **ALL 4 STOCK EXCHANGES** | **3,207** | **1.08** |

Source: author's own estimation based on Euronext data.

The closing prices is the last price observed in the market (which may differ from the mean bid/ask quoted) and it will be used as a proxy for the underlying stock price when the market closes and bid and ask quoted prices are collected. In such a case it is unreasonable that closing prices stand outside of the spread. From Table 1.1 we conclude that there is a large difference among stock exchanges. The average for the entire sample was 1 minute and 5 seconds[4], but while the five biggest of Paris trades on average every 4 seconds, the five smallest of Lisbon trades every 6 minutes and 5 seconds. Therefore, it is expected that for less liquid stocks, the assumption that closing prices stand for the underlying (unobservable "true") stock price at the close of the market is less probable. However, at the same time, it is also expected that less liquid stocks present a wider spread and, therefore, closing prices are more likely to fall within the spread when the market closes.

The sample is composed by 33,048 observed closing prices. Table 1.2 shows the distribution of these closing prices, comparing them with bid and ask prices. As was explained before, a trade always occur at the bid or the ask price. However, when an order is sent to the system, fulfilling the stated quantity for a price (bid or ask), the trade occurs and the best bid (ask) becomes the next bid (ask) standing in the system. If this process occurs near to the end of the trading session, the closing price does not coincide with the bid or the ask price.

**Table 1.2. Closing Prices comparing with bid and ask prices**

| | Amsterdam | | Brussels | | Lisbon | | Paris | | TOTAL | |
|---|---|---|---|---|---|---|---|---|---|---|
| S < Bid | 18 | 0.2% | 3 | 0.0% | 22 | 0.3% | 14 | 0.2% | 57 | 0.2% |
| S = Bid | 3,448 | 39.5% | 2,928 | 36.2% | 3,539 | 44.0% | 3,810 | 46.5% | 13,725 | 41.5% |
| Bid < S < Ask | 1,386 | 15.9% | 1.785 | 22.0% | 688 | 8.6% | 559 | 6.8% | 4,418 | 13.4% |
| S = Ask | 3,856 | 44.2% | 3,377 | 41.7% | 3,771 | 46.9% | 3,806 | 46.5% | 14,810 | 44.8% |
| S > Ask | 12 | 0.1% | 4 | 0.0% | 26 | 0.3% | 2 | 0.0% | 44 | 0.1% |
| **ALL** | **8,720** | **100.0%** | **8,097** | **100.0%** | **8,046** | **100.0%** | **8,191** | **100.0%** | **33,054** | **100.0%** |

Source: author's own estimation based on Euronext data.

[4] This is used as a proxy for the time to maturity of the options strangle.

Euro Overnight Index Average (EONIA) were used as a proxy for the daily risk free interest rate and collected from the site www.euribor.org.

We also collected daily trading volumes of the number of shares traded for each stock, adjusted for the number of shares listed on the exchange. A Turnover Rate was created to express a relative measure of trading frequency:

$$Turnover\ Rate = \frac{\#\,\text{Shares Traded}}{\#\,\text{Outstanding Listed Shares}} \qquad \textbf{(1.20)}$$

The total amount of outstanding shares was adjusted for stock splits, capital adjustments and other effects that could change the significance of the ratio along time. The data on these effects was collected from Euronext Data.

The spreads observed for the stocks collected vary widely as is shown in Table 1.3. On average, spreads are 0,226% of the closing price when market closes, although the average spread for the 5 smallest of Lisbon was 0,665% that represents 10 times the observed spread on the 5 biggest of Paris (0,065%). We also can conclude that the biggest firms have lower average spread than the smallest firms and biggest stock exchange have lower average spread than smallest stock exchange.

**Table 1.3. Average SPREAD**

| | Average Spread |
|---|---|
| **AMSTERDAM** | |
| 5 Biggest | 0.115% |
| 5 Smallest | 0.167% |
| **All Amsterdam Sample** | **0.141%** |
| **BRUSSELS** | |
| 5 Biggest | 0.151% |
| 5 Smallest | 0.273% |
| **All Brussels Sample** | **0.212%** |
| **LISBON** | |
| 5 Biggest | 0.272% |
| 5 Smallest | 0.665% |
| **All Lisbon Sample** | **0.469%** |
| **PARIS** | |
| 5 Biggest | 0.065% |
| 5 Smallest | 0.099% |
| **All Paris Sample** | **0.082%** |
| **ALL** | **0.226%** |

Source: author's own estimation based on Euronext data.

In the previous section we raised the hypothesis that in equilibrium, the spread should be related to the value of the strangle offered by the dealer to the market, as shown in equation (1.21).

$$\frac{1-P_I}{P_I}0.3\frac{K_B}{K_A}(K_A - K_B) = c(K_A) + p(K_B) \qquad \textbf{(1.21)}$$

From this equation we estimated the volatilities implied in the strangle assuming the value of the call and the put option as given by the Black and Scholes (1973) equation. The values obtained by the equation (1.21) are shown in the Table 1.4 and we can conclude that Stock Exchanges that are more liquids have implied volatilities smallest than Stock Exchanges that are fewer liquids.

**Table 1.4. Implied Volatility**

| | Mean | σ |
|---|---|---|
| **AMSTERDAM** | | |
| 5 Biggest | 0.771 | 1.554 |
| 5 Smallest | 1.137 | 0.977 |
| **All Amsterdam Sample** | **0.952** | **1.314** |
| **BRUSSELS** | | |
| 5 Biggest | 1.028 | 0.796 |
| 5 Smallest | 1.701 | 1.260 |
| **All Brussels Sample** | **1.334** | **1.086** |
| **LISBON** | | |
| 5 Biggest | 1.887 | 1.033 |
| 5 Smallest | 4.502 | 3.847 |
| **All Lisbon Sample** | **3.220** | **3.127** |

| | | |
|---|---|---|
| **PARIS** | | |
| 5 Biggest | 0.082 | 0.076 |
| 5 Smallest | 0.682 | 0.635 |
| **All Paris Sample** | **0.405** | **0.556** |

Source: author's own estimation based on Euronext data.

We first hypothesised that volatilities implied in closing bid and ask prices of day t ($\sigma_{\text{imp}_t}$) are linearly related to the Garman and Klass (1980) historical volatility for that day ($\sigma_{\text{hist}_t}$)[5]:

$$\sigma_{\text{imp}_t} = \beta_0 + \beta_1 \sigma_{\text{hist}_t} + \varepsilon_t \qquad \textbf{(1.22)}$$

Secondly we tested if the implied volatility in equation (1.21) for day t-1 is a good forecast for the future volatility observed during the following day t, also using Garman and Klass (1980) as an unbiased estimator of future volatility.

$$\sigma_{\text{hist}_t} = \beta_0 + \beta_1 \sigma_{\text{imp}_{t-1}} + \varepsilon_t \qquad \textbf{(1.23)}$$

For each hypothesis we run a first test for the entire sample, and a second test for each Stock Exchange.

[5] Garman and Klass [1980] show that their suggested estimator is by far more efficient than other extreme value estimators, since it captures information from the opening, closing, day high and day low observed prices.

## *5. Empirical Results*

The results of testing equation (1.22) applied to the entire sample (first rows of Table 1.5) show an apparent linear relation between implied volatilities in stock spreads observed in closing prices and the historical volatility observed during the trading day. As we can see, the biggest are the Stock Exchange, lowest are the $\beta_0$ and the $\beta_1$.

**Table 1.5. Results of Equation 1.22**

| COUNTRY | # Obs | $R^2$ adj. | $\beta_0$ | $\beta_1$ | F | DW |
|---|---|---|---|---|---|---|
| TOTAL SAMPLE | 33,048 | 0.008 | 1.206** | 1.221** | 262.52 | |
| t-value | | | 61.877 | 16.202 | | |
| Paris | 8,190 | 0.001 | 0.395** | 0.0624 | 1.253 | 1.290 |
| t-value | | | 29.243 | 1.119 | | |
| Amsterdam | 8,718 | 0.037 | 0.595** | 1.581** | 337.648 | 1.198 |
| t-value | | | 24.958 | 18.375 | | |
| Brussels | 8,097 | 0,005 | 1.294** | 0.206** | 5.310 | 1.502 |
| t-value | | | 60.976 | 2.304 | | |
| Lisbon | 8,043 | 0,023 | 2.702** | 2.545** | 185.84 | 0.797 |
| t-value | | | 52.655 | 13.632 | | |

** significance level 1%

Source: author's own estimation based on Euronext data.

When equation (1.23) was tested the results were substantially poorer as is shown in Table 1.6. For both cases (Table 1.5 and 1.6) the results are already corrected for autocorrelation.

**Table 1.6. Results of Equation 1.23**

| COUNTRY | # Obs | $R^2$ adj. | $\beta_0$ | $\beta_1$ | F | DW |
|---|---|---|---|---|---|---|
| TOTAL SAMPLE | 33,167 | 0.002 | 0.270** | -0.014** | 70.014 | |
| t-value | | | 127.621 | -8.367 | | |
| Paris | 8,181 | 0.000 | 0.213** | 0.002 | 0.921 | 1.026 |
| t-value | | | 140.896 | 0.959 | | |
| Amsterdam | 8,708 | 0.035 | 0.204** | 0.238** | 316.841 | 0.995 |
| t-value | | | 97.370 | 18.416 | | |
| Brussels | 8,087 | 0.001 | 0.190** | 0.004** | 7.585 | 0.910 |
| t-value | | | 80.11 | 2.754 | | |
| Lisbon | 8,033 | 0.178 | 0.175** | 0.009** | 145.466 | 0.863 |
| t-value | | | 59.851 | 13.603 | | |

** significance level 1%

Source: author's own estimation based on Euronext data.

In summary, it seems that implied volatilities extracted from closing bid and ask prices are linearly related to the historical (observed) volatilities during the day, and are a weak forecast for future volatilities for the following day. These conclusions observed in general for the entire sample are extendable to a country by country analysis. As a result, when volatility is high during the day, with a wide difference between the day maximum and minimum, a high implied volatility is expected in the closing spread, and therefore, liquidity traders are expected to pay a higher price for liquidity. As a conclusion, liquidity traders should avoid trading at the close of the market in order to avoid higher liquidity premiums. Assuming that implied volatilities are then related with historical prices, we tried to observe the influence of trading frequency, measured by the ratio we called *rotation* on such relation. Table 1.7 shows the descriptive statistics on the Turnover Rate for entire sample.

**Table 1.7. Turnover Rate**

| COUNTRY | Mean | Std Deviation |
|---|---|---|
| Paris | 0.00564 | 0.0045 |
| Amsterdam | 0.00274 | 0.0036 |
| Brussels | 0.00586 | 0.0070 |
| Lisbon | 0.00412 | 0.0086 |

Source: author's own estimation based on Euronext data.

Then we calculated the residuals given by the regression equation (1.22). We tested whether these residuals are linearly related to the rotation ratio. Assuming $\varepsilon_{i,t}$ as the residual obtained from regression equation (1.22) and $Rot_{i,t}$ as the rotation ratio for country i in day t, we hypothesised:

$$\varepsilon_t = \beta_0 + \beta_1 \text{Rot}_t + \varsigma_t \qquad (1.24)$$

The results are summarised in Table 1.8.

**Table 1.8. Results of Equation 1.24**

| COUNTRY | # Obs | $R^2$ adj. | $\beta_0$ | $\beta_1$ | F | DW |
|---|---|---|---|---|---|---|
| TOTAL SAMPLE | 33,048 | 0.007 | 0.126** | -27.174** | 236.00 | |
| t-value | | | 8.998 | -15.362 | | |
| Paris | 8,190 | 0.029 | -0.119** | 21.268** | 249.07 | 1.35 |
| t-value | | | -12.342 | 15.781 | | |
| Amsterdam | 8,718 | 0.019 | 0.147** | -25.056** | 165.53 | 1.21 |
| t-value | | | 8.245 | -12.866 | | |
| Brussels | 8,097 | 0.001 | -0.028* | 10.181** | 9.38 | 1.51 |
| t-value | | | -1.845 | 3.066 | | |
| Lisbon | 8,033 | 0.006 | 0.115** | -27.599** | 48.077 | 0.80 |
| t-value | | | 3.007 | -6.934 | | |

** significance level 1%; * significance level 5%

Source: author's own estimation based on Euronext data.

The results seem to show that stocks exchanges with higher frequency trading tend to present lower residuals observed in equation (1.22). This supports the raised hypothesis that the relation between implied volatility observed in spreads and historical volatilities is closer for frequently traded stocks than for thinly traded stocks. The negative relation observed in $\beta_1$ shows that as turnover rate (frequency) increases the errors observed in regression equation (1.22) tend to decrease.

For the last, we also test the implied volatility of each Stock Exchange. We estimate the linear regression with Dummies.

$$\sigma_{imp_t} = \beta_0 + \beta_1 D_1 \sigma_{histt} + \beta_2 D_2 \sigma_{histt} + \beta_3 D_3 \sigma_{histt} + \varepsilon_t \quad \textbf{(1.25)}$$

Where $D_1$ is Amsterdam, $D_2$ is Brussels and $D_3$ is Paris.

**Table 1.9. Results of equation 1.25**

| | # Obs | $R^2$ adj. | $\beta_0$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | F |
|---|---|---|---|---|---|---|---|
| TOTAL SAMPLE | 33,048 | 0.47 | 1.44 | 0.60** | -0.34** | 0.07 | 750.56 |
| t-value | | | 99.48 | 5.69 | -2.78 | 0.49 | |

** significance level 1%

Source: author's own estimation based on Euronext data.

As we stated in the beginning of the paper we rejected some observations that were inconsistent with the underlying "true" stock price hypothesis. However, it may well be that all observations become possibly reasonable if instead of an implied volatility parameter we select an implied probability for the dealer to expect a liquidity or an informed trader.

## *6. Conclusion*

The paper addresses the dealer problem facing informed and uninformed traders. In equilibrium the dealer sets the bid ask spread in order to maximize his expected profit. This is obtained by reducing the spread to a minimum in order to promote trading from liquidity traders, but increasing the spread to a point where the probability of facing informed traders is reduced.

The spread can also be viewed as a written strangle on the underlying stock price. Bid prices can be seen as the price by which dealers are obliged to buy (exercise price of a written call option) and the ask prices can be seen as the prices by which they are obliged to sell (exercise price of a written put option) whenever traders arrive and close the deal (exercise the option).

We developed an equilibrium condition for the theoretical value of the spread from which we derived implied volatilities for the underlying stocks.

Then we tested and concluded that it seems to exist a linear relation between implied volatilities, extracted from closing bid ask spreads, and historical volatilities, observed in the market during the trading day. This relation holds for every stock in the sample but seems to be dependent on liquidity: more liquid stocks present lower residuals.

Then implied volatilities were used as a forecasting tool of future volatility. However, the results failed to support such a relation.

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## *Essay Two: Illiquidity on Eurozone countries during the 2007-08 financial crisis*


# *Abstract*

Using a sample of 11 Eurozone countries, we evaluate the impact of the financial crisis, on (il)liquidity of stock markets. We analyse some indicators of liquidity, like the bid-ask spread, turnover rate and Amihud Illiquidity measure.

The financial crisis of 2007/2008 was one of the greatest financial crises since Great Depression and restructured the world of finance. There were many events during financial crisis, but was in September 2008, that crisis hit its most critical period. We use the 15th of September of 2008, the day that Lehman Brothers filed for Chapter 11 bankruptcy protection, to separate two periods of the financial crisis. His assumed that was after the bankrupting of the Lehman Brothers, that the stock markets fall deeper.

We examine countries in deeper trouble (Portugal, Ireland, Italy, Greece and Spain - PIIGS) versus the other countries in the Eurozone.

We concluded that the liquidity is more related with the size of the market then the fact of the country belongs to PIIGS or the others Eurozone countries.

# *1. Introduction*

A stock is liquid if it can be traded quickly and in a big quantity, at a lower price and with a few impacts in price. The stocks with little liquidity have a bigger transaction costs and to be more attractive should offer a bigger return.

In periods of crises and instability, the level of trust in market will decrease making a significant impact in liquidity of the stocks. Traders will allocate their resources in assets more liquid, making a decreasing in market liquidity.

Any investor needs to consider the liquidity risk in their decision and in periods of financial crisis, most of the emergent countries and some developed countries show stock markets decreasing and low liquidity, compromising the stock trades (Pasquariello, 2008).

To measure liquidity, we consider three alternative proxies for liquidity. The first one is the most popular measure of liquidity that has been used in other studies, the Bid-ask spread. We use directly observable trading activity variables to calculate the relative bid-ask spread in a basis day by day. The others two proxies is the turnover rate - the total number of shares traded over a period divided by the average number of shares outstanding for the period, and the Amihud illiquidity measure - the average between the absolute returns and its trading volumes for a certain stock over a time period.

We use a database of the prices, spreads, turnover and market value of the stocks of eleven Eurozone countries (Portugal, Ireland, Italy, Greece and Spain – PIIGS, and German, France, Netherlands, Austria, Belgium and Finland – Others countries) between $14^{th}$ March of 2008 and $15^{th}$ March of 2009. We assumed that the day that Lehman Brothers filed for Chapter 11 bankruptcy protection, $15^{th}$ of September of 2008, is the separate day of the period before and after the stock markets fall deeper during the financial crisis.

Our main research question is whether the effect of liquidity is stronger in times of crises and if stock markets of the countries known as PIIGS had more problems of liquidity then the others Eurozone countries.

We expect an illiquidity strong effect after the $15^{th}$ September 2008, when the crisis of stock market started on financial crisis 2007-08, the capital constraints became binding and inventory holding costs and search costs rose dramatically for all market participants.

The next section reviews some of the relevant literature on liquidity in periods of crisis. Section three presents the data and preliminary findings and section four describes the results. Section five presents the conclusions.

## *2. Literature Review*

In finance, the bid-ask spread is related with liquidity and information asymmetry. Higher information asymmetry and lower liquidity imply bigger spread. Demsetz (1968) was one of the first researchers to investigate the bid-ask spread. He considered that analysing bid-ask spread is similar to analysing the immediate supply and demand. Demsetz argued that a trader can pay a price for immediacy, so the bid-ask spread reflects total trading cost that trader will pay for transacting.

Amihud and Mendelson (1986) developed a model to analyze the relationship between return and risk (beta) and bid-ask spread. They demonstrated that return is an increasing function of risk and illiquidity. The greater the difference between the ask price and the bid price, the greater the illiquidity, indicating a premium requested from the seller to execute the order immediately. The increased liquidity of stocks traded, allows its holders sell them at a higher price. They also found that in equilibrium, liquidity is correlated with the frequency of trading.

Amihud and Mendelson (1989) found evidence that the return is negatively correlated with spread, and is affected by the size of the firm, January effect and by the sensitivity of the variations of the market index. Eleswarapu (1997) tested if abnormal returns of NASDAQ stocks were negative correlated with spread. He divided the stocks in 49 portfolios and used the Fama-Macbeth regression and SUR (seemingly unrelated regression). He found a negative correlation between spread and returns.

In another article Amihud and Mendelson (1991), analyzes the decrease in liquidity of the stocks during the stock market crash in New York in 1987, and they showed that there was also a reduction in prices. They found that the bid-ask increased during the crisis and liquidity and prices decrease. After the crisis, with decreased of bid-ask spread, the prices increased, corroborating the hypothesis that liquidity affects prices positively.

Cao and Petrasek (2013) studied what factors affect the relative performance of stocks during liquidity crises. In an event-study they used some market liquidity measures to identify the liquidity crisis. They concluded that abnormal stock returns during liquidity crises are strongly negatively related to liquidity risk, measured by the co-movement of stock returns with market liquidity.

Chordia, Sarkar and Subrahmanyam (2003) studied the commonality in liquidity for stocks and bonds markets. They concluded that the correlation between stock and bond market liquidity sharply increases during periods of crises and that the loss of liquidity in times of crisis is systemic. Tinic and West (1972), Stoll (1978) and Jegadeesh and Subrahmanyam (1993) find that spreads are correlated negatively with the price, volume and the number of market makers, and positively with volatility. Atkins and Dyl (1997), Glosten and Harris (1988), and Menyah and Paudyal (2000), showed that factors that determine the bid-ask spread are trading volume, volatility and market value.

Datar, Naik and Radcliffe (1998) use another proxy to measure the liquidity - turnover rate (stocks traded by stocks outstanding). They used no-financial firms listed in NYSE between 1963 and 1991 and they found that the returns are negative related with turnover rate, confirming the hypothesis that illiquidity stock offer bigger returns. They provided evidence for a negative correlation between liquidity and stock returns. Chordia *et al.* (2001), Marshall and Young (2003), Chan and Faff (2003) and Jun *et al.* (2003) also used the turnover rate to measure the liquidity and the conclusions are identical do Datar et al (1998).

Brennan, Chordia and Subrahmanyam (1998) used as proxy of liquidity the volume in money to identify the determinants of abnormal returns of stocks. They studied if returns are explained by characteristics of the firm like book value/market value, size, dividend and liquidity. They observed a significant and negative relation between return and volume in money for the stocks of NYSE and NASDAQ that is consistent with the liquidity premium in asset pricing. Chordia, Subrahmanyam and Anshuman (2001) made a similar study to Brennan, Chordia and Subrahmanyam (1998) but included another proxy – turnover rate.

Amihud (2002) studied the relation between illiquidity and return. He developed a measure to compare the absolute return with the volume in dollars. He concluded that illiquidity has a significant and positive effect over the expected returns.

Liu (2006) developed a model that incorporates the liquidity in CAPM. He used as a liquidity *proxy* the turnover standardized and adjusted for the number of days without negotiation. He shows that this model catches a liquidity premium that CAPM can't catch, and he also notes that the greatest liquidity declines in American stock market occur during the greatest economic and financial shocks.

Chen and Poon (2008) used the measure propose by Amihud in 2002 and they studied the liquidity in 37 countries. They concluded that periods of high volatility drives to situations of illiquidity. They also show that in periods of financial crises the illiquidity rises.

Duffie, Garleanu, and Pedersen (2007) show that in crisis period, liquidity have a bigger role, because inventory holding costs is higher. The investors more risk averse could shift their portfolio from illiquid to liquid assets.

Koutmos and Martin (2011) evaluate the impact of the Asian crisis on bid-ask spread of 21 emerging and developed country currencies. They concluded that for Asian emerging markets the spread widened instead the developed countries that spread narrowed. They detect incremental spread effects for smallest markets that could be attributed to an environment characterized by information asymmetries stemming from unstable economies.

Poon et al (2013) studies the effect of institutional investors trading on the liquidity characteristics of S&P 1500 stock during the 2007-2008 financial crisis. The mean quoted spread firms was 50% higher than for the three years preceding the crisis. There was a switch from buy position during the pre-crisis period to sell position during the crisis.

## *3. Data and Preliminary Findings*

The Eurozone currently consists in 18 European Union (EU) countries that have adopted the euro (€) as their common currency. On January 1, 1999 the euro was officially launched with 11 countries (Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain) and the Greece joined on January 1, 2001. The member countries replace their national currencies for euros on January 1, 2002.

The monetary policy of each Eurozone country is submitted to the European Central Bank (ECB). There were some rules to maintain the stability of euro that was imposed to the countries. Some countries such as Greece, Portugal, Ireland, Italy, and Spain (often referred to as "PIIGS") have more problems than the Others Eurozone Countries (OEC) during the Eurozone Crisis. We want to understand if the liquidity of stock markets of these countries (PIIGS) had different behaviour than the others Eurozone countries, on financial crisis of 2008. For this study we included 11 of the countries that start with the euro on January 1, 2002 (we only excluded the Luxembourg).

The data of the eleven countries was collected on Datastream for the period of 15$^{th}$ March 2008 until 15$^{th}$ March 2009 (252 days). We collected data of the 10 biggest firms of each country (we do not included firms quoted in stock exchange but not belong's of the stock exchange's country) and we calculate the market capitalization by country only for the 10 firms. We also calculate the average return, volatility and volume in 000€ to compare the differences between countries.

**Table 2.1. Mean of Return, Market Capitalization, Volatility and Volume in €**

| Country | Return | Market Cap. (M€) | Volatility | Volume in 000€ |
|---|---|---|---|---|
| | 15/03/08 to 15/03/09 | 15/03/08 to 15/03/09 | 15/03/08 to 15/03/09 | 15/03/08 to 15/03/09 |
| **PIIGS** | | | | |
| Portugal | -0.001878 | 37,489 | 0.401374 | 18,418 |
| Spain | -0.002243 | 298,499 | 0.433914 | 411,416 |
| Italy | -0.003136 | 278,922 | 0.466487 | 360,824 |
| Ireland | -0.002954 | 30,738 | 0.657425 | 23,565 |
| Greece | -0.003291 | 43,898 | 0.538908 | 14,311 |
| **OEC** | | | | |
| Germany | -0.001737 | 335,276 | 0.487647 | 322,083 |
| Netherlands | -0.002287 | 268,275 | 0.509538 | 215,028 |
| France | -0.001438 | 412,066 | 0.487093 | 275,829 |
| Finland | -0.003069 | 104,839 | 0.516268 | 227,164 |
| Belgium | -0.002439 | 99,730 | 0.565210 | 50,373 |
| Austria | -0.002442 | 43,348 | 0.599535 | 23,908 |

Source: author's own estimation based on Datastream data.

The return in all countries are negative during the period studied and the volatility is high. Table 2.1 demonstrate that there are big and small stock markets on PIIGS and Others Eurozone Countries. We can designate Portugal, Ireland, Greece and Austria as Small Stock Exchanges, Finland and Belgium as Medium Stock Exchanges, and Spain, Italy, Germany, France and Netherlands as Large Stock Exchanges.

## 3.1. Liquidity proxies

For this research, we use three liquidity proxies. The bid-ask spread (BAS), turnover rate and Amihud Illiquidity Ratio.

The bid-ask spread was calculated relative to its midpoint:

$$BAS_i = \frac{ask_i - bid_i}{(ask_i + bid_i)/2} \times 100 \qquad \textbf{(2.1)}$$

where $ask_i$ is the closing ask price in Euros at day *i*, and $bid_i$ is the closing bid price in Euros at day *i*.

There are many calculations of the bid-ask spread (effective, quoted, proportional), but we prefer the relative bid-ask spread because is the most liquidity measure studied because it makes spreads of different stocks comparable to each other.

Table 2.2 reports the relative bid-ask spread mean for the entire examination period and for two sub-periods. The first sub-period, 15th March 2008 to 14th September 2008 and the second sub-period, 15th September 2008 to 15th March 2009.

**Table 2.2. Relative Bid-Ask Spread Mean**

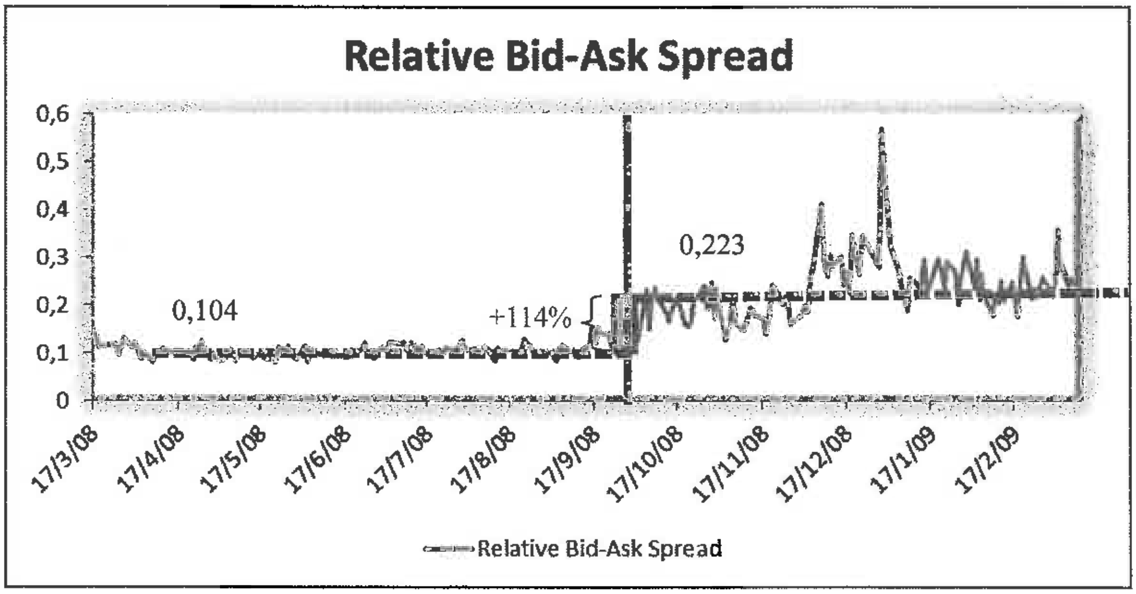
| Country | Full Period | Pre | Post | Variation |
|---|---|---|---|---|
| | 15/03/2008 | 15/03/2008 | 15/09/2008 | |
| | 15/03/2009 | 14/09/2008 | 15/03/2009 | Pre - Post |
| **PIIGS** | | | | |
| Portugal | 0.324839 | 0.290533 | 0.359695 | 23.81% |
| Spain | 0.109668 | 0.086677 | 0.133028 | 53.47% |
| Italy | 0.310851 | 0.069697 | 0.555864 | 697.54% |
| Ireland | 0.689344 | 0.543850 | 0.837167 | 53.93% |
| Greece | 0.780319 | 0.637296 | 0.925630 | 45.24% |
| **OEC** | | | | |
| Germany | 0.093460 | 0.061998 | 0.125424 | 102.30% |
| Netherlands | 0.086484 | 0.067761 | 0.105507 | 55.70% |
| France | 0.068324 | 0.055352 | 0.081505 | 47.25% |
| Finland | 0.141775 | 0.110023 | 0.174035 | 58.18% |
| Belgium | 0.207688 | 0.139726 | 0.276738 | 98.06% |
| Austria | 0.470776 | 0.262277 | 0.682612 | 160.26% |
| **All** | **0.163317** | **0.104211** | **0.223369** | **114.34%** |

Source: author's own estimation based on Datastream data.

It can be seen in Table 2.2 that all BAS increases in the second sub-period, after the Lehman Brothers bankrupting. This is consistent with the theory that in periods of crisis, the bid-ask spread increases.

On Figure 2.1, we demonstrate for all 11 countries the rise of the BAS after the 15th September 2008. The variation between the two sub-periods was more than 114%.

**Figure 2.1. Relative bid-ask spread of all 11 countries of Eurozone, before and after the 15/09/2008**




Source: author's own estimation based on Datastream data.

The Turnover Rate is calculated by dividing the total number of shares traded over a period by the average number of shares outstanding for the period. We used the logarithmic of Turnover Rate like Datar et al (1998) had used. In Table 2.3

$$\text{Turnover Rate}_t = \frac{\#\text{shares traded in t}}{\#\text{shares outstanding in t}} \qquad \textbf{(2.2)}$$

**Table 2.3. Turnover Rate Mean**

| Country | Full Period<br>15/03/2008<br>15/03/2009 | Pre<br>15/03/2008<br>14/09/2008 | Post<br>15/09/2008<br>15/03/2009 | Variation<br><br>Pre - Post |
|---|---|---|---|---|
| **PIIGS** | | | | |
| Portugal | 0.00373 | 0.004121 | 0.003334 | -19.11% |
| Spain | 0.009684 | 0.008951 | 0.010429 | 16.52% |
| Italy | 0.008708 | 0.009301 | 0.008105 | -12.87% |
| Ireland | 0.004065 | 0.003851 | 0.004282 | 11.19% |
| Greece | 0.003148 | 0.002995 | 0.003304 | 10.30% |
| **OEC** | | | | |
| Germany | 0.009012 | 0.008537 | 0.009494 | 11.21% |
| Netherlands | 0.006172 | 0.005545 | 0.006810 | 22.82% |
| France | 0.005549 | 0.005055 | 0.006051 | 19.71% |
| Finland | 0.006650 | 0.006366 | 0.006938 | 8.99% |
| Belgium | 0.003832 | 0.003099 | 0.004577 | 47.69% |
| Austria | 0.003607 | 0.003516 | 0.003699 | 5.21% |
| **ALL** | 0.007130 | 0.006754 | 0.007513 | 11.24% |

Source: author's own estimation based on Datastream data.

The lagest stock markets have a bigger turnover rate wich demonstrates they have more liquidity. Only in Portugal and Italy the turnover rate decreases in the second sub-period.

Amihud (2002) proposed a liquidity proxy based on Kyle (1985) measure. The Amihud illiquidity measure relates the return to the trade volume measure in dollars (in this case, euros). The Amihud illiquidity measure is defined as the average between the absolute returns ($r_j$) and its trading volumes ($v_j$), a t day for a certain stock over a time period with N observed, i.e.,

$$Amihud_t = \frac{1}{N_t}\sum_{j=1}^{N_t}\frac{|r_j|}{v_j} \qquad \textbf{(2.3)}$$

A higher Amihud illiquidity measure means that trading a stock causes its price to move more in response to a given volume of trading, in turn, implying lower liquidity. We use the daily volume and daily price to generate the returns and calculate the Amihud illiquidity measure on a day-by-day basis.

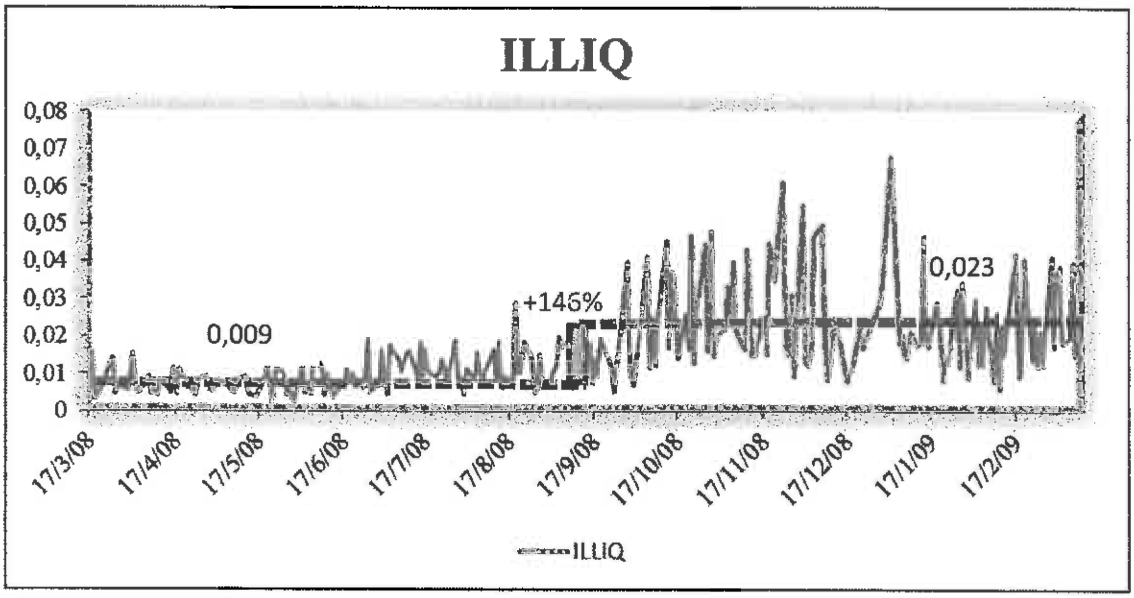
**Table 2.4. Amihud Illiquidity Measure Mean**

| Country | Full Period | Pre | Post | Variation |
|---|---|---|---|---|
| | 15/03/2008 | 15/03/2008 | 15/09/2008 | |
| | 15/03/2009 | 14/09/2008 | 15/03/2009 | Pre - Post |
| **PIIGS** | | | | |
| Portugal | 0.088977 | 0.053504 | 0.125018 | 133.66% |
| Spain | 0.004817 | 0.003203 | 0.006457 | 101.61% |
| Italy | 0.006630 | 0.002742 | 0.010581 | 285.88% |
| Ireland | 0.116244 | 0.102134 | 0.130580 | 27.85% |
| Greece | 0.074904 | 0.045316 | 0.104965 | 131.63% |
| **OEC** | | | | |
| Germany | 0.005634 | 0.002483 | 0.008835 | 255.80% |
| Netherlands | 0.009984 | 0.004463 | 0.015593 | 249.40% |
| France | 0.006915 | 0.004118 | 0.009757 | 136.91% |
| Finland | 0.010427 | 0.004995 | 0.015946 | 219.21% |
| Belgium | 0.045628 | 0.027665 | 0.063878 | 130.90% |
| Austria | 0.143747 | 0.044848 | 0.244227 | 444.56% |
| **ALL** | 0.016059 | 0.009324 | 0.022903 | 145.63% |

Table 2.4 show us that in small markets the Amihud illiquidity measure is bigger (less liquidity) and after 15th September 2008 the Amihud illiquidity measure increase significantly in all countries. This is consistent with the theory that in periods of crisis, the illiquidity increases.

In Figure 2.2, we demonstrate for all 11 countries the rise of the Amihud illiquidity measure after the 15th September 2008. The variation between the two sub-periods was more than 146%.

**Figure 2.2. Amihud Illiquidty Measure of all 11 countries of Eurozone, before and and after the 15/09/2008**




Source: author's own estimation based on Datastream data.

### 3.2. Sample correlations

Table 2.5 presents a summary of the time-series average of cross-sectional correlation between all variables used in regressions. Some features are worthy of mention. In most of countries, the BAS is negative correlated with the return, what is consistent with other studies. The variable more correlated with the BAS and Amihud illiquidity measure is the Market Value. For the full period, this correlation is always negative, but in the two sub-periods is not consistent.

Most of the variables have small correlation between them and there are no significant differences in results between PIIGS and the others Eurozone countries.

# Table 2.5. Correlations between variables

| | Full Period 15-03-2008 to 15-03-2009 | | | | | | | Pre 15-03-2008 to 15-03-2009 | | | | | | | Post 15-03-2008 to 15-03-2009 | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Portugal** | | | | | | | | | | | | | | | | | | | | | |
| | **baspor** | **retpor** | **ilpor** | **lnturnor** | **lnvopor** | **lnvolatpor** | **lnmvpor** | **baspor** | **retpor** | **ilpor** | **lnturnor** | **lnvopor** | **lnvolatpor** | **lnmvpor** | **baspor** | **retpor** | **ilpor** | **lnturnor** | **lnvopor** | **lnvolatpor** | **lnmvpor** |
| **baspor** | 1,0000 | | | | | | | 1,0000 | | | | | | | 1,0000 | | | | | | |
| **retpor** | -0,1746 | 1,0000 | | | | | | 0,0584 | 1,0000 | | | | | | -0,2467 | 1,0000 | | | | | |
| **ilpor** | 0,3145 | -0,1339 | 1,0000 | | | | | 0,0735 | -0,0905 | 1,0000 | | | | | 0,2638 | -0,1449 | 1,0000 | | | | |
| **lnturnpor** | 0,1422 | -0,0088 | -0,1228 | 1,0000 | | | | 0,1280 | 0,0113 | -0,0547 | 1,0000 | | | | 0,3079 | -0,0337 | 0,0215 | 1,0000 | | | |
| **lnvopor** | 0,0889 | 0,0143 | -0,1236 | 0,9494 | 1,0000 | | | 0,0833 | -0,0023 | -0,0336 | 0,9396 | 1,0000 | | | 0,2867 | 0,0068 | 0,0617 | 0,9519 | 1,0000 | | |
| **lnvolatpor** | 0,3203 | 0,0081 | 0,3372 | 0,0785 | 0,0221 | 1,0000 | | 0,3490 | 0,1639 | 0,1918 | 0,1445 | 0,1109 | 1,0000 | | 0,1640 | -0,0203 | 0,1408 | 0,3731 | 0,3692 | 1,0000 | |
| **lnmvpor** | -0,2692 | -0,0419 | -0,4357 | 0,3613 | 0,4267 | -0,4899 | 1,0000 | -0,2765 | -0,0466 | -0,4390 | -0,0153 | 0,0568 | -0,5034 | 1,0000 | 0,0782 | -0,2194 | -0,0467 | 0,4427 | 0,4302 | 0,2412 | 1,0000 |
| **Spain** | | | | | | | | | | | | | | | | | | | | | |
| | **basspa** | **retspa** | **ilspa** | **lnturnspa** | **lnvospa** | **lnvolatspa** | **lnmvspa** | **basspa** | **retspa** | **ilspa** | **lnturnspa** | **lnvospa** | **lnvolatspa** | **lnmvspa** | **basspa** | **retspa** | **ilspa** | **lnturnspa** | **lnvospa** | **lnvolatspa** | **lnmvspa** |
| **basspa** | 1,0000 | | | | | | | 1,0000 | | | | | | | 1,0000 | | | | | | |
| **retspa** | -0,1022 | 1,0000 | | | | | | -0,1782 | 1,0000 | | | | | | -0,0823 | 1,0000 | | | | | |
| **ilspa** | 0,4356 | -0,0518 | 1,0000 | | | | | 0,2637 | -0,0540 | 1,0000 | | | | | 0,2227 | -0,0325 | 1,0000 | | | | |
| **lnturnspa** | 0,1661 | -0,0395 | 0,1290 | 1,0000 | | | | -0,0943 | 0,0518 | -0,0505 | 1,0000 | | | | 0,0731 | -0,0652 | 0,0989 | 1,0000 | | | |
| **lnvospa** | -0,3187 | 0,0547 | -0,1600 | 0,5626 | 1,0000 | | | -0,1494 | 0,0786 | -0,0182 | 0,6223 | 1,0000 | | | -0,0595 | 0,0273 | -0,0208 | 0,7585 | 1,0000 | | |
| **lnvolatspa** | 0,5800 | 0,0175 | 0,4483 | 0,3254 | -0,1188 | 1,0000 | | 0,1286 | 0,1269 | 0,2984 | 0,0847 | 0,1324 | 1,0000 | | 0,0046 | 0,0693 | 0,2264 | 0,3762 | 0,3587 | 1,0000 | |
| **lnmvspa** | -0,7642 | 0,0978 | -0,4312 | -0,1058 | 0,4328 | -0,6691 | 1,0000 | -0,2675 | 0,0848 | -0,3919 | 0,0498 | 0,1153 | -0,4925 | 1,0000 | -0,4345 | 0,1253 | -0,1223 | 0,1954 | 0,3502 | 0,1970 | 1,0000 |
| **Italy** | | | | | | | | | | | | | | | | | | | | | |
| | **basita** | **retita** | **ilita** | **lnturnta** | **lnvoita** | **lnvolatita** | **lnmvita** | **basita** | **retita** | **ilita** | **lnturnta** | **lnvoita** | **lnvolatita** | **lnmvita** | **basita** | **retita** | **ilita** | **lnturnta** | **lnvoita** | **lnvolatita** | **lnmvita** |
| basita | 1,0000 | | | | | | | 1,0000 | | | | | | | 1,0000 | | | | | | |
| retita | -0,0441 | 1,0000 | | | | | | -0,1397 | 1,0000 | | | | | | 0,0146 | 1,0000 | | | | | |
| ilita | 0,4351 | -0,0836 | 1,0000 | | | | | 0,2254 | -0,0717 | 1,0000 | | | | | 0,1678 | -0,0436 | 1,0000 | | | | |
| lnturnita | -0,4085 | -0,0581 | -0,2171 | 1,0000 | | | | -0,1185 | 0,0162 | -0,2781 | 1,0000 | | | | -0,4551 | -0,1132 | -0,0992 | 1,0000 | | | |
| lnvoita | 0,0298 | -0,0603 | 0,2768 | 0,4138 | 1,0000 | | | 0,0832 | -0,0254 | 0,1275 | 0,4048 | 1,0000 | | | -0,4118 | -0,0343 | 0,1004 | 0,6759 | 1,0000 | | |
| lnvolatita | 0,4061 | -0,0125 | 0,5604 | -0,2659 | 0,4109 | 1,0000 | | 0,3490 | 0,1416 | 0,2652 | -0,1384 | 0,1263 | 1,0000 | | -0,3518 | 0,1091 | 0,2734 | -0,1564 | 0,1930 | 1,0000 | |
| lnmvita | -0,7228 | 0,0946 | -0,5136 | 0,3121 | -0,2806 | -0,7234 | 1,0000 | -0,5265 | 0,0838 | -0,2979 | 0,3180 | -0,0254 | -0,3758 | 1,0000 | -0,5124 | 0,0335 | -0,1152 | 0,2308 | 0,2404 | 0,2346 | 1,0000 |

| | Full Period 15-03-2008 to 15-03-2009 | | | | | | | Pre 15-03-2008 to 15-03-2009 | | | | | | | Post 15-03-2008 to 15-03-2009 | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Ireland** | | | | | | | | | | | | | | | | | | | | | |
| | **basire** | **retire** | **ilire** | **lnturnire** | **lnvoire** | **lnvolatire** | **lnmvire** | **basire** | **retire** | **ilire** | **lnturnire** | **lnvoire** | **lnvolatire** | **lnmvire** | **basire** | **retire** | **ilire** | **lnturnire** | **lnvoire** | **lnvolatire** | **lnmvire** |
| basire | 1,0000 | | | | | | | 1,0000 | | | | | | | 1,0000 | | | | | | |
| retire | 0,1136 | 1,0000 | | | | | | 0,1435 | 1,0000 | | | | | | 0,1461 | 1,0000 | | | | | |
| ilire | 0,2170 | -0,2448 | 1,0000 | | | | | 0,1592 | -0,1377 | 1,0000 | | | | | 0,1788 | -0,3143 | 1,0000 | | | | |
| lnturnire | 0,1617 | 0,0900 | -0,0872 | 1,0000 | | | | 0,1038 | 0,1211 | -0,1552 | 1,0000 | | | | 0,1435 | 0,0714 | -0,0592 | 1,0000 | | | |
| lnvoire | -0,0008 | -0,0458 | -0,0470 | 0,4379 | 1,0000 | | | -0,1603 | 0,0772 | -0,2298 | 0,2878 | 1,0000 | | | 0,1558 | -0,1418 | 0,1161 | 0,6122 | 1,0000 | | |
| lnvolatire | 0,5011 | 0,0484 | 0,3205 | 0,1756 | -0,1782 | 1,0000 | | 0,3075 | 0,1127 | 0,4362 | 0,0868 | -0,4722 | 1,0000 | | 0,2970 | 0,0375 | 0,1425 | 0,2599 | 0,3089 | 1,0000 | |
| lnmvire | -0,5192 | 0,0338 | -0,2151 | -0,0384 | 0,2208 | -0,6841 | 1,0000 | -0,2181 | 0,0442 | -0,4289 | -0,1338 | 0,4431 | -0,7349 | 1,0000 | -0,0523 | 0,0359 | 0,0365 | 0,5047 | 0,3720 | 0,4041 | 1,0000 |

| **Greece** | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **basgre** | **retgre** | **ilgre** | **lnturngre** | **lnvogre** | **lnvolatgre** | **lnmvgre** | **basgre** | **retgre** | **ilgre** | **lnturngre** | **lnvogre** | **lnvolatgre** | **lnmvgre** | **basgre** | **retgre** | **ilgre** | **lnturngre** | **lnvogre** | **lnvolatgre** | **lnmvgre** |
| **basgre** | 1,0000 | | | | | | | 1,0000 | | | | | | | 1,0000 | | | | | | |
| **retgre** | -0,1212 | 1,0000 | | | | | | -0,0478 | 1,0000 | | | | | | -0,0836 | 1,0000 | | | | | |
| **ilgre** | 0,2721 | -0,1342 | 1,0000 | | | | | 0,0468 | -0,1170 | 1,0000 | | | | | 0,0593 | -0,0947 | 1,0000 | | | | |
| **lnturngre** | 0,1149 | -0,0014 | -0,1180 | 1,0000 | | | | 0,0343 | 0,1845 | -0,0389 | 1,0000 | | | | 0,1810 | -0,1604 | -0,2847 | 1,0000 | | | |
| **lnvogre** | 0,0808 | -0,0940 | -0,2043 | 0,3465 | 1,0000 | | | 0,2395 | -0,0675 | 0,1054 | 0,2921 | 1,0000 | | | -0,0055 | -0,1192 | -0,3893 | 0,4497 | 1,0000 | | |
| **lnvolatgre** | 0,4046 | -0,0496 | 0,2726 | 0,1913 | 0,1284 | 1,0000 | | 0,2189 | 0,1572 | 0,0637 | 0,0403 | 0,2654 | 1,0000 | | -0,0313 | -0,0236 | -0,0340 | 0,4142 | 0,1124 | 1,0000 | |
| **lnmvgre** | -0,5291 | 0,1104 | -0,4472 | 0,0332 | 0,1072 | -0,5986 | 1,0000 | -0,2675 | 0,2394 | -0,2180 | 0,1384 | -0,1366 | -0,0180 | 1,0000 | 0,0634 | -0,0774 | -0,1662 | 0,4173 | 0,6110 | 0,1656 | 1,0000 |

| | Full Period 15-03-2008 to 15-03-2009 | | | | | | | Pre 15-03-2008 to 15-03-2009 | | | | | | | Post 15-03-2008 to 15-03-2009 | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Germany** | | | | | | | | | | | | | | | | | | | | | |
| | basger | retger | ilger | lnturnger | lnvoger | lnvolatger | lnmvger | basger | retger | ilger | lnturnger | lnvoger | lnvolatger | lnmvger | basger | retger | ilger | lnturnger | lnvoger | lnvolatger | lnmvger |
| basger | 1,0000 | | | | | | | 1,0000 | | | | | | | 1,0000 | | | | | | |
| retger | -0,1496 | 1,0000 | | | | | | 0,0531 | 1,0000 | | | | | | -0,1652 | 1,0000 | | | | | |
| ilger | 0,4045 | 0,0023 | 1,0000 | | | | | -0,0004 | -0,1188 | 1,0000 | | | | | 0,0780 | 0,0586 | 1,0000 | | | | |
| lnturnger | 0,3676 | -0,0591 | 0,1402 | 1,0000 | | | | 0,3115 | 0,0186 | 0,1742 | 1,0000 | | | | 0,4529 | -0,0745 | 0,1017 | 1,0000 | | | |
| lnvoger | 0,2952 | -0,0703 | 0,1275 | 0,4979 | 1,0000 | | | 0,1862 | 0,1191 | -0,1166 | 0,3573 | 1,0000 | | | 0,3161 | -0,1277 | 0,1009 | 0,5910 | 1,0000 | | |
| lnvolatger | 0,6507 | 0,0252 | 0,5561 | 0,1893 | 0,2456 | 1,0000 | | 0,2428 | 0,1916 | 0,0811 | 0,2059 | 0,0035 | 1,0000 | | 0,3250 | 0,1115 | 0,2529 | 0,1902 | 0,3289 | 1,0000 | |
| lnmvger | -0,5771 | 0,0611 | -0,5853 | 0,0235 | -0,0947 | -0,7682 | 1,0000 | -0,1333 | 0,1529 | -0,1560 | -0,1532 | 0,1178 | 0,1033 | 1,0000 | 0,0299 | 0,0017 | -0,2280 | 0,3730 | 0,1331 | -0,2256 | 1,0000 |
| **Netherlands** | | | | | | | | | | | | | | | | | | | | | |
| | basnet | retnet | ilnet | lnturnnet | lnvonet | lnvolatnet | lnmvnet | basnet | retnet | ilnet | lnturnnet | lnvonet | lnvolatnet | lnmvnet | basnet | retnet | ilnet | lnturnnet | lnvonet | lnvolatnet | lnmvnet |
| basnet | 1,0000 | | | | | | | 1,0000 | | | | | | | 1,0000 | | | | | | |
| retnet | -0,0737 | 1,0000 | | | | | | -0,0568 | 1,0000 | | | | | | -0,0494 | 1,0000 | | | | | |
| ilnet | 0,4666 | -0,0245 | 1,0000 | | | | | 0,2581 | -0,0679 | 1,0000 | | | | | 0,3355 | 0,0155 | 1,0000 | | | | |
| lnturnnet | 0,1858 | -0,1039 | 0,0925 | 1,0000 | | | | -0,0305 | -0,1259 | -0,0527 | 1,0000 | | | | 0,1331 | -0,0884 | -0,0416 | 1,0000 | | | |
| lnvonet | 0,2892 | -0,0712 | 0,1880 | 0,8380 | 1,0000 | | | -0,0594 | -0,0792 | -0,0545 | 0,7600 | 1,0000 | | | 0,2570 | -0,0491 | 0,0415 | 0,8666 | 1,0000 | | |
| lnvolatnet | 0,4620 | -0,0112 | 0,4848 | 0,3363 | 0,4508 | 1,0000 | | 0,0946 | 0,0663 | 0,1384 | 0,2486 | 0,1058 | 1,0000 | | 0,2015 | 0,0839 | 0,1533 | 0,2361 | 0,4083 | 1,0000 | |
| lnmvnet | -0,4060 | -0,0154 | -0,4688 | -0,1676 | -0,2539 | -0,8181 | 1,0000 | -0,1710 | -0,0304 | -0,1652 | -0,2643 | -0,1200 | -0,8656 | 1,0000 | 0,0508 | -0,2140 | -0,0390 | 0,3470 | 0,2634 | -0,0801 | 1,0000 |
| **France** | | | | | | | | | | | | | | | | | | | | | |
| | basfra | retfra | ilfra | lnturnfra | lnvofra | lnvolatfra | lnmvfra | basfra | retfra | ilfra | lnturnfra | lnvofra | lnvolatfra | lnmvfra | basfra | retfra | ilfra | lnturnfra | lnvofra | lnvolatfra | lnmvfra |
| basfra | 1,0000 | | | | | | | 1,0000 | | | | | | | 1,0000 | | | | | | |
| retfra | 0,0803 | 1,0000 | | | | | | 0,0598 | 1,0000 | | | | | | 0,1130 | 1,0000 | | | | | |
| ilfra | 0,4604 | -0,0042 | 1,0000 | | | | | 0,2747 | 0,0132 | 1,0000 | | | | | 0,3905 | 0,0252 | 1,0000 | | | | |
| lnturnfra | 0,0406 | 0,0335 | 0,1098 | 1,0000 | | | | 0,0146 | 0,1094 | 0,1623 | 1,0000 | | | | -0,0466 | 0,0277 | -0,0267 | 1,0000 | | | |
| lnvofra | -0,0355 | -0,0161 | 0,0705 | 0,8994 | 1,0000 | | | -0,0383 | 0,0192 | 0,1299 | 0,8676 | 1,0000 | | | -0,1503 | -0,0106 | -0,0888 | 0,9097 | 1,0000 | | |
| lnvolatfra | 0,3130 | 0,0118 | 0,4424 | 0,2974 | 0,3171 | 1,0000 | | 0,0113 | 0,2008 | 0,2209 | 0,2030 | 0,2173 | 1,0000 | | 0,0983 | 0,0853 | 0,1179 | 0,2427 | 0,2492 | 1,0000 | |
| lnmvfra | -0,2997 | -0,0311 | -0,4207 | -0,0976 | -0,0788 | -0,6839 | 1,0000 | -0,0647 | -0,1045 | -0,2538 | -0,3528 | -0,2585 | -0,5150 | 1,0000 | -0,0528 | -0,1474 | -0,0704 | 0,3262 | 0,3747 | 0,2554 | 1,0000 |

| | Full Period<br>15-03-2008 to 15-03-2009 | | | | | | | Pre<br>15-03-2008 to 15-03-2009 | | | | | | | Post<br>15-03-2008 to 15-03-2009 | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Finland** | | | | | | | | | | | | | | | | | | | | | |
| | **basfin** | **retfin** | **ilfin** | **lnturnfin** | **lnvofin** | **lnvolatfin** | **lnmvfin** | **basfin** | **retfin** | **ilfin** | **lnturnfin** | **lnvofin** | **lnvolatfin** | **lnmvfin** | **basfin** | **retfin** | **ilfin** | **lnturnfin** | **lnvofin** | **lnvolatfin** | **lnmvfin** |
| **basfin** | 1,0000 | | | | | | | 1,0000 | | | | | | | 1,0000 | | | | | | |
| **retfin** | -0,0917 | 1,0000 | | | | | | -0,1975 | 1,0000 | | | | | | -0,0368 | 1,0000 | | | | | |
| **ilfin** | 0,3523 | -0,0666 | 1,0000 | | | | | 0,1221 | 0,0047 | 1,0000 | | | | | 0,0840 | -0,0558 | 1,0000 | | | | |
| **lnturnfin** | -0,0610 | -0,0465 | -0,0432 | 1,0000 | | | | -0,1101 | -0,0706 | -0,0621 | 1,0000 | | | | -0,2164 | -0,0237 | -0,1712 | 1,0000 | | | |
| **lnvofin** | 0,0170 | 0,0319 | 0,0923 | 0,6912 | 1,0000 | | | -0,0584 | -0,0219 | 0,0139 | 0,5733 | 1,0000 | | | -0,1965 | 0,0758 | -0,0481 | 0,7842 | 1,0000 | | |
| **lnvolatfin** | 0,3964 | 0,0306 | 0,3496 | 0,2415 | 0,2980 | 1,0000 | | -0,0727 | 0,1178 | -0,1272 | 0,2252 | -0,0048 | 1,0000 | | -0,0395 | 0,0999 | -0,0011 | 0,2015 | 0,3209 | 1,0000 | |
| **lnmvfin** | -0,5623 | 0,0783 | -0,5059 | -0,0408 | -0,0879 | -0,5240 | 1,0000 | -0,1068 | 0,1194 | -0,1710 | 0,0853 | -0,0785 | 0,3902 | 1,0000 | -0,1087 | 0,0528 | -0,1429 | 0,2548 | 0,4716 | 0,5339 | 1,0000 |
| **Belgium** | | | | | | | | | | | | | | | | | | | | | |
| | **basbel** | **retbel** | **ilbel** | **lnturnbel** | **lnvobel** | **lnvolatbel** | **lnmvbel** | **basbel** | **retbel** | **ilbel** | **lnturnbel** | **lnvobel** | **lnvolatbel** | **lnmvbel** | **basbel** | **retbel** | **ilbel** | **lnturnbel** | **lnvobel** | **lnvolatbel** | **lnmvbel** |
| **basbel** | 1,0000 | | | | | | | 1,0000 | | | | | | | 1,0000 | | | | | | |
| **retbel** | -0,1147 | 1,0000 | | | | | | -0,1375 | 1,0000 | | | | | | -0,1099 | 1,0000 | | | | | |
| **ilbel** | 0,3709 | -0,1318 | 1,0000 | | | | | 0,0679 | -0,0186 | 1,0000 | | | | | 0,2416 | -0,1501 | 1,0000 | | | | |
| **lnturnbel** | 0,3383 | -0,0664 | 0,1765 | 1,0000 | | | | -0,0057 | -0,0710 | 0,0383 | 1,0000 | | | | 0,1660 | -0,0509 | -0,0225 | 1,0000 | | | |
| **lnvobel** | 0,0582 | -0,0471 | 0,0507 | 0,3570 | 1,0000 | | | -0,0946 | 0,1245 | -0,0894 | 0,1913 | 1,0000 | | | 0,0726 | -0,1487 | 0,0802 | 0,5286 | 1,0000 | | |
| **lnvolatbel** | 0,5467 | 0,0127 | 0,4639 | 0,5199 | 0,0098 | 1,0000 | | 0,0000 | 0,1135 | 0,4002 | 0,1872 | -0,2051 | 1,0000 | | 0,3716 | 0,0644 | 0,2642 | 0,2894 | 0,0224 | 1,0000 | |
| **lnmvbel** | -0,5396 | -0,0528 | -0,4224 | -0,4503 | 0,0295 | -0,8897 | 1,0000 | -0,0342 | -0,0591 | -0,4654 | -0,2517 | 0,2488 | -0,8143 | 1,0000 | -0,2898 | -0,2375 | -0,1263 | 0,0321 | 0,1916 | -0,4751 | 1,0000 |
| **Austria** | | | | | | | | | | | | | | | | | | | | | |
| | **basaus** | **retaus** | **ilaus** | **lnturnaus** | **lnvoaus** | **lnvolataus** | **lnmvaus** | **basaus** | **retaus** | **ilaus** | **lnturnaus** | **lnvoaus** | **lnvolataus** | **lnmvaus** | **basaus** | **retaus** | **ilaus** | **lnturnaus** | **lnvoaus** | **lnvolataus** | **lnmvaus** |
| **basaus** | 1,0000 | | | | | | | 1,0000 | | | | | | | 1,0000 | | | | | | |
| **retaus** | -0,1828 | 1,0000 | | | | | | -0,0277 | 1,0000 | | | | | | -0,2349 | 1,0000 | | | | | |
| **ilaus** | 0,5559 | -0,0759 | 1,0000 | | | | | -0,1436 | -0,2115 | 1,0000 | | | | | 0,2622 | -0,0384 | 1,0000 | | | | |
| **lnturnaus** | -0,0272 | 0,0241 | -0,1938 | 1,0000 | | | | -0,0587 | -0,0459 | 0,0116 | 1,0000 | | | | -0,0507 | 0,0478 | -0,3123 | 1,0000 | | | |
| **lnvoaus** | 0,4707 | 0,0646 | 0,3716 | 0,1684 | 1,0000 | | | 0,0243 | -0,0589 | -0,1049 | 0,4515 | 1,0000 | | | 0,0078 | 0,1832 | 0,0232 | 0,1014 | 1,0000 | | |
| **lnvolataus** | 0,6978 | -0,0286 | 0,6053 | 0,0431 | 0,4868 | 1,0000 | | -0,0960 | -0,0100 | 0,1443 | 0,1699 | -0,1675 | 1,0000 | | 0,2322 | 0,0745 | 0,2842 | 0,0206 | -0,1512 | 1,0000 | |
| **lnmvaus** | -0,6770 | 0,0584 | -0,5818 | 0,0485 | -0,5742 | -0,8388 | 1,0000 | 0,3111 | 0,1313 | -0,1766 | -0,1892 | 0,2484 | -0,4413 | 1,0000 | 0,0378 | -0,0784 | -0,1551 | 0,5439 | -0,0444 | 0,2166 | 1,0000 |

## *4. Methodology and Results*

To analyse the illiquidity effects we use three liquidity proxies, the bid-ask spread, turnover rate and Amihud illiquidity measure. To explain those proxies, we use two variables that was more common in literature – return (Ret) and logarithmic volume (LnVol), and we use also the logarithmic market value (LnMV) and logarithmic volatility (LnVolat) that was used in less studies. We calculated the results for all sample and for the two sub-periods.

Furthermore, before employing the variables in econometric modeling, we check the stationarity condition for the time series of stock returns, trading volume and bid-ask spreads using the augmented Dickey-Fuller (ADF) test. Our results (not shown here) reveal that all three time series can be considered stationary.

We use a multiple regression for the three proxies.

$$BAS_t = \beta_0 + \beta_1 \mathrm{Ret}_t + \beta_2 LnVolat_t + \beta_3 LnVol_t + \beta_4 LnMV_t + \varepsilon_t \quad \textbf{(2.4)}$$

$$Turnover\ Rate_t = \beta_0 + \beta_1 \mathrm{Ret}_t + \beta_2 LnVolat_t + \beta_3 LnVol_t + \beta_4 LnMV_t + \varepsilon_t \quad \textbf{(2.5)}$$

$$Amihud\ Illiquidity_t = \beta_0 + \beta_1 \mathrm{Ret}_t + \beta_2 LnVolat_t + \beta_3 LnVol_t + \beta_4 LnMV_t + \varepsilon_t \quad \textbf{(2.6)}$$

Tables 2.6 to 2.14 show us the results of the equation 2.4 to 2.6.

## Table 2.6. Results of equation 2.4 (BAS) for full period (15/03/2008 to 15/03/2009)

| | # Observ. | $R^2$ | Return | LnVol | LnVolat. | LnMV | Constant |
|---|---|---|---|---|---|---|---|
| **PIIGS** | | | | | | | |
| Portugal | 252 | 0,1841 | -1,1317 | 0,0540 *** | 0,0618 ** | -0,1592 *** | 1,5757 *** |
| | | | (-1,59) | (3,42) | (2,35) | (-3,52) | (3,93) |
| Spain | 252 | 0,5942 | -0,0455 | -0,0009 | 0,0102 ** | -0,1054 *** | 1,4529 *** |
| | | | (-0,62) | (-0,25) | (2,07) | (-10,46) | (12,97) |
| Italy | 252 | 0,5700 | 0,4089 * | -0,1676 ** | -0,1519 ** | -1,3515 *** | 1,8866 *** |
| | | | (0,52) | (-2,25) | (-2,54) | (-15,81) | (14,49) |
| Ireland | 252 | 0,3391 | 1,0160 ** | 0,1027 *** | 0,2055 *** | -0,3266 *** | 3,0598 *** |
| | | | (2,12) | (2,81) | (4,00) | (-5,44) | (4,76) |
| Greece | 252 | 0,3080 | -0,6538 | 0,0597 ** | 0,0974 | -0,3781 *** | 4,3927 *** |
| | | | (-0,74) | (2,27) | (1,37) | (-6,92) | (8,44) |
| **OEC** | | | | | | | |
| Germany | 252 | 0,4823 | -0,2693 ** | 0,0183 *** | 0,0473 *** | -0,0428 *** | 0,5468 *** |
| | | | (-2,13) | (2,86) | (4,49) | (-2,74) | (2,69) |
| Netherlands | 252 | 0,2308 | -0,0896 | 0,0132 | 0,0279 ** | -0,0194 | 0,2294 *** |
| | | | (-0,66) | (1,32) | (2,47) | (-1,16) | (1,03) |
| France | 252 | 0,1314 | 0,1048 | -0,0147 | 0,0251 *** | -0,0294 | 0,6049 ** |
| | | | (0,82) | (-1,03) | (2,79) | (-1,58) | (2,11) |
| Finland | 252 | 0,3388 | -0,1174 | -0,0118 ** | 0,0326 *** | -0,0945 *** | 1,3571 *** |
| | | | (-1,04) | (-1,23) | (2,95) | (-8,87) | (9,01) |
| Belgium | 252 | 0,3334 | -0,6588 * | 0,0197 | 0,0941 ** | -0,1242 * | 1,5032 ** |
| | | | (-1,72) | (1,3) | (2,03) | (-1,82) | (2,00) |
| Austria | 252 | 0,5522 | -1,4956 *** | 0,0709 *** | 0,2935 *** | -0,1584 ** | 1,8147 ** |
| | | | (-2,91) | (2,73) | (4,85) | (-2,45) | (2,43) |

***, **, * denotes significance at 1%, 5% and 10% respectively. Robust t statistic in parenthesis

**Table 2.7. Results of equation 2.4 (BAS) for pre-period (15/03/2008 to 14/09/2008)**

| | # Observ. | $R^2$ | Return | LnVol. | LnVolat. | LnMV | Constant |
|---|---|---|---|---|---|---|---|
| **PIIGS** | | | | | | | |
| Portugal | 127 | 0,1392 | 0,0444 | 0,0121 | 0,0936 ** | -0,1171 | 1,5435 * |
| | | | (0,09) | (0,85) | (2,47) | (-1,44) | (1,79) |
| Spain | 127 | 0,1104 | -0,1345 * | -0,0040 | 0,0039 | -0,0352 ** | 0,5771 *** |
| | | | (-1,79) | (-1,06) | (0,64) | (-2,05) | (2,72) |
| Italy | 127 | 0,3223 | -0,1207 | 0,0018 | 0,0137 ** | -0,0671 *** | 0,9225 *** |
| | | | (-1,58) | (0,54) | (2,55) | (-5,54) | (5,86) |
| Ireland | 127 | 0,1017 | 0,7938 | -0,0203 | 0,1462 ** | -0,0082 | 0,9491 |
| | | | (0,95) | (-0,41) | (2,39) | (-0,04) | (0,55) |
| Greece | 127 | 0,141 | -0,0755 | 0,0574 ** | 0,1784 ** | -0,4696 *** | 0,5472 *** |
| | | | (-0,08) | (1,77) | (1,92) | (-3,01) | (3,23) |
| **OEC** | | | | | | | |
| Germany | 127 | 0,1266 | 0,0093 | 0,0078 *** | 0,0173 *** | -0,0618 ** | 0,8268 ** |
| | | | (0,08) | (2,87) | (2,99) | (-2,10) | (2,18) |
| Netherlands | 127 | 0,0507 | -0,1149 | -0,0072 | -0,0348 | -0,1191 * | 1,6063 * |
| | | | (-0,66) | (-0,93) | (-1,01) | (-1,80) | (1,94) |
| France | 127 | 0,0111 | 0,0837 | -0,0041 | -0,0040 | -0,0343 | 0,5366 |
| | | | (0,60) | (-0,50) | (-0,32) | (-0,91) | (1,08) |
| Finland | 127 | 0,0511 | -0,2853 | -0,0072 | -0,0028 | -0,0312 | 0,5374 |
| | | | (-2,15) | (-0,77) | (-0,17) | (-0,89) | (1,21) |
| Belgium | 127 | 0,0267 | -0,3335 | -0,0080 | -0,0129 | -0,0299 | 0,5542 *** |
| | | | (-0,99) | (-0,83) | (-0,38) | (-0,47) | (0,79) |
| Austria | 127 | 0,1075 | -0,4996 | -0,0206 | 0,0337 | 0,2932 | -2,7773 *** |
| | | | (-0,87) | (-0,62) | (0,56) | (3,35) | (-2,94) |

***, **, * denotes significance at 1%, 5% and 10% respectively. Robust t statistic in parenthesis

**Table 2.8. Results of equation 2.4 (BAS) for post-period (15/09/2008 to 15/03/2009)**

| | # Observ. | $R^2$ | Return | LnVol. | LnVolat. | LnMV | Constant |
|---|---|---|---|---|---|---|---|
| **PIIGS** | | | | | | | |
| Portugal | 125 | 0,1618 | -1,5921 | 0,1033 *** | 0,0290 | -0,2094 | 1,6357 |
| | | | (-1,81) | (3,34) | (0,88) | (-1,27) | (1,00) |
| Spain | 125 | 0,2035 | -0,0238 | 0,0048 | 0,0063 | -0,1071 *** | 1,4245 *** |
| | | | (-0,27) | (0,81) | (0,77) | (-5,23) | (5,77) |
| Italy | 125 | 0,3916 | 0,4657 | -0,3650 *** | -0,2869 *** | -1,1236 *** | 1,8150 *** |
| | | | (0,54) | (-2,69) | (-3,92) | (-6,18) | (9,30) |
| Ireland | 125 | 0,1665 | 1,2430 ** | 0,1280 ** | 0,4956 *** | -0,5890 *** | 5,5048 *** |
| | | | (2,15) | (2,07) | (3,45) | (-3,1) | (3,2) |
| Greece | 125 | 0,016 | -0,8341 | -0,0413 | -0,0455 | 0,2072 | -0,9143 |
| | | | (-0,78) | (-0,74) | (-0,53) | (0,94) | (-0,46) |
| **OEC** | | | | | | | |
| Germany | 125 | 0,1873 | -0,2649 *** | 0,0229 *** | 0,0448 ** | 0,0299 | -0,3923 |
| | | | (-2,06) | (2,04) | (2,92) | (1,03) | (-1,05) |
| Netherlands | 125 | 0,0797 | -0,0588 | 0,0251 | 0,0179 | -0,0015 | -0,1024 |
| | | | (-0,4) | (1,5) | (1,28) | (-0,06) | (-0,33) |
| France | 125 | 0,052 | 0,1369 | -0,0228 | 0,0255 | -0,0021 | 0,3330 |
| | | | (0,90) | (-0,99) | (1,94) | (-0,06) | (0,9) |
| Finland | 125 | 0,041 | -0,0426 | -0,0279 | 0,0133 | -0,0163 | 0,6107 |
| | | | (-0,31) | (-1,5) | (0,46) | (-0,27) | (1,04) |
| Belgium | 125 | 0,1896 | -0,7814 | 0,0353 | 0,1785 *** | -0,2004 | 2,2281 |
| | | | (-1,47) | (1,1) | (3,16) | (-1,63) | (1,68) |
| Austria | 125 | 0,1284 | -1,6441 *** | 0,0445 | 0,2733 *** | -0,0916 | 1,3384 |
| | | | (-2,71) | (1,32) | (2,82) | (-0,42) | (0,59) |

***, **, * denotes significance at 1%, 5% and 10% respectively. Robust t statistic in parenthesis

**Table 2.9. Results of equation 2.5 (Turnover Rate) for full period (15/03/2008 to 15/03/2009)**

| | # Observ. | $R^2$ | Return | LnVolume | LnVolatility | LnMV | Constant |
|---|---|---|---|---|---|---|---|
| **PIIGS** | | | | | | | |
| Portugal | 252 | 0,9056 | -0,4798 | 0,8462 *** | 0,0494 * | -0,0551 | -12,5914 *** |
| | | | (-1,51) | (42,73) | (1,77) | (-0,97) | (-24,85) |
| Spain | 252 | 0,5012 | -0,7090 | 0,4728 *** | 0,1973 *** | -0,3705 *** | -4,0598 *** |
| | | | (-1,17) | (12,15) | (4,84) | (-3,83) | (-3,82) |
| Italy | 252 | 0,4271 | -0,5960 | 0,6623 *** | -0,2638 *** | 0,3293 *** | -1,6208 *** |
| | | | (-1,26) | (10,77) | (-5,04) | (4,00) | (-14,01) |
| Ireland | 252 | 0,2695 | 1,3624 | 0,6165 *** | 0,3772 *** | 0,0755 | -12,7319 *** |
| | | | (1,92) | (5,97) | (3,87) | (0,76) | (-9,66) |
| Greece | 252 | 0,1551 | 0,6015 | 0,3347 *** | 0,4636 *** | 0,2347 | -10,6713 *** |
| | | | (0,33) | (3,64) | (4,45) | (2,17) | (-6,74) |
| **OEC** | | | | | | | |
| Germany | 252 | 0,294 | -0,7550 | 0,3712 *** | 0,2323 *** | 0,4880 *** | -13,3981 *** |
| | | | (-0,72) | (3,58) | (3,39) | (4,45) | (-9,99) |
| Netherlands | 252 | 0,7065 | -0,4878 | 0,7961 *** | -0,0212 | 0,0311 | -12,8458 *** |
| | | | (-1,2) | (14,78) | (-0,48) | (0,44) | (-14,2) |
| France | 252 | 0,8118 | 0,5993 | 0,8835 *** | -0,0093 | -0,0676 | -12,5724 *** |
| | | | (1,55) | (23,51) | (-0,28) | (-0,73) | -11,54) |
| Finland | 252 | 0,4869 | -0,9637 | 0,6750 *** | 0,0900 | 0,0801 | -11,8416 *** |
| | | | (-1,11) | (14,52) | (1,35) | (1,04) | (-14,22) |
| Belgium | 252 | | -0,7801 | 0,3232 *** | 0,4455 *** | -0,0194 | -8,2163 *** |
| | | 0,3974 | (-1,1) | (6,09) | (4,87) | (-0,15) | (-5,1) |
| Austria | 252 | 0,0828 | -0,1462 | 0,1797 *** | 0,2267 *** | 0,4228 *** | -11,3325 *** |
| | | | (-0,15) | (3,51) | (2,59) | (3,69) | (-7,91) |

***, **, * denotes significance at 1%, 5% and 10% respectively. Robust t statistic in parenthesis

**Table 2.10. Results of equation 2.5 (Turnover Rate)for pre-period (15/03/2008 to 14/09/2008)**

| | # Observ. | $R^2$ | Return | LnVol. | LnVolat. | LnMV | Constant |
|---|---|---|---|---|---|---|---|
| **PIIGS** | | | | | | | |
| Portugal | 127 | 0,8877 | 0,2371 | 0,8178 *** | 0,0083 | -0,2362 * | -10,4448 *** |
| | | | (0,03) | (27,56) | (0,14) | (-1,79) | (-8,34) |
| Spain | 127 | 0,388 | 0,1200 | 0,4245 *** | -0,0185 | -0,1038 | -7,3095 |
| | | | (0,12) | (7,63) | (-0,19) | (-0,37) | (-2,1) |
| Italy | 127 | 0,2772 | 0,3296 | 0,4454 *** | -0,1480 | 1,1602 *** | -24,3713 *** |
| | | | (0,19) | (6,2) | (-1,04) | (2,77) | (-4,72) |
| Ireland | 127 | 0,1814 | 1,4981 | 0,5786 *** | 0,1158 | -0,7925 ** | -3,3410 |
| | | | (1,40) | (3,07) | (0,71) | (-2,6) | (-0,92) |
| Greece | 127 | 0,1512 | 0,7650 ** | 0,4340 ** | -0,2829 | 0,9556 | -19,9888 ** |
| | | | (2,00) | (2,34) | (-1,26) | (1,57) | (-2,53) |
| **OEC** | | | | | | | |
| Germany | 127 | 0,219 | -0,9702 | 0,2761 *** | 0,2962 *** | -1,3643 *** | 11,2283 * |
| | | | (-0,49) | (2,07) | (2,92) | (-2,99) | (1,8) |
| Netherlands | 127 | 0,6152 | -1,6715 | 0,7130 *** | 0,1705 | -0,4008 | -6,3818 |
| | | | (-1,43 | (8,09) | (0,85) | (-1,04) | (-1,35) |
| France | 127 | 0,782 | 1,7736 | 0,8405 *** | -0,1295 ** | -0,8570 | -2,0097 *** |
| | | | (2,06 | (13,74) | (-2,09) | (-4,07) | (-0,69) |
| Finland | 127 | 0,3907 | -1,622 | 0,713 *** | 0,376 | 0,253 | -13,935 |
| | | | (-0,6 | (8,52) | (2,31) | (0,73) | (-3,38) |
| Belgium | 127 | 0,1479 | -1,8643 | 0,1789 *** | -0,0324 | -0,8341 *** | 2,2419 |
| | | | (-1,45 | (3,15) | (-0,21) | (-2,81) | (0,65) |
| Austria | 127 | 0,3183 | 0,3861 | 0,5030 *** | 0,2597 * | -0,5880 *** | -2,4427 |
| | | | (0,26 | (4,29) | (1,79) | (-3,31) | (-1,32) |

***, **, * denotes significance at 1%, 5% and 10% respectively. Robust t statistic in parenthesis

**Table 2.11. Results of equation 2.5 (Turnover Rate)for post-period (15/09/2008 to 15/03/2009)**

| | # Observ. | $R^2$ | Return | LnVol. | LnVolat. | LnMV | Constant |
|---|---|---|---|---|---|---|---|
| **PIIGS** | | | | | | | |
| Portugal | 125 | 0,909 | -0,5507 | 0,8672 *** | 0,0245 | 0,1288 | -14,6907 *** |
| | | | (-1,69) | (3,78) | (0,80) | (1,00) | (-1,20) |
| Spain | 125 | 0,6019 | -0,8371 | 0,5500 *** | 0,1467 | -0,2258 | -6,5424 *** |
| | | | (-1,34) | (11,16) | (2,06) | (-1,40) | (-3,32) |
| Italy | 125 | 0,5629 | -0,5931 | 0,8347 *** | -0,3880 *** | 0,3490 | -18,3522 *** |
| | | | (-1,24) | (8,77) | (-5,69) | (1,78) | (-8,71) |
| Ireland | 125 | 0,4822 | 1,6512 | 0,6433 *** | -0,0853 *** | 1,1812 | -24,1901 *** |
| | | | (2,16) | (5,48) | (-0,54) | (4,28) | (-8,81) |
| Greece | 125 | 0,3653 | -1,7403 | 0,2602 *** | 0,6352 *** | 0,5445 * | -13,1941 *** |
| | | | (-1,14) | (3,27) | (5,08) | (1,80) | (-4,81) |
| **OEC** | | | | | | | |
| Germany | 125 | 0,445 | -0,2371 | 0,4849 *** | 0,1112 *** | 1,0186 | -20,8725 *** |
| | | | (-0,22) | (6,61) | (1,40) | (4,50) | (-7,62) |
| Netherlands | 125 | 0,7775 | -0,1216 | 0,8427 *** | -0,1344 *** | 0,2587 ** | -16,0968 *** |
| | | | (-0,30) | (12,84) | (-2,88) | (2,39) | (-13,93) |
| France | 125 | 0,8294 | 0,3653 | 0,8894 *** | 0,0238 | -0,0461 | -12,8916 *** |
| | | | (0,82) | (17,75) | (0,41) | (-0,31) | (-7,54) |
| Finland | 125 | 0,6387 | -0,8476 | 0,7585 *** | 0,0288 | -0,3521 *** | -7,7482 *** |
| | | | (-1,47) | (14,2) | (0,36) | (-2,69) | (-6,22) |
| Belgium | 125 | 0,3627 | 0,2889 | 0,5121 *** | 0,4811 *** | 0,2077 | -12,5445 *** |
| | | | (0,37) | (5,74) | (4,26) | (1,14) | (-6,19) |
| Austria | 125 | 0,3248 | 0,7904 | 0,0713 | -0,1528 *** | 2,1702 | -28,5016 *** |
| | | | (0,92) | (1,04) | (-1,27) | (7,40) | (-9,29) |

***, **, * denotes significance at 1%, 5% and 10% respectively. Robust t statistic in parenthesis

**Table 2.12. Results of equation 2.6 (Amihud Illiquidity) for full period (15/03/2008 to 1503/2009)**

| | # Observ. | $R^2$ | Return | LnVolume | LnVolatility | LnMV | Constant |
|---|---|---|---|---|---|---|---|
| **PIIGS** | | | | | | | |
| Portugal | 252 | 0,2338 | -0,6183 | 0,0073 | 0,0344 ** | -0,1533 *** | 1,6676 *** |
| | | | (-1,06) | (0,71) | (2,00) | (-6,22) | (7,01) |
| Spain | 252 | 0,234 | -0,0053 | -0,0003 | 0,0030 *** | -0,0043 ** | 0,0644 *** |
| | | | (-0,37) | (-0,46) | (3,33) | (-2,21) | (2,95) |
| Italy | 252 | 0,3445 | -0,0145 | 0,0013 | 0,0060 *** | -0,0064 *** | 0,0785 ** |
| | | | (-0,48) | (1,04) | (4,05) | (-2,72) | (2,55) |
| Ireland | 252 | 0,1715 | -0,8259 *** | -0,0011 | 0,1030 *** | 0,0135 | 0,0360 |
| | | | (-2,87) | (-0,06) | (4,24) | (0,48) | (0,12) |
| Greece | 252 | 0,2379 | -0,356 | -0,026 *** | 0,016 | -0,082 *** | 1,163 *** |
| | | | (-1,31) | (-2,99) | (0,95) | (-4,41) | (5,46) |
| **OEC** | | | | | | | |
| Germany | 252 | 0,3716 | 0,0050 | 0,0004 | 0,0028 | -0,0099 | 0,1310 |
| | | | (0,22) | (0,63) | (2,42) ** | (-4,09) *** | (4,41) *** |
| Netherlands | 252 | 0,2515 | -0,0093 | -0,0003 | 0,0076 *** | -0,0087 | 0,1266 ** |
| | | | (-0,24) | (-0,12) | (2,65) | (-1,91) | (2,12) |
| France | 252 | 0,2239 | -0,0035 | -0,0008 | 0,0046 *** | -0,0079 ** | 0,1199 *** |
| | | | (-0,16) | (-0,58) | (3,31) | (-2,37) | (2,79) |
| Finland | 252 | 0,2674 | -0,0146 | 0,0007 | 0,0043 * | -0,0175 *** | 0,2089 *** |
| | | | (-0,38) | (0,31) | (1,75) | (-6,13) | (5,35) |
| Belgium | 252 | 0,2371 | -0,2403 | 0,0050 | 0,0441 *** | -0,0120 | 0,1637 |
| | | | (-1,4) | (0,79) | (2,88) | (-0,59) | (0,68) |
| Austria | 252 | 0,3889 | -0,3045 | 0,0183 | 0,1582 *** | -0,0955 | 1,1198 |
| | | | (-0,64) | (0,78) | (3,76) | (-1,88) | (1,79) |

***, **, * denotes significance at 1%, 5% and 10% respectively. Robust t statistic in parenthesis

**Table 2.13. Results of equation 2.6 (Amihud Illiquidity) for pre-period (15/03/2008 to 14/09/2008)**

| | # Observ. | $R^2$ | Return | LnVol. | LnVolat. | LnMV | Constant |
|---|---|---|---|---|---|---|---|
| **PIIGS** | | | | | | | |
| Portugal | 127 | 0,2054 | -0,3096 | -0,0006 | -0,0031 | -0,1850 *** | 2,0336 *** |
| | | | (-0,96) | (-0,08) | (-0,19) | (-5,02) | (5,33) |
| Spain | 127 | 0,1703 | -0,0069 | 0,00001 | 0,0017 * | -0,0093 *** | 0,1244 *** |
| | | | (-0,44) | (0,02) | (1,77) | (-3,56) | (3,62) |
| Italy | 127 | 0,1317 | -0,0144 | 0,0008 | 0,0026 ** | -0,0068 ** | 0,0845 ** |
| | | | (-0,67) | (1,15) | (2,25) | (-2,17) | (2,11) |
| Ireland | 127 | 0,2420 | -0,5486 | 0,0055 | 0,0834 *** | -0,1114 ** | 1,2777 ** |
| | | | (-1,24) | (0,25) | (2,63) | (-2,12) | (2,42) |
| Greece | 127 | 0,0603 | -0,1556 | 0,0037 | 0,0102 | -0,0651 ** | 0,7364 ** |
| | | | (-0,63) | (0,87) | (0,66) | (-2,18) | (2,24) |
| **OEC** | | | | | | | |
| Germany | 127 | 0,0547 | -0,0179 | -0,0004 | 0,0010 | -0,0059 * | 0,0821 * |
| | | | (-0,83) | (-1,27) | (1,42) | (-1,79) | (1,95) |
| Netherlands | 127 | 0,0392 | -0,0202 | -0,0009 | -0,0001 | -0,0081 | 0,1161 |
| | | | (-0,68) | (-0,68) | (-0,03) | (-1,01) | (1,14) |
| France | 127 | 0,0796 | -0,0063 | 0,0006 | 0,0020 | -0,0096 * | 0,1257 * |
| | | | (-0,25) | (0,6) | (1,3) | (-1,85) | (1,75) |
| Finland | 127 | 0,0345 | 0,0062 | 0,0000 | -0,0014 | -0,0072 | 0,0885 |
| | | | (0,29) | (0,03) | (-0,86) | (-1,4) | (1,35) |
| Belgium | 127 | 0,2218 | -0,0704 | 0,0019 | 0,0077 | -0,0828 *** | 0,9898 *** |
| | | | (-0,47) | (0,49) | (0,56) | (-3,1) | (3,26) |
| Austria | 127 | 0,0802 | -0,3737 | -0,0077 | 0,0166 | -0,0213 | 0,3473 |
| | | | (-1,72) | (-0,83) | (0,91) | (-0,81) | (1,27) |

***, **, * denotes significance at 1%, 5% and 10% respectively. Robust t statistic in parenthesis

**Table 2.14. Results of equation 2.6 (Amihud Illiquidity) for post-period (15/09/2008 to 15/03/2009)**

| | # Observ. | $R^2$ | Return | LnVol. | LnVolat. | LnMV | Constant |
|---|---|---|---|---|---|---|---|
| **PIIGS** | | | | | | | |
| Portugal | 125 | 0,0579 | -0,6917 | 0,0164 | 0,0399 * | -0,1581 * | 1,6427 * |
| | | | (-0,9) | (0,81) | (1,67) | (-1,73) | (1,83) |
| Spain | 125 | 0,0848 | -0,0043 | -0,0007 | 0,0044 *** | -0,0059 * | 0,0887 ** |
| | | | (-0,25) | (-0,71) | (3,14) | (-1,67) | (2,2) |
| Italy | 125 | 0,1211 | -0,0157 | 0,0024 | 0,0088 *** | -0,0120 *** | 0,1367 *** |
| | | | (-0,47) | (0,99) | (4,43) | - (2,72) | (2,76) |
| Ireland | 125 | 0,1238 | -0,9396 ** | 0,0102 | 0,0897 | -0,0249 | 0,2914 |
| | | | (-2,44) | (0,36) | (1,65) | -(0,31) | (0,39) |
| Greece | 125 | 0,1798 | -0,4789 | -0,0848 *** | -0,0010 | 0,0719 | 0,0285 |
| | | | (-1,43) | (-4,7) | (-0,04) | (1,33) | (0,06) |
| **OEC** | | | | | | | |
| Germany | 125 | 0,1006 | 0,0091 | 0,0011 | 0,0033 * | -0,0100 ** | 0,1267 *** |
| | | | (0,36) | (0,78) | (1,73) | (-2,45) | (2,76) |
| Netherlands | 125 | 0,0245 | -0,0012 | -0,0006 | 0,0067 * | -0,0021 | 0,0494 |
| | | | (-0,03) | (-0,15) | (1,9) | (-0,29) | (0,62) |
| France | 125 | 0,0333 | -0,0001 | -0,0019 | 0,0046 ** | -0,0045 | 0,0877 |
| | | | (0) | (-0,84) | (2) | (-0,86) | (1,49) |
| Finland | 125 | 0,0318 | -0,0223 | 0,0006 | 0,0070 | -0,0179 ** | 0,2152 *** |
| | | | (-0,43) | (0,16) | (1,26) | (-2,07) | (2,64) |
| Belgium | 125 | 0,1031 | -0,288 | 0,009 | 0,055 *** | -0,021 | 0,226 |
| | | | (-1,27) | (0,65) | (2,64) | (-0,66) | (0,63) |
| Austria | 125 | 0,1434 | -0,4662 | 0,0303 | 0,2790 *** | -0,4362 *** | 4,5470 *** |
| | | | (-0,87) | (0,84) | (3,91) | (-3,49) | (3,46) |

***, **, * denotes significance at 1%, 5% and 10% respectively. Robust t statistic in parenthesis

The results of testing equation 2.4 (BAS) applied to the entire sample (Table 2.6) demonstrate that for most of the countries, returns is significant and is negative, consistent with the theory. Only Italy, Ireland and France had a positive impact. Market Value in all countries presents a negative impact and Volatility presents a positive impact (except Italy). The variable Volume is not consistent in all countries. Volume and Volatility present a small significance.

Table 2.7 and 2.8 show us the results of equation 2.4 before and after the 15th September of 2008 (crisis) for BAS. The impact of the return and Market Value increases in most of the cases after the crisis and is negative significant in majority. Volatility and Volume also increases after the crisis. We cannot distinguish some standard between PIIGS and the others Eurozone countries. For example, the impacts of Returns is more significant in small stock exchanges with less liquidity, like Portugal, Ireland and Greece (PIIGS) and Austria (OCE).

For the second proxy, Turnover Rate, Table 2.9 to 2.11 present the results of equation 2.5. The Volume is the variable with more impact and is significantly positive. Returns, in most of the cases had a negative impact. Volatility and Market Value are not consistent. When we compare the pre and post period we do not find a standard for the countries in any variable.

The results of equation 2.6 (Amihud Illiquidity Measure) are shown in Table 2.12, 2.13 and 2.14. For the entire period, the variable return had a bigger impact and is negative (except Germany), that is consistent with the theory. The Return had more impact in countries with less liquidity, like Portugal, Greece and Ireland (PIIGS) and Austria (OCE). Volume and Volatility had a small impact and Market Value had a negative impact and is higher for the small stock exchanges. Comparing the pre and post crisis period, we can found some evidence that the impact of the variables are higher.

For all the three proxies, there is not a difference of behaviour of the PIIGS countries and the other Eurozone countries. The difference is between the countries with big stock exchanges (more liquid) and the countries with small stock exchanges (less liquid) that liquidity have more impact after the crisis.

## *5. Conclusions*

This study presents first evidence of the impact of Eurozone Stock Exchanges in financial crisis of 2008, essentially before and after the 15th September 2008, when the crisis of stock market started. We separate the Eurozone countries in two: the countries who had bigger problems later on Eurozone crisis (PIIGS) and the other Eurozone countries, to understand if the liquidity of stock markets of these countries (PIIGS) had different behaviour than the others.

We found an evidence of illiquidity effect after the 15th September 2008, for all the 11 countries, but this illiquidity is stronger for the countries with small stock exchanges (Portugal, Ireland, Greece and Austria). The size of the market is more important to the liquidity of the stock exchange.

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## *Essay Three: Dilution and Dividend Effects on the Portuguese Equity Warrant Market*

### *Abstract*

The aim of this study is to analyse the impact of dilution and dividends on the goodness of fit of warrant pricing valuation models, to a illiquidity financial product in a illiquidity market - the Portuguese warrants market. In order to avoid modelling bias over the research design, and to test dividend and dilution effects we decided to keep this empirical research under the Black-Scholes framework. Therefore, four pricing models were used: the original Black-Scholes model and three derivations. Using these four models we empirically estimate values for actual warrant prices, computing the mean percentage error for each (the difference between model prices and market prices). We found that the original Black-Scholes model when adjusted to account for dilution as well as for dividends works best in the Portuguese market. The analysis uses data collected from the Euronext - Lisbon, between 1998 and 2000.

## *1. Introduction*

Since the early sixties equity warrants have been under a constant interest of research in finance. With the seminal paper of Black-Scholes (1973) the warrants pricing literature had a new blow of a powerful methodology and got a strong framework to find whether other effects are actually observed. As long term options, warrants are expected to suffer the impact of dividends as well the so-called dilution effect.

Although dilution and dividends have been a constant concern of researchers, only in literature we find sophisticated methods to deal with these problems, namely in Merton (1973), Roll (1977), Galai and Schneller (1978), Geske (1979, 1981), Whaley (1981), Lauterbach and Schultz (1990) or Schulz and Trautmann (1989 and 1994). These authors show that dilution and dividends have some impact on market prices for warrants. But are there similar effects in illiquid markets? The bias introduced by thin trading is so strong that it may be plausible that the typical effects that we notice in other warrant markets, namely the dilution and dividend effects, may not be observed in illiquid markets. We thought that it could be interesting to check empirically whether dividends and dilution have some impact on warrants market prices, using a quite illiquid market as the Portuguese.

In order to avoid modelling bias over the research design and in order to test only, dividend and dilution effects, we decided to develop our research exclusively within the Black-Scholes framework. We used four warrants pricing models: the original Black-Scholes model and three of its derivations. Using these four models we empirically estimate values for actual warrant prices, computing the mean percentage error, as the difference between model prices and market prices. It is supposed that the most efficient model shows the smallest percentage error. The analysis uses data collected from the Euronext - Lisbon[6], between 1998 and 2000.

We concluded that there is a need for adjusting the original Black-Scholes model to dilution and to dividends. Concerning the dilution effect, we used the adjusted Black-Scholes formula proposed by Lauterbach and Schultz (1990) and discuss the possibility of the warrant market price to already include this effect, as supported by Crouhy and Galai (1991).

In order to test the need for dividend adjustments we used two models: the adjustment of the dividends in a discrete way and the adjustment to the payment of dividends proposed by Merton (1973).

The paper is organized as follows: in section 2 we review the literature, next we present the methodology and the data used and finally we present the empirical results and the conclusions.

## *2. Literature Review*

Until 1973, the literature on contingent stocks was substantially devoted to warrants. Over-the-counter (OTC) options didn't receive a lot of attention either by academics or research departments of financial institutions. This trend changed with the start of traded options in the floor of some American stock exchanges and with the publication of the seminal paper of Black and Scholes (1973). However, in their article the empirical validation of the equation was based on warrants. The shift of attention from

[6] Although the official name Portuguese stock exchange at the time when this research was carried out was BVLP - Bolsa de Valores de Lisboa e Porto, after the recent merger it became part of the Euronext group, and adopted the formal name of Euronext - Lisbon.

warrants to options is partially explained by the additional difficulties that the study of warrants incorporates such as dilution and dividends.

Early warrant studies, such as Sprenkle (1961), Samuelson (1965), Chen (1970) and Bierman (1973), ignored the dilution effect and considered warrants equivalent to call options. After Black and Scholes (1973) there have been many empirical studies on option pricing valuation, but very few empirical studies on warrant markets.

Ferri, Kremer and Oberhelman (1986) studied the goodness of fit of the pricing models, using a sample of 50 warrants traded in the U.S.A. for nine days between 1983 and 1984. Lauterbach and Schultz (1990) compared the Black-Scholes model with the Constant Elasticity of Variance model (CEV), observing the daily price of 39 warrants between 1979 and 1980 in the U.S.A. Schulz and Trautmann (1989) studied 49 German warrants from 1979 to 1986, using weekly data. Stucki and Wasserfallen (1991) used five pricing options models to study equity warrants, through a sample of 2.100 weekly observations for 44 Swiss warrants from January 1986 to February 1987.

In the empirical studies on warrant pricing several alternative models have been used. Veld (2003) refers that these models try to overcome some of the assumptions used by Black-Scholes, such as:

- No dividend payments - Merton (1973) adjusted the Black-Scholes equation allowing the model to be used in dividend paying firms;
- The early exercise before maturity - Merton (1973) showed that the rational investors must only use the possibility of early exercise for call options just before the ex-dividend date. Other models such as Black (1975), Roll (1977), Geske (1979, 1981) and Whaley (1981), and the American Constant Variance diffusion model (Schulz and Trautmann, 1994) try to overcome both problems (early exercise and dividend effect);

- Constant volatility – in the Constant Elasticity of Variance (CEV) model the assumption of constant volatility is replaced by the constant elasticity of volatility assumption. In this model it is assumed, that the elasticity factor is defined in such a way that the volatility decreases as the price of the underlying stock increases.

Sidenius (1996) refers some examples of the difficulties of warrant valuation. One example is the constant volatility assumption, which is not very realistic for a warrant with duration of several years. The volatility of a warrant with long maturity is likely to change and therefore, the valuation model should be adjusted to take into account these changes on volatility.

In the pioneering studies on warrant pricing, it was assumed that the firm was completely financed by shares and warrants. As a consequence, Galai and Scheneller (1978) and Crouhy and Galai (1991) priced warrants and shares based on the total value of the firm. Leonard and Solt (1990), Lauterbach and Schultz (1990) and Schulz and Trautman (1994) tested the Black-Scholes model in pricing warrants while Noreen and Wolfson (1981), Ferri, Kremer and Oberhelman (1986), Sisson (1987), Lauterbach and Schultz (1990) and Hauser and Lauterbach (1996 and 1997) compared the results obtained by the Black-Scholes model with the results of other pricing models. They concluded that the Black-Scholes model is the most representative and it is more precise than other models (including the Constant Elasticity of Variance). In spite of similar results, the studies differ in their treatment of dividends and dilution effects.

Emanuel (1983) and Constantinides (1984) derived a valuation model and an optimal exercise strategy for the American warrants with payment of dividends. Cox and Rubinstein (1985) and Spatt and Sterbenz (1988) considered the hypothesis of a potential expansion of the firm, deriving the optimal exercise strategy for American warrants.

Kim and Young (1980) studied the efficiency of the warrants market observing the relationship between warrants and shares. The study involved a hedging strategy, which was based on a long position on the underlying stock and a short position on warrants. This strategy tends to minimize profits below a target rate instead of maximizing them. They developed a model for determining the optimal hedge ratio, which was based on the probability of the price of the share having a certain value in the warrant's expiration date. This probability is defined by a function, which considers the current prices of the share and the warrant and the exercise price. The empirical study considered 18 warrants traded between 1962 and 1977. They concluded that the profit of a hedging strategy with warrants is superior to the profit with a buy and hold strategy.

Wei (1995) evaluated the Nikkei 225 put warrants. He studied warrants traded in the Toronto Stock Exchange (Canada) and used several pricing models proposed by Dravid, Richardson and Sun (1993), Reiner (1991) and Wei (1992). Wei concluded that these models tend to undervalue the warrants relatively to their market prices. This under valuation tends to be stronger when warrants are deep in-the-money, the volatility of the underlying Index is high and the trading volume is also high.

Huang and Chen (2002) applied the stochastic volatility option pricing model of Hull and White to the covered warrants traded on the Taiwan Stock Exchange (TSE). They concluded that the HW model with implied volatility outperforms others in predicting the warrant prices, indicating that the pricing model incorporated with stochastic volatility feature can improve the pricing of warrants. Ukhov (2004) developed an algorithm for pricing warrants using stock prices, an observable variable and stock return variance. The algorithm is based on the variables used in the Black-

Scholes option pricing formula, the number of shares outstanding, the number of warrants issued, and the number of shares of stock that each warrant entitles the owner to receive when exercised.

Lim and Terry (2003) created a formula do evaluate multiple series of warrants and compared the theoretical warrant price from their model with existing models, like Black-Scholes (1973) and Galai-Schneller (1978). They found a subtle slippage effect and also a cross dilution effect that caused the existing models, to be inappropriate for pricing such classes of multiple warrants.

Horst and Veld (2003) used the Black-Scholes, the Square Root model version of the CEV, and the Binomial model to price the call warrants based on long-term call options and found that call warrants are overvalued between 25 and 30 percent for all three models. The authors considered "that the overvaluation can be attributed to a behavioral preference of private investors for call warrants"

Loudon and Nguyen (2006) concluded that are a large excess warrant premium and provided evidence that it is significantly related to the identity of the warrant issuer, even after taking into account important liquidity and hedging factors.

## *3. Methodology*

In this study we are particularly interested on how strong are the empirical impacts of dividends and dilution effect on pricing warrants. In order to keep the research design out of any other effect we decided to use a very simple model as the Black and Scholes (1973). We could then obtain a minimum of implied parameters from the basic model, and introduce the referred effects: dividends and dilution. Of course the same approach could be developed using a different model, but as the Black-Scholes model keeps the number of unobservable parameters to a minimum, and because it is a benchmark on pricing in similar studies, we also decided to use it as a reference model.

The methodology of this study consists of obtaining theoretical values for the four pricing models selected (Black-Scholes and three of its derivations) and to compute a mean percentage error for each one of them relative to the observed market prices[7]. We assume that the best model to price equity warrants is the one that presents the smallest percentage error. This criterion is a common procedure to previous warrant pricing studies.

To test the need for modelling adjustments when assuming dividend paying firms, two derivations of the black-scholes model were used:

1. adjustment to dividends in a discrete way, that is, the adjustment for dividends is made by replacing the price of the underlying stock, s, for the price of the stock minus the net present value of the dividends that will be paid until maturity, $s_d$, as it was done by lauterbach and schultz (1990);
2. adjustment according to merton (1973) model where the dividends are supposed to be paid continuously until maturity according to a constant dividend yield.

in terms of the dilution effect, a derivation of the black-scholes model proposed by lauterbach and schultz (1990) was applied.

Stucki and wasserfallen (1991) applied the arbitrage conditions, in order to prevent arbitrage opportunities in the database. Warrant prices should satisfy the same arbitrage conditions that are applied to call option prices, and pricing models are only meaningful

[7] Since the number of warrants considered in the study is very small (only six warrants), we study the entire set of warrants as well as each one of them.

when those conditions are not violated. There exist at least three conditions for the minimal value of a warrant that should be tested:

1 – The value of the warrant ($W$) should be at least equal to the maximum between zero and the difference between the current underlying stock price ($S$) and the exercise price ($X$):

$$W \geq \max(S\text{-}X,\ 0) \tag{3.1}$$

This equation defines lower bound for an American contract.

2 - The value of the warrant should be equal or greater than the maximum between zero and the difference between the current price of the underlying stock and the present value of the exercise price:

$$W \geq \max\left(S\text{-}Xe^{-r(T-t)},\ 0\right) \tag{3.2}$$

where $T$ is the exercise date and $r$ is the risk-free interest rate. This equation defines lower bound for an European contract.

3 - Taking into account the effects of dividends on the price of the underlying stock one can impose stricter limits on the previous condition. Replacing the price of the underlying stock, $S$, for the price of the stock minus the net present value of the dividends that will be paid until maturity, $S_d$: , we will get:

$$W \geq \max\left(S_d - Xe^{-r(T-t)},\ 0\right) \tag{3.3}$$

In order to select warrant prices for this study we tested whether these three arbitrage conditions were violated. Whenever we detected some violation, the corresponding observation was excluded from the sample.

As explained previously, we used four warrant pricing models in order to test the goodness of fit for the Portuguese warrants market:

*1. The Black-Scholes model – BS:*

$$W = \left[SN(d_1) - Xe^{-r(T-t)}N(d_2)\right] \times \gamma \qquad (3.4)$$

$$d_1 = \frac{\ln\frac{S}{X} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}} \qquad (3.5)$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)} \qquad (3.6)$$

where:

$W$ = value of the warrant;

$S$ = stock price;

$X$ = exercise price;

$\gamma$ = exercise ratio = number shares of the underlying stock that can be bought / sold with each warrant;

$r$ = risk-free interest rate;

$T$ = expiration date of the warrant;

$(T-t)$ = time-to-maturity of the warrant;

$\sigma$ = stock return volatility;

N(.) = cumulative standard normal distribution function.

*2. Black-Scholes model adjusted for dividends in the discrete form (adjusting the underlying stock price with the net present value of the dividends) – Bsdiv:*

$$W = \left[S_d N\left(d_1^{div}\right) - Xe^{-r(T-t)}N\left(d_2^{div}\right)\right] \times \gamma \qquad (3.7)$$

$$d_1^{div} = \frac{\ln \frac{S_d}{X} + \left( r + \frac{\sigma^2}{2} \right)(T-t)}{\sigma \sqrt{T-t}} \tag{3.8}$$

$$d_2^{div} = d_1^{div} - \sigma \sqrt{T-t} \tag{3.9}$$

where:

$$S_d = S - \sum_{i=1}^{n} D_i e^{-r(T_i - t)} \tag{3.10}$$

$D_i$ = the $i^{th}$ dividend;

$T_i$ = time moment when the $i^{th}$ dividend is paid;

3. *Black-Scholes model adjusted for dividends in the continuous form, proposed by Merton (1973) – BS-M;*

$$W = \left[ Se^{-d(T-t)} N\left(d_1^M\right) - Xe^{-r(T-t)} N\left(d_2^M\right) \right] \times \gamma \tag{3.11}$$

$$d_1^M = \frac{\ln \frac{S}{X} + \left( r - d + \frac{\sigma^2}{2} \right)(T-t)}{\sigma \sqrt{T-t}} \tag{3.12}$$

$$d_2^M = d_1^M - \sigma \sqrt{T-t} \tag{3.13}$$

where:

$d$ = dividend yield. (3.14)

4. *The Lauterbach and Schultz (1990) - Black-Scholes Dilution-Adjusted model – BSAD.*

Some authors multiply the Black-Scholes formula by the dilution factor ($N/((N/\gamma)+M)$), where $M$ is the number of warrants issued and $N$ is the number of shares outstanding. However, such a procedure is naive and assumes that market prices for both markets (the warrants markets as well the underlying security market) are absolutely segregated, which is not the case. Some other modifications are needed. Lauterbach and Schultz (1990) suggested replacing the underlying stock price, $S$, by the equity value per share of common stock, $S_v$; replacing the volatility, $\sigma$, by the equity volatility, $\sigma_v$; and finally multiplying the result by the dilution factor. According to Lauterbach and Schultz (1990) the value of the equity warrant is then given by:

$$W = \frac{N}{N/\gamma + M}\left[\left(S_d + \frac{M}{N}W\right)N(d_1^{AD}) - Xe^{-r(T-t)}N(d_2^{AD})\right] \quad (3.15)$$

$$d_1^{AD} = \frac{\ln\left[\frac{\left(S_d + \frac{M}{N}W\right)}{X}\right] + \left(r + \frac{\sigma_v^2}{2}\right)(T-t)}{\sigma_v\sqrt{(T-t)}} \quad (3.16)$$

$$d_2^{AD} = d_1^{AD} - \sigma_v\sqrt{(T-t)} \quad (3.17)$$

where:

$\left(S_d + \frac{M}{N}W\right) = S_v$ = equity value that the stock price is adjusted to the dividends;

$\sigma_v$ = the equity volatility that equals the volatility of the total asset instead of the volatility of the underlying share price.

It is clear that for firms that do not pay dividends, the warrant theoretical prices were equivalent in the first three models, since dividends is the only variable that differs.

In order to estimate the volatility, we started by computing the implied volatility from each warrant market price. As in other studies, we assumed that the Black-Scholes model holds and both, the stock and options markets, are efficient. Then we used the implied volatility as an appropriated estimator for future market volatility. The implied volatility was calculated using the Newton-Raphson iterative process. The iterative process stopped whenever the simulated warrant price differed less than 0.001% from the warrant market price. We did not remove observations with very low market prices, very deep in-the-money or out-of-the-money or close to the maturity, because in Duque (1994) they were found to have significant information.

A similar process was developed using the four models, which lead us to compute four different implied volatilities per trading day.

Then we averaged the past five days implied volatility observations in order to feed the model when forecasting the future warrant price. This procedure was done for each of the models under study. Whenever the arbitrage conditions were not satisfied, implied volatilities could not be computed because the algorithm does not converge to a stable figure.

$$\sigma_t^k = \frac{\sum_{i=1}^{5} \sigma_{imp_{t-1}}^k}{5} \tag{3.18}$$

As equation (3.18) documents, for each day $t$ the volatility to be fed into the model $k$ ($\sigma_t^k$) equals the average implied volatility observed in the previous 5 days ($\sigma_{imp_{t-1}}^k$), being these implied volatilities computed by using model $k$ [8].

The performance of each model is measured by *percentage error*:

$$\text{Percentage Error} = \frac{\text{model value - market price}}{\text{market price}} \times 100\% \tag{3.19}$$

The prices of the models are calculated for each warrant and for each daily observation. The mean of the percentage error is useful to determine if a model

[8] We are using 4 different models. Therefore $k = 1, 2, 3$ or $4$.

systematically undervalues or overvalues the prices observed for the warrants. A positive value indicates that the model overvalues the warrant market price, while a negative value indicates that the model undervalues the warrant market price. To decide what is the most efficient model we use the *Absolute Percentage Error*:

$$\text{Absolute Percentage Error} = \left| \frac{\text{model value - market price}}{\text{market price}} \right| \times 100\% \qquad (3.20)$$

The methodology used is similar to other studies such as Noreen and Wolfson (1981), Ferri, Kremer and Oberhelman (1986), Schulz and Trautmann (1989 and 1994), Lauterbach and Schultz (1990), Stucki and Wasserfallen (1991), Veld (1992), Kremer and Roenfeldt (1992), Hauser and Lauterbach (1997), Shastri and Sirodom (1995) or Low (2000).

# *4. Data*

## 4.1. Characteristics of equity warrants traded on Euronext - Lisbon

### *4.1.1. Issuing and Listing Date*

Up to 1999, the Portuguese law only allowed warrants to be issued when linked to bonds. After the issue, the warrants were typically split from the bond and listed as an independent financial asset.

The time period between the issue and the listing on the stock exchange varied from 1.71 months (in the case of Jerónimo Martins) to 12 months (in the case of Banco Comercial de Macau (BCM)). In average warrants took 4 months to be listed.

*4.1.2. Exercise Ratio*

We could not detect any particular pattern when observing the exercise ratio of the issues of warrants in Portugal. Four issues show a exercise ratio smaller than 1[9], four issues show a ratio greater than 1 and the remaining three have a ratio equal to 1. The smallest exercise ratio (0.3333) belongs to Efacec and Cofina, while the largest exercise ratio (4) belongs to Sonae Indústria.

The initial exercise ratio of Jerónimo Martins was equal to 1. However, after having decided a stock split that occurred on November 26, 1997, with a ratio of 2.5 shares for each existing share, the initial conditions had to be adjusted. In such circumstances, there are two alternatives: (i) to adjust the exercise price to 16.36€ (that is, 40.90€/2.5); (ii) to adjust the exercise ratio. Jerónimo Martins decided to change the exercise ratio, but it did not use the ratio of 1 for 2.5. The holder that exercised the warrant would get one share (the initial right) plus a free share and a partial right corresponding to a half of a share. This equals a split in the warrants' exercise ratio from 1 to 2.5.

*4.1.3. Exercise Period*

Most of the Portuguese equity warrants are Pseudo-American (Bermuda)[10], while the remaining ones are European. Six warrants could be exercised during first year following the issue. The warrants of BCM, Somague, Inparsa A and B, and Modelo Continente have a precise exercise time period, which is 1 year (for BCM) and 1 month for the remaining issues.

[9] This means that one needs more than one warrant to exercise.

[10] Even showing these Pseudo-American characteristics, the Black-Scholes still holds since no dividends are paid when these warrants start to be American type.

### *4.1.4. Expiration Date*

By the time we started to study these warrants (December 31, 2000), the warrants BCM, Tertir, Efacec, Inparsa A and B, Sonae Indústria and Modelo Continente had already expired. The warrants Somague, Engil, Cofina and Jerónimo Martins were still listed in Euronext - Lisbon and were expected to continue up to 2003.

### *4.1.5. Moneyness Degree (S/X)*

In most of the cases (6), the price of the underlying stock, *S*, is greater than the exercise price, *X*, on the day when they are listed, that is, the warrants are typically issued in-the-money. The moneyness ratio *S/X* varies between 0.52 (Engil) to 2.7 (Inparsa A and B). The average moneyness ratio of these issues was 1.31 (in-of-the-money). The warrants Inparsa A and B were the only issued quite deep in-the-money. Veld (1992) argues that when warrants are issued deep in-the-money, this tends to guarantee they will be sold signalling a firm's need of rising capital.

### *4.1.6. Initial time to maturity when listing*

The initial time to maturity when listing (the remaining life of the warrant at the moment of the listing in the stock exchange) is the time period between the moment when warrants are listed and the expiration date. The first warrant issued in Portugal (Banco Comercial de Macau) was listed in 1991 and had a time to maturity of 1.69

years (the BCM warrant was admitted to listing only one year after the issue, jointly with a bond).

The shortest maturity was 1.15 years, for the Modelo Continente warrant, and the longest maturity was 6.59 years, for the Jerónimo Martins' warrant. The average initial maturity of the Portuguese warrants is 3.41 years.

### *4.1.7. Trading Frequency*

The equity warrants market in Portugal has been quite illiquid. The average trading frequency ratio is 47.09%. The most dramatic case was Tertir that traded only 14 days out of the 926 in which it was quoted (1.51%)!

## Table 3.1. Equity Warrants traded on Euronext-Lisbon

a) All equity warrants in Portugal were jointly issued with bonds, with the exception of the issue made by Futebol Clube do Porto - Futebol SAD. However, this issue occurred under a new legal regime that authorised the issue of the warrants independently. After the issue, all the warrants were split from the bonds and listed, independently; b) Tertir issued in June of 1992, 1 500 000 bonds with attached warrants. These were listed on 26/03/1993. At the issuing date there were five possible quarters to exercise the warrants: the 3rd quarter of each year from 1992 to 1996. However, the exercise price was a stochastic variable with a floor of 4.99€ because it depends on the stock price path of the underlying security; c) The exercise ratio of Jerónimo Martins was changed from 1 to 2.5 shares; d) Each Engil warrant had two rights: one to be exercised on 0.425 shares of Engil during the month of August 2000, with an exercise price of 10.42€, and another right to be exercised on 0.425 shares of Engil during the month of August 2003 with an exercise price of 12.06€; e) Inparsa issued bonds with warrants in November 1998. Each bond had attached 2 callable warrants (A and B), each one of them giving an exercise right of 2 Inparsa shares with an exercise price of 7.48€ per share. The warrant A was expected to expire within 2 years, and the warrant B was expected to expire within 3 years. However, they were called just 4 months after being listed on February 1, 1999. On the April 27, 1999 the warrants A and B were merged into one single warrant Inparsa. The exercise period occurred between May 1 and 31, 1999 and the warrants stopped to be quoted on May 24, 1999.

| *Underlying Stock* | *Number of warrants Issued* | *Issuing Date* a) | *Listing Date* | *Exercise Ratio* | *Exercise Price* | *Exercise Period* | *Expiration Date* |
|---|---|---|---|---|---|---|---|
| BCM | 1 875 000 | 22-10-90 | 22-10-91 | 1 | 14.96€ | 01/07/92 to 30/06/93 | 30-06-1993 |
| Tertir | 1 500 000 | 26-06-92 | 26-03-93 | 1 | b) | Last quarter of 92, 93, 94, 95, 96 | 30-09-1996 |
| Jer. Martins | 2 281 761 | 23-12-96 | 13-02-97 | 1 c) | 40.90€ | 15/08 to 15/09 of 97 until 03 | 15-09-2003 |
| Efacec | 4 500 000 | 06-12-96 | 14-02-97 | 0.3333 | 5.32€ | Feb. and Aug. of 97, 98 and 99 and Nov. 99 | 12-11-1999 |
| Sonae Indústr. | 8 000 000 | 23-02-98 | 14-05-98 | 4 | 9.60€ | Nov. 98; May and Nov. 99-00 | 20-11-2000 |
| Somague | 10 000 000 | 05-05-98 | 03-09-98 | 0.5 | 12.47€ | 14/05 to 16/06 of 2003 | 16-06-2003 |
| Cofina | 3 000 000 | 03-08-98 | 14-10-98 | 0.3333 | 17.46€ | During July 2001 and 2003 | 31-07-2003 |
| Engil d) | 7 000 000 | 11-08-98 | 16-10-98 | 0.425 | 10.42€<br>12.06€ | August 2000<br>August 2003 | 31-08-2000<br>31-08-2003 |
| Inparsa A e) | 10 000 000 | 05-11-98 | 01-02-99 | 2 | 7.48€ | December 2000 | 04-01-2001 |
| Inparsa B e) | 10 000 000 | 05-11-98 | 01-02-99 | 2 | 7.48€ | December 2001 | 04-01-2002 |
| Modelo Cont. | 5 000 000 | 09-08-99 | 21-10-99 | 1.5 | 5.00€ | 15/11 to 15/12 of 2000 | 15-12-2000 |

## Table 3.2. Statistics of equity warrants when first listed

a) The Trading Frequency represents the percentage of days that the warrants were traded, relative to the number of trading days.

| *Underlying Stock* | *Shares outstanding on listing date* | *Dilution Ratio* $(M\gamma)/(M\gamma+N)$ | *Moneyness Degree S/X* | *Initial Time to Maturity (in years)* | *Trading Frequency* a) | *Type of Warrant* |
|---|---|---|---|---|---|---|
| BCM | 6 500 000 | 22.39% | 1.09 | 1.69 | 9.64% | Pseudo-American |
| Tertir | 6 400 000 | 18.99% | - | 3.52 | 1.51% | Pseudo-American |
| Jer. Martins | 26 412 612 | 7.95% | 1.17 | 6.59 | 27.47% | Pseudo-American |
| Efacec | 10 131 580 | 12.75% | 1.33 | 2.74 | 21.86% | Pseudo-American |
| Sonae Indústria | 30 600 000 | 51.12% | 1.45 | 2.52 | 85.78% | Pseudo-American |
| Somague | 17 100 100 | 22.62% | 0.65 | 4.79 | 52.78% | European |
| Cofina | 5 000 000 | 16.67% | 0.71 | 4.80 | 49.82% | Pseudo-American |
| Engil | 16 100 600 | 15.60% | 0.52 | 4.84 | 31.14% | Pseudo-American |
| Inparsa A | 58 500 000 | 25.48% | 2.70 | 1.92 | 100.00% | European |
| Inparsa B | 58 500 000 | 25.48% | 2.70 | 2.92 | 100.00% | European |
| Modelo Contin. | 150 000 000 | 4.76% | 0.77 | 1.15 | 37.94% | European |
| **Average** | | **20.35%** | **1.31** | **3.41** | **47.09%** | |

On the other extreme we have the Sonae Indústria's warrants that traded during 487 days out of the 574 in which they were quoted (85.85%). Another extreme case of liquidity was the Inparsa A and B warrants. They were traded every day but were called for exercise by the issuer, only 74 days after the issue.

Table 3.1 and Table 3.2 show all the relevant information for all the issues occurred in Portugal for equity warrants, including: amount issued, issuing date, listing date, exercise price, exercise period, expiration date, exercise ratio, exercise conditions, issue price, the first trading day, the dilution degree, the trading frequency, the moneyness degree, the time to the maturity and the warrant type.

Until 1998 there were few trades and consequently, we had few observations of warrant trading prices. Therefore we decided to select the time period between 1998 and 2000. During this time period, nine equity warrants were listed in Portugal. Two of the issues were on the same underlying firm (Inparsa) and they were issued together with the same bond. In other words, the buyer of one bond would get the bond plus two warrants Inparsa (A and B). Engil's warrant had two different exercise prices for two different periods, allowing the double exercise (exercise on both periods). That is, each warrant were in fact two warrants. This means that Engil's warrant was really a portfolio of two independent warrants, quite distinct from a typical warrant. For these reasons, we did consider neither the Inparsa A, Inparsa B nor Engil's warrants in our research. We used the remaining six listed equity warrants during 1998 to 2000. Table 3.3 describes and synthesizes the sample of the equity warrants.

**Table 3.3 - Characteristics of the sample of the equity warrants**

| *Underlying Stock* | *Observ.* | *Sampline Period* | *Date of the 1st observation* | *Warrants value* | | | *Average Time to Maturity (in years)* | *Dilution Ratio (mean)* |
|---|---|---|---|---|---|---|---|---|
| | | | | *Mean* | *Minimum* | *Maximum* | | |
| Jer. Martins | 167 | 02-01-98 / 29-12-00 | 09-01-1998 | 52.73€ | 23,99€ | 101.00€ | 4.80 | 6.78% |
| Efacec | 36 | 02-01-98 / 10-12-99 | 12-06-1998 | 0.68€ | 0,18€ | 1.40€ | 0.51 | 11.05% |
| Sonae Ind. | 345 | 14-05-98 / 24-11-00 | 31-08-1998 | 1.95€ | 0,01€ | 14.49€ | 0.83 | 43.28% |
| Somague | 304 | 03-09-98 / 29-12-00 | 04-09-1998 | 0.23€ | 0,02€ | 0.51€ | 3.54 | 22.62% |
| Cofina | 122 | 14-10-98 / 29-12-00 | 16-10-1998 | 0.61€ | 0,05€ | 5.00€ | 4.11 | 16.67% |
| Model Cont. | 107 | 21-10-99 / 11-12-00 | 25-10-1999 | 0.96€ | 0,01€ | 2.30€ | 0.55 | 4.76% |
| **All** | 1 081 | | | 9.02€ | 0,01€ | 101.00€ | 2.94 | 23.94% |

We managed to collect a total of 1 481 observations, corresponding to 6 issues of warrants. From this set, 400 daily observations were excluded. For one of these observations, the underlying stock (Efacec) did not trade on that day, preventing the calculation of the implied volatility and, consequently, of the pricing models. All other 399 observations were excluded due to violations of the arbitrage conditions. The number of excluded observations in the sample means a substantial percentage on the starting sample.

To summarize, we selected 1 081 observations of the warrants daily closing prices, concerning to 6 issues of equity warrants, which, from now on, will be called the entire sample. Table 3.3 describes the sample characteristics, particularly the number of observations, sampling period, the date for the first observation, the warrants price, the time to maturity and dilution ratio.

# *5. Empirical results*

As stated before, our aim was to analyze the impact of dilution and dividends on the goodness of fit of warrant pricing valuation models, to a illiquidity financial product in a illiquidity market - the Portuguese warrants market. We started by testing for violations to the arbitrage conditions. Then, the four models were calculated and the mean percentage errors were computed.

The greatest difficulty in our empirical study was the shortage of trustworthy data. Since the frequency trading of equity warrants in Portugal is very thin, we had to work with discontinuous time series and with very different time intervals in-between trades and closing prices (for instance, Jerónimo Martins traded infrequently and time gaps between two consecutive closing prices go from 1 to 102 days). Although in similar studies, authors reject these observations of thinly traded warrants, we kept them, firstly because one of our goals was to check whether thinly traded markets could show a different pattern of price behaviour and secondly because of the shortage of data.

Probably, as a result, our conclusions are somehow contradictory. But as we compare different models with the same dataset we may argue that this data shortage problem is common to all of them, which in terms of comparisons should not be significant.

## *5.1. Testing for Arbitrage Conditions*

First we tested for violations on any of the arbitrage conditions. Table 3.4 shows the number of occurrences as well as their significance.

**Table 3.4 – Number of Observations Violating the Arbitrage Conditions[11]**

# represents the total number of observations for which the corresponding arbitrage condition was violated and % means its weight on the total number of observations per firm.

| Warrant | $W \geq S - X$ | | $W \geq S - Xe^{-r(T-t)}$ | | $W \geq S_d - Xe^{-r(T-t)}$ | |
|---|---|---|---|---|---|---|
| | # | % | # | % | # | % |
| *Jerónimo Martins* | 0 | 0.0% | 0 | 0.0% | 0 | 0.0% |
| *Efacec* | 38 | 43.7% | 50 | 57.5% | 25 | 28.7% |
| *Sonae Indústria* | 121 | 22.3% | 197 | 36.3% | 197 | 36.3% |
| *Somague* | 0 | 0.0% | 0 | 0.0% | 0 | 0.0% |
| *Cofina* | 136 | 49.8% | 150 | 54.9% | 148 | 54.2% |
| *Modelo Continente* | 0 | 0.0% | 0 | 0.0% | 0 | 0.0% |
| *Total* | 295 | 19.9% | 397 | 26.8% | 370 | 25.0% |

For three out of six warrants (Jerónimo Martins, Somague and Modelo Continente) we found no violations to any of the arbitrage conditions. The other three violate the 1st condition between 22% and 50% of the times, the 2nd condition between 36% and 58% of the times and the 3rd condition between 29% and 54% of the times. The 2nd condition shows the largest percentage of violations. When the stock price is adjusted to the dividends, $S_d$, the percentage of violations decreases. These very high percentages may lead us to infer for a serious warrant mispricing. However, non-synchronous data between the spot market and the warrant market may justify some of the violations of the boundary conditions. But there are also other reasons for this apparent high percentage in breaking boundary conditions (27% of the original sample). Warrants were relatively recent in the Portuguese market and, therefore, few investors were aware of warrant pricing theory; the market was very thin with public orders pending in the board for long time. Additionally we see no decrease in the number of violations, meaning that the market does not seem to have learnt with the passage of time. The boundary conditions were tested without considering any transaction costs.

[11] Having that: $W_{american} \geq W_{bermudian} \geq W_{european}$, and $W_{european} \geq S_d - Xe^{-r(T-t)} \Leftrightarrow W_{european} \geq S - Xe^{-r(T-t)}$, then: $W_{bermudian} \geq S_d - Xe^{-r(T-t)}$ and $W_{bermudian} \geq S - Xe^{-r(T-t)}$

### *5.2. Testing the Performance of Warrant Pricing Models*

We measured the performance of models by the goodness of fit of the forecasted warrant price to market prices (assuming actual underlying stock prices, actual interest rates and recently averaged historical implied volatilities). When taken in relative terms we called it percentage error and percentage absolute error.

In order to avoid mixing any other effect, we compared the performance of the four valuation models according to different dimensions: moneyness degree, time to maturity and dividends. We also compared the models for the entire sample.

#### *5.2.1. Testing Performance According to Moneyness Degree*

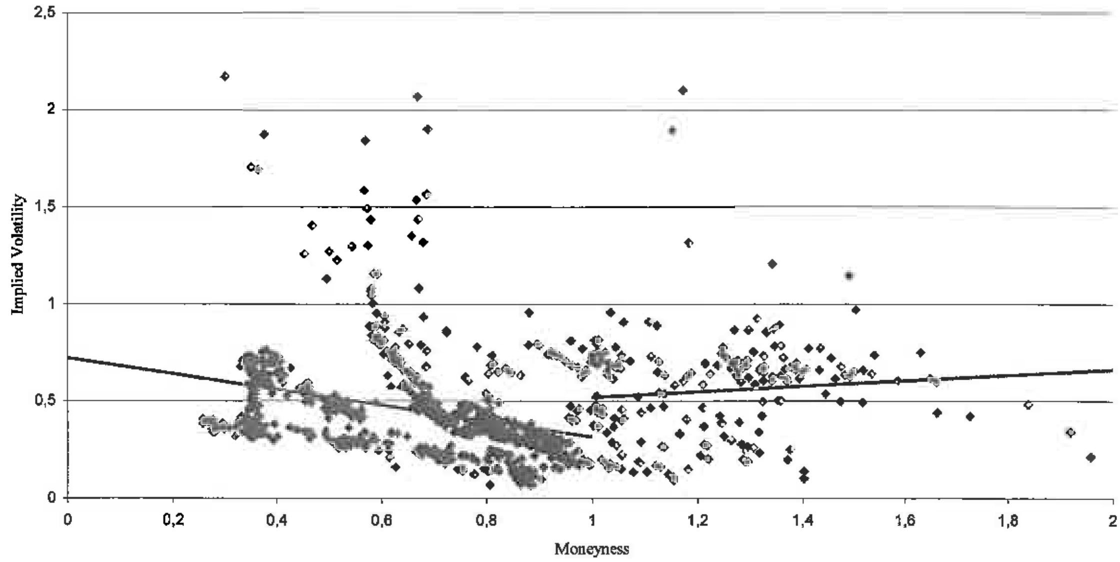
We defined Moneyness degree as the ratio of the underlying stock price deducted by the net present value of the dividends paid until maturity, $S_d$, relative to the net present value of the exercise price, that is:

$$Mnss = \frac{S_d}{Xe^{-r(T-t)}} \tag{3.21}$$

According to the literature, implied volatility estimated from at-the-money stock options tends to differ from implied volatility of in-the-money and out-of-the-money options. Our treatment of the data follows approximately the same procedure to the one used by Duque and Lopes (2003) where the effect of the moneyness degree was tested as an explanatory factor of the differences found on implied volatilities.

We started by plotting implied volatilities estimated for each model for each of the warrant issued (please refer to Figures A.3.1 to A.3.6 in the appendix). We also plotted the entire sample in a single chart (Figure 3.1 bellow) in order to present a broad idea of any possible bias known as the smile effect. In a first glance we spot a significant number of observations with extremely high implied volatilities (well above 100%). However, we do not observe any of these high implied volatilities when we restrict the analysis to the at-the-money observation. We also observe visually a bias related to the moneyness degree (smile effect).

**Figure 3.1. The Four Black and Scholes Implied Volatilities of the Entire Sample According to Moneyness**

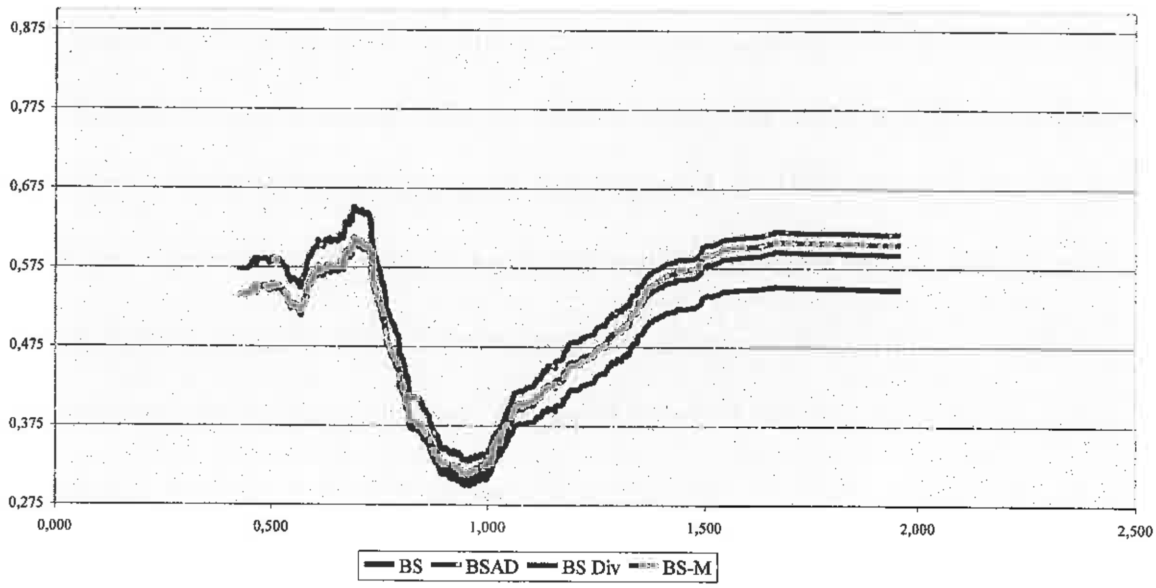



In some observations we observe some extremely high values for implied volatility figures, which may well represent strongly overpriced warrants.

The smile effect presented in the entire literature on option pricing bias is somehow more evident in Figure 3.2 after having sorted the entire sample by moneyness degree and, for each observation, having averaged the previous 200 observations.

From Figure 3.2 we draw two main conclusions: first, different models tend to show the same kind of pattern, although the Black and Scholes model adjusted for dilution (BSAD) seems to present systematically higher implied volatilities; second, the smile effect is asymmetric (out-of-the-money options seem to be far more sensitive to changes in moneyness than in-the-money options).

**Figure 3.2. Implied Volatility Smiles Considering the Entire Sample**




In order to test the statistical significance of the findings just presented we considered the following regression equations using only the Black-Scholes implied volatility dataset:

$$\sigma_{imp,in} = \beta_0 + \beta_1(Mnss), \qquad \text{for Mnss} > 1 \qquad (3.22)$$

$$\sigma_{imp,out} = \beta_0 + \beta_1(Mnss), \qquad \text{for Mnss} < 1 \qquad (3.23)$$

These regression equations were first calculated for each firm and next to the entire sample (Table 3.5).

As a very general comment we may say that implied volatilities in the Portuguese warrants market tend to consistently show the well-known smile effect, although they are more evident and statistically significant for out-of-the-money warrants. In Table

3.5 while all $\beta_1$ are statistically significant for out-of-the-money options, only for Cofina $\beta_1$ was statistically significant in the case of in-the-money options. This lack of significance for in-the-money regression equation parameters is not a consequence of the number of observations. From all the regression equations estimated in Table 3.5, Cofina is the case for which we got the smallest number of data points.

In addition, the away-from-the-money bias is more sensitive for out-of-the-money options than for in-the-money options, which is consistent with the findings of Duque and Lopes (2003) when studying equity options traded in LIFFE. The absolute value of $\beta_1$ is far higher for all the out-of-the-money regression equations than for the in-the-money regression equations. Therefore, the same change in the underlying stock price moving away from the money has a higher impact on the out-of-the-money options mispricing than on in-the-money options mispricing. A similar conclusion can be obtained by comparing the absolute value of the slopes of the regression equations estimated to entire sample. Apart from being statistically insignificant the $\beta_1$ for in-the-money regression equation (0.142) was significantly lower than the corresponding out-of-the-money parameter (0.406)[12]. Figure 3.1 shows the regression lines for both in and out-of-the-money warrants estimated to the entire sample according to Ordinary Least Squares.

[12] At a 90% confidence level the slopes of both regressions and their corresponding confidence intervals are the following: $\left|\beta_2^{ITM}\right| = +0.142 \quad \in [-0.040; +0.322]$ and $\left|\beta_2^{OTM}\right| = +0.142 \quad \in [+0.340; +0.471]$.

**Table 3.5 - Results of Linear Regressions**

Each cell represents the estimated parameters and the corresponding t-statistic of regression equation $\sigma_{imp,in} = \beta_0 + \beta_1(Mnss)$. * Means that values are significant at a 95% confidence level.

To calculate the implied volatility, we use $\sigma_t^k = \frac{\sum_{i=1}^{5} \sigma_{imp_{t-1}}^k}{5}$

| | Mnss < 1 *(out-of-the-money)* | | | | Mnss > 1 *(in-the-money)* | | | |
|---|---|---|---|---|---|---|---|---|
| | $\beta_0$ | $\beta_1$ | $R^2$ Adj. | N. Obs. | $\beta_0$ | $\beta_1$ | $R^2$ Adj. | N. Obs. |
| *Jerónimo Martins* | 2.233* (26.207) | -1.733* (-16.884) | 0.840 | 55 | 0.509 (4.805) | 0.117 (1.382) | 0.008 | 112 |
| *Cofina* | 0.626* (22.551) | (-0.559)* (-15.652) | 0.711 | 100 | -0.005 (-0.053) | 0.186* (2.813) | 0.248 | 22 |
| *Modelo Continente* | 2.677* (12.108) | -2.742* (-9.964) | 0.481 | 107 | | | | |
| *Efacec* | | | | | 0.451 (1.170) | 0.187 (0.604) | -0.019 | 35 |
| *Sonae Indústria* | 1.663* (31.801) | -1.583* (-23.448) | 0.633 | 319 | 0.375 (2.200) | -0.143 (-0.977) | -0.002 | 26 |
| *Somague* | 0.722* (21.849) | -0.610* (-7.896) | 0.168 | 304 | | | | |
| ***Total Sample*** | 0,721 (26.489) | -0,406* (-10.203) | 0,104 | 886 | 0,377 (2.750) | 0,142 (1.292) | 0,003 | 195 |

Additionally we split the entire sample into three groups (in, at and out-of-the-money) instead of the two just presented (Table 3.6) in order to observe in more detail the asymmetry of the smile effect for the entire sample. However, given the short number of observations, we did not compute the regression equations for individual firms as previously done in Table 3.5.

**Table 3.6 - Results of Linear Regression Applied to the Entire Sample**

Each cell represents the estimated parameters and the corresponding t-statistic of regression equation $\sigma_{imp,in} = \beta_0 + \beta_1(Mnss)$.

* Means that values are significant at a 95% confidence level.

| | Mnss < 0.7 *(Out-of-the-money)* | 0.07 < Mnss < 1.3 *(At-the-money)* | Mnss > 1.3 *(In-the-money)* |
|---|---|---|---|
| $\beta_0$ | 0.732* (24.508) | 0.693 (-2.030) | 0,430 (1.958) |
| $\beta_1$ | -0.426* (-9.262) | 1.129 (3.254) | 0,106 (0,646) |
| $R^2$ Adj. | 0.096 | 0.062 | -0.004 |
| N. Obs. | 798 | 145 | 138 |

The conclusions drawn based upon the figures of Table 3.5 are reinforced by the figures presented in Table 3.6. We underline the most significant conclusions. First, the u-shaped form of the smile obtained from Table 3.6 is clearer than the one extracted previously from Table 3.5. The slope for the in-the-money observations is now statistically significant as for out-of-the-money options and the difference between both betas is now more significant. Second, as we concluded from Table 3.5, the results presented in Table 3.6 also document that away-from-the-money bias is more sensitive for out-of-the-money options than for in-the-money options. Lastly, the v-shape obtained from the slopes of the regression equation presented in Table 3.6 is widen than the corresponding v-shape form obtained from Table 3.5[13].

Then we calculated the average implied volatilities but the conclusions did not change (see Table 3.7).

Generally speaking, average far-from-the-money implied volatilities are higher than at-the-money implied volatility figures whatever the model under use. However, when implied volatility is extracted from the original Black-Scholes model it is always smaller than when it is obtained by the others models. This was previously found by when composing Figure 3.2 and is now confirmed by averaging implied volatilities that were grouped by moneyness. Maybe opposing to what we could expect, if the average Black and Scholes' implied volatility figures are always lower than other models' implied volatilities, this may mean that when introducing new parameters in warrant price modelling we are introducing new sources of uncertainty, increasing the degree of residual risk that is observable in the only residual variable implicitly estimated.

[13] The comparison of the arctangent of the slopes (Table 5.2 and 5.3) may prove the following: $acr\tan gent(0.106) - acr\tan gent(-1.426) = 0.474$; $acr\tan gent(0.142) - acr\tan gent(-1.406) = 0.491$.

**Table 3.7 - Comparison of the Percentage Error with the Moneyness Degree**

The Average Implied Volatility was calculated from the implied volatilities extracted for each model under consideration. The Percentage Error and the Percentage Absolute Error are estimated according to the following expressions:

$$PercentageError = \frac{\text{model value} - \text{market price}}{\text{market price}} \times 100\%;$$

$$AbsolutePercentageError = \left| \frac{\text{model value} - \text{market price}}{\text{market price}} \right| \times 100\%$$

The number of observations droped to 1.051 because don't have implied volatility to put in the first five observations of each warrants.

| *Warrant Pricing Model* | *No. of Observ.* | *Average Implied Volatility* | *Percentage Error* Mean | *Percentage Error* $\sigma$ | *Absolute Percentage Error* Mean | *Absolute Percentage Error* $\sigma$ |
|---|---|---|---|---|---|---|
| *In-the-Money* | | | | | | |
| *Black-Scholes* | 128 | 57,0% | 0,98% | 16,2% | 9,36% | 13,2% |
| *BSdiv* | 128 | 60,6% | 0,66% | 16,5% | 9,68% | 13,3% |
| *BS-M* | 128 | 63,0% | 0,30% | 16,4% | 9,82% | 13,1% |
| *BSAD* | 128 | 63,1% | 0,33% | 16,5% | 9,91% | 13,2% |
| *At-the-Money* | | | | | | |
| *Black-Scholes* | 140 | 42,2% | 2,32% | 19,9% | 12,81% | 15,3% |
| *BSdiv* | 140 | 44,8% | 2,07% | 19,7% | 12,80% | 15,1% |
| *BS-M* | 140 | 45,4% | 2,09% | 19,6% | 12,94% | 14,9% |
| *BSAD* | 140 | 47,2% | 2,36% | 20,5% | 13,59% | 15,5% |
| *Out-of-the-Money* | | | | | | |
| *Black-Scholes* | 783 | 46,2% | -2,50% | 34,9% | 19,86% | 28,8% |
| *BSdiv* | 783 | 46,5% | -2,57% | 34,8% | 19,86% | 28,6% |
| *BS-M* | 783 | 46,8% | -2,37% | 34,9% | 19,77% | 28,9% |
| *BSAD* | 783 | 49,8% | -2,59% | 34,2% | 19,38% | 28,3% |

When the comparison between models is based on terms of the percentage error taking into account the moneyness degree, we found that warrants in-the-money tend to show better performances (the results are presented in Table 3.7). This could be seen as a general tendency for the models under study to price better in-the-money warrants than at or out-of-the-money warrants. This is somehow unexpected since the literature shows that models tend to perform better for at-the-money options. Therefore the conclusions should be carefully read. In this study (the methodology used is common to all the empirical studies carried out on warrant pricing) we estimate volatility to calculate the theoretical warrant price based on the most recent implied volatility estimates. For each day (observation) the volatility is calculated as the average of the previous 5-days implied volatilities. So, instead of speaking of models that are "better to price" we rather prefer to speak about models that are "less sensitive to changes in parameters" (in the present case we are talking about volatility). And in fact, the lambda of in-the-money options is significantly lower than the lambda of at-the-money options.

Table 3.7 also shows that out-of-the-money warrants present negative percentage error, while in-the-money or at-the-money warrants present positive percentage errors. This means that while out-of-the-money warrants tend to be undervalued, in-the-money and at-the-money warrants tend to be overvalued.

A last word should be addressed to the comparative performance of the models under study (Table 3.7). And it is clear that the performance of each model depends on the moneyness degree. For warrants in-the-money the best model seems to be the original Black-Scholes model, for warrants out-of-the-money the best model seems to be the BSAD, while for warrants at-the-money the best model seems to be the BSdiv[14]. However differences are not significant. The errors inside of each moneyness degree are very similar and the bigger difference between the different models is 0,79% in at-the-money.

### *5.2.2. Testing Performance According to Time to Maturity*

Table 3.8 presents percentage errors and implied volatility figures when warrants are segregated by time to maturity. We found that as maturity approaches, models tend to decrease its performance, which is particularly poor for short term warrants (with less than two months to maturity). This decrease in terms of performance is also matched by the increase on implied volatilities, similar to what was found in Duque and Paxson (1997). But, although this seems to be the general pattern, there is a notorious difference

[14] The word "best" is used as a synonymous of most stable according to our previous explanation above.

for warrants maturing between 1 and 2 years. Those warrants are the best performers (with lower percentage and absolute percentage errors) and show a significantly lower implied volatility.

When trying to spot differences among models we observed that no model performs systematically better than the others. However, when observing the percentage absolute error the BSAD is the best performing model either for very short term warrants, either for longer term warrants. Nevertheless, there are never great differences among the models[15].

**Table 3.8. Comparison of the percentage error with the Time to the Maturity**

The Average Implied Volatility was calculated from the implied volatilties extracted for each model under consideration. TTM = Time to Maturity (0,167 years = 2 months). The Percentage Error and the Percentage Absolute Error are estimated according to the following expressions:

$$\text{PercentageError} = \frac{\text{model value - market price}}{\text{market price}} \times 100\% ;$$

$$\text{AbsolutePercentageError} = \left| \frac{\text{model value - market price}}{\text{market price}} \right| \times 100\%$$

| Model | No. of Observ. | Implied Volatility | Percentage Error Mean | Percentage Error σ | Absolute Percentage Error Mean | Absolute Percentage Error σ |
|---|---|---|---|---|---|---|
| *TTM > 2 years* | | | | | | |
| *Black-Scholes* | 587 | 47,87% | -0,12% | 25,8% | 16,38% | 19,9% |
| BSdiv | 587 | 48,38% | -0,17% | 25,8% | 16,38% | 19,9% |
| *BS-M* | 587 | 50,20% | -0,17% | 25,8% | 16,36% | 19,9% |
| *BSAD* | 587 | 50,73% | -0,53% | 25,1% | 15,90% | 19,4% |
| *TTM 1 to 2 years* | | | | | | |
| *Black-Scholes* | 121 | 27,85% | 0,47% | 19,5% | 13,90% | 14,2% |
| *BSdiv* | 121 | 27,95% | 0,38% | 19,3% | 13,82% | 14,0% |
| *BS-M* | 121 | 27,97% | 0,34% | 19,2% | 13,78% | 13,9% |
| *BSAD* | 121 | 31,91% | 0,61% | 20,5% | 15,06% | 14,7% |
| *TTM 0,167 to 1 year* | | | | | | |
| *Black-Scholes* | 283 | 44,55% | -1,29% | 22,8% | 15,28% | 17,2% |
| BSdiv | 283 | 46,83% | -1,44% | 23,0% | 15,44% | 17,2% |
| *BS-M* | 283 | 45,48% | -1,23% | 23,0% | 15,37% | 17,3% |
| *BSAD* | 283 | 50,75% | -1,23% | 23,0% | 15,34% | 17,2% |
| *TTM < 0,167 years* | | | | | | |
| *Black-Scholes* | 60 | 87,54% | -18,79% | 78,2% | 48,73% | 63,9% |
| BSdiv | 60 | 89,77% | -19,68% | 77,1% | 48,78% | 62,9% |
| *BS-M* | 60 | 87,78% | -18,66% | 78,4% | 48,72% | 64,1% |
| *BSAD* | 60 | 93,73% | -17,82% | 77,0% | 47,47% | 63,2% |

[15] This waving behaviour may show a positive hope for mean reverting stochastic volatility models. However, this is out of the scope of this paper.

### *5.2.3. Testing Performance Considering Dividend Paying Firms*

Only two out of six warrants were issued by dividend paying shares. Therefore, as two of the models were adjusted for dividends we restricted our analysis to those firms that paid dividends during the sampling time period. We dropped those non-dividend paying companies from our sample in order to compare the relative performance of the models under scope. Table 3.9 presents the results:

**Table 3.9 – Percentage Error of Warrants of Dividend Paying Firms**

The Average Implied Volatility was calculated from the implied volatilities extracted for each model under consideration. The sample was reduced to firms that paid dividends during the time period of analysis. Values in brackets represent the t-values. * Means that value are significant at a 99% confidence level. The Percentage Error and the Percentage Absolute Error are estimated according to the following expressions:

$$\text{PercentageError} = \frac{\text{model value} - \text{market price}}{\text{market price}} \times 100\%;$$

$$\text{AbsolutePercentageError} = \left|\frac{\text{model value} - \text{market price}}{\text{market price}}\right| \times 100\%$$

| *Model* | *No. of Observ.* | *Implied Volatility* | *Percentage Error* | | *Absolute Percentage Error* | |
|---|---|---|---|---|---|---|
| | | | *Mean* | *σ* | *Mean* | *σ* |
| *Black-Scholes* | 412 | 50,4% | 0,52%<br>(0,26) | 40,4% | 20,1%*<br>(11,63) | 35,1% |
| *Bsdiv* | 412 | 53,1% | 0,20%<br>(0,10) | 40,2% | 20,2%*<br>(11,80) | 34,7% |
| *BS-M* | 412 | 54,5% | 0,48%<br>(0,24) | 40,4% | 20,1%*<br>(11,62) | 35,1% |
| *BSAD* | 412 | 54,4% | 0,17%<br>(0,09) | 40,1% | 20,1%*<br>(11,53) | 34,7% |

We found that although the models performance is very similar, it is the BSAD that better performs particularly when using the percentage error for performance indicator. Therefore, when firms having warrants, pay dividends, it seems significant to account for it in warrant price and modelling. It is also interesting to spot that among the models that exclusively consider dividend paying adjustments, the discrete model seems to outperform the Merton (1973) model. This is logical since the Merton model is expected to perform better for stock indices or stocks that pay dividends several times along the year, which is not the case for Portuguese stocks. As a final remark we may say that adjusting for dilution and dividends (particularly the discrete model) seems to result in lower implied volatility variation.

### *5.2.4. Testing Performance with the Entire Sample*

In this item we used the entire sample with no segregations to examine whether one of the models systematically out or underperforms the others. Tables 3.10 and 3.11 summarize the empirical results.

**Table 3.10. Percentage Error of All the Warrants**

The Average Implied Volatility was calculated from the implied volatilties extracted for each model under consideration. Values in brackets represent the t-values. * Means that values are significant at a 99% confidence level. The Percentage Error and the Percentage Absolute Error are estimated according to the following expressions:

$$\text{PercentageError} = \frac{\text{model value} - \text{market price}}{\text{market price}} \times 100\%;$$

$$\text{AbsolutePercentageError} = \left| \frac{\text{model value} - \text{market price}}{\text{market price}} \right| \times 100\%$$

| Model | No. of Observ. | Implied Volatility | Percentage Error Mean | Percentage Error $\sigma$ | Absolute Percentage Error Mean | Absolute Percentage Error $\sigma$ |
|---|---|---|---|---|---|---|
| *Black-Scholes* | 1.051 | 47,0% | -1,43% (-1,472) | 31,6% | 17,65%* (21,85) | 26,2% |
| *Bsdiv* | 1.051 | 48,0% | -1,56% (-1,610) | 31,4% | 17,69%* (22,05) | 26,0% |
| *BS-M* | 1.051 | 48,6% | -1,45% (-1,491) | 31,6% | 17,65%* (21,84) | 26,2% |
| *BSAD* | 1.051 | 51,1% | -1,57% (-1,639) | 31,1% | 17,45%* (21,99) | 25,7% |

The t-statistic for matched samples (Table 3.11) shows that differences among the means are not statistically significant for most of the cases (at 1% and 5% significance level). However, when the BSAD model is compared with other models, differences start to be significant. In Table 3.10 the BASD model is the model that shows lower Absolute Percentage Error and differences are strongly emphasized in Table 3.11. This means that we found empirical evidence for the dilution effect in the Portuguese equity warrants market that is consistent with the findings of Hauser and Lauterbach (1997) and Low (2000). Therefore, we strongly recommend for warrant price modelling the use of models that accommodate both dilution and dividend paying effect, particularly in the discrete form.

**Table 3.11. Implied Volatility Differences Using two Models and Matched Samples**

The Average Implied Volatility was calculated from the difference of implied volatilities extracted for a pair of models (under consideration). The Percentage Error and the Percentage Absolute Error are estimated according to the following expressions:

$$PercentageError = \frac{\text{model value} - \text{market price}}{\text{market price}} \times 100\%;$$

$$AbsolutePercentageError = \left|\frac{\text{model value} - \text{market price}}{\text{market price}}\right| \times 100\%$$

| | *Percentage Error* | | | | | *Percentage Absolute Error* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *Mean* | *σ* | *T-statistic* | *Df* | *Sig (2 tailed)* | *Mean* | *σ* | *T-statistic* | *Df* | *Sig (2 tailed)* |
| *BS <-> BSDIV* | 0,128% | 0,91% | 4,584 | 1050 | 0,000 | -0,039% | 0,82% | -1,547 | 1050 | 0,122 |
| *BS <-> BS-M* | 0,019% | 1,32% | 0,456 | 1050 | 0,649 | 0,002% | 1,28% | 0,042 | 1050 | 0,967 |
| *BS <-> BSAD* | 0,138% | 3,27% | 1,366 | 1050 | 0,172 | 0,193% | 2,91% | 2,155 | 1050 | 0,031 |
| *BSDIV <-> BS-M* | -0,109% | 1,54% | -2,299 | 1050 | 0,022 | 0,041% | 1,44% | 0,917 | 1050 | 0,359 |
| *BSDIV <-> BSAD* | 0,010% | 3,16% | 0,101 | 1050 | 0,920 | 0,232% | 2,81% | 2,677 | 1050 | 0,008 |
| *BS-M <-> BSAD* | 0,119% | 3,26% | 1,187 | 1050 | 0,236 | 0,192% | 2,90% | 2,143 | 1050 | 0,032 |

## *6. Conclusions*

Only recently the literature on equity warrants presented sophisticated methods to deal with dilution or dividend paying stocks. The main contributions to the topic have been Merton (1973), Roll (1977), Galai and Schneller (1978), Geske (1979, 1981), Whaley (1981), Lauterbach and Schultz (1990) or Schulz and Trautmann (1989 and 1994). It is expected that dilution and dividends have some impact on market prices for warrants. However, do we still notice analogous effects in illiquid markets? Thin trading introduces pricing bias that may well absorb all typical effects that we see in other warrant markets, namely the dilution and dividend effects. It could be interesting to check empirically whether dividends and dilution have some impact on warrants market prices, using a quite illiquid market as the Portuguese.

In order to avoid any other undesirable effects we developed our research exclusively within the Black-Scholes framework. We chose four warrant pricing models: the original Black-Scholes model and three of its derivations. Using these four models we empirically estimated values for actual warrant prices, and calculated the mean percentage error for each, as the difference between model prices and market prices. We assumed that the most efficient model shows the smallest percentage error. We used data supplied from the Euronext - Lisbon from 1998 to 2000.

Implied volatility extracted by the models, was higher for warrants in-the-money and out-of-the-money and lower for the warrants at-the-money, showing signs of the so called "smile effect". This effect seems not to be symmetric since we found that out-of-the-money warrants are more sensitive to the exercise bias than in-the-money warrants. Additionally, we found a strong pattern for the increase of implied volatility as time to maturity decreases. This resembles stock option markets and supports the findings of Duque and Lopes (2003).

Although there are no strong differences among the models the results lead us to conclude that the in the Portuguese warrants market from 1998 to 2000, the BSAD model, which accounts for the dilution effect and net present value of dividends, outperforms the others. These results are evident after comparing the models based on the Mean Absolute Error methodology. The BSAD model shows the lowest absolute percentage error and its standard deviation among all the models. When the comparison is between pairs of models, the results show irrelevant differences among them, except when comparing the BSAD model with every single one. In these cases the differences between each model and the BSAD model turns strongly significant, showing this model to be apparently the most appropriated model to price equity warrants, as expected by the theoretical literature.

Although this research strongly supports the appropriateness of the BSAD model to value equity warrants these results are still restricted by some limitations. Firstly, the equity warrants under scope are typically Bermudian style options that are not appropriately value by European style models that were used in this research. Secondly, the models used assumed dividends as known and certain. Stochastic dividend models should alternatively be used, although out of the scope of this paper.

## APPENDIX

**Figure A.3.1** - Implied Volatility versus Moneyness for Jerónimo Martins

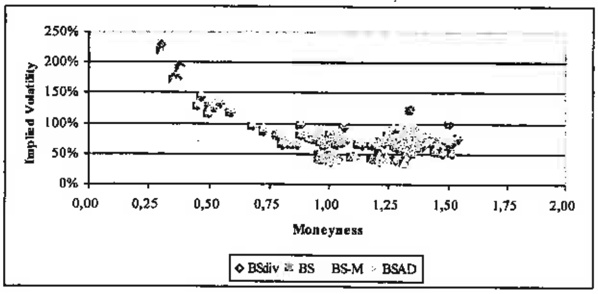



**Figure A.3.2** - Implied Volatility versus Moneyness for Cofina

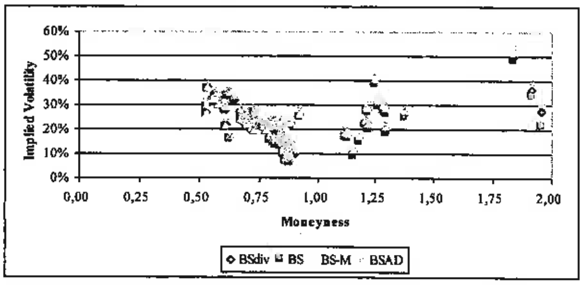



**Figure A.3.3** - Implied Volatility versus Moneyness for Modelo Continente

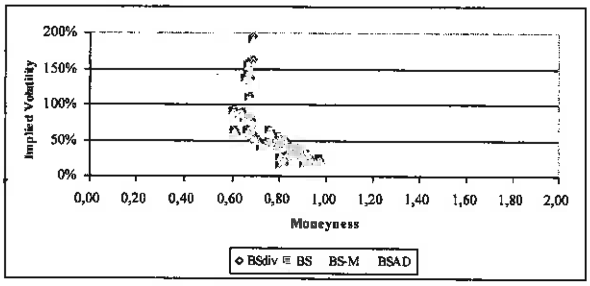



**Figure A.3.4** - Implied Volatility versus Moneyness for Efacec

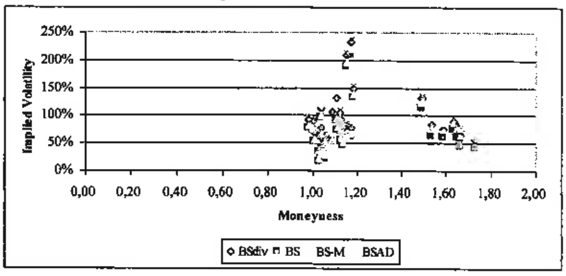



**Figure A.3.5** - Implied Volatility versus Moneyness for Sonae Indústria

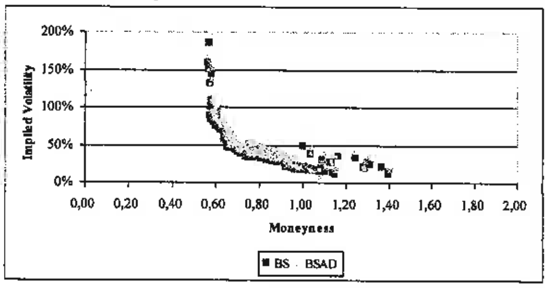
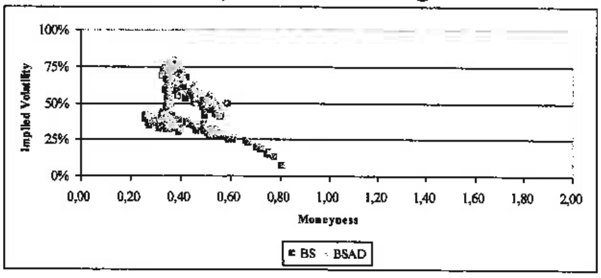



**Figure A.3.6** - Implied Volatility versus Moneyness for Somague



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