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INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

Evaluation of ruin probabilities for surplus processes with credibility and surplus dependent premiums

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(Mestre)

Dissertação para obtenção do Grau de Doutor em
Matemática Aplicada à Economia e Gestão

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Doutor Hans Ulrich Gerber

Fevereiro 2008

Dedicated to Mariana and Nelson

Acknowledgements

Apart from staring at 200 blank pages at the beginning of the writing process, this is the most difficult part to write. There were many people, from the tallest (Nelson) to the shortest (Mariana), who supported me in several ways during the past three and half years.

My first thoughts are for my supervisors Professor Alfredo Egídio dos Reis and Professor Howard Waters for their generous support, valuable guidance and specially for their friendship during this project. Many thanks for having accepted me as a Ph.D. student. I've increased my knowledge in so many different areas.

I would like to thank Professor João Tiago Mexia for introducing me to the academic career. I am grateful to Centro de Matemática e Aplicações of FCT/UNL for the financial support for my working visits to Heriot-Watt University and to present part of this work at the IME 2007 conference, to Fundação Calouste Gulbenkian for financial support to present part of this work at the IME 2006 conference, to Faculdade de Ciências e Tecnologia and CEMAPRE-ISEG for allowing me to use their servers to run the C++ programs. I would also like to thank two people whom I know only through e-mail, Dr. Kenneth Wilder of the University of Chicago for helping me to change his C++ random number generator class in order to run in 64 bit machines and David Guilherme for his help in solving some problems concerning C++.

To my friends Ana Luísa Batista, Carolann Waters, David Pontes, Fátima Miguens, Isabel Gomes, Gracinda Guerreiro, Pedro Corte Real, Rui Cardoso, Teresa Afonso, Vitor Chagas and all the others who are not mentioned here, I thank you for your support and specially for your friendship.

My utmost gratitude goes to my parents who encouraged me and made every possible effort to support me in whatever way they could, without having the slightest idea of the meaning of ruin theory or credibility theory. From all my heart I'm grateful to my daughter Mariana (who was born at the beginning of this project) and my husband Nelson, to whom I proudly dedicate this work. They have taught me the meaning of unconditional love.

Despite all the work involved in the project, the bad nights sleep, the extra white hairs, all the chips, popcorn and chewing gum eaten, I must confess that I had much fun, learned a lot, had the opportunity of knowing other cultures and people and enjoyed very much doing it. Thank you all for having made it possible.

Resumo

É proposto um método para o cálculo da probabilidade de ruina em tempo contínuo e horizonte finito para um processo de Poisson composto onde o prémio é constante ao longo de cada período de tempo (ano), mas depende da informação passada de indemnizações agregadas anuais. Em função disso, o prémio é ajustado anualmente, passando a ser variável de período para período.

Um dos grandes contributos deste trabalho é o facto da metodologia apresentada ser facilmente aplicável a carteiras de grande dimensão. O método é baseado na simulação das indemnizações agregadas anuais e no calculo da probabilidade de ruína dado um determinado montante de reserva no início e no fim do período. Este cálculo da probabilidade de ruína é aproximado de duas formas: primeiro usando um movimento Browniano adequado e depois uma aproximação à distribuição gama deslocada.

A coerência dos resultados produzidos pelo modelo é testada comparando os resultados produzidos para o modelo clássico de risco com o modelo-base e com os resultados exactos obtidos por Wikstad (1971) e por Seal (1978), em tempo contínuo e horizonte finito.

O método é aplicado a três modelos de risco diferentes em que o prémio é actualizado no ínicio do ano. Para cada modelo as indemnizações agregadas seguem uma distribuição de Poisson composta em que processo do número de sinistros tem o parâmetro de Poisson fixo ou variável.

No primeiro modelo o prémio é definido como função do nível de reserva em algum momento anterior. O coeficiente de carga para o prémio anual é determinado em cada caso de forma à probabilidade em horizonte infinito, partindo da reserva incial considerada, ser aproximadamente um valor pré-definido para o modelo clássico. Para tal, é utilizada a aproximação de De Vylder (1978). No segundo e terceiro modelos considera-se uma carteira que satisfaz as hipóteses dos modelos de credibilidade de Bühlmann e Bühlmann-Straub sendo o prémio anual actualizado de acordo com estes modelos.

Palavras-Chave: Probabilidade de Ruína, Movimento Browniano, Aproximação à Gama deslocada, Simulação, Prémios dependentes da reserva, Prémios de Credibilidade.

Abstract

In this dissertation we present a method for the numerical evaluation of the ruin probability in continuous and finite time for a classical risk process where the premium can change from year to year. A major consideration in the development of this methodology is that it should be easily applicable to large portfolios. Our method is based on the simulation of the annual aggregate claims and then on the calculation of the ruin probability for a given surplus at the start and at the end of each year. We calculate the within-year ruin probability assuming first a Brownian motion approximation and, secondly, a translated gamma distribution approximation for aggregate claim amounts.

We will check the accuracy of our method by comparing our results applied to the classical risk process with the results of Wikstad (1971) and Seal (1978b) in finite and continuous time. We also check its accuracy in the case of exponential and mixed exponential claim amounts by choosing a very long time horizon and comparing results with exact results for infinite time ruin.

We apply our method to three different risk models where the premium is set at the start of each year but can change from year to year. For each model aggregate claims have a compound Poisson distribution with either a fixed or a variable Poisson parameter for the claim number process. For the first model the premium in each year is a function of the surplus level at the start of that, or an earlier, year. The premium rate is set so that the

probability of ultimate ruin from that time is approximately equal to a pre-determined value. We will use De Vylder's (1978) approximation to achieve that. For the second and third models we consider a portfolio of risks which satisfy the assumptions of the Bühlmann or Bühlmann-Straub credibility models with the pure premium updated each year in accordance with these models.

KEYWORDS:Ruin probabilities, Brownian motion approximation, translated gamma approximation, simulation, Surplus dependent premiums, Credibility premiums.

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Chapter 1

Introduction

The aim of this thesis is to present a method for calculating the probability of ruin in finite time for a compound Poisson risk process where the premium rate is constant throughout the year but depends on the past aggregate annual claims experience and hence changes each year.

Ruin theory has its roots in the beginning of the twentieth century when fundamental ideas were published by Lundberg (1903). Since then many studies have dealt with the exact or approximate computation of ruin probabilities, mainly for the classical risk process.

The problem of calculating the probability of ruin when the premium is a function of the surplus level at the end of the year has been studied by many authors, in most of the cases in infinite time. For example, Davidson (1969) let the safety loading decrease with an increasing risk reserve. Taylor (1980) and Jasiulewicz (2001) consider the case where the premium rate varies continuously as a function of the surplus. Petersen (1989) illustrates with a simple numerical method how the probability of ruin can be calculated when the general premium rate depends on the reserve. Dickson (1991) considers the case where the premium rate changes when the surplus crosses an upper barrier. More recently, Cardoso and Waters (2005) presented a numerical method for calculating finite time ruin probabilities for the same problem.

Credibility theory development began in the early twentieth century. The work of

Mowbray (1914) raised the question of what premium should be used if the individual risk experience for a single contract is found to be insufficient. Whitney (1918) suggested using a weighted average of the individual and collective premium as a solution. Most authors have the opinion that it was Bühlmann who supplied a theoretical background to this problem in his papers Bühlmann (1967) and Bühlmann (1969). Bühlmann and Straub (1970) recognised that contracts often have different underlying risk exposures and extended Bühlmann's model.

The problem of calculating the probability of ruin for a risk process where the premium is updated according to a credibility model was considered by Dubey (1977) and Tsai and Parker (2004). The former contains some interesting theoretical results; the latter paper is focused on numerical results for a discrete time model where the premium is updated according to the Bühlmann credibility model.

The use of simulation to estimate ruin probabilities is not new. Dufresne and Gerber (1989) observe that the probability of ruin is related to the stationary distribution of a certain associated process and estimate it using simulation. Michaud (1996) simulated the jumps and interjump times for two models in order to approximate the probability of ultimate ruin. In his first model the surplus earns interest; in his second model the premium depends on the level of the surplus.

Our method involves simulating the aggregate claims for each year, calculating the premium to be charged each year given the past aggregate claim amounts, and then calculating the within year probability of ruin assuming either a Brownian motion approximation to the surplus process or a translated gamma distribution approximation for aggregate claim amounts. The Brownian motion approximation is well established, see for example Sections 8.6 and 8.7 of Klugman et al. (2004), and should work well if the expected number of claims in each year is reasonably large. The translated gamma approximation uses ideas which go back to Seal (1978a) and which have recently been used by Dickson and Waters (2006). We would expect the latter approximation to be better than the former since it is based on three rather than two moments.

In Chapter 2 we set out our model and general procedure for calculating the ruin probabilities in finite time which we will use in the following chapters. Details of the approximations and of our simulation procedure are given. We will check the accuracy of our method by comparing our results applied to the classical risk process with the results of Wikstad (1971) and Seal (1978b) in finite and continuous time. We also compare our results with the results for ultimate ruin probabilities in cases where claim amounts are exponentially distributed and where they are a mixture of exponentials (Gerber (1979)).

In Chapter 3 we consider the problem of calculating the probability of ruin for a risk process where the premium is set at the start of the year. We consider the premium calculated according to the level of surplus at the beginning of the year. In practice it may not be possible to achieve this since there may be some delay between the end of the year and the new premium being introduced. For this reason we consider the case where the premium in the coming year depends on the surplus one year ago. We also consider the classical case where the premium depends on the initial surplus and is constant throughout the remaining period. The premium rate is set in each case so that the probability of ultimate ruin from that time is approximately equal to a pre-determined value (0.005 or 0.01). We will use De Vylder's (1978) approximation to achieve that.

Chapter 4 is a step forward. We deal with the calculation of the probability of ruin for a portfolio of risks where the premium for each individual risk is updated each year based on the experience for all the risks according to the Bühlmann credibility model. In order to understand the impact of the credibility premium on the ruin probability we will consider also the premium calculated using the conditional expected value of the aggregate claim amount. The safety loading will be calculated using the same approach as in the previous chapter. The methodology is similar to the one used before with some minor adjustments.

In Chapter 5 we consider the Bühlmann-Straub credibility model. This model allows for variation in exposure or size. In this chapter we will use a fixed safety loading and compare the results using the Bühlmann-Straub credibility premium with the ones using the conditional expected value of aggregate claims.

In Chapters 3 to 5 we also consider two cases for the Poisson parameter, the classical one (constant throughout the period) and a varying one.

Numerical examples are discussed at the end of each chapter. Figures and statistics are shown to make it easier to understand the results. Some conclusions and comments on further research are set out in Chapter 6.

Much of the work of Chapters 2 and 3 of this thesis is presented in the paper by Afonso et al. (2007a), and the work of Chapters 4 and 5 is presented in the paper by Afonso et al. (2007b).

Chapter 2

The basic algorithm

In this Chapter we set out our model and general procedure for the numerical evaluation of the ruin probability in continuous and finite time for a classical risk process where the premium can change from year to year. Our method is based on the simulation of the annual aggregate claims and then on the numerical calculation of the ruin probability for a given surplus at the start and at the end of each year. We calculate the within-year ruin probability assuming first a Brownian motion process approximation and, secondly, a translated gamma distribution approximation.

In Section 2.1 we set out our model and general procedure for calculating ruin probabilities in continuous and finite time. In Sections 2.2 and 2.3 we give details of the Brownian motion and the translated gamma approximations we use to calculate the probability of ruin within each year, given the surplus at the start and at the end of the year. Details of our procedure to simulate ruin probabilities are given in Section 2.4. Wikstad (1971) and Seal (1978b) give values for the probability of ruin in finite and continuous time for some examples of a classical risk process (with constant premiums). In Section 2.5 we check the accuracy of our methodology by applying it to these examples and comparing our values with theirs. We also compare our results for a long period, say 1 000 years, with the results of Gerber (1979) for the ultimate ruin probability when claim amounts have an exponential distribution and also a mixture of exponentials distribution. This last example can also be found in Bowers et al. (1997) (Example 13.6.2). Finally in Section 2.6 we set out some conclusions.

2.1 The model

With no loss of generality consider time to be measured in years. Consider a risk process over an n-year period which is described by

$$U(t) = u + \sum_{j=1}^{i-1} P_j + (t - i + 1)P_i - S(t), \quad 0 \le t \le n \tag{2.1}$$

where i is such that $t \in [i-1, i)$, $i = 1, 2, ..., n$, and where $\sum_{j=1}^{0} P_j = 0$.

$U(t)$ is the insurer's surplus at time t, $0 \le t \le n$,

u is the insurer's initial surplus ($= U(0)$) and is assumed to be known,

P_i is the premium charged in year i, where year i means time $i-1$ to i,

$S(t)$ is the aggregate claims up to time t so that $S(0) = 0$.

We define $Y_i = S(i) - S(i-1)$, $i = 1, 2, \ldots, n$ so that Y_i is the aggregate claims in year i. We assume that $\{Y_i\}_{i=1}^{n}$ is a sequence of *i.i.d.* random variables, each with a compound Poisson distribution whose first three moments exist. This assumption will be modified in Chapters 4 and 5 to allow the distribution of aggregate claims to depend on the value of a risk parameter, θ, say.

$\{S(t)\}_{t=0}^{\infty}$ is assumed to be a compound Poisson process. The Poisson parameter, and hence the expected number of claims each year, is λ.

We also assume that premiums are received continuously at a constant rate throughout each year and that the initial premium, P_1, is known. For $i = 2, \ldots, n$, we assume that P_i is a function of $\{Y_j\}_{j=1}^{i-1}$, the aggregate claims in the preceding years and is updated at the beginning of the year.

For $i \ge 2$, the premium P_i and surplus level $U(i)$ are random variables since they both depend on the claims experience in previous years. Where we wish to refer to a particular realisation of these variables, we will use the lower case letters p_i and $u(i)$.

The probability of ruin in continuous time within n years is denoted by $\psi(u, n)$ and defined as:

$$\psi(u, n) \stackrel{def}{=} \Pr(U(t) < 0 \text{ for some } t \in (0, n])$$

Let $\psi(u(i-1), 1, u(i))$ be the probability of ruin within year i, given the surplus $u(i-1)$ at the start of the year and the surplus $u(i)$ at the end. To approximate this probability we will use Brownian motion process and translated gamma distribution approximations.

2.2 The Brownian motion process approximation

In this section we give an approximation, using a Brownian motion process with drift, to calculate the ruin probability within one year, given the surplus at the start and at the end of the year.

Definition 2.2.1 A continuous-time stochastic process $\{W(s); s \geq 0\}$ is a Brownian motion process with drift coefficient μ and variance per unit time σ^2 if:

(i) $W(0) = 0$;

(ii) $\{W(s); s \geq 0\}$ has stationary and independent increments;

(iii) $W(s)$ is normally distributed (with mean μs and variance $\sigma^2 s$).

Let $\{W(s); s \geq 0\}$ be a Brownian motion process. For the given values $u(i-1)$ and p_i, we approximate the surplus process, $\{U(t)\}$ over the time interval $i-1 \leq t \leq i$ by the Brownian motion process $\{u(i-1) + W(s)\}$ over the time interval $0 \leq s \leq 1$, with $s = t - i + 1$ and with:

$$\mu = p_i - \mathrm{E}[Y_i] \quad \text{and} \quad \sigma^2 = \mathrm{Var}[Y_i]$$

so that for $0 \le s \; (= t - i + 1) \le 1$:

$$\mathrm{E}[u(i-1) + W(s)] = u(i-1) + s(p_i - \mathrm{E}[Y_i]) = \mathrm{E}[U(t) \mid U(i-1) = u(i-1)]$$

$$\mathrm{Var}[u(i-1) + W(s)] = \sigma^2 s = \mathrm{Var}[U(t)] \mid U(i-1) = u(i-1)]$$

Let T denote the time until ruin for this approximating process, so that:

$$T = \inf(s > 0 : u(i-1) + W(s) < 0)$$

with the convention that $T = \infty$ if ruin never occurs. Klugman et al. (2004), Corollary 8.25, show that the probability that ruin ever occurs, denoted $\psi_{BM}(u(i-1))$, is:

$$\psi_{BM}(u(i-1)) = \exp\left(-\frac{2\mu u(i-1)}{\sigma^2}\right) \tag{2.2}$$

and in Corollary 8.27 show that the probability density of the time to ruin, given that ruin occurs, denoted $f_T(s)$, is:

$$f_T(s) = \frac{u(i-1)}{\sqrt{2\pi\sigma^2}} s^{-3/2} \exp\Big(- \frac{(u(i-1) - \mu s)^2}{2\sigma^2 s}\Big), \quad s > 0 \tag{2.3}$$

Hence, the probability density of the time to ruin, for finite s, without conditioning on whether ruin occurs, is the product of (2.2) and (2.3), that is:

$$f_T(s)\psi_{BM}(u(i-1)) = \frac{u(i-1)}{\sqrt{2\pi\sigma^2}} s^{-3/2} \exp\Big(- \frac{(u(i-1) - \mu s)^2 + 4\mu s u(i-1)}{2\sigma^2 s}\Big) \tag{2.4}$$

Klugman et al. (2004) also show (page 259) that, for $0 < s < 1$, the conditional probability density of $u(i-1) + W(1)$ at y, given that $T = s$, denoted $f(y|T = s)$, is:

$$f(y|T = s) = \exp\Big(\frac{y\mu}{\sigma^2}\Big) \frac{\exp\Big(- \frac{y^2 + \mu^2(1-s)^2}{2\sigma^2(1-s)}\Big)}{\sqrt{2\pi\sigma^2(1-s)}} \tag{2.5}$$

Hence, for $0 < T < 1$, the joint probability density of $u(i-1) + W(1)$ and T is given by the product of (2.4) and (2.5) and the conditional probability density of T, given that $u(i-1) + W(1) = u(i)$, denoted $f_W(s)$, is the product of (2.4) and (2.5) divided by the (marginal) density of $u(i-1) + W(1)$ at the point $u(i)$. Since $W(1)$ has a normal distribution, we have:

$$f_W(s) = \frac{\exp\left(\frac{u(i)\mu}{\sigma^2}\right)\frac{u(i-1)s^{-3/2}}{2\pi\sigma^2}\frac{1}{\sqrt{1-s}}\exp\left(-\frac{u(i)^2+\mu^2(1-s)^2}{2\sigma^2(1-s)}-\frac{(u(i-1)-\mu s)^2}{2\sigma^2 s}-\frac{2\mu u(i-1)}{\sigma^2}\right)}{n(u(i)-u(i-1),\mu,\sigma^2)}$$

where $n(\cdot,\mu,\sigma^2)$ is the density function of the normal distribution.

Finally, the probability of ruin in the year, given that the surplus at the end of the year is $u(i)$, is given by the integral of this last conditional density from $s = 0$ to $s = 1$. We denote this probability $\psi_{BM}(u(i-1), 1, u(i))$, so that:

$$\psi_{BM}(u(i-1), 1, u(i)) = \int_{s=0}^{1} f_W(s)\, ds \tag{2.6}$$

For now on we will use $\psi_{BM}(u(i-1), 1, u(i))$ as an approximation to $\psi(u(i-1), 1, u(i))$.

2.3 The translated gamma distribution approximation

We now return to the (compound Poisson) surplus process, $\{U(t)\}$ described in (2.1). We consider the time interval $[i-1, i)$ and we assume we know the history of the process up to time $i-1$. Hence, the premium income in the year, p_i, is known.

We are interested in $\psi(u(i-1), 1, u(i))$, the probability of ruin within the year given the starting and final values for $U(t)$. We will develop a formula for $\psi(u(i-1), 1, u(i))$ following methods in Dickson and Waters (2006, Section 3.2).

Let $\Delta(u(i-1), 1, y)$ denote the probability that, starting from a surplus of $u(i-1)$, ruin does not occur in the year and the surplus at the end of the year is greater than y. Let $f(\cdot, s)$ and $F(\cdot, s)$ denote the density function and the distribution function of the aggregate claims in a time interval of length s. Then:

$$\Delta(u(i-1), 1, y) = \int_y^{\infty} (1 - \psi(u(i-1), 1, z))\, f(u(i-1) + p_i - z, 1)\, dz$$

and so:

$$\psi(u(i-1), 1, y) = 1 + \frac{1}{f(u(i-1) + p_i - y, 1)}\frac{d}{dy}\Delta(u(i-1), 1, y)$$

Let $\delta(u(i-1),t,y)dy$ denote the probability that, starting from initial surplus $u(i-1)$, ruin does not occur before time t and the surplus at time t is between y and $y+dy$. Then:

$$\delta(u(i-1),t,y) = -\frac{d}{dy}\Delta(u(i-1),t,y)$$

Using formula (3.13) from Dickson and Waters (2006):

$$\begin{aligned}\delta(u(i-1),1,y) &= f\left(u(i-1)+p_i-y,1\right) - f\left(u(i-1)+p_i-y,1-y/p_i\right)\exp(-\lambda y/p_i) \\ &\quad -p_i\int_{s=0}^{1-y/p_i} f\left(u(i-1)+p_i s,s\right)\delta(0,1-s,y)ds\end{aligned}$$

and formula (3.11) from the same reference:

$$\delta(0,t,y) = \frac{y}{p_i t} f\left(p_i t-y,t\right)$$

and writing $y=u(i)$ we have:

$$\begin{aligned}\psi(u(i-1),1,u(i)) &= \frac{\int_{s=0}^{1-u(i)/p_i} f(u(i-1)+p_i s,s)\frac{u(i)}{(1-s)}f(p_i(1-s)-u(i),1-s)ds}{f(u(i-1)+p_i-u(i),1)} \\ &\quad +\frac{f(u(i-1)+p_i-u(i),1-u(i)/p_i)\exp(-\lambda u(i)/p_i)}{f(u(i-1)+p_i-u(i),1)} \qquad (2.7)\end{aligned}$$

Formula (2.7) is an exact expression for $\psi(u(i-1),1,u(i))$, but it is not easy to evaluate it since it requires values of the *pdf* $f(\cdot,s)$ for values of s from 0 to 1. Although these values can be calculated using well known recursive formulas, the number of values required can be prohibitively large, particularly if λ is large, and so some approximate method of calculation is required. To evaluate formula (2.7) we assume that the probability densities can be approximated by the densities of translated gamma random variables, matched by moments. This idea goes back at least to Seal (1978a) and has been used more recently by Dickson and Waters (1993, 2006). It has its roots in Bohman and Esscher (1963, 1964).

Let $H(s)$ be a random variable with a gamma distribution with parameters αs and β (so that its mean is $\alpha s/\beta$). Let $f_G(x;\alpha s,\beta)$ be its probability density function and $F_G(x;\alpha s,\beta)$ denote its cumulative distribution function. Let κ and s be constants. Then $H(s)+\kappa s$ has a translated gamma distribution with probability density function $f_G(x-\kappa s;\alpha s,\beta)$. Let α, β and κ be chosen so that $H(1)+\kappa$ has the same mean, variance and coefficient of skewness as $S(1)$ ($\equiv Y_i$). Then it is well known that $H(s)+\kappa s$ has a translated gamma distribution with parameters αs, β and κs and hence the same first three moments as $S(s)$. Let μ'_k denote the k-th moment about zero of the individual claim amount distribution. We have:

$$
\begin{aligned}
\alpha &= \frac{4\lambda\mu_2'^3}{\mu_3'^2}\\
\beta &= \frac{2\mu_2'}{\mu_3'}\\
k &= \lambda\left(\mu-\frac{2\mu_2'^2}{\mu_3'}\right)
\end{aligned}
\tag{2.8}
$$

Hence, $f_G(x-\kappa s;\alpha s,\beta)$ can be regarded as an approximation for $f(x,s)$ and we can approximate formula (2.7) by replacing each compound Poisson *pdf* by the *pdf* of $H(s)+\kappa s$, with the appropriate value of s. In particular,we need to replace:

$$
\begin{aligned}
f(x,s) &\quad \text{by} \quad f_G(x-\kappa s;\alpha s,\beta)\\
\exp(-\lambda t) &\quad \text{by} \quad F_G(-\kappa t;\alpha t,\beta)
\end{aligned}
$$

For this last relationship, note that for the compound Poisson process $\exp(-\lambda t)$ is the probability of no claims in a time interval of length t. We approximate this by the probability that $H(t)+\kappa t$ is negative, which is $F_G(-\kappa t;\alpha t,\beta)$.

Our translated gamma approximation to $\psi(u(i-1),1,u(i))$, which we denote by $\psi_{TG}(u(i-1),1,u(i))$, is given by:

$$
\begin{aligned}
&\psi_{TG}(u(i-1),1,u(i)) =\\
&= \frac{\int_{s=0}^{1-\frac{u(i)}{p_i}} f_G\left(u(i-1)+p_i s-\kappa s;\alpha s,\beta\right)\frac{u(i)}{(1-s)} f_G\left(p_i(1-s)-u(i)-\kappa(1-s);\alpha(1-s),\beta\right) ds}{f_G\left(u(i-1)+p_i-u(i)-\kappa;\alpha,\beta\right)}\\
&+ \frac{f_G\left(u(i-1)+p_i-u(i)-\kappa(1-\frac{u(i)}{p_i});\alpha(1-\frac{u(i)}{p_i}),\beta\right) F_G\left(-\kappa\frac{u(i)}{p_i};\alpha\frac{u(i)}{p_i},\beta\right)}{f_G\left(u(i-1)+p_i-u(i)-\kappa;\alpha,\beta\right)}
\end{aligned}
\tag{2.9}
$$

The advantage of using $\psi_{TG}(u(i-1),1,u(i))$ as an approximation to $\psi(u(i-1),1,u(i))$ is that there are well established and fast algorithms for calculating gamma densities so that the former can be calculated far more quickly and easily than the latter.

2.4 The simulation procedure

Our goal is to estimate $\psi(u,n)$. To achieve this we will simulate N paths of the surplus process (2.1). Each path starts at u $(= U(0))$. We simulate the aggregate claims in each year and calculate the respective premium in order to calculate the probability of ruin given the surplus at the start and at the end of each year. If the surplus at the end of the year is negative, ruin has occurred, we stop this run, set as an estimate for the probability of ruin the value 1 and we start another run.

We need to use numerical integration for the ruin probabilities within each year using formulas (2.6) and (2.9). As we are going to use simulation we need to pay attention to computer run time. We must pay attention also to accuracy because some of the values are very small and if we do not have a good numerical approximation routine we may have errors bigger than the result itself. With that in mind we used the adaptive Simpson quadrature presented in Section 3 of Gander and Gautschi (2000) to numerically approximate the integrals in formulas (2.6) and (2.9).

Let $\psi_{jBM}(u,n)$ and $\psi_{jTG}(u,n)$, $j = 1,2,\ldots,N$, denote the estimate of $\psi(u,n)$ from the $j-th$ run for the Brownian motion and translated gamma approximations respectively. We will use $\psi_{j\cdot}(u,n)$ as a generic notation for these two approximations. Our procedure for calculating $\psi_{j\cdot}(u,n)$ is as follows:

(i) Simulate the values of $\{Y_i\}_{i=1}^{n}$. To do this we assume each Y_i is approximately distributed as $H(1)+\kappa$, where $H(1) \sim \Gamma(\alpha,\beta)$, so that Y_i follows a translated gamma distribution with parameters α, β and κ defined as in (2.8).

(ii) From the simulated values of $\{Y_i\}_{i=1}^{n}$, say $\{y_i\}_{i=1}^{n}$, calculate the premium each year, say $\{p_i\}_{i=1}^{n}$, and the surplus at the end of each year, say $\{u(i)\}_{i=1}^{n}$ $\Big(u(i) = u(i-1) + p_i - y_i, \quad 1 \le i \le n\Big)$.

Note: The model (2.1) is a continuous one. In our simulation procedure p_i is known at the beginning of the year. It depends on the past simulated values of y_k, $k = 1, ..., i-1$. The first premium may not depend on the past simulated values as we will see in Chapter 3.

(iii) If $u(i) < 0$ for any $i, i = 1, 2, \ldots, n$, then we set the estimate of $\psi_{j\cdot}(u, n)$ from this simulation to 1 and we start another one.

(iv) If $u(i) \geq 0$ for all $i, i = 1, 2, \ldots, n$, we calculate $\psi(u(i-1), 1, u(i))$, using either a Brownian motion or translated gamma approximation.

(a) Brownian motion approximation:

We set $\mu = p_i - \mathrm{E}[Y_i]$ and $\sigma^2 = \mathrm{Var}[Y_i]$ and $\psi(u(i-1), 1, u(i))$ is approximated by (2.6).

(b) Translated gamma approximation:

If $u(i-1) + p_i - y_i \geq p_i$ then ruin cannot have occurred and we set $\psi_{TG}(u(i-1), 1, u(i)) = 0$. This particular situation will happen if $y_i < 0$, which can happen if $\kappa < 0$. If $0 < u(i-1) + p_i - y_i < p_i$ then the probability of ruin, $\psi(u(i-1), 1, u(i))$, is approximated by (2.9).

(iv) Our estimate of the ruin probability within n years using the Brownian motion and translated gamma approximations is then:

$$\begin{aligned}\psi_{jBM}(u,n) &= 1 - \prod_{i=1}^{n}(1 - \psi_{BM}(u(i-1), 1, u(i))) \\ \psi_{jTG}(u,n) &= 1 - \prod_{i=1}^{n}(1 - \psi_{TG}(u(i-1), 1, u(i)))\end{aligned} \tag{2.10}$$

(v) Carry out the next run up to a total of N.

The mean of our N estimates, $\{\psi_{j\cdot}(u,n)\}_{j=1}^{N}$, is then our estimate of $\psi(u,n)$ and we can use the sample standard deviation of the N estimates to calculate approximate confidence intervals for the estimate. We will denote our estimates $\hat{\psi}_{BM}(u,n)$ and $\hat{\psi}_{TG}(u,n)$. In all the examples in this chapter and throughout we use 50 000 simulations. For a given combination of term (n), claim size distribution and target probability of ruin (see Section 3.3) we use the same 50 000 sets of simulated aggregate annual claims. This makes it easier to compare results within each example.

This procedure was implemented in C++. For the generation of random numbers, in particular for the generation of $\{y_i\}_{i=1}^n$, we used the C++ random number generator class code produced by Wilder (2006). The method for generating gamma variables appears in Marsaglia and Tsang (2000).

2.5 Classical model - comparison with published results

Our methodology for estimating the probability of ruin in finite and continuous time is based on two approximations:

(i) We simulate the annual aggregate claims using a translated gamma approximation.

(ii) We estimate the within year probability of ruin, given the starting and final surplus, using a Brownian motion or a translated gamma approximation.

We would expect both to be reasonable approximations if the expected number of claims each year, λ, is large and the individual claim size distribution does not have too fat a tail. Note that if $u = 0$ we cannot use the Brownian motion approximation for the within year probability of ruin since each simulation will give $\psi_{BM}(0, 1, u(1)) = 1$.

Wikstad (1971) and Seal (1978a) provide values of ruin probabilities in finite and continuous time for some compound Poisson risk processes, in all cases with a fixed premium rate. Gerber (1979) provides an exact formula to calculate ruin probabilities in infinite time. We can test the accuracy of our method by applying it to their examples. The ruin probabilities in their examples range from practically zero to almost 1. Although the values of practical interest are probabilities of ruin between 0.001 and 0.05 we also compare some other cases.

We use the procedure defined in Section 2.4 to obtain the estimates for $\psi(u,n)$ and for $\psi(u)$.

The classical surplus process (see for instance Bowers et al. (1997) or Klugman et al. (2004)) is given by:

$$U(t) = u + ct - S(t), \quad 0 \le t \le n$$

$U(t)$ is the insurer's surplus at time t, $0 \le t \le n$,

u is the insurer's initial surplus ($= U(0)$) and is assumed to be known,

c is the premium charged in year i,

$S(t)$ is the aggregate claims up to time t, $S(t) = \sum_{i=1}^{N(t)} Z_i$,

$\{Z_i\}_{i=1}^{\infty}$ is a sequence of i.i.d. random variables,

$p(z)$ is the density function of Z_i,

$\{S(t)\}_{t=0}^{\infty}$ is assumed to be a compound Poisson process. The Poisson parameter, and the expected number of claims each year, is λ. $Z_1, Z_2, ...$ are identically distributed random variables and the random variables $N, Z_1, Z_2, ...$ are mutually independent.

The premiums are received continuously at a constant rate c per year (unit time) thus the total net premium in $(0,t]$ is ct. The net premium has a positive loading, ζ so that $c = (1+\zeta)\mathrm{E}[S(1)]$, where $\zeta > 0$.

2.5.1 Seal's results

Seal (1978b), Table (2.4), gives values of the probability of ruin in continuous and finite time, say $\psi(u,n)$, for various combinations of initial reserve (u) and time (n). The premium loading factor is ζ=0.1, for the compound Poisson risk process with exponentially distributed individual claims with mean $1/\nu$, with $\lambda = 1$ and $\nu = 1$. The premiums are received continuously at a constant rate $c = 1.1$.

We set, in our model, the premium in each year p_i constant and equal to c. The drift and variance of the Brownian motion process are:

$$\mu = p_i - \mathrm{E}[Y_i] = \frac{(1+\zeta)\lambda}{\nu} - \frac{\lambda}{\nu} = 0.1 \quad \text{and} \quad \sigma^2 = \mathrm{Var}[Y_i] = \frac{2\lambda}{\nu^2} = 2. \qquad (2.11)$$

The parameters for the translated gamma approximation α, β and κ are given by:

$$\alpha = \frac{4\lambda\mu_2'^3}{\mu_3'^2} = \frac{2^3}{3^2}\lambda, \quad \beta = \frac{2\mu_2'}{\mu_3'} = \frac{2\nu}{3} \quad \text{and} \quad \kappa = \lambda\left(\mu - \frac{2\mu_2'^2}{\mu_3'}\right) = -\frac{\lambda}{3\nu}. \qquad (2.12)$$

Table 2.1 show the values from Seal (1978b) for selected cases and our estimates, $\hat{\psi}_{BM}(u,n)$ and $\hat{\psi}_{TG}(u,n)$, of these values together with the standard errors of these estimates.

n	u	$\psi(u,n)$	$\psi_{TG}(u,n)$	SD$[\hat{\psi}_{TG}(u,n)]$	$\psi_{BM}(u,n)$	SD$[\hat{\psi}_{BM}(u,n)]$	$\frac{\psi_{TG}(u,n)}{\psi(u,n)}$	$\frac{\psi_{BM}(u,n)}{\psi(u,n)}$
10	6	0.13688	0.13220	0.00147	0.14759	0.00152	0.96581	1.07827
	8	0.06776	0.06658	0.00108	0.07453	0.00113	0.98265	1.09998
	10	0.03190	0.03105	0.00075	0.03491	0.00079	0.97339	1.09439
50	6	0.36173	0.35583	0.00210	0.37853	0.00211	0.98370	1.04644
	8	0.26015	0.25446	0.00192	0.27131	0.00194	0.97811	1.04288
	10	0.18369	0.18062	0.00169	0.19291	0.00172	0.98328	1.05017
	22	0.01562	0.01448	0.00052	0.01577	0.00054	0.92696	1.00951
	44	0.00004	0.00004	0.00003	0.00004	0.00003	1.00000	1.07362
	66	0	0.00000	0.00000	0.00000	0.00000	-	-
600	22	0.11628	0.11757	0.00143	0.12186	0.00145	1.01112	1.04795
	44	0.01348	0.01328	0.00051	0.01379	0.00052	0.98530	1.02280
	66	0.00135	0.00162	0.00018	0.00172	0.00018	1.19859	1.27661

Table 2.1: Values and estimates of $\psi(u,n)$: Exponentially distributed claim amounts. Seal (1978b).

2.5.2 Wikstad's results

Wikstad (1971) in his case IA considered exponentially distributed individual claims with mean 1 and with one claim expected each year ($\lambda = 1$). Table 2.2 show the values from Wikstad, case IA, for selected cases and our estimates, $\hat{\psi}_{BM}(u,n)$ and $\hat{\psi}_{TG}(u,n)$, of these values together with the standard errors of these estimates.

The parameters for the Brownian motion and translated gamma approximation are the same as (2.11) and (2.12).

n	ζ	u	$\psi(u,n)$	$\psi_{TG}(u,n)$	SD$[\hat{\psi}_{TG}(u,n)]$	$\psi_{BM}(u,n)$	SD$[\hat{\psi}_{BM}(u,n)]$	$\frac{\psi_{TG}(u,n)}{\psi(u,n)}$	$\frac{\psi_{BM}(u,n)}{\psi(u,n)}$
1	0.05	1	0.2420	0.23456	0.00174	0.39019	0.00149	0.96926	1.61237
		10	0.0003	0.00052	0.00010	0.00049	0.00010	1.72405	1.61874
	0.15	1	0.2342	0.22641	0.00171	0.36959	0.00148	0.96674	1.57811
		10	0.0003	0.00040	0.00009	0.00036	0.00008	1.34985	1.21374
	0.25	1	0.2268	0.21536	0.00166	0.34626	0.00147	0.94955	1.52674
		10	0.0003	0.00038	0.00008	0.00033	0.00008	1.25846	1.10873
10	0.05	1	0.6376	0.62548	0.00176	0.78667	0.00111	0.98099	1.23379
		10	0.0367	0.03487	0.00075	0.03621	0.00076	0.95014	0.98669
	0.15	1	0.5882	0.57766	0.00176	0.73823	0.00118	0.98207	1.25507
		10	0.0277	0.02832	0.00067	0.02808	0.00067	1.02244	1.01362
	0.25	1	0.5414	0.52794	0.00174	0.68532	0.00124	0.97514	1.26584
		10	0.0209	0.02011	0.00056	0.01897	0.00055	0.96216	0.90774
100	0.05	1	0.8433	0.84433	0.00091	0.91687	0.00050	1.00122	1.08724
		10	0.3464	0.34440	0.00178	0.35470	0.00179	0.99422	1.02396
	0.15	1	0.7451	0.74572	0.00100	0.84908	0.00061	1.00083	1.13956
		10	0.1920	0.19103	0.00138	0.18271	0.00138	0.99494	0.95160
	0.25	1	0.6510	0.65227	0.00099	0.77581	0.00064	1.00195	1.19173
		10	0.1016	0.10191	0.00097	0.08426	0.00093	1.00307	0.82933

Table 2.2: Values and estimates of $\psi(u,n)$: Exponentially distributed claim amounts. Wikstad (1971).

In Wikstad's case IIA, he considered a compound Poisson surplus model where individual claim amounts have the following distribution:

$$P(z) = 1 - 0.0039793\exp(-0.014631z) - 0.1078392\exp(-0.19206z) - 0.8881815\exp(-5.514588z)$$

This is described by Wikstad as a 'rather crude attempt' to model Swedish non-industrial fire insurance data from 1948-1951. He also describes the distribution as 'extremely skew'.

Table 2.3 shows the values from Wikstad's example IIA for selected cases and shows our estimates, $\hat{\psi}_{BM}(u,n)$ and $\hat{\psi}_{TG}(u,n)$, of these values together with the standard errors of these estimates. The drift and variance of the Brownian motion process are:

$$\mu = \zeta\lambda \quad \text{and} \quad \sigma^2 = 43.0837\lambda.$$

The parameters for the translated gamma approximation α, β and κ are given by:

$$\alpha = \lambda/186, \quad \beta = 2/179 \quad \text{and} \quad \kappa = 287\lambda/559.$$

n	ζ	u	$\psi(u,n)$	$\psi_{TG}(u,n)$	SD$[\hat{\psi}_{TG}(u,n)]$	$\psi_{BM}(u,n)$	SD$[\hat{\psi}_{BM}(u,n)]$	$\frac{\psi_{TG}(u,n)}{\psi(u,n)}$	$\frac{\psi_{BM}(u,n)}{\psi(u,n)}$
1	0.05	1	0.0841	0.01758	0.00059	0.93296	0.00004	0.20907	11.09343
		10	0.0190	0.00831	0.00041	0.01706	0.00042	0.43726	0.89781
		100	0.0009	0.00108	0.00015	0.00108	0.00015	1.20000	1.20001
	0.15	1	0.0832	0.01928	0.00061	0.92890	0.00005	0.23169	11.16462
		10	0.0188	0.00935	0.00043	0.01787	0.00044	0.49736	0.95074
		100	0.0009	0.00092	0.00014	0.00093	0.00014	1.02260	1.03123
	0.25	1	0.0824	0.01871	0.00060	0.92469	0.00005	0.22706	11.22191
		10	0.0187	0.00861	0.00041	0.01672	0.00042	0.46032	0.89425
		100	0.0009	0.00094	0.00014	0.00094	0.00014	1.04478	1.04737
10	0.05	1	0.3964	0.13992	0.00155	0.99997	0.00000	0.35297	2.52264
		10	0.1445	0.08276	0.00123	0.12480	0.00132	0.57271	0.86367
		100	0.0094	0.01124	0.00047	0.01217	0.00049	1.19588	1.29486
	0.15	1	0.3787	0.13062	0.00151	0.99989	0.00000	0.34491	2.64032
		10	0.1374	0.07864	0.00120	0.11537	0.00128	0.57236	0.83965
		100	0.0093	0.00876	0.00042	0.00950	0.00043	0.94205	1.02177
	0.25	1	0.3623	0.12556	0.00148	0.99966	0.00000	0.34656	2.75922
		10	0.1308	0.07575	0.00118	0.10892	0.00126	0.57915	0.83272
		100	0.0092	0.00908	0.00042	0.00983	0.00044	0.98726	1.06806
100	0.05	1	0.6846	0.48304	0.00223	0.99999	0.00000	0.70557	1.46069
		10	0.4625	0.38526	0.00218	0.45350	0.00213	0.83300	0.98053
		100	0.0896	0.09109	0.00129	0.09867	0.00132	1.01666	1.10121
	0.15	1	0.6377	0.44342	0.00222	0.99995	0.00000	0.69535	1.56805
		10	0.4164	0.35476	0.00214	0.41399	0.00212	0.85196	0.99421
		100	0.0833	0.08564	0.00125	0.09277	0.00129	1.02803	1.11365
	0.25	1	0.5955	0.41706	0.00220	0.99981	0.00000	0.70035	1.67894
		10	0.3780	0.33113	0.00210	0.38479	0.00209	0.87601	1.01797
		100	0.0777	0.08090	0.00122	0.08783	0.00126	1.04117	1.13031

Table 2.3: Values and estimates of $\psi(u,n)$: Swedish fire insurance claim amounts. Wikstad (1971).

2.5.3 Gerber's exact formula for infinite time ruin probability

Gerber (1979) gives exact formulas for the ultimate ruin probabilities in the cases where claim amounts are exponentially distributed and where they are a mixture of exponentials. Although the aim of our research is to develop a model for the calculation of finite time ruin probabilities, by choosing a very long time interval we can test its accuracy by comparing values with exact values for infinite time ruin probabilities.

Exponential claim amounts distribution

If the claim amount distribution is exponential with parameter $\nu > 0$ the probability of ruin is an exponential function of the initial surplus measured in mean claim amounts (see formula (3.14) Chapter 8 of Gerber (1979)).

$$\psi(u) = \frac{1}{1+\zeta} \exp\left(-\frac{\nu\zeta}{1+\zeta}u\right), u \geq 0 \tag{2.13}$$

Table 2.4 shows the values of the probability of ruin in continuous and infinite time for selected cases and our estimates $\hat{\psi}_{BM}(u, 1\,000)$ and $\hat{\psi}_{TG}(u, 1\,000)$, for the compound Poisson risk process with exponentially distributed individual claims. The premium is set constant and equals $c = (1+\zeta)\lambda/\nu = (1+\zeta)20\,000$, with $\lambda = 1\,000$ and $\nu = 0.05$. We show also the standard errors of these estimates.

The drift and variance of the Brownian motion process are:

$$\mu = \frac{\zeta\lambda}{\nu} \quad \text{and} \quad \sigma^2 = \frac{2\lambda}{\nu^2}.$$

The parameters for the translated gamma approximation α, β and κ are given by:

$$\alpha = \frac{2^3}{3^2}\lambda, \quad \beta = \frac{2\nu}{3} \quad \text{and} \quad \kappa = -\frac{\lambda}{3\nu}.$$

Mixtures of exponential claim amounts distribution

If the claim amounts distribution is a mixture of exponentials of the form $p(z) = \sum_{i=1}^{n-i} A_i\nu_i e^{-\nu_i z}, z > 0, \nu_i > 0, A_i > 0, A_1 + \ldots + A_n = 1$ the probability of ruin is given by (see formula (13.6.13) of Bowers et al. (1997)):

ζ	u	$\psi(u)$	$\psi_{TG}(u, 1000)$	SD$[\hat{\psi}_{TG}(u, 1000)]$	$\psi_{BM}(u, 1000)$	SD$[\hat{\psi}_{BM}(u, 1000)]$	$\frac{\psi_{TG}(u,1000)}{\psi(u)}$	$\frac{\psi_{BM}(u,1000)}{\psi(u)}$
0.05	300	0.466230	0.467444	0.001315	0.473160	0.001293	1.00260	1.01486
	500	0.289597	0.291546	0.001369	0.288694	0.001362	1.00673	0.99688
	700	0.179882	0.182033	0.001249	0.177128	0.001242	1.01196	0.98469
	900	0.111733	0.113708	0.001074	0.109278	0.001066	1.01768	0.97803
	1100	0.069402	0.070960	0.000893	0.067671	0.000885	1.02245	0.97506
	1300	0.043109	0.044273	0.000729	0.042056	0.000722	1.02701	0.97558
0.15	300	0.122912	0.122910	0.000407	0.105787	0.000366	0.99998	0.86067
	500	0.033352	0.033448	0.000217	0.023904	0.000179	1.00286	0.71671
	700	0.009050	0.009124	0.000110	0.005506	0.000087	1.00814	0.60844
	900	0.002456	0.002500	0.000057	0.001304	0.000044	1.01795	0.53093
	1100	0.000666	0.000691	0.000032	0.000321	0.000026	1.03656	0.48245
	1300	0.000181	0.000197	0.000022	0.000089	0.000020	1.09083	0.49027
0.25	300	0.039830	0.039976	0.000126	0.023567	0.000081	1.00368	0.59171
	500	0.005390	0.005463	0.000033	0.001953	0.000015	1.01340	0.36239
	700	0.000730	0.000746	0.000008	0.000164	0.000003	1.02199	0.22537
	900	0.000099	0.000102	0.000002	0.000014	0.000001	1.03127	0.14378
	1100	1.34E-05	1.39E-05	5.43E-07	1.26E-06	9.48E-08	1.04382	0.09466
	1300	1.81E-06	1.92E-06	1.31E-07	1.15E-07	1.50E-08	1.05982	0.06357

Table 2.4: Values and estimates of $\psi(u)$: Exponentially distributed claim amounts. Gerber (1979).

$$\psi(u) = C_i e^{-r_i u}, u \geq 0 \tag{2.14}$$

were C_i is given by the solution of the equation

$$\frac{1}{1+\zeta} \frac{\zeta(M_Z(r) - 1)}{1 + (1+\zeta)\mathrm{E}[Z]r - M_Z(r)} = \sum_{i=1}^{n} \frac{C_i r_i}{r_i - r} \tag{2.15}$$

We are going to compare the results of formula (2.14) with our model with time $n = 1\,000$.

Table 2.5 shows the values of the probability of ruin in continuous and infinite time for selected cases and our estimates $\hat{\psi}_{BM}(u, 1\,000)$ and $\hat{\psi}_{TG}(u, 1\,000)$, for a premium loading factor $\zeta = 2/5$, for the compound Poisson risk process with distribution of individual claims being the mixture of exponential of example 3.2 of Gerber (1979) or more recently example 13.6.2 of Bowers et al. (1997):

$$g(z) = 3/2 e^{-3z} + 7/2 e^{-7z}, z > 0$$

$$\psi(u) = \frac{24}{35} e^{-u} + \frac{1}{35} e^{-6u}, u \geq 0 \tag{2.16}$$

The premium is set constant and equals $c = (1+\zeta)\lambda \mathrm{E}[Z] = 5 \times 10^6$, with $\lambda = 1\,000$. We show also the standard errors of these estimates. The drift and variance of the Brownian motion process are:

$$\mu = \frac{5\zeta\lambda}{21} \quad \text{and} \quad \sigma^2 = \frac{58\lambda}{441}.$$

The parameters for the translated gamma approximation α, β and κ are given by:

$$\alpha = \frac{195\,112}{308\,025}\lambda, \quad \beta = \frac{406}{185} \quad \text{and} \quad \kappa = \frac{589}{11\,655}\lambda.$$

ζ	u	$\psi(u)$	$\psi_{TG}(u,1000)$	SD$[\hat{\psi}_{TG}(u,1000)]$	$\psi_{BM}(u,1000)$	SD$[\hat{\psi}_{BM}(u,1\,000)]$	$\frac{\psi_{TG}(u,1000)}{\psi(u)}$	$\frac{\psi_{BM}(u,1000)}{\psi(u)}$
0.4	3	0.03414	0.03428	7.92E-05	0.01298	3.30E-05	1.00424	0.38016
	4	0.01256	0.01272	3.94E-05	0.00305	1.10E-05	1.01286	0.24313
	5	0.00462	0.00472	1.88E-05	0.00072	3.54E-06	1.02132	0.15566
	6	0.00170	0.00175	8.74E-06	0.00017	1.12E-06	1.02945	0.09979

Table 2.5: Values and estimates of $\psi(u)$: Individual claim amounts being a mixture of exponentials. Gerber (1979).

2.6 Comments on results

We can see from Section 2.5 that $\psi_{TG}(u,n)$ is generally closer to $\psi(u,n)$ than $\psi_{BM}(u,n)$. This is as expected since the former approximation is based on matching three moments and the latter is based on matching only two.

The exception is Wikstad case IIA, Table 2.3. We have here an individual claim size distribution which is "extremely skew". In some examples ($n = 10$) the Brownian motion approximations works better than the translated gama approximation. We can also observe that the results improve for large values of u. The case where $n = 1$ is an extreme case since one claim is expected each year and for instance for $u = 10$ the probability that this claim on its own exceeds the initial surplus is 0.0192, which is almost the same as the probability of ruin over 10 years in each case.

The standard errors of our estimates are almost identical for the two approximations to the within year probability of ruin. This is not surprising since the major source of randomness comes from the simulation of the aggregate annual claims and the same simulations are used for the two approximations.

Our algorithm is quite fast. For instance the results for $n = 600$ of Table 2.1 took approximately 19 hours and results for $n = 100$ of Table 2.3 took approximately 20 hours in a Linux Server (Debian Stable) with 4 processors AMD Opteron of 64 bits with 2 200Mhz and with 4GB RAM.

Since $\psi_{TG}(u, n)$ generally produces more accurate values than $\psi_{BM}(u, n)$, with no significant difference in the standard errors, we will use the former approximation throughout the rest of this thesis.

Now that we checked that our algorithm produces good results when compared with the classical risk process, we will use it in the following chapters to calculate the ruin probability in finite and continuous time in some cases where the premium varies from year to year.

Chapter 3

Premium as a function of the surplus

In this chapter we will consider the problem of calculating the probability of ruin in continuous and finite time when the premium is a function of the surplus level. We apply our method to a risk process where the premium at the start of each year depends on the current or past levels of the surplus. The higher the surplus the lower will be the premium. This way, the company can take advantage of a greater surplus to lower its premium rates and try to be more competitive, as well as benefiting its clients.

We will consider several cases of premium rating. We start by assuming the classical case where the premium for the coming year depends on the initial surplus and is constant throughout the remaining periods. Secondly, the premium in each year depends on the surplus at the end of the preceding year, *i.e.* on the current surplus. This is intuitively appealing but may not be practicable, since it requires the insurer to determine and charge the new premium instantaneously. In practice there may be some delay in setting a new premium rate so in a third case we consider that the premium in the coming year depends on the surplus one year ago. In all these cases, the higher the surplus, the lower will be the premium. The premium rate is set each year so that the probability of ultimate ruin from that time is always (approximately) equal to a pre-determined value. We do this using De Vylder's (1978) approximation; details are given in Section 3.1. In Section 3.2 we will consider different models for the claim number process. Some applications are presented in Section 3.3. We will use three different claim amount distributions, exponential, lognormal and gamma. We also consider the premium varying by layers as in Michaud (1996). Some considerations and comments are set out in Section 3.4.

3.1 The model

Recall that $\{Y_i\}_{i=1}^n$ is the sequence of *i.i.d.* random variables for the aggregate claims in one year, with a common distribution whose first three moments are known. We assume that the premium at the start of each year depends on the level of the surplus at a given moment. For $i \geq 1$ we write P_i, the premium rate to be charged in the *i-th* year, as $h(u_{\tau_i})$, where h is some function which we will specify below and u_{τ_i} takes one of three values:

$$u_{\tau_i} = u(0); \quad \text{or } u_{\tau_i} = u(i-1); \quad \text{or } u_{\tau_i} = u(\max(i-2, 0)).$$

In the first case, P_i is fixed throughout the n years at a level depending on the initial surplus; in the second case P_i depends on the surplus at the end of the preceding year, *i.e.* at the current time. This is the most intuitively appealing case. However, it may not be possible for an insurer to adjust the premium rate instantaneously as this case requires. The last case allows for this by determining the premium as a function of the level of surplus one year earlier.

Consider a risk process over a period of n years and let $\mu'_k = \mathrm{E}[Z_i^k]$, $k = 1, 2, \cdots$. The Poisson parameter for the number of claims is, for now, λ, and the premium is calculated using the expected value principle:

$$P_i = (1 + \zeta(u_{\tau_i}, \omega))\lambda\mu'_1, \ \zeta > 0 \tag{3.17}$$

Given the surplus u_{τ_i}, we will determine the safety loading, $\zeta(u_{\tau_i}, \omega))$, so that the probability of ultimate ruin, assuming the premium rate does not change, is approximately some pre-determined level, ω, for example 0.01. We will use De Vylder's (1978) approximation to achieve this. Let:

$$\tilde{a} = \frac{3\mu_2'}{\mu_3'}, \quad \tilde{\lambda} = \frac{9\lambda {\mu_2'}^3}{2{\mu_3'}^2}, \quad \text{and} \quad \tilde{P} = P_i - \lambda\mu_1' + \frac{\tilde{\lambda}}{\tilde{a}}$$

Then De Vylder's approximation to the probability of ultimate ruin given initial surplus u_{τ_i}, denoted $\psi_{DV}(u_{\tau_i})$, is given by:

$$\psi_{DV}(u_{\tau_i}) = \frac{\tilde{\lambda}}{\tilde{a}\tilde{P}} \exp\left\{-\left(\tilde{a} - \frac{\tilde{\lambda}}{\tilde{P}}\right) u_{\tau_i}\right\} \tag{3.18}$$

Given ω, a pre-determined value for $\psi_{DV}(u_{\tau_i})$, we can calculate numerically the corresponding value of $\tilde{P}$ hence P_i and hence $\zeta(u_{\tau_i}, \omega)$.

Formula (3.18) does not give a closed form solution for P_i. Since we are going to have to calculate the premium for each year of each (of many) simulations, it is convenient to have a simple formula for the safety loading in terms of u_{τ_i}. We achieve this by calculating the value of u_{τ_i} for a range of values of the safety loading using formula (3.18) and then fitting a power curve to these values using the tool *Add trendline* of Excel. The fitted curve will be in the format:

$$\zeta(u_{\tau_i}, \omega) = A u_{\tau_i}^B \tag{3.19}$$

There are two points to note about this procedure:

(i) For small values of u_{τ_i} the De Vylder's approximation can give uncomfortably large values for the premium loading factor. De Vylder (1978, page 118) says that for very small values of u 'the accuracy *(of his approximation)* is not so good'.

(ii) When ω is, for instance, 0.01 or 0.005 the upper bound for the safety loading will be 99 and 199 respectively. No insurer will apply such safety loadings.

For this reasons we consider an upper bound of 100% on the premium loading factor, so that equation (3.17) will be:

$$P_i = h(u_{\tau_i}) = \Big(1 + \min\left(A u_{\tau_i}^B, 1\right)\Big)\lambda\mu_1' \tag{3.20}$$

We are going to illustrate the method using one of our applications.

Example: Lognormal claim amount

Let the individual claim amount have a lognormal distribution with parameters $\mu = \sigma^2/2$, $\sigma^2 = \ln(4)$ and let the Poisson parameter $\lambda = 1\,000$. The De Vylder's approximation parameters are:

$$\tilde{a} = 0.1875, \quad \tilde{\lambda} = 70.3125 \quad \text{and} \quad \tilde{P}_i = 1\,875.0.$$

Let the pre-determined probability of ultimate ruin be $\omega = 0.01$. For each value of the safety loading we will find using De Vylder's approximation the corresponding $u : \psi(u) = \omega$. Table 3.6 shows the results for $\zeta = 0.01, 0.02, ..., 1.50$.

ζ	u	ζ	u	ζ	u	ζ	u	ζ	u	ζ	u
0.01	940.19	0.26	53.12	0.51	34.67	0.76	27.86	1.01	24.12	1.26	21.68
0.02	479.60	0.27	51.76	0.52	34.29	0.77	27.67	1.02	24.01	1.27	21.60
0.03	326.03	0.28	50.50	0.53	33.92	0.78	27.48	1.03	23.89	1.28	21.52
0.04	249.21	0.29	49.31	0.54	33.56	0.79	27.30	1.04	23.78	1.29	21.44
0.05	203.09	0.3	48.21	0.55	33.21	0.8	27.13	1.05	23.67	1.3	21.36
0.06	172.33	0.31	47.17	0.56	32.87	0.81	26.95	1.06	23.56	1.31	21.28
0.07	150.33	0.32	46.20	0.57	32.55	0.82	26.79	1.07	23.45	1.32	21.21
0.08	133.82	0.33	45.28	0.58	32.23	0.83	26.62	1.08	23.35	1.33	21.13
0.09	120.97	0.34	44.41	0.59	31.93	0.84	26.46	1.09	23.24	1.34	21.06
0.1	110.68	0.35	43.59	0.6	31.63	0.85	26.30	1.1	23.14	1.35	20.98
0.11	102.24	0.36	42.82	0.61	31.34	0.86	26.14	1.11	23.04	1.36	20.91
0.12	95.21	0.37	42.08	0.62	31.06	0.87	25.99	1.12	22.94	1.37	20.84
0.13	89.24	0.38	41.38	0.63	30.79	0.88	25.84	1.13	22.84	1.38	20.77
0.14	84.13	0.39	40.72	0.64	30.53	0.89	25.69	1.14	22.74	1.39	20.70
0.15	79.68	0.4	40.09	0.65	30.27	0.9	25.55	1.15	22.65	1.4	20.63
0.16	75.79	0.41	39.48	0.66	30.02	0.91	25.41	1.16	22.55	1.41	20.56
0.17	72.35	0.42	38.90	0.67	29.78	0.92	25.27	1.17	22.46	1.42	20.49
0.18	69.28	0.43	38.35	0.68	29.55	0.93	25.13	1.18	22.37	1.43	20.42
0.19	66.54	0.44	37.82	0.69	29.32	0.94	25.00	1.19	22.28	1.44	20.36
0.2	64.06	0.45	37.32	0.7	29.09	0.95	24.87	1.2	22.19	1.45	20.29
0.21	61.81	0.46	36.83	0.71	28.87	0.96	24.74	1.21	22.10	1.46	20.23
0.22	59.77	0.47	36.37	0.72	28.66	0.97	24.61	1.22	22.02	1.47	20.16
0.23	57.90	0.48	35.92	0.73	28.45	0.98	24.49	1.23	21.93	1.48	20.10
0.24	56.18	0.49	35.49	0.74	28.25	0.99	24.36	1.24	21.85	1.49	20.03
0.25	54.59	0.5	35.07	0.75	28.05	1	24.24	1.25	21.76	1.5	19.97

Table 3.6: Pairs (ζ, u): Lognormal claims, $\omega = 0.01$.

We plotted these values (dots) in Figure 3.1. We also show the fitted power function (line). $A = 95.87145$ and $B = -1.44538$ in formula (3.19).

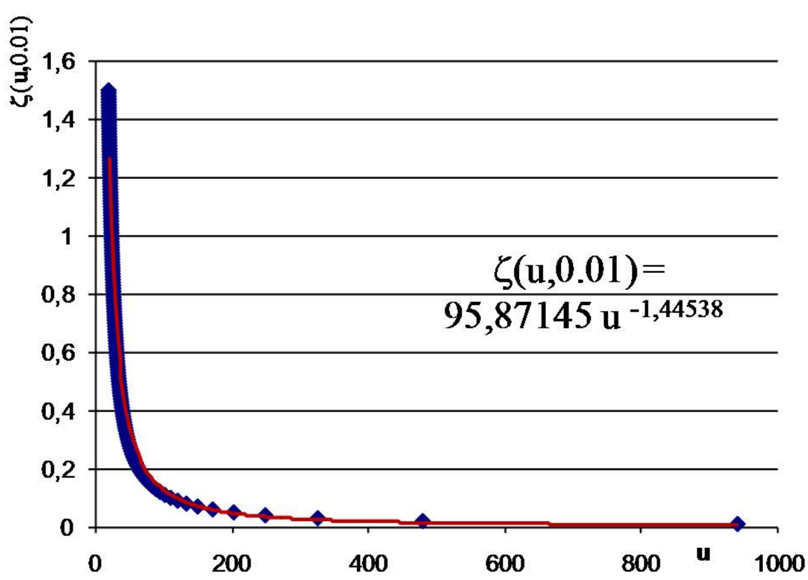


Figure 3.1: Fitted power function: Lognormal claims, $\omega = 0.01$.

So the premium will be given by:

$$P_i = h(u_{\tau_i}) = \Big(1 + \min\left(95.87145\ u_{\tau_i}^{-1.44538}, 1\right)\Big)\lambda\mu_1'.$$

3.2 Claim number process

In all our numerical examples we will assume that the number of claims each year has a Poisson distribution. However, we will use two different models for the parameter of the Poisson distribution. These two models, which we will denote N1 and N2, are defined as follows:

N1 The Poisson parameter, denoted λ, is constant and equal to $1\,000$ each year.

N2 The Poisson parameter in year j, denoted λ_j, is a random variable and $\{\lambda_j\}_{j=1}^n$ is a set of *i.i.d.* random variables, each with a $U(800, 1\,200)$ distribution.

In this case, the premium is calculated using the mean value of λ, so that equation (3.20) will be:

$$P_i = h(u_{\tau_i}) = (1 + \min(Au_{\tau_i}^B, 1))\mathrm{E}[\lambda]\mu_1' \tag{3.21}$$

Model N1 is the classical model for claim numbers. However, Daykin et al. (1996) suggest that this model may not capture the full variability of the claim number process in practice. They say on page 329, *'each (claim number) process is* *a superimposition of trends, cycles and short–term fluctuations'*, and also that, *'It seems clear that business cycles are so common in general insurance, and their impact so profound, that any risk theory model which claims to describe real–life situations must permit the user to evaluate the impact of any cycles which may be present.'* Their Figure 12.2.1 shows some examples of the variability of the claim ratio (claims/premiums) for general insurance. Our model N2 is a simple attempt to produce this variability through a variable Poisson parameter.

3.3 Numerical examples

In our applications in this section we will use three different individual claim size distributions: exponential, lognormal and gamma with mean, variance and skewness shown in Table 3.7 and *cdf* as in Figure 3.2.

	Exponential	Lognormal	Gamma
Mean	1	1	1
Variance	1	3	3
Skewness	2	10.39	3.46

Table 3.7: Mean, variance and skewness: Exponential, lognormal and gamma.

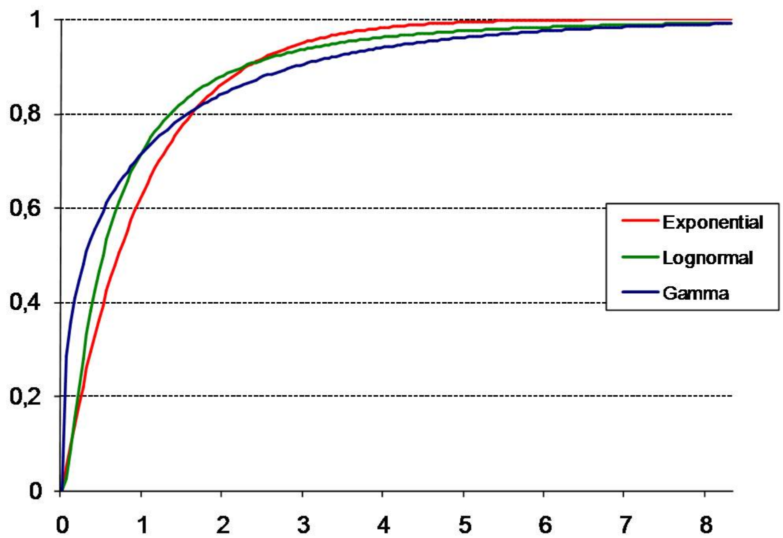


Figure 3.2: *Cdf*: Exponential, lognormal and gamma.

3.3.1 Notation

Let us define the notation that will be used throughout this chapter and in some cases throughout the thesis.

We will use two 'target' probabilities of ultimate ruin, which we label T1 and T2, as follows:

T1 $\psi(u) = 0.005$;

T2 $\psi(u) = 0.01$,

two cases for the Poisson parameter mentioned on Section 3.2:

N1 the Poisson parameter, denoted λ, is constant and equal to 1 000 each year.

N2 the Poisson parameter in year j, denoted λ_j, is a random variable and $\{\lambda_j\}_{j=1}^{n}$ is a set of *i.i.d.* random variables, each with a $U(800, 1\,200)$ distribution,

and three cases for the premium:

P1 constant premium as a function of the initial surplus $P_i = h(u(0))$;

P2 premium as a function of the surplus at the beginning of the year $P_i = h(u(i-1)), i \geq 1$;

P3 premium as a function of the surplus of the year before $P_i = h\left(u(\max(i-2, 0))\right), i \geq 2$.

Given the conclusion in Section 2.6, in all our examples we only present results for the translated gamma approximation obtained with 50 000 simulations for the tables and 10 000 simulations for the figures. For each Poisson parameter the simulated set of aggregate claims are the same in each simulation for the three cases of premium and for the different surpluses. We present for selected cases our estimate of the ruin probability, $\hat{\psi}(u, n)$, and the standard errors of the estimates for each initial surplus. The time period is 10 years ($n = 10$). Ten years has been chosen because it is a reasonable planning horizon in practice. The premium in each year is given by (3.20) or (3.21) depending on the Poisson parameter.

3.3.2 Exponential claim amounts

Consider that the individual claim amounts are exponentially distributed with mean 1 (variance 1 and skewness 2).

Table 3.8 shows for each of the two target probabilities of ultimate ruin the values of the parameters A and B to be used in formula (3.19).

target	A	B
T1	15.38387	-1.24137
T2	12.26914	-1.22917

Table 3.8: Exponential: Parameters for the power function for formula (3.19).

For the chosen initial surplus, Table 3.9 shows values of the safety loading obtained using De Vylder's approximation ($\zeta(u_{\tau_i}, \omega)$) and given by the fitted power function ($Au_{\tau_i}^B$). For small initial surpluses in Table 3.9 the fitted formula gives values for the safety loading higher than the De Vylder's formula. For higher initial surpluses the fitted values are lower than De Vylder's. The ruin probabilities will be affected by this as we can see in Table 3.10 for $\psi(u)$ and with more impact in Tables 3.11 to 3.14 for $\psi(u, 10)$ as we are dealing with finite time.

u	T1		T2	
	$\zeta(u_{\tau_i}, \omega)$	$Au_{\tau_i}^B$	$\zeta(u_{\tau_i}, \omega)$	$Au_{\tau_i}^B$
40	0.1481	0.1579	0.1263	0.1317
50	0.1158	0.1197	0.0992	0.1001
60	0.0950	0.0954	0.0816	0.0800
70	0.0806	0.0788	0.0693	0.0662
80	0.0700	0.0668	0.0603	0.0562
90	0.0618	0.0577	0.0533	0.0486

Table 3.9: Exponential: Safety loading obtained by De Vylder's formula *vs* fitted power function.

Table 3.10 show the results for the (approximate) probability of ultimate ruin using formula (3.18) for selected values of the initial surplus, u, for the safety loading of Table 3.9. The results should be close to the target value for $\psi(u)$, either 0.005 (T1) or 0.01 (T2) especially for $\zeta(u_{\tau_i}, \omega)$. The differences between the target and calculated probabilities of ultimate ruin arise from the inaccuracy of the fit of the two parameter power function as shown in Table 3.9.

u	T1 $\psi(u)$		T2 $\psi(u)$	
	$\zeta(u_{\tau_i}, \omega)$	$Au_{\tau_i}^B$	$\zeta(u_{\tau_i}, \omega)$	$Au_{\tau_i}^B$
40	0.005	0.0037	0.01	0.0084
50	0.005	0.0043	0.01	0.0096
60	0.005	0.0049	0.01	0.0109
70	0.005	0.0056	0.01	0.0121
80	0.005	0.0063	0.01	0.0134
90	0.005	0.0070	0.01	0.0147

Table 3.10: Exponential: Values for $\psi(u)$ calculated using De Vylder's formula and safety loadings of Table 3.9.

Tables 3.11 to 3.14 show numerical results for different combinations of T and N. These tables have the same format. Estimated values of $\psi_{TG}(u, 10)$, together with the standard error of each estimate, are shown for various values of the initial surplus, u and for three cases for the premium (P1, P2, P3).

u	P1		P2		P3	
	$\psi_{TG}(u, 10)$	SD[$\psi_{TG}(u, 10)$]	$\psi_{TG}(u, 10)$	SD[$\psi_{TG}(u, 10)$]	$\psi_{TG}(u, 10)$	SD[$\psi_{TG}(u, 10)$]
40	0.00370	3.42E-09	0.00418	8.63E-09	0.00388	6.22E-09
50	0.00422	1.13E-08	0.00496	1.82E-08	0.00467	1.67E-08
60	0.00497	2.94E-08	0.00543	3.00E-08	0.00584	4.03E-08
70	0.00569	5.04E-08	0.00532	3.80E-08	0.00693	6.52E-08
80	0.00630	6.75E-08	0.00473	3.91E-08	0.00769	8.36E-08
90	0.00686	8.20E-08	0.00389	3.50E-08	0.00804	9.33E-08

Table 3.11: Exponential: Estimates and standard deviations of $\psi(u, 10)$, T1 N1.

The values of $\psi_{TG}(u, 10)$ for case N1 and $\tau_i = 0$, so that the premium is constant throughout the term (P1), should be close to the target probability of ultimate ruin. In many cases they are close, but in some cases they are not. Reasons for the differences are;

(i) We are simulating annual aggregate claims and simulation induces an error. However, the standard deviations, which indicate this error, are all very small.

u	P1 $\psi_{TG}(u,10)$	P1 SD$[\psi_{TG}(u,10)]$	P2 $\psi_{TG}(u,10)$	P2 SD$[\psi_{TG}(u,10)]$	P3 $\psi_{TG}(u,10)$	P3 SD$[\psi_{TG}(u,10)]$
40	0.11270	1.48E-06	0.27753	3.27E-06	0.23432	3.03E-06
50	0.18125	2.53E-06	0.31909	3.62E-06	0.30875	3.74E-06
60	0.23619	3.24E-06	0.34073	3.82E-06	0.35653	4.12E-06
70	0.27984	3.71E-06	0.34818	3.90E-06	0.38332	4.29E-06
80	0.31357	4.01E-06	0.34834	3.93E-06	0.39867	4.38E-06
90	0.33766	4.21E-06	0.34342	3.93E-06	0.40581	4.43E-06

Table 3.12: Exponential: Estimates and standard deviations of $\psi(u, 10)$, T1 N2.

u	P1 $\psi_{TG}(u,10)$	P1 SD$[\psi_{TG}(u,10)]$	P2 $\psi_{TG}(u,10)$	P2 SD$[\psi_{TG}(u,10)]$	P3 $\psi_{TG}(u,10)$	P3 SD$[\psi_{TG}(u,10)]$
40	0.00848	1.79E-08	0.01038	3.77E-08	0.00942	3.16E-08
50	0.00976	5.02E-08	0.01202	6.79E-08	0.01177	7.78E-08
60	0.01116	9.28E-08	0.01236	9.17E-08	0.01419	1.33E-07
70	0.01247	1.32E-07	0.01144	9.87E-08	0.01606	1.78E-07
80	0.01394	1.70E-07	0.00985	9.30E-08	0.01715	2.07E-07
90	0.01532	2.04E-07	0.00808	8.03E-08	0.01736	2.19E-07

Table 3.13: Exponential: Estimates and standard deviations of $\psi(u, 10)$, T2 N1.

(ii) We are comparing a finite time probability of ruin, $\psi(u, 10)$, with a target probability of ultimate ruin, $\psi(u)$. However, since $\lambda = 1\,000$, we expect 10 000 claims in the 10 year term so there should be little difference between these probabilities.

(iii) The premium, calculated using formula (3.19), does not give exactly the target probability of ruin, as can be seen in Table 3.10. It can be seen that the values for N1 and P1 in Tables 3.11 and 3.13 are consistent with those in Table 3.10 columns 3 and 5 respectively.

The more interesting aspects of Tables 3.11 to 3.14, are the effect of adjusting the premium at the start of each year (P2) and the effect of the variability of a key parameter, λ (N2).

We make the following additional comments about the results in Tables 3.11 to 3.14:

u	P1 $\psi_{TG}(u,10)$	P1 SD$[\psi_{TG}(u,10)]$	P2 $\psi_{TG}(u,10)$	P2 SD$[\psi_{TG}(u,10)]$	P3 $\psi_{TG}(u,10)$	P3 SD$[\psi_{TG}(u,10)]$
40	0.17737	2.37E-06	0.35190	3.78E-06	0.31620	3.74E-06
50	0.24598	3.28E-06	0.38497	4.01E-06	0.38072	4.22E-06
60	0.29748	3.82E-06	0.39903	4.12E-06	0.41523	4.41E-06
70	0.33850	4.16E-06	0.40168	4.16E-06	0.43396	4.49E-06
80	0.36714	4.37E-06	0.39799	4.18E-06	0.44354	4.53E-06
90	0.38733	4.49E-06	0.39043	4.17E-06	0.44648	4.56E-06

Table 3.14: Exponential: Estimates and standard deviations of $\psi(u,10)$, T2 N2.

(iv) In the cases where the Poisson parameter is fixed (N1), the effect of adjusting the premium is generally to reduce $\psi_{TG}(u,10)$ for larger values of u and to increase it for smaller values of u. See, for example, $\psi_{TG}(40,10)$ in Table 3.11 and $\psi_{TG}(90,10)$ in Table 3.13 or Figures 3.3(a) and 3.3(b). The explanation for this is probably that, starting from a low initial surplus, the surplus is likely to drift upwards and so the effect of adjusting the premium (downwards) will be to reduce the premium, thereby increasing the probability of ruin. On the other hand, starting from a larger initial surplus, some sample paths, which would lead to ruin with a fixed premium (P1) will drift downwards first, which will lead to higher adjusted premiums and hence ruin may be averted.

(v) The effect of the variability of λ is considerable: $\psi_{TG}(u,10)$ increases, in some cases by a factor greater than 40. In almost every case, adjusting the premiums leads to a higher probability of ruin. This is presumably because a major cause of ruin will be a high value of λ in a year, and hence an inadequate premium. If the surplus has drifted to a level higher than u, so that the adjusted premium is smaller than its initial value, the effect of this increase in expected claims will be amplified.

Tables 3.15 and 3.16 show some statistical information for the paths where at the end of the year we have a surplus below zero (end of year ruin).

We recorded for each simulation where $u(i) < 0$ for some i the following information for different combinations of T, N and P, and for the range of initial surplus:

P is the premium scenario.

NRuins is the number of end year ruin.

Prop is the proportion of the probability of ruin in Table 3.11 to 3.14 attributable to within year ruin.

Avg i is the mean of the year in which end of year ruin occurs.

Avg $u(i)$ is the mean of the surplus at the end of the year of ruin.

Avg $u(i-1)$ is the mean of the surplus at the start of the year of ruin.

Avg λ is the mean Poisson parameter in the year of ruin. This is always 1 000 in case N1.

Avg p_i is the mean premium in the year of ruin.

Avg y_i is the mean aggregate claims in the year of ruin.

These results were obtained for different values of u and for the different cases of premium (P1, P2, P3) and correspond to the results of Tables 3.11 to 3.14 in the sense that they were produced with the same simulations, 50 000 for case N1 and 20 000 for case N2. This is due to the size of the file. In the case N2 we will have many more cases where ruin occurs. We also present the standard deviation (SD) of the results.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	SD $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg p_i	SD p_i	Avg y_i	SD y_i
40	1	0	1.000	-	-	-	-	-	-	1157.88	-	0.00	-
40	2	8	0.962	4.63	3.46	-2.99	2.81	125.32	25.82	1040.60	11.90	1168.91	14.52
40	3	3	0.985	7.33	1.53	-31.70	31.84	102.48	18.03	1020.96	0.39	1155.13	13.44
50	1	5	0.976	1.00	0.00	-14.86	7.10	50.00	0.00	1119.68	0.00	1184.54	7.10
50	2	20	0.919	3.00	2.66	-9.67	7.54	98.02	41.97	1065.58	34.56	1173.27	21.70
50	3	10	0.957	4.40	3.75	-30.78	27.38	61.23	16.63	1074.54	47.66	1166.54	21.43
60	1	14	0.944	1.14	0.53	-14.21	12.42	57.50	9.36	1095.44	0.00	1167.15	16.25
60	2	30	0.889	2.37	2.28	-12.11	10.93	91.83	38.14	1067.98	27.69	1171.92	17.96
60	3	29	0.901	4.21	3.36	-18.58	23.25	67.36	22.81	1062.42	31.99	1148.36	29.67
70	1	39	0.863	1.56	0.68	-12.86	12.29	53.94	26.72	1078.82	0.00	1145.62	28.48
70	2	40	0.850	2.13	2.07	-12.63	11.91	94.03	33.91	1062.43	20.42	1169.08	17.81
70	3	58	0.833	3.38	2.84	-15.86	18.45	61.21	28.68	1061.71	23.29	1138.78	30.66
80	1	65	0.794	1.69	0.79	-14.07	12.46	60.37	29.88	1066.78	0.00	1141.22	27.52
80	2	52	0.780	1.88	1.86	-10.97	11.95	95.43	27.89	1058.25	14.68	1164.65	17.88
80	3	85	0.779	2.98	2.53	-15.82	17.01	64.65	29.76	1056.70	16.14	1137.17	30.51
90	1	87	0.746	2.00	1.05	-15.16	14.03	55.56	33.34	1057.69	0.00	1128.41	34.01
90	2	47	0.758	1.98	1.94	-11.45	11.99	103.07	25.79	1051.97	12.17	1166.49	17.85
90	3	107	0.734	2.92	2.27	-16.34	16.74	57.99	33.88	1051.35	12.71	1125.68	34.62

Table 3.15: Exponential: Statistical information for ruin cases, T1 N1.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	SD $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg λ	SD λ	Avg p_i	SD p_i	Avg y_i	SD y_i
40	1	1225	0.457	1.27	0.65	-31.24	26.06	39.99	15.52	1170.07	25.26	1157.88	0.00	1229.10	28.59
40	2	4172	0.248	3.86	2.73	-36.08	29.66	101.00	54.81	1164.55	28.98	1077.26	52.65	1214.34	36.59
40	3	3465	0.261	4.59	3.02	-47.95	42.89	76.22	55.93	1153.16	41.17	1071.51	63.55	1195.68	54.06
50	1	2613	0.279	1.47	0.93	-37.88	30.64	50.02	20.35	1162.89	30.57	1119.68	0.00	1207.58	34.20
50	2	4916	0.230	3.29	2.66	-37.97	30.35	92.92	51.45	1163.03	29.53	1078.10	40.67	1208.99	35.28
50	3	4928	0.202	3.71	2.88	-47.39	40.58	72.08	49.85	1153.87	38.98	1072.54	47.42	1192.01	48.16
60	1	3830	0.189	1.72	1.31	-42.92	33.93	58.10	24.28	1158.10	33.10	1095.44	0.00	1196.47	37.75
60	2	5448	0.201	3.02	2.60	-39.65	31.72	92.02	46.28	1160.76	30.83	1072.11	31.32	1203.78	36.30
60	3	6010	0.157	3.38	2.76	-48.15	39.68	73.85	46.09	1152.35	38.72	1065.18	35.68	1187.17	46.75
70	1	4789	0.144	1.98	1.60	-45.86	36.72	65.71	29.44	1154.94	35.34	1078.82	0.00	1190.39	41.08
70	2	5690	0.183	2.89	2.56	-40.76	32.50	94.99	42.55	1159.79	31.20	1065.52	26.10	1201.27	37.26
70	3	6601	0.139	3.22	2.68	-49.84	40.03	76.51	43.72	1150.63	39.48	1057.77	28.04	1184.11	47.13
80	1	5535	0.117	2.26	1.89	-48.02	38.06	72.22	33.11	1153.18	37.12	1066.78	0.00	1187.02	42.87
80	2	5762	0.173	2.85	2.54	-41.22	32.68	99.14	39.43	1159.38	31.51	1060.03	23.52	1200.38	37.49
80	3	6895	0.135	3.14	2.62	-51.30	40.74	80.02	43.15	1150.07	39.92	1051.40	23.06	1182.71	47.34
90	1	6090	0.098	2.48	2.04	-49.62	39.47	77.01	36.47	1151.70	38.46	1057.69	0.00	1184.32	44.74
90	2	5739	0.164	2.87	2.55	-40.96	32.59	104.07	37.40	1159.52	31.41	1055.63	22.90	1200.67	37.36
90	3	7058	0.130	3.13	2.60	-52.16	41.45	83.54	43.42	1149.60	40.29	1046.19	19.60	1181.89	47.77

Table 3.16: Exponential: Statistical information for ruin cases, T1 N2.

Tables 3.15 and 3.16 illustrate the comments made previously. The tables for target T2 have the same behavior as these ones and are not going to be presented. From these tables we can add:

(i) Ruin occurs mainly in the first years, apart from the cases where the initial surplus is lower. In this situation the safety loading is higher and ruin will occur later.

(ii) On average the aggregate claims in the year ruin occurred (around 1 150 for N1 and 1 200 for N2) is higher than the premium (around 1 070 for each case) and is higher than $\mathrm{E}[Y] = 1\,000$.

(iii) The average premium in the year of ruin, around 1,070, is comparable for all three premium scenarios.

(iv) In the cases P2 and P3 the average premium is much lower for lower values of initial surplus in the case N1. This probably has to do with the fact that the first premium is very high. This will produce a bigger surplus in the second year and that will allow the premium to go down in the next years. In case N2 the fact that the number of claims is not constant may influence the result for the average premium.

(v) The average surplus at the start of the year of ruin is comparable for P1 and P3, but noticeably higher for P2: around 55 to 60 for P1 and P3 but 100 for P2 in case N1 and around 50 to 80 for P1 and P3 but still 100 for P2 in case N2.

(vi) We can see from Table 3.16 that the average value of the Poisson parameter in the year of ruin is around 1,160, which is near the upper end of its range.

(vii) The proportion of within ruin probability is around 0.81 for case N1 and 0.17 for case N2.

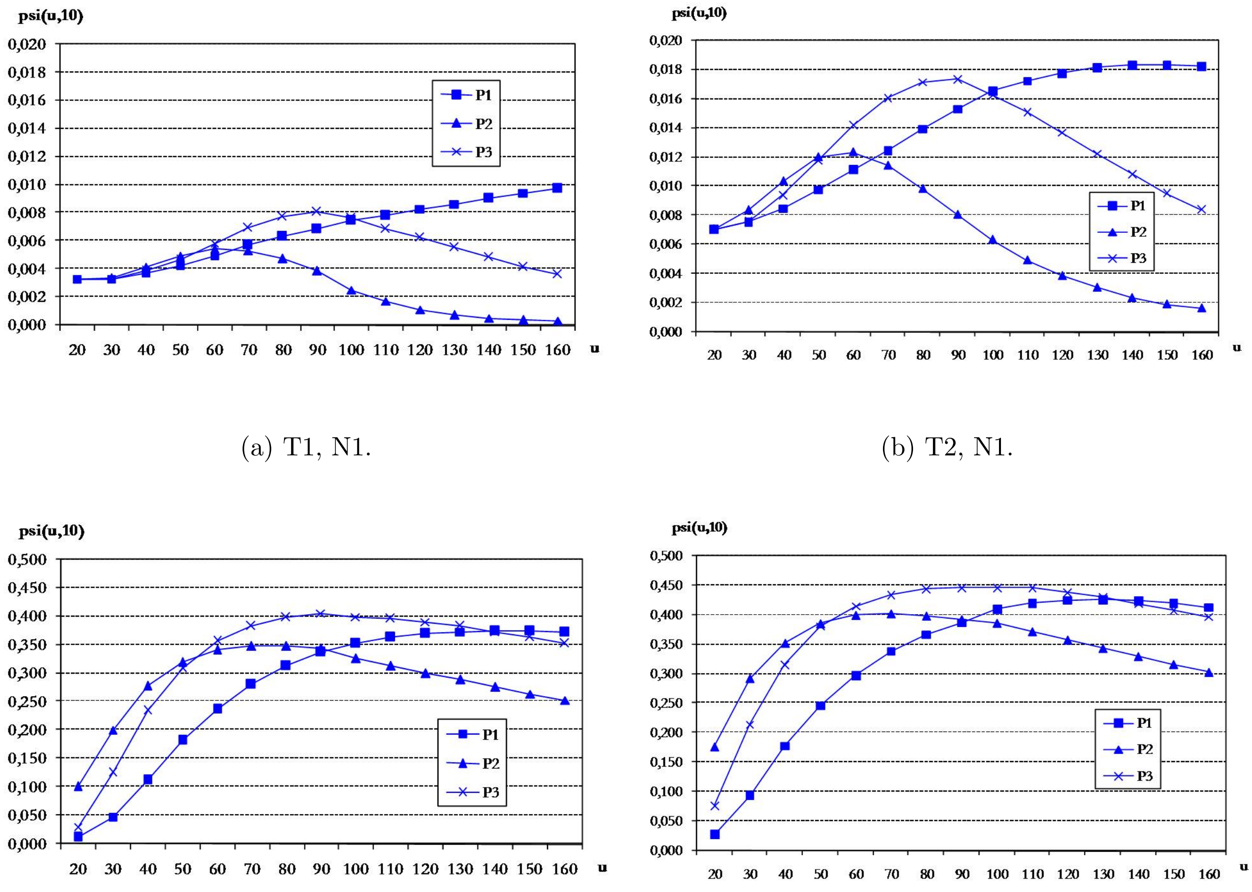
These statistics suggest that for P2 a major factor causing ruin is a relatively high surplus at the start of the year, and hence a low premium, followed by heavier than expected aggregate claims. For P3 it seems that a major cause of ruin is a relatively high value for $u(i-2)$, and hence a low value for p_i, followed by heavier than expected aggregate claims in years $i-1$ (so that $u(i-1)$ is lower than $u(i-2)$) and i. For scenario N2, aggregate claims are increased by a high Poisson parameter in the year of ruin.

As we saw in Tables 3.15 and 3.16 the proportion of end of year ruin is different for N1 and N2. For N1 it is at most 30% and for N2 it is at least 80%. Just to give an idea of how end of ruin is happening Table 3.17 shows Pearson's correlation coefficient (the estimator of ρ, the correlation coefficient) between the claim amount y_i and y_{i-1} in the cases where $u(i) < 0$. These results do not include the cases where ruin occurred in year 1 because y_0 does not exist. For each initial surplus and type of premium we calculate for each combination of T and N the number of records used to obtain the correlation (Nr) and the estimator of the correlation coefficient ($r_{y_i,y_{i-1}}$). These results were obtained with 200 000 simulations. We tested the hypotheses $H_0 : \rho = 0$ (y_i is linearly independent of y_{i-1}) *vs* $H_1 : \rho \neq 0$. The significance level used is $\alpha = 5\%$. When the hypothesis H_0 is accepted the result for $r_{y_i,y_{i-1}}$ is in italic font. We also calculate the confidence interval for the correlation coefficient with $\alpha = 5\%$ using the Fisher's Z transformation (not shown) to help us understand the results. For instance for case (u=90, P2, T1, N1), $r_{y_i,y_{i-1}}$ =-0.4 and the confidence interval goes from -0.57 to -0.19, and case (u=90, P2, T1, N2), $r_{y_i,y_{i-1}}$ =-0.28 and the confidence interval goes from -0.29 to -0.27. For large values of Nr (Nr>1 000) we have a small interval.

From Table 3.17 we can observe that there is a negative correlation between y_i and y_{i-1}. In Tables 3.15 and 3.16 we can see that Avg y_i is higher that $\mathrm{E}[Y] = 1\,000$. That seems to indicate that lower claims in year $i-1$ will lead in cases P2 and P3 to lower premiums and then high claims in year i will lead to end of year ruin. In case P1 ruin occurs mainly in the first year due to high claims. As we already suspected.

		T1 N1		T1 N2		T2 N1		T2 N2	
u	P	Nr	$r_{y_i,y_{i-1}}$	Nr	$r_{y_i,y_{i-1}}$	Nr	$r_{y_i,y_{i-1}}$	Nr	$r_{y_i,y_{i-1}}$
40	1	0	-	0	-	0	-	0	-
40	2	29	-0.50	2 223	-0.45	2 223	-0.45	2 223	-0.45
40	3	8	*-0.44*	28 426	-0.23	28 434	-0.23	28 978	-0.23
50	1	1	-	1	-	56	-0.42	33 100	-0.26
50	2	52	-0.47	7 145	-0.37	7 146	-0.37	33 088	-0.32
50	3	37	-0.57	28 117	-0.21	28 136	-0.21	29 364	-0.22
60	1	8	-0.86	8	-0.86	132	-0.45	32 937	-0.25
60	2	67	-0.43	13 232	-0.32	13 236	-0.32	43 847	-0.36
60	3	72	-0.68	27 718	-0.20	27 779	-0.20	29 716	-0.21
70	1	38	-0.76	38	-0.76	265	-0.59	33 100	-0.25
70	2	78	-0.40	19 470	-0.28	19 482	-0.28	53 409	-0.34
70	3	130	-0.73	27 482	-0.19	27 653	-0.19	30 324	-0.21
80	1	101	-0.63	101	-0.63	460	-0.67	33 521	-0.25
80	2	78	-0.40	25 461	-0.27	25 485	-0.27	61 731	-0.33
80	3	198	-0.76	27 443	-0.20	27 806	-0.19	31 164	-0.21
90	1	209	-0.58	209	-0.58	733	-0.70	34 022	-0.25
90	2	78	-0.40	30 978	-0.28	31 019	-0.28	69 226	-0.32
90	3	271	-0.78	27 548	-0.19	28 122	-0.18	32 083	-0.20

Table 3.17: Exponential: Correlation between y_i and y_{i-1}.



(a) T1, N1.

(b) T2, N1.

(c) T1, N2.

(d) T2, N2.

Figure 3.3: Exponential: $\psi(u, 10)$ for several values of initial surplus.

The graphics in Figure 3.3 show for different combinations of T, N and different cases of premium P1, P2, P3 the ruin probabilities over a 10 year period for a range of initial surpluses. The results for this figure were obtained with 10 000 simulations.

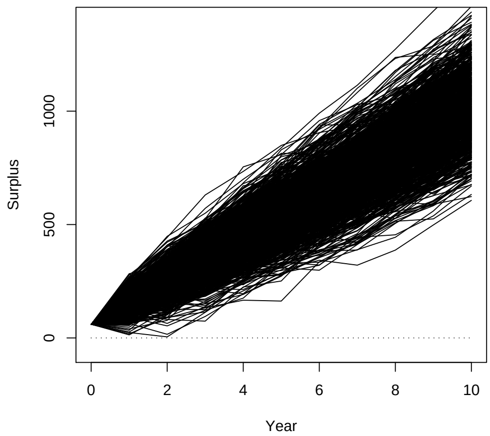
From Figures 3.3(a) and 3.3(b), the case where the number of claims is constant (N1) we may see that at some point the influence of the initial surplus on the ruin probabilities is higher than the safety loading adjustments. This is due to the finite time period. For instance in Figure 3.3(a) we can see that for P2 for initial surplus grater than 70, $\psi(u, 10)$ starts decreasing. This is expected because in a ten year period it is more difficult to erode a high surplus even with low premiums. In case P3 we can see the same type of curve with higher ruin probabilities for high initial surpluses. The case P1 takes more initial surplus to start decreasing the ruin probabilities. In the case N2 we have the same behavior for a different level of ruin probabilities.

Figures 3.4 to 3.6 show 1 000 paths of the surplus over a 10 year period, starting from initial surplus 60, 120 and 200 for each set of figures. The Poisson parameter for the number of claims is constant (N1) and the target is $\psi(u) = 0.005$ (T1). We have a set of figures for each type of premium (P1, P2, P3). All these figures where obtained with the same set of simulated aggregate claims.

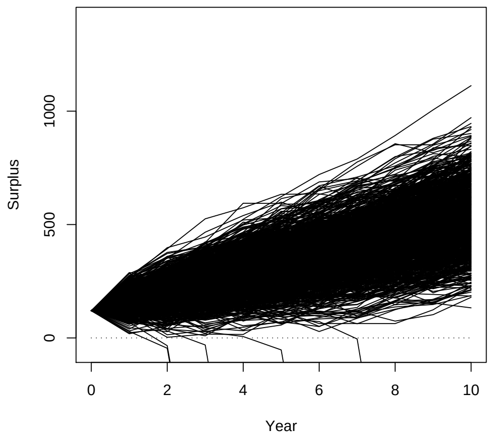
Figure 3.4 gives evidence of the behavior of the ruin probability over a range of initial surpluses when the premium is constant (P1). When the initial surplus is lower the safety loading is higher and the surplus process is pushed upwards. If ruin does not occur in the first years then it is likely it will never occur. If the initial surplus is higher the safety loading is lower and the surplus process is pushed downwards and it is more probable that ruin occurs.

Figure 3.5 shows some paths for the case where the premium is updated as a function of the surplus at the end of the previous year (P2). When the initial surplus is lower the surplus process may take some values near zero (but positive). In these cases the response of the premium is immediate. We see some paths drifting up considerably. That does not happen so often when the initial surplus is high.

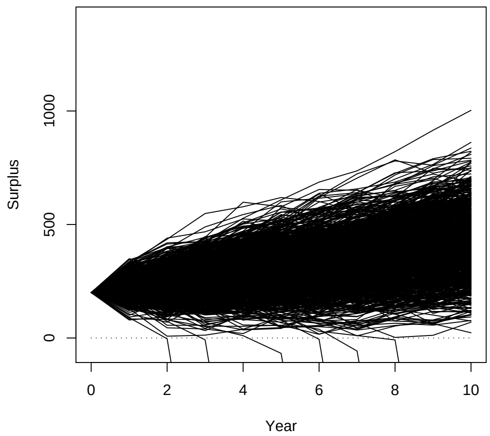
Figure 3.6 shows also the same features as Figure 3.5. The paths are not lines as it is printed in Figures 3.5 and 3.6. If we print the figures with dots we lose the information of the trend of the path.



(a) u=60

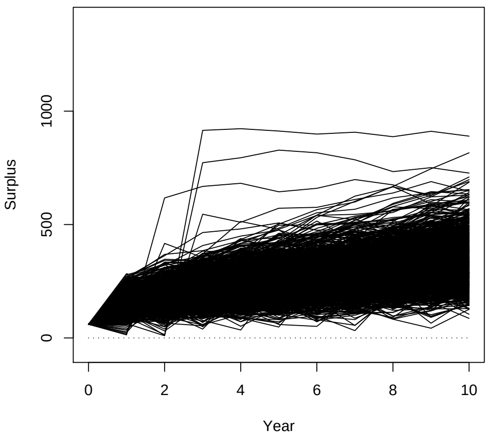


(b) u=120

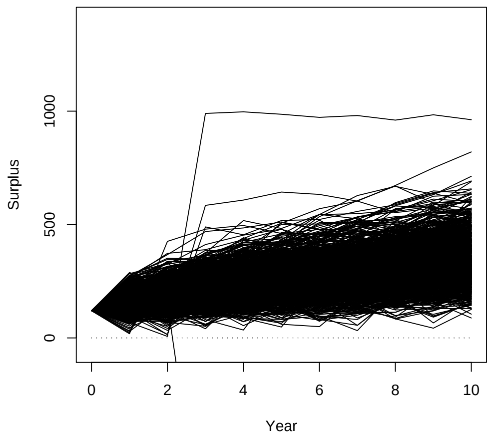


(c) u=200

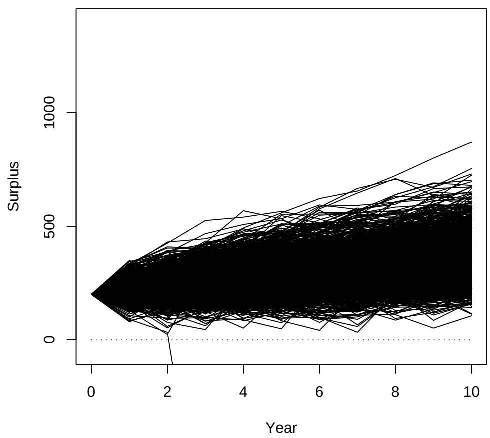
Figure 3.4: Exponential: Simulated paths, case P1 N1 T1.



(a) u=60

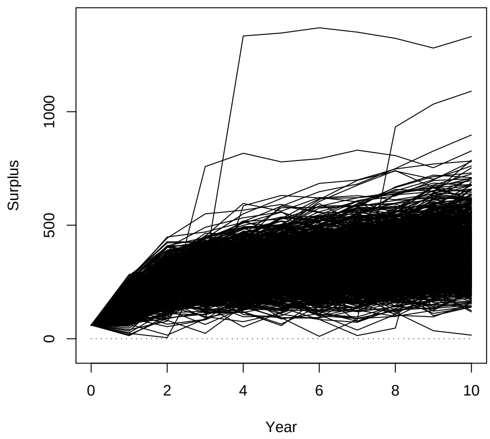


(b) u=120

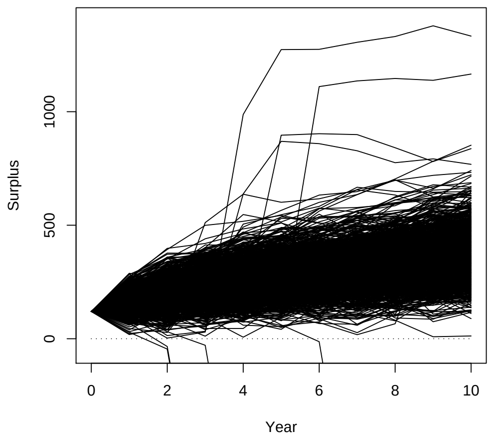


(c) u=200

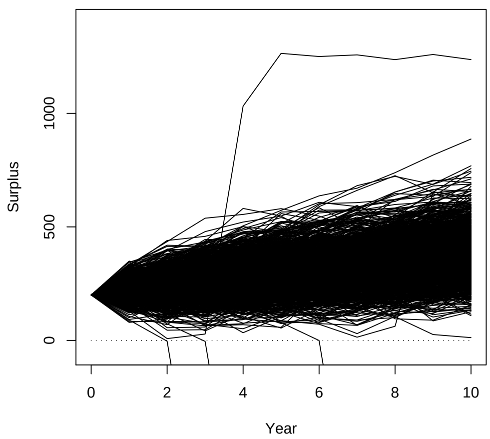
Figure 3.5: Exponential: Simulated paths, case P2 N1 T1.



(a) u=60



(b) u=120



(c) u=200

Figure 3.6: Exponential: Simulated paths, case P3 N1 T1.

3.3.3 Lognormal claim amounts

Consider that the individual claim amounts are lognormal distributed with mean 1, variance 3 and skewness 10.39.

Table 3.18 shows for each of the two target probabilities of ultimate ruin the values of the parameters A and B to be used in formula (3.19).

target	A	B
T1	141.02398	-1.47958
T2	95.87145	-1.44538

Table 3.18: Lognormal: Parameters for the power function for formula (3.19).

For the chosen initial surplus, Table 3.19 show values of the safety loading obtained using De Vylder's approximation ($\zeta(u_{\mathcal{T}_i}, \omega)$) and given by the fitted power function ($Au_{\mathcal{T}_i}^B$). For almost all values in Table 3.19 the fitted formula gives values for the safety loading higher than De Vylder's formula. An exception is $u = 170$ for T1. The ruin probabilities will be affected by this as we can see in Table 3.20 for $\psi(u)$ and with more impact in Tables 3.21 to 3.24 for $\psi(u, 10)$ as we are dealing with finite time.

u	T1		u	T2	
	$\zeta(u_{\mathcal{T}_i}, \omega)$	$Au_{\mathcal{T}_i}^B$		$\zeta(u_{\mathcal{T}_i}, \omega)$	$Au_{\mathcal{T}_i}^B$
120	0.1084	0.1183	80	0.1492	0.1702
130	0.0984	0.1051	90	0.1286	0.1436
140	0.0901	0.0942	100	0.1130	0.1233
150	0.0830	0.0850	110	0.1007	0.1074
160	0.0770	0.0773	120	0.0909	0.0947
170	0.0718	0.0707	130	0.0827	0.0844

Table 3.19: Lognormal: Safety loading obtained by De Vylder's formula *vs* fitted power function.

Table 3.20 shows the results for the (approximate) probability of ultimate ruin using formula (3.18) for selected values of the initial surplus, u, for the safety loading of Table 3.19. The results should be close to the target value for $\psi(u)$, either 0.005 (T1) or 0.01 (T2) especially for $\zeta(u_{\mathcal{T}_i}, \omega)$. The differences between the target and calculated probabilities of ultimate ruin arise from the inaccuracy of the fit of the two parameter power function as shown in Table 3.19.

u	T1 $\psi(u)$ $\zeta(u_{\tau_i}, \omega)$	T1 $\psi(u)$ $Au^B_{\tau_i}$	u	T2 $\psi(u)$ $\zeta(u_{\tau_i}, \omega)$	T2 $\psi(u)$ $Au^B_{\tau_i}$
120	0.005	0.0034	80	0.01	0.0064
130	0.005	0.0038	90	0.01	0.0068
140	0.005	0.0041	100	0.01	0.0073
150	0.005	0.0045	110	0.01	0.0079
160	0.005	0.0049	120	0.01	0.0085
170	0.005	0.0054	130	0.01	0.0093

Table 3.20: Lognormal: Values for $\psi(u)$ calculated using formula (3.18) and safety loading (3.19).

Tables 3.21 to 3.24 show numerical results for different combinations of T and N. Estimated values of $\psi_{TG}(u, 10)$, together with the standard error of each estimate, are shown for various values of the initial surplus, u and for three cases for the premium, (P1, P2, P3).

u	P1 $\psi_{TG}(u,10)$	P1 SD[$\psi_{TG}(u,10)$]	P2 $\psi_{TG}(u,10)$	P2 SD[$\psi_{TG}(u,10)$]	P3 $\psi_{TG}(u,10)$	P3 SD[$\psi_{TG}(u,10)$]
120	0.00342	2.98E-08	0.00344	2.64E-08	0.00451	4.34E-08
130	0.00370	3.76E-08	0.00325	2.78E-08	0.00488	5.14E-08
140	0.00401	4.44E-08	0.00298	2.72E-08	0.00518	5.79E-08
150	0.00441	5.19E-08	0.00266	2.55E-08	0.00540	6.27E-08
160	0.00489	6.10E-08	0.00233	2.31E-08	0.00551	6.57E-08
170	0.00542	7.16E-08	0.00201	2.00E-08	0.00553	6.73E-08

Table 3.21: Lognormal: Estimates and standard deviations of $\psi(u, 10)$, T1 N1.

u	P1 $\psi_{TG}(u,10)$	P1 SD$[\psi_{TG}(u,10)]$	P2 $\psi_{TG}(u,10)$	P2 SD$[\psi_{TG}(u,10)]$	P3 $\psi_{TG}(u,10)$	P3 SD$[\psi_{TG}(u,10)]$
120	0.09461	1.46E-06	0.10225	1.46E-06	0.14942	2.17E-06
130	0.11109	1.73E-06	0.10321	1.49E-06	0.16070	2.32E-06
140	0.12593	1.95E-06	0.10245	1.50E-06	0.16866	2.43E-06
150	0.13953	2.16E-06	0.10026	1.48E-06	0.17373	2.50E-06
160	0.15173	2.34E-06	0.09717	1.44E-06	0.17654	2.54E-06
170	0.16210	2.48E-06	0.09349	1.40E-06	0.17764	2.56E-06

Table 3.22: Lognormal: Estimates and standard deviations of $\psi(u, 10)$, T1 N2.

u	P1 $\psi_{TG}(u,10)$	P1 SD$[\psi_{TG}(u,10)]$	P2 $\psi_{TG}(u,10)$	P2 SD$[\psi_{TG}(u,10)]$	P3 $\psi_{TG}(u,10)$	P3 SD$[\psi_{TG}(u,10)]$
80	0.00678	2.72E-08	0.00833	4.13E-08	0.00776	3.99E-08
90	0.00720	4.37E-08	0.00876	5.51E-08	0.00878	6.35E-08
100	0.00773	6.30E-08	0.00892	6.68E-08	0.00993	9.09E-08
110	0.00834	8.18E-08	0.00874	7.46E-08	0.01095	1.15E-07
120	0.00906	1.01E-07	0.00825	7.65E-08	0.01177	1.34E-07
130	0.00985	1.20E-07	0.00758	7.46E-08	0.01238	1.48E-07

Table 3.23: Lognormal: Estimates and standard deviations of $\psi(u, 10)$, T2 N1.

u	P1 $\psi_{TG}(u,10)$	P1 SD$[\psi_{TG}(u,10)]$	P2 $\psi_{TG}(u,10)$	P2 SD$[\psi_{TG}(u,10)]$	P3 $\psi_{TG}(u,10)$	P3 SD$[\psi_{TG}(u,10)]$
80	0.06606	8.90E-07	0.13088	1.71E-06	0.13885	1.93E-06
90	0.08909	1.29E-06	0.14188	1.88E-06	0.16751	2.33E-06
100	0.11166	1.66E-06	0.14912	1.99E-06	0.18880	2.61E-06
110	0.13368	2.00E-06	0.15283	2.07E-06	0.20444	2.79E-06
120	0.15357	2.30E-06	0.15344	2.11E-06	0.21603	2.93E-06
130	0.17115	2.54E-06	0.15155	2.10E-06	0.22389	3.03E-06

Table 3.24: Lognormal: Estimates and standard deviations of $\psi(u, 10)$, T2 N2.

The comments about the results in Tables 3.21 to 3.24 are the same as we made for Tables 3.11 to 3.14 ((i) to (v) in Section 3.3.2). The comment (iv) is not obvious from the tables we study, specially in case T1, a range of surpluses that may be considered high. This comment may be confirmed by Figure 3.7.

Tables 3.25 and 3.26 show some statistical information for the paths where at the end of the year we have a surplus below zero (end of year ruin).

We recorded for each simulation where $u(i) < 0$ for some i, the same set of results as in the exponential example. These results were obtained for different values of u and for the different cases of premium (P1, P2, P3) and correspond to the results of Tables 3.21 and 3.22 in the sense that they were produced with the same simulations. 50 000 for case N1 and 20 000 for case N2.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	SD $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg p_i	SD p_i	Avg y_i	SD y_i
120	1	23	0.866	1.26	0.54	-19.03	21.26	106.32	31.47	1118.30	0.00	1243.65	41.48
120	2	29	0.831	2.17	1.93	-21.87	21.72	146.30	50.31	1100.45	30.68	1268.62	29.79
120	3	41	0.818	3.12	2.45	-27.02	24.07	109.01	50.13	1089.38	34.50	1225.40	53.78
130	1	36	0.806	1.53	0.77	-18.41	18.93	103.68	41.38	1105.09	0.00	1227.17	48.01
130	2	33	0.797	2.03	1.85	-21.28	21.68	149.20	44.27	1094.27	24.96	1264.75	29.81
130	3	54	0.778	2.98	2.33	-24.78	22.98	107.08	47.55	1085.96	26.99	1217.81	53.42
140	1	50	0.751	1.78	0.89	-19.05	18.71	106.98	45.75	1094.17	0.00	1220.20	48.99
140	2	34	0.771	2.00	1.83	-21.29	21.66	155.24	40.82	1087.34	21.91	1263.87	29.80
140	3	66	0.745	2.89	2.18	-23.83	22.85	106.59	47.02	1082.06	21.41	1212.49	53.73
150	1	58	0.737	2.05	1.30	-23.38	19.75	103.22	51.35	1085.03	0.00	1211.63	51.60
150	2	33	0.752	2.03	1.85	-21.32	21.68	162.53	39.02	1080.91	20.43	1264.75	29.81
150	3	72	0.733	2.86	2.08	-25.26	22.71	104.82	51.25	1077.00	18.93	1207.09	55.50
160	1	66	0.730	2.35	1.42	-25.80	22.55	98.98	57.43	1077.29	0.00	1202.07	57.17
160	2	31	0.734	2.10	1.89	-21.12	21.70	170.22	38.31	1075.30	19.80	1266.63	29.79
160	3	75	0.728	2.92	2.03	-26.39	23.00	104.61	53.30	1071.73	18.49	1202.72	56.67
170	1	79	0.708	2.76	1.59	-24.51	22.31	90.92	58.91	1070.66	0.00	1186.09	62.60
170	2	26	0.741	2.31	2.00	-22.84	21.56	178.73	40.27	1070.48	20.58	1272.05	29.60
170	3	75	0.729	3.03	1.99	-27.66	23.21	101.51	56.36	1067.97	18.52	1197.14	59.06

Table 3.25: Lognormal: Statistical information for ruin cases, T1 N1.

u	p	NRuins	Prop	Avg i	SD i	Avg $u(i)$	SD $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg λ	SD λ	Avg p_i	SD p_i	Avg y_i	SD y_i
120	1	1307	0.3093	1.95	1.31	-45.53	39.91	96.08	44.75	1159.02	36.26	1118.30	0.00	1259.91	53.01
120	2	1357	0.3365	2.82	2.45	-38.29	34.08	151.99	51.77	1163.24	32.51	1097.14	35.97	1287.42	40.54
120	3	2077	0.3050	3.74	2.83	-51.20	46.37	107.51	57.93	1152.89	40.50	1083.85	40.26	1242.56	64.46
130	1	1606	0.2772	2.18	1.50	-47.30	41.54	98.22	48.89	1156.44	37.55	1105.09	0.00	1250.61	56.84
130	2	1410	0.3169	2.78	2.45	-38.25	34.33	156.09	49.08	1162.77	32.98	1091.26	32.03	1285.60	40.94
130	3	2281	0.2903	3.63	2.76	-51.84	46.69	109.12	58.14	1152.37	40.54	1078.81	35.12	1239.78	63.51
140	1	1884	0.2520	2.40	1.71	-49.57	43.51	100.36	53.97	1154.13	39.37	1094.17	0.00	1244.10	60.80
140	2	1425	0.3045	2.75	2.43	-38.26	34.41	160.40	45.99	1162.59	33.05	1086.21	30.09	1284.88	40.91
140	3	2433	0.2787	3.55	2.71	-52.05	47.22	111.71	59.44	1152.02	40.83	1073.63	30.31	1237.39	63.90
150	1	2174	0.2210	2.65	1.87	-49.91	44.68	103.54	57.99	1151.54	41.47	1085.03	0.00	1238.48	64.05
150	2	1411	0.2963	2.78	2.45	-38.25	34.34	165.76	44.78	1162.76	32.97	1081.57	29.32	1285.59	40.93
150	3	2551	0.2658	3.52	2.67	-52.31	47.45	113.51	60.72	1150.88	41.40	1069.18	27.44	1235.00	64.43
160	1	2417	0.2035	2.89	1.99	-52.38	46.49	104.06	60.18	1149.87	42.67	1077.29	0.00	1233.73	65.83
160	2	1373	0.2935	2.82	2.45	-38.29	34.15	170.50	43.60	1163.06	32.71	1078.23	30.43	1287.01	40.62
160	3	2621	0.2577	3.52	2.64	-53.04	47.99	114.78	62.81	1150.20	41.89	1065.19	25.53	1233.01	65.43
170	1	2623	0.1909	3.12	2.14	-54.44	47.64	103.80	63.48	1148.46	43.25	1070.66	0.00	1228.90	68.56
170	2	1323	0.2925	2.87	2.47	-38.23	33.96	175.30	42.74	1163.32	32.73	1075.00	30.24	1288.52	40.27
170	3	2658	0.2519	3.54	2.62	-53.48	48.54	116.26	64.77	1149.56	42.30	1061.48	23.64	1231.23	66.79

Table 3.26: Lognormal: Statistical information for ruin cases, T1 N2.

Tables 3.25 and 3.26 illustrate the comments made previously. The tables for target T2 have the same behavior as these ones and are not going to be presented. From these tables we can add the same comments made for the exponential case, differing only in the values:

(i) Ruin occurs mainly in the first years, apart from the cases where the initial surplus is lower. In this situation the safety loading is higher and ruin will occur later.

(ii) On average the aggregate claims in the year ruin occurred (around 1 220 for N1 and 1 250 for N2) is higher than the premium (around 1 080 for each case) and is higher than $\mathrm{E}[Y] = 1\,000$.

(iii) The average premium in the year of ruin, around 1,080, is comparable for all three premium scenarios.

(iv) In the cases P2 and P3 the average premium is lower (but not so low as in the exponential case) for lower values of initial surplus in the case N1.

(v) The average surplus at the start of the year of ruin is comparable for P1 and P3, but noticeably higher for P2: around 100 to 105 for P1 and P3 but 160 for P2 in case N1 and around 100 to 115 for P1 and P3 but still 160 for P2 in case N2.

(vi) We can see from Table 3.26 that the average value of the Poisson parameter in the year of ruin is around 1,150, which is the upper end of its range.

(vii) The proportion of within year ruin probability is around 0.75 for case N1 and 0.27 for case N2.

These statistics suggest the same major factors causing ruin as in the exponential case.

Table 3.27 shows the Pearson's correlation coefficient between the claim amount y_i and y_{i-1} in the cases where $u(i) < 0$. These results were obtained in the same way as in the exponential case. In this case the hypotheses $H_0 : \rho = 0$ is rejected in all cases. These results were also obtained with 200 000 simulations. For the case T1 N1 the confidence intervals are quite large because of the low number of observations. For instance in case (u=150, P1, T1, N1), $r_{y_i,y_{i-1}}$ =-0.66 and the confidence interval goes from -0.75 to -0.56.

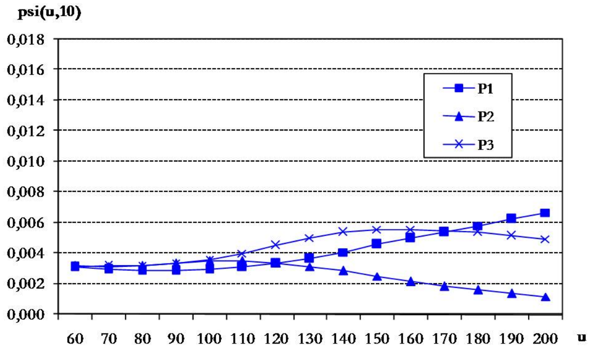
		T1 N1		T1 N2		T2 N1		T2 N2	
u	P	Nr	$r_{y_i,y_{i-1}}$	Nr	$r_{y_i,y_{i-1}}$	Nr	$r_{y_i,y_{i-1}}$	Nr	$r_{y_i,y_{i-1}}$
120	1	27	-0.73	6 760	-0.47	6 762	-0.47	8 387	-0.43
120	2	64	-0.45	7 559	-0.25	7 687	-0.24	19 193	-0.25
120	3	107	-0.75	15 108	-0.51	15 178	-0.50	28 461	-0.45
130	1	53	-0.71	9 428	-0.44	9 438	-0.44	13 198	-0.43
130	2	65	-0.43	7 546	-0.24	7 694	-0.23	19 406	-0.24
130	3	146	-0.74	16 737	-0.51	16 858	-0.51	32 762	-0.47
140	1	86	-0.71	12 300	-0.41	12 339	-0.41	18 949	-0.42
140	2	66	-0.44	7 539	-0.24	7 698	-0.23	19 434	-0.23
140	3	186	-0.73	18 194	-0.51	18 387	-0.51	36 802	-0.49
150	1	142	-0.66	15 224	-0.38	15 320	-0.38	25 325	-0.40
150	2	65	-0.43	7 544	-0.24	7 710	-0.23	19 489	-0.22
150	3	228	-0.77	19 482	-0.51	19 760	-0.51	40 239	-0.49
160	1	206	-0.58	18 140	-0.36	18 319	-0.36	31 910	-0.37
160	2	64	-0.45	7 554	-0.24	7 724	-0.23	19 468	-0.22
160	3	262	-0.78	20 458	-0.51	20 833	-0.51	43 236	-0.49
170	1	284	-0.55	20 882	-0.34	21 155	-0.34	38 377	-0.36
170	2	62	-0.44	7 554	-0.25	7 724	-0.24	19 468	-0.22
170	3	292	-0.80	21 296	-0.51	21 748	-0.51	45 760	-0.49

Table 3.27: Lognormal: Correlation between y_i and y_{i-1}.

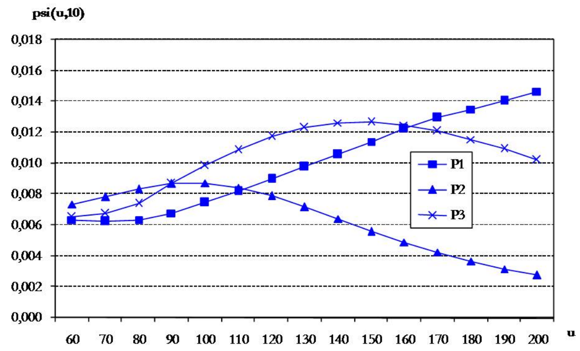
From Table 3.27 we can observe that there is a negative correlation between y_i and y_{i-1}. The conclusions for the lognormal case are the same as the exponential.

The graphics in Figure 3.7 show for different combinations of T, N and different cases of premium P1, P2, P3 the ruin probabilities over a 10 year period for a range of initial surpluses. The results for this figure were obtained with 10 000 simulations as in the previous example. The same comments as for the exponential case may be made for this figure as they have the same behavior.

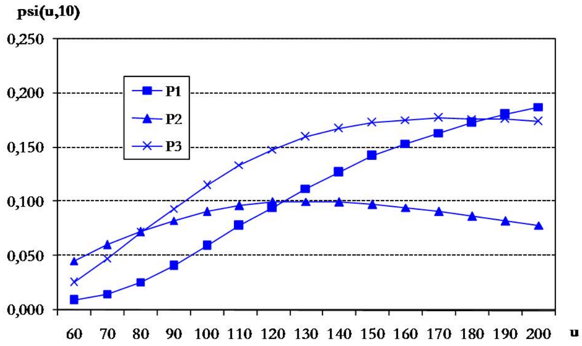
Figures 3.8 to 3.10 show 1 000 paths of the surplus over a 10 year period, starting from initial surplus 60, 120 and 200 for each set of figures. The Poisson parameter for the number of claims is constant (N1) and the target is $\psi(u) = 0.005$ (T1). We



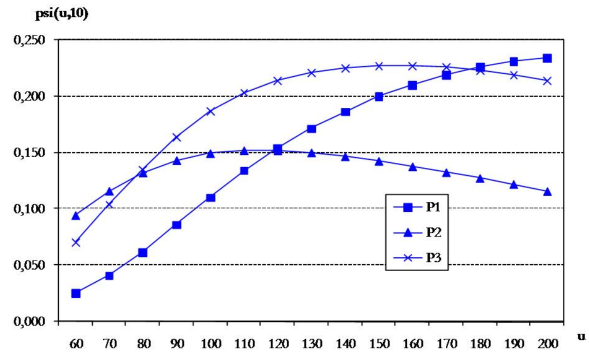
(a) $T1, N1$.



(b) $T2, N1$.



(c) $T1, N2$.

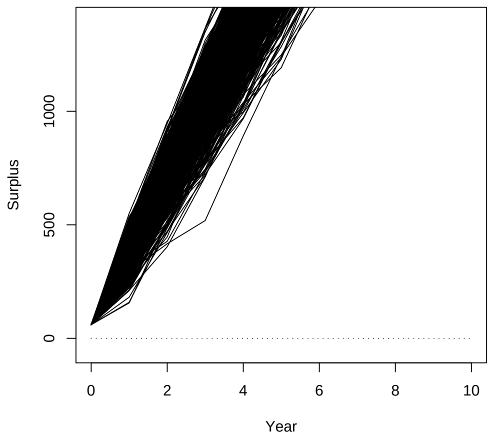


(d) $T2, N2$.

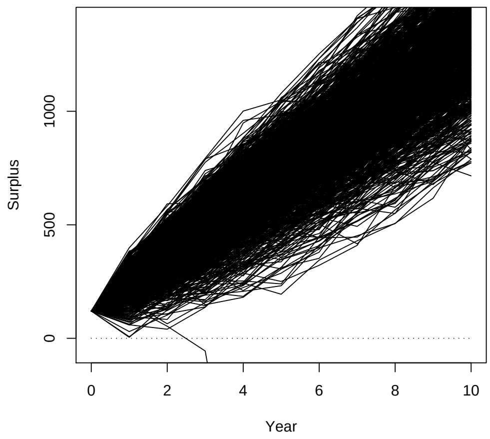
Figure 3.7: Lognormal: $\psi(u, 10)$ for several values of initial surplus.

have a set of figures for each type of premium (P1, P2, P3). All these figures were obtained with the same set of simulated aggregated claims.

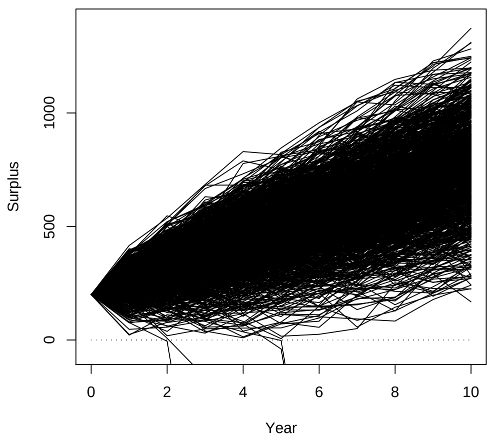
In Figure 3.8 we have the same feature as the exponential case except that with lower initial surpluses the surplus process is pushed upwards significantly. In cases P2 and P3 (Figures 3.9 and 3.10) it happens only in first year and first two years respectively then the surplus process is kept stable in a range of approximately 700 units. With high values of initial surplus the case P3 has more visible jumps on the surplus.



(a) u=60

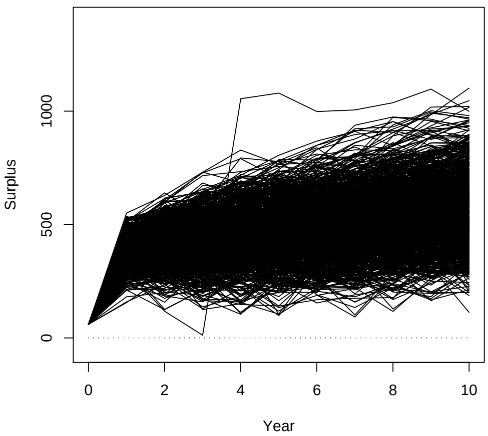


(b) u=120

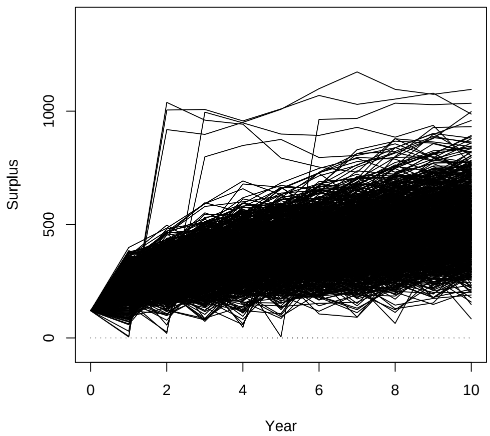


(c) u=200

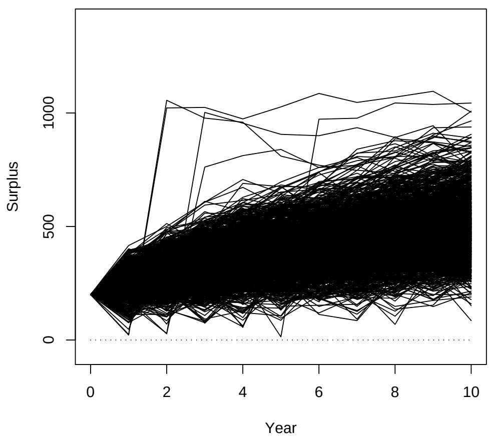
Figure 3.8: Lognormal: Simulated paths, case P1 N1 T1.



(a) u=60

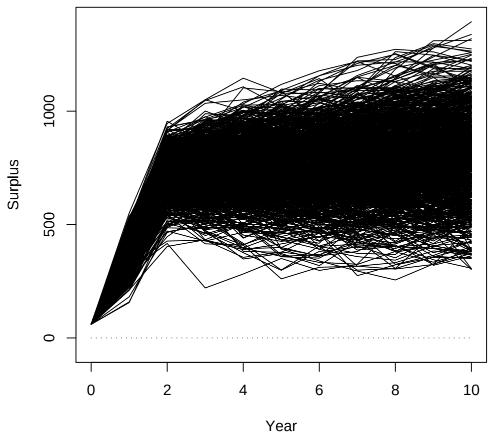


(b) u=120

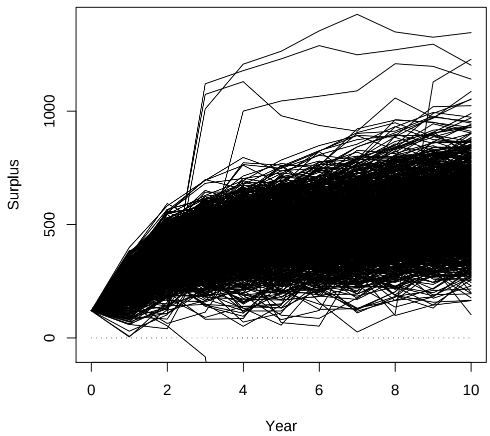


(c) u=200

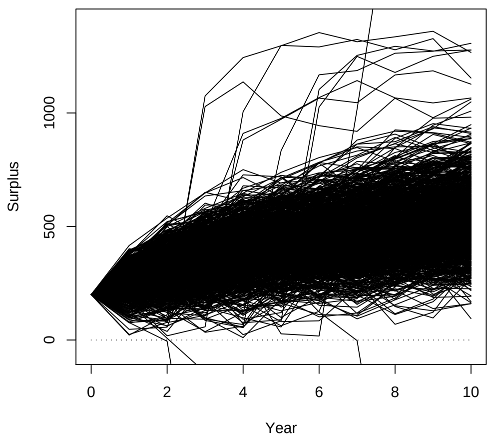
Figure 3.9: Lognormal: Simulated paths, case P2 N1 T1.



(a) u=60



(b) u=120



(c) u=200

Figure 3.10: Lognormal: Simulated paths, case P3 N1 T1.

3.3.4 Gamma claim amounts

Consider that the individual claim amounts are gamma distributed with mean 1, variance 3 and skewness 3.46.

Table 3.28 shows for each of the two target probabilities of ultimate ruin the values of the parameters A and B to be used in formula (3.19).

target	A	B
T1	42.79712	-1.27121
T2	33.33404	-1.25689

Table 3.28: Gamma: Parameters for the power function for formula (3.19).

For the chosen initial surplus, Table 3.29 show values of the safety loading obtained using De Vylder's approximation ($\zeta(u_{\tau_i}, \omega)$) and given by the fitted power function ($Au_{\tau_i}^B$). For case T1 the majority of u have the fitted formula giving higher values than De Vylder's formula in Table 3.29. For case T2 the first half of u have the fitted formula giving values for the safety loading higher than De Vylder's formula. In the remaining values we have lower values for the fitted formula than De Vylder's formula. The ruin probabilities will be affected by this the same way as in the previous examples.

u	T1		u	T2	
	$\zeta(u_{\tau_i}, \omega)$	$Au_{\tau_i}^B$		$\zeta(u_{\tau_i}, \omega)$	$Au_{\tau_i}^B$
120	0.0962	0.0974	80	0.1284	0.1352
130	0.0882	0.0879	90	0.1127	0.1166
140	0.0815	0.0800	100	0.1004	0.1021
150	0.0757	0.0733	110	0.0906	0.0906
160	0.0706	0.0675	120	0.0825	0.0812
170	0.0662	0.0625	130	0.0757	0.0734

Table 3.29: Gamma: Safety loading obtained by De Vylder's formula *vs* fitted power function.

Table 3.30 shows the results for the (approximate) probability of ultimate ruin using formula (3.18) for selected values of the initial surplus, u, for the safety loading of Table 3.29. The results should be close to the target value for $\psi(u)$, either 0.005

(T1) or 0.01 (T2) especially for $\zeta(u_{\tau_i}, \omega)$. The differences between the target and calculated probabilities of ultimate ruin arise from the inaccuracy of the fit of the two parameter power function as shown in Table 3.29 as in the previous examples.

u	T1 $\psi(u)$ $\zeta(u_{\tau_i}, \omega)$	T1 $\psi(u)$ $Au_{\tau_i}^B$	u	T2 $\psi(u)$ $\zeta(u_{\tau_i}, \omega)$	T2 $\psi(u)$ $Au_{\tau_i}^B$
120	0.0050	0.0047	80	0.0100	0.0081
130	0.0050	0.0051	90	0.0100	0.0087
140	0.0050	0.0054	100	0.0100	0.0093
150	0.0050	0.0058	110	0.0100	0.0100
160	0.0050	0.0062	120	0.0100	0.0107
170	0.0050	0.0066	130	0.0100	0.0113

Table 3.30: Gamma: Values for $\psi(u)$ calculated using formula (3.18) and safety loading (3.19).

Tables 3.31 to 3.34 show numerical results for different combinations of T and N. Estimated values of $\psi_{TG}(u, 10)$, together with the standard error of each estimate, are shown for various values of the initial surplus, u and for three cases for the premium.

u	P1 $\psi_{TG}(u,10)$	P1 SD$[\psi_{TG}(u,10)]$	P2 $\psi_{TG}(u,10)$	P2 SD$[\psi_{TG}(u,10)]$	P3 $\psi_{TG}(u,10)$	P3 SD$[\psi_{TG}(u,10)]$
120	0,00493	5,17E-08	0,00370	3,12E-08	0,00595	6,22E-08
130	0,00527	6,03E-08	0,00325	2,91E-08	0,00617	6,77E-08
140	0,00558	6,75E-08	0,00278	2,58E-08	0,00626	7,08E-08
150	0,00591	7,48E-08	0,00232	2,20E-08	0,00621	7,16E-08
160	0,00624	8,20E-08	0,00189	1,77E-08	0,00605	7,07E-08
170	0,00660	9,00E-08	0,00150	1,35E-08	0,00578	6,81E-08

Table 3.31: Gamma: Estimates and standard deviations of $\psi(u, 10)$, T1 N1.

	P1		P2		P3	
u	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$
120	0,14893	2,22E-06	0,14487	1,97E-06	0,20305	2,79E-06
130	0,16322	2,43E-06	0,14204	1,96E-06	0,20865	2,87E-06
140	0,17487	2,60E-06	0,13771	1,92E-06	0,21148	2,91E-06
150	0,18443	2,73E-06	0,13242	1,86E-06	0,21222	2,93E-06
160	0,19247	2,84E-06	0,12664	1,80E-06	0,21132	2,93E-06
170	0,19877	2,93E-06	0,12064	1,73E-06	0,20894	2,91E-06

Table 3.32: Gamma: Estimates and standard deviations of $\psi(u, 10)$, T1 N2.

	P1		P2		P3	
u	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$
80	0,00840	5,05E-08	0,01015	6,60E-08	0,01004	7,14E-08
90	0,00904	7,34E-08	0,01013	7,88E-08	0,01121	1,01E-07
100	0,00966	9,47E-08	0,00965	8,45E-08	0,01219	1,26E-07
110	0,01029	1,15E-07	0,00885	8,42E-08	0,01289	1,46E-07
120	0,01087	1,32E-07	0,00790	7,99E-08	0,01327	1,59E-07
130	0,01149	1,47E-07	0,00691	7,31E-08	0,01331	1,65E-07

Table 3.33: Gamma: Estimates and standard deviations of $\psi(u, 10)$, T2 N1.

	P1		P2		P3	
u	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$
80	0,12681	1,79E-06	0,19298	2,40E-06	0,21022	2,77E-06
90	0,15168	2,17E-06	0,20046	2,52E-06	0,23212	3,04E-06
100	0,17381	2,50E-06	0,20308	2,58E-06	0,24694	3,21E-06
110	0,19262	2,76E-06	0,20210	2,60E-06	0,25681	3,33E-06
120	0,20870	2,97E-06	0,19852	2,59E-06	0,26269	3,40E-06
130	0,22132	3,13E-06	0,19304	2,55E-06	0,26531	3,44E-06

Table 3.34: Gamma: Estimates and standard deviations of $\psi(u, 10)$, T2 N2.

The comments about the results in Tables 3.31 to 3.34 are the same as we made for the exponential example, Tables 3.11 to 3.14 ((i) to (v) in Section 3.3.2). The comment (iv) has the same feature as the lognormal example. It is not obvious from the Tables we study, specially in case T1, a range of surpluses that may be considered high. This comment may be confirmed by Figure 3.11.

Tables 3.35 and 3.36 show some statistical information for the paths where at the end of the year we have a surplus below zero (end of year ruin). They were obtained in the same way as for the exponential and lognormal examples.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	SD $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg p_i	SD p_i	Avg y_i	SD y_i
120	1	52	0.789	1.38	0.63	-20.44	19.98	99.83	36.21	1097.35	0.00	1217.62	39.16
120	2	43	0.768	1.51	1.52	-19.28	19.83	125.72	22.00	1094.42	14.39	1239.42	23.08
120	3	68	0.771	2.44	2.22	-23.03	20.65	102.59	42.00	1084.23	21.93	1209.86	43.86
130	1	66	0.750	1.77	1.05	-20.30	19.75	96.93	42.35	1087.93	0.00	1205.17	44.26
130	2	43	0.736	1.51	1.52	-18.79	19.83	134.14	20.03	1086.50	13.06	1239.42	23.08
130	3	76	0.754	2.51	2.19	-23.40	21.48	103.27	44.53	1077.67	17.75	1204.34	45.68
140	1	75	0.731	2.09	1.30	-22.90	20.27	91.75	49.60	1080.03	0.00	1194.68	49.42
140	2	38	0.726	1.55	1.61	-19.43	19.88	141.27	16.73	1080.53	12.13	1241.23	23.40
140	3	81	0.741	2.58	2.10	-23.61	22.14	102.47	48.74	1072.13	14.31	1198.21	48.66
150	1	91	0.692	2.53	1.52	-23.30	21.85	86.65	52.23	1073.31	0.00	1183.26	51.40
150	2	34	0.706	1.62	1.69	-18.84	19.99	149.90	17.17	1074.81	12.71	1243.54	23.70
150	3	80	0.743	2.68	2.09	-24.70	22.56	102.87	51.15	1067.09	12.64	1194.66	50.24
160	1	93	0.702	2.96	1.68	-27.45	22.98	80.44	51.49	1067.53	0.00	1175.43	50.68
160	2	24	0.746	1.88	1.96	-22.73	19.68	157.80	20.53	1070.66	15.02	1251.19	24.45
160	3	72	0.762	2.90	2.10	-28.27	22.58	97.49	53.87	1063.04	11.88	1188.80	52.35
170	1	109	0.669	3.32	1.78	-26.03	23.88	81.17	49.35	1062.52	0.00	1169.72	49.56
170	2	19	0.747	2.11	2.16	-24.66	18.12	164.77	23.64	1067.42	16.58	1256.85	24.52
170	3	73	0.747	2.19	2.19	-28.04	22.64	96.71	53.83	1059.73	11.40	1184.49	51.54

Table 3.35: Gamma: Statistical information for ruin cases, T1 N1.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	SD $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg λ	SD λ	Avg p_i	SD p_i	Avg y_i	SD y_i
120	1	2168	0.272	2.21	1.63	-47.65	41.74	92.79	45.30	1155.89	38.12	1097.35	0.00	1237.79	54.18
120	2	1998	0.310	2.90	2.58	-39.47	34.70	142.49	46.40	1163.87	31.27	1087.93	30.01	1269.90	40.44
120	3	2930	0.278	3.46	2.72	-52.48	46.12	104.75	57.36	1152.77	41.54	1075.21	30.61	1232.43	59.81
130	1	2452	0.249	2.43	1.79	-49.10	43.35	95.78	50.17	1153.74	39.92	1087.93	0.00	1232.81	57.75
130	2	1987	0.301	2.90	2.58	-39.42	34.67	147.24	44.30	1163.96	31.16	1083.45	28.88	1270.10	40.39
130	3	3070	0.264	3.44	2.69	-52.81	46.65	106.45	58.27	1151.78	42.34	1070.44	26.96	1229.70	61.21
140	1	2724	0.221	2.66	1.94	-50.21	44.02	97.80	54.17	1151.37	42.25	1080.03	0.00	1228.03	60.27
140	2	1950	0.292	2.94	2.59	-39.16	34.56	151.65	42.55	1164.06	31.10	1080.06	29.12	1270.87	39.95
140	3	3147	0.256	3.45	2.66	-53.10	46.96	109.09	59.91	1150.95	42.98	1066.00	24.52	1228.19	62.26
150	1	2947	0.201	2.90	2.08	-51.31	45.03	99.75	56.85	1149.55	43.23	1073.31	0.00	1224.36	62.18
150	2	1889	0.287	3.00	2.61	-38.87	34.39	156.59	42.47	1164.45	30.73	1077.03	29.57	1272.48	39.57
150	3	3194	0.247	3.49	2.64	-53.30	47.40	110.95	61.76	1150.10	43.29	1062.31	22.74	1226.56	63.84
160	1	3141	0.184	3.12	2.20	-52.76	46.30	100.48	60.26	1148.10	44.85	1067.53	0.00	1220.77	64.68
160	2	1794	0.292	3.06	2.63	-38.91	34.14	161.01	42.39	1164.75	30.89	1074.61	30.22	1274.53	39.18
160	3	3200	0.243	3.53	2.63	-53.92	48.08	112.34	63.88	1149.29	44.21	1059.12	21.44	1225.39	65.02
170	1	3319	0.165	3.31	2.28	-53.88	47.67	101.71	62.56	1147.01	45.81	1062.52	0.00	1218.11	65.75
170	2	1730	0.283	3.14	2.67	-38.15	33.94	165.74	43.06	1165.28	30.48	1072.20	30.15	1276.09	38.90
170	3	3196	0.235	3.58	2.62	-53.84	48.58	114.55	65.87	1148.87	44.68	1056.18	20.31	1224.57	66.00

Table 3.36: Gamma: Statistical information for ruin cases, T1 N2.

Tables 3.35 and 3.36 illustrate the comments made previously. The tables for target T2 have the same behavior as these ones and are not going to be presented. From these tables we can add the same comments made for the previous examples (exponential and lognormal), differing only in the values:

(i) Ruin occurs mainly in the first years, apart from the cases where the initial surplus is lower. In this situation the safety loading is higher and ruin will occur later.

(ii) On average the aggregate claims in the year ruin occurred (around 1 200 for N1 and 1 240 for N2) is higher than the premium (around 1 070 for each case) and is higher than $\mathrm{E}[Y] = 1\,000$.

(iii) The average premium in the year of ruin, around 1,070, is comparable for all three premium scenarios.

(iv) In the cases P2 and P3 the average premium has a similar behavior as the lognormal case.

(v) The average surplus at the start of the year of ruin is comparable for P1 and P3, but noticeably higher for P2: around 87 to 100 for P1 and P3 but 140 for P2 in case N1 and around 98 to 110 for P1 and P3 but still 150 for P2 in case N2.

(vi) We can see from Table 3.26 that the average value of the Poisson parameter in the year of ruin is around 1,150, which is the upper end of its range.

(vii) The proportion of within year ruin probability is around 0.73 for case N1 and 0.25 for case N2.

These statistics suggest the same major factors causing ruin as in the exponential and lognormal cases.

Table 3.37 shows the Pearson's correlation coefficient between the claim amounts y_i and y_{i-1} in the cases where $u(i) < 0$. These results were obtained in the same

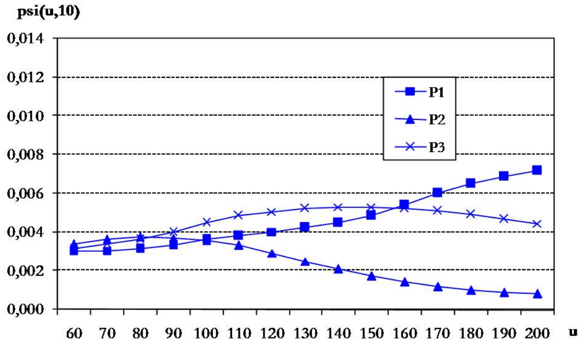
		T1 N1		T1 N2		T2 N1		T2 N2	
u	P	Nr	$r_{y_i,y_{i-1}}$	Nr	$r_{y_i,y_{i-1}}$	Nr	$r_{y_i,y_{i-1}}$	Nr	$r_{y_i,y_{i-1}}$
120	1	95	-0.65	12 377	-0.39	12 389	-0.39	17 695	-0.38
120	2	50	-0.37	10 032	-0.23	10 159	-0.23	24 888	-0.24
120	3	165	-0.72	19 921	-0.47	20 041	-0.47	38 508	-0.45
130	1	141	-0.56	15 514	-0.37	15 550	-0.37	24 096	-0.37
130	2	50	-0.37	10 015	-0.23	10 161	-0.22	24 923	-0.23
130	3	196	-0.73	21 276	-0.47	21 448	-0.47	42 507	-0.47
140	1	191	-0.48	18 415	-0.35	18 495	-0.35	30 673	-0.36
140	2	49	-0.39	10 011	-0.23	10 162	-0.22	24 969	-0.23
140	3	226	-0.72	22 462	-0.48	22 712	-0.48	45 959	-0.47
150	1	248	-0.45	21 146	-0.34	21 316	-0.34	37 084	-0.35
150	2	49	-0.39	10 025	-0.23	10 176	-0.22	24 965	-0.23
150	3	247	-0.72	23 407	-0.48	23 740	-0.47	48 785	-0.47
160	1	314	-0.33	23 797	-0.33	24 074	-0.33	43 483	-0.34
160	2	47	-0.42	10 021	-0.24	10 171	-0.23	24 964	-0.23
160	3	268	-0.73	24 192	-0.48	24 607	-0.48	51 232	-0.47
170	1	394	-0.32	26 184	-0.32	26 602	-0.32	49 497	-0.33
170	2	47	-0.44	10 028	-0.24	10 176	-0.23	24 974	-0.23
170	3	275	-0.73	24 796	-0.48	25 290	-0.48	53 260	-0.46

Table 3.37: Gamma: Correlation between y_i and y_{i-1}.

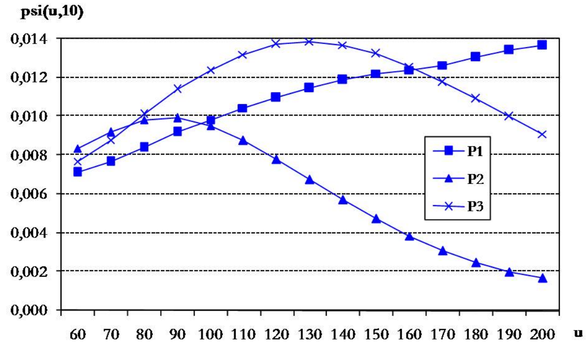
way as the previous examples. In this case all the tests made rejected $H_0 : \rho = 0$ for all cases.

From Table 3.37 we can observe that there is a negative correlation between y_i and y_{i-1}. The conclusions for the gamma example are the same as the exponential example.

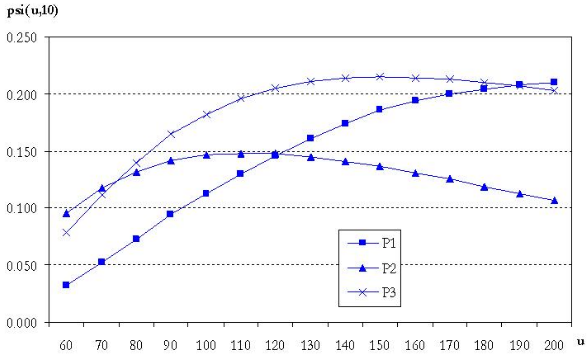
The graphics in Figure 3.11 show for different combinations of T, N and different cases of premium P1, P2, P3 the ruin probabilities over a 10 year period for a range of initial surpluses. The results for this figure were also obtained with 10 000 simulations. The same comments as for the exponential and lognormal examples may be made for this figure as they have the same behavior. They are much more like the lognormal in shape and values than like the exponential example.



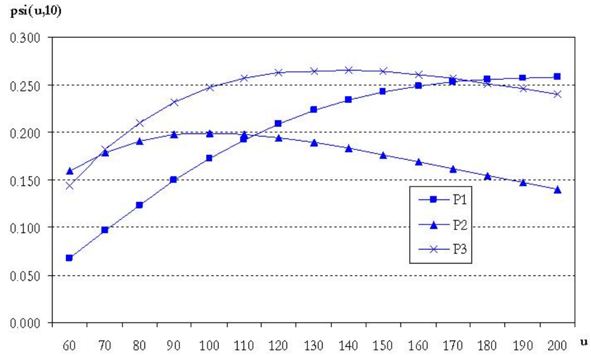
(a) $T1, N1$.



(b) $T2, N1$.



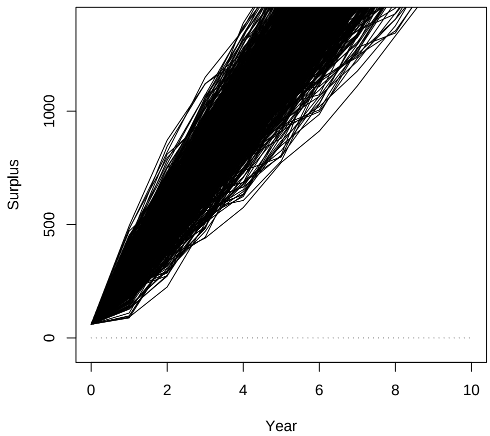
(c) $T1, N2$.



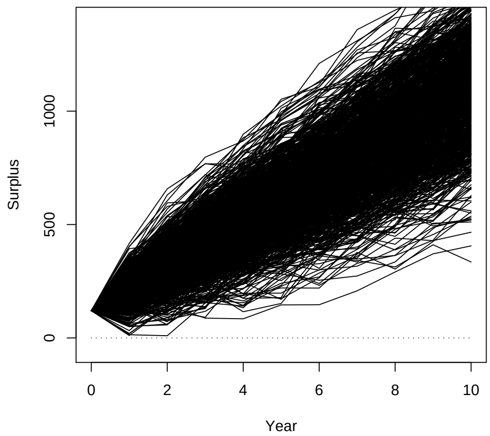
(d) $T2, N2$.

Figure 3.11: Gamma: $\psi(u, 10)$ for several values of initial surplus.

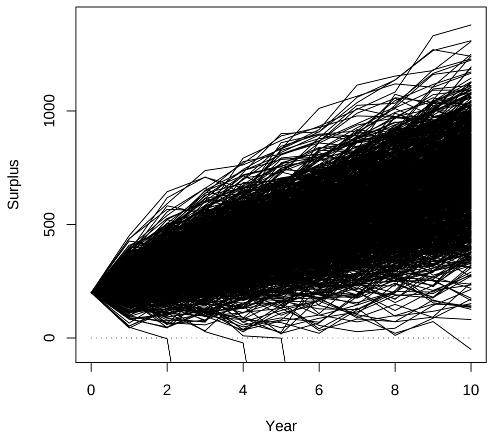
Figures 3.12 to 3.14 show 1 000 paths of the surplus over a 10 year period, starting from initial surplus 60, 120 and 200 for each set of figures. The Poisson parameter for the number of claims is constant (N1) and the target is $\psi(u) = 0.005$ (T1). We have a set of figures for each type of premium (P1, P2, P3). All these figures were obtained with the same set of simulated aggregated claims. They have the same features as the lognormal example.



(a) u=60

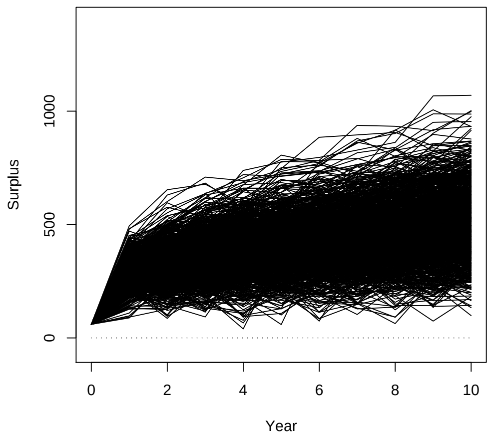


(b) u=120

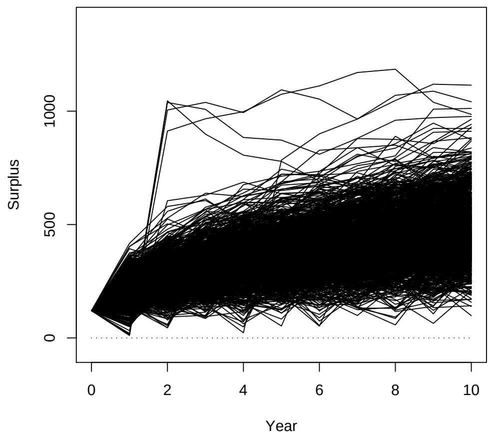


(c) u=200

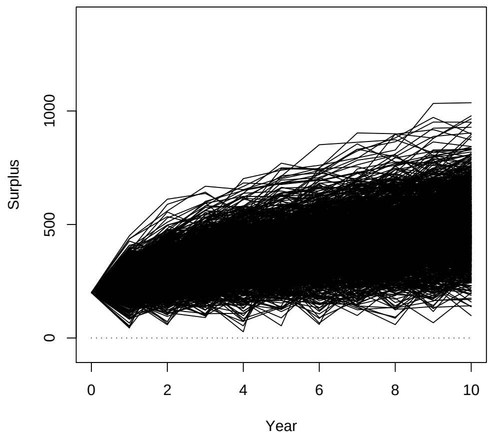
Figure 3.12: Gamma: Simulated paths, case P1 N1 T1



(a) u=60

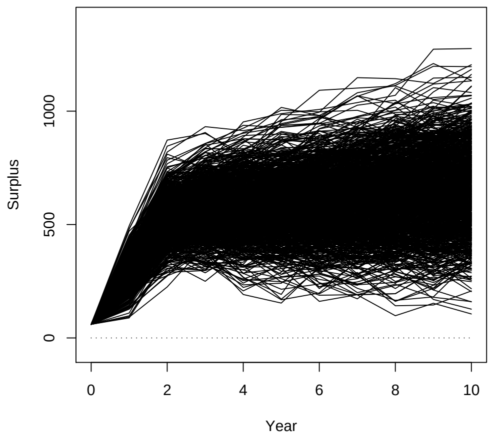


(b) u=120

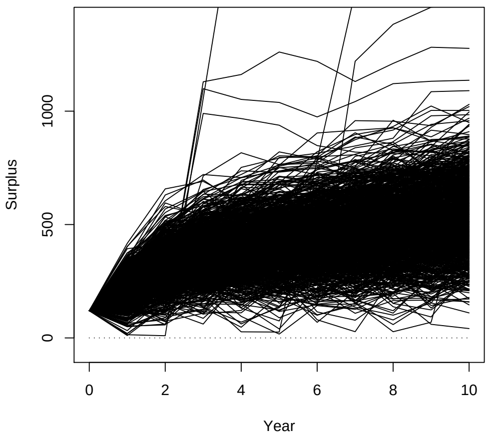


(c) u=200

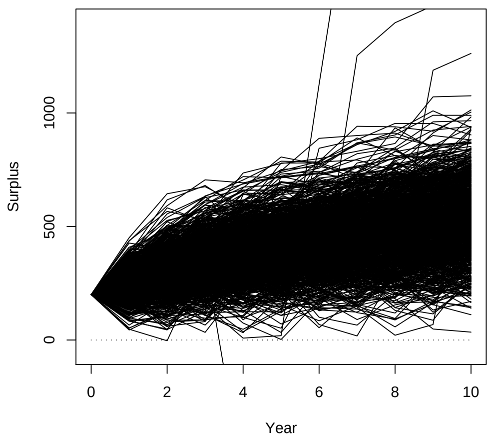
Figure 3.13: Gamma: Simulated paths, case P2 N1 T1



(a) u=60



(b) u=120



(c) u=200

Figure 3.14: Gamma: Simulated paths, case P3 N1 T1

3.3.5 Michaud's example

Michaud (1996) considered an example where the premium varies in layers according to the level of the current surplus. In his simulation procedure the premium is updated immediately using (3.22) after the claim occurs. x indicates the value of the current surplus.

$$P_i = \begin{cases} 1.7, & 0 < x \leq 2; \\ 1.6, & 2 < x \leq 4; \\ 1.5, & 4 < x \leq 6; \\ 1.4, & 6 < x \leq 8; \\ 1.3, & 8 < x \leq 10; \\ 1.2, & x > 10. \end{cases} \tag{3.22}$$

Let us also consider that the premiums vary according to the level of the surplus in a layers basis. We will use the same parameters as in Subsection 5.2 of Michaud (1996).

The Poisson parameter λ is 1. The claims are distributed according to a translated gamma distribution (claims > -1) with mean equal to 1 and variance equal to 2. The premiums are as follows:

$$P_i = \begin{cases} 1.7, & 0 < u_{\tau_i} \leq 2; \\ 1.6, & 2 < u_{\tau_i} \leq 4; \\ 1.5, & 4 < u_{\tau_i} \leq 6; \\ 1.4, & 6 < u_{\tau_i} \leq 8; \\ 1.3, & 8 < u_{\tau_i} \leq 10; \\ 1.2, & u_{\tau_i} > 10. \end{cases} \tag{3.23}$$

Michaud's example differs from ours in the following aspects:

(i) his results are for ultimate ruin, our results are for finite time;

(ii) his premium rates change in continuous time, our premium is updated at the beginning of each year;

(iii) his claims can take negative values, our method of evaluating the within-year ruin probabilities of ruin using the translated gamma approximation does not allow for negative claims.

Using the procedure of Section 2.4 we may obtain the estimates for $\psi(u, n)$

The parameters for the translated gamma approximation α, β and κ are given by:

$$\alpha = \frac{108}{121}\lambda, \quad \beta = \frac{6}{11} \quad \text{and} \quad \kappa = -\frac{7}{11}\lambda.$$

Tables 3.38 and 3.39 show numerical results for the estimates of the probability of ruin and the standard deviations of these estimates for $n = 10\,000$ and for:

(i) initial surplus $u = 2, 4, 6, 8, 10, 12, 14$;

(ii) three methods for adjusting the premium, $u_{\tau_i} = u(0)$, so that the premium is fixed throughout the n years (P1), $u_{\tau_i} = u(i-1)$, the premium depends on the surplus at the beginning of the year (P2), and $u_{\tau_i} = u(max(i-2, 0))$ there is a delay of one year (P3),

(iii) two algorithms for calculating the Poisson parameter for the expected number of claims. These are:

(a) $\lambda = 1$ so that the Poisson parameter is constant from year to year.

(b) $\lambda \sim U[0.8, 1.2]$ so that the Poisson parameter varies from year to year.

These results are based on 100 000 simulations. For a given algorithm for the Poisson parameter, the same simulations of the aggregate annual claims are used for the different initial surpluses and the different ways of updating the premium. Each table took approximately 5 hours of computation time.

	P1		P2		P3	
u	$\psi_{TG}(u, 10000)$	SD[$\psi_{TG}(u, 10000)$]	$\psi_{TG}(u, 10000)$	SD[$\psi_{TG}(u, 10000)$]	$\psi_{TG}(u, 10000)$	SD[$\psi_{TG}(u, 10000)$]
2	0.33028	1.97E-06	0.46595	2.24E-06	0.46172	2.25E-06
4	0.20447	1.48E-06	0.32349	1.99E-06	0.32366	2.00E-06
6	0.14540	1.17E-06	0.23343	1.66E-06	0.23691	1.68E-06
8	0.12556	1.05E-06	0.17839	1.37E-06	0.18334	1.41E-06
10	0.13155	1.10E-06	0.14129	1.14E-06	0.14692	1.19E-06
12	0.17931	1.44E-06	0.11223	9.44E-07	0.11801	9.90E-07
14	0.14070	1.18E-06	0.08886	7.69E-07	0.09316	8.05E-07

Table 3.38: Values of $\psi_{TG}(u, 10\,000)$: $\lambda = 1$.

	P1		P2		P3	
u	$\psi_{TG}(u, 10000)$	SD[$\psi_{TG}(u, 10000)$]	$\psi_{TG}(u, 10000)$	SD[$\psi_{TG}(u, 10000)$]	$\psi_{TG}(u, 10000)$	SD[$\psi_{TG}(u, 10000)$]
2	0.32808	1.96E-06	0.46556	2.24E-06	0.46113	2.24E-06
4	0.20357	1.48E-06	0.32511	2.00E-06	0.32524	2.01E-06
6	0.14625	1.18E-06	0.23749	1.68E-06	0.24079	1.71E-06
8	0.12584	1.05E-06	0.18222	1.40E-06	0.18728	1.43E-06
10	0.13258	1.11E-06	0.14384	1.16E-06	0.14952	1.20E-06
12	0.18066	1.45E-06	0.11428	9.58E-07	0.12006	1.00E-06
14	0.14319	1.20E-06	0.08965	7.74E-07	0.09416	8.12E-07

Table 3.39: Values of $\psi_{TG}(u, 10\,000)$: $\lambda \sim U[0.8, 1.2]$.

The results for variable λ (Table 3.39) are very close to those for the constant λ (Table 3.38). This contrasts with the results in Sections 3.3.2, 3.3.3 and 3.3.4. This happens because we have a small λ.

Keeping in mind the differences between the two methodologies we are going to show both results in Table 3.40. $\psi(u)$ denotes the ruin probability obtained by Michaud (1996, Table 7) using 1 000 000 simulated claims and $\psi_{TG}(u, 10\,000)$ denote the ruin probabilities obtained by our method with $u_{\tau_i} = u(i-1)$ and $\lambda = 1$.

The results for $\psi_{TG}(u, 10\,000)$ are in general greater that $\psi(u)$. Let us consider for instance for $u = 12$. The premium is 1.2 and we expect one claim to occur, but more than one may occur . In our model the premium is updated only at the end of the year. In Michaud's model the premium is updated after the claim occurs and that will lead to lower ruin probabilities.

u	$\psi(u)$	$\psi_{TG}(u, 10\,000)$
2	0.501131	0.465951
4	0.305775	0.323488
6	0.176930	0.233434
8	0.098507	0.178394
10	0.053432	0.141294
12	-	0.112228
14	-	0.088861

Table 3.40: Estimates of ruin probabilities for Michaud's and for our methodologies.

3.4 Comments on results

Throughout this chapter we saw the results of applying our model when premiums can change from year to year depending on the surplus at some time. We used three different distributions (exponential, lognormal and gamma), two approaches to the Poisson parameter (constant and variable), three ways of calculating the premium (always depending on the surplus), two target probabilities of ruin and a fixed finite term of ten years.

We also calculated results for $\lambda = 100$ and $\lambda = 10\,000$ and the Poisson parameter varying between 900 and 1 000, but these results are not shown in detail. For different values of λ the behavior is the same as presented in Figures 3.3, 3.7 and 3.11. For the case N2 the ruin probabilities increase very much with λ. For instance consider the lognormal claim amount example with initial surplus $u = 140$ and P1. $\psi_{TG}(140, 10, \lambda = 100) = 0.00549$, $\psi_{TG}(140, 10, \lambda = 1\,000) = 0.12593$ and $\psi_{TG}(140, 10, \lambda = 10\,000) = 0.32680$. In the case $U(900, 1\,100)$ the absolute values of ruin probabilities are much lower (for instance the table corresponding to Table 3.22 has values varying from 0.093 to 0.176) but all the features and relations between the results are the same.

We make the following general comments about the results:

(i) Formula (3.20) is an attempt to control the surplus process by adjusting the premium each year so that the probability of ultimate ruin has a given target value. This is necessarily a somewhat crude attempt since a two parameter function does not fully reflect the behavior of the premium loading, $\zeta(u_{\tau_i}, \omega)$, as the surplus varies. This is specially true for large values of u as the fitted function is based on few values. We have checked the accuracy of this formula by calculating the (approximate) probability of ultimate ruin using formula (3.18) for selected values of the initial surplus, u, and the fitted premium loading, $\zeta(u)$ $(= Au^B)$. The results should be close to the target value for $\psi(u)$, either 0.01 or 0.005. Some results are shown in Tables 3.10, 3.20 and 3.30 for some values of u of interest in our examples. The differences between the target and calculated probabilities of ultimate ruin arise from the inaccuracy of the fit of the two parameter power function (over a large range of values of u) and the sensitivity of $\psi(u)$ to the premium loading, $\zeta(u_{\tau_i}, \omega)$.

(ii) Varying the Poisson parameter, scenario N2, increases the probability of ruin considerably, as we would expect.

We gained some insight on the causes of end year ruin by recording some results for each simulation where $u(i) < 0$ for some i. That allowed us to comment:

(iii) The average aggregate claims in the year of ruin is significantly higher than the expected aggregate claims.

(iv) The average surplus at the start of the year of ruin is comparable for P1 and P3, but noticeably higher for P2.

(v) The average premium in the year of ruin, around 1 070, is comparable for all three premium and three distributions scenarios.

We plotted $\psi(u, 10)$ for a large range of values for the initial surplus, combining T and N for P1, P2 and P3.

(vi) For lower values of u we have the following relation for the ruin probabilities as functions of the type of premium: P2>P3>P1. At some u that depends on the target, on the Poisson parameter and distribution of claims (and also on λ) we have P3>P2>P1, then P3>P1>P2 and finally for high values of u we have P1>P3>P2.

(vii) For high values of u the ruin probabilities for case P2 decrease very much, followed by case P3 and, last, for case P1.

Finally we also plotted some simulated paths for different combinations of P, N and T for initial surpluses of 60, 120 and 200. We can see:

(viii) In the case P1 with low u the surplus process is pushed upwards in the first years (more intensively in the lognormal and gamma cases). If ruin does not occur in the first years then it is likely it will never occur.

(ix) Cases P2 and P3 have a similar behavior. For paths that have the surplus process near zero, but positive, the immediate response of the premium is to push the surplus upwards considerably.

We can also add:

(x) The standard deviations of our estimates $\psi_{TG}(u, 10)$ are all very small.

(xi) The results for the lognormal distribution are very similar to the results for the gamma distribution.

(xii) The correlation between y_i and y_{i-1} is negative. The test of hypotheses $H_0 : \rho = 0$ *vs* $H_1 : \rho \neq 0$ rejects the hypotheses H_0 in all cases, except for a single case that we suppose it is for lack of information. From the Tables of the statistics we can see that Avg y_i is higher that $\mathrm{E}[Y] = 1\,000$. That seems to indicate that lower claims in year $i-1$ will lead in cases P2 and P3 to lower premiums and then high claims in year i will lead to end of year ruin. In case P1 ruin occurs mainly in the first year due to high claims.

Chapter 4

Premium using the Bühlmann credibility model

In this chapter we study the behavior of ruin probabilities within n years when the premium is updated according to the Bühlmann (1967, 1969) credibility model. We are going to compare these results with the ruin probabilities within n years when the premium is calculated using the collective premium (μ_0). We will use two different approaches to calculate the safety loading: first it depends on the initial surplus and is fixed throughout the n-years period and, secondly, it depends on the surplus at the beginning of each year. In Section 4.1 we will define Bühlmann's model briefly. The reader may find more information in Bühlmann (1967, 1969), or, for instance, in Bühlmann and Gisler (2005) and Klugman et al. (2004). We will apply our method to estimate ruin probabilities in continuous and finite time using only the translated gamma approximation and one target for the ultimate ruin probability (T2, $\omega = 0.01$). The methodology will be described in Section 4.2. In Section 4.3 we will illustrate it with a numerical example. First we illustrate our method using one single simulation, presenting all calculations so that the reader can understand all the steps needed to produce the final results presented in the next subsection. Some comments are set out in Section 4.4.

4.1 The Bühlmann model

We consider that we have a portfolio of risks. By a risk we mean a single policy or a group of policies. We assume we have observed m years of past claim amounts for each risk. It is usual that for a given risk, its past experience (for instance the mean, $\overline{Y}_k$) will be different from the collective premium (μ_0). Each risk has its own behavior.

Let us assume that the risk level of each risk may be characterized by a risk parameter θ, and this parameter may be different from risk to risk. We assume that the actuary does not know the value of the risk parameter for any given risk. The actuary assumes that θ is a realization of a random variable Θ, whose distribution he knows. Let $U(\theta)$ be this distribution which is known as the structural distribution.

Before defining the assumptions and the model we need to settle the notation. We use a general notation for a single risk and for the portfolio.

4.1.1 Notation

Let us define the notation that will be used throughout the chapter.

Port. is the portfolio (when needed we will use Pt.),

r is the number of risks in the portfolio,

k is the risk, $k = 1, \ldots, r$,

m is the number of years of past observed data,

n is the evaluation future periods (years) or evaluation horizon,

i is the year $i = 1, \ldots, m + n$,

θ_k is the risk parameter of risk k,

$\zeta(u_{\tau_i}, \omega)$ is the safety loading as defined in Section 3.1,

$Y_{ki}(\theta_k)$ is the aggregate claims for risk k in year i, and follows a compound Poisson distribution conditional on the risk parameter. We denote the distribution function $F(\theta)$. Having in mind that, in this chapter, the aggregate claims depend always on a risk parameter, we will use the notation Y_{ki} for simplicity,

$Y_{\bullet i}$ is the sum of the aggregate claims for year i, $Y_{\bullet i} = \sum_{k=1}^{r} Y_{ki}(\theta_k)$,

P_{ki}^C is the Bühlmann credibility pure premium for risk k, and year i, defined in Section 4.1.3,

$P_{ki}^E = \mu_0 = \mathrm{E}[\mathrm{E}[Y_{ki}|\Theta_k = \theta]]$ is the pure collective premium for risk k, and year i,

P_{ki} is the premium for risk k in year i. $P_{ki} = (1 + \zeta(u_{\tau_i}, \omega))P_{ki}^{\bullet}$, with $i = m+1, \ldots, m+n$ and $P_{ki}^{\bullet}$ equal to P_{ki}^E or P_{ki}^C.

We are going to consider four different approaches to the premium:

- P1 constant premium as a function of the initial surplus $P_{ki} = (1 + \zeta(u(0), \omega))P_{ki}^E$,
- P2 premium as a function of the surplus at the beginning of the year $P_{ki} = (1 + \zeta(u(i-1), \omega))P_{ki}^E$,
- P4 credibility updated premium with constant safety loading as a function of the initial surplus $P_{ki} = (1 + \zeta(u(0), \omega))P_{ki}^C$,
- P5 credibility updated premium with safety loading as a function of the surplus at the beginning of the year $P_{ki} = (1 + \zeta(u(i-1), \omega))P_{ki}^C$,

P1 and P2 are similar to the premiums defined in Chapter 3. We will not use in this chapter the premium type P3. The notation P3 is not going to be used in this chapter because it is associated with a premium updated with a delay of one year as in the previous chapter. P4 and P5 are the credibility premiums.

$P_{\bullet i} = \sum_{k=1}^{r} P_{ki}$ is the premium for the portfolio in year i.

4.1.2 Assumptions

Following Model Assumptions 3.6 of Bühlmann and Gisler (2005) we have for the Bühlmann model the following assumptions:

B1: The random variables Y_{ki} $(i = 1, \ldots, m+n)$ are, conditional on $\Theta_k = \theta$, independent with the same distribution function $F(\theta)$ and conditional moments

$$\mu(\theta) = \mathrm{E}[Y_{ki}|\Theta_k = \theta]$$
$$\sigma^2(\theta) = \mathrm{Var}[Y_{ki}|\Theta_k = \theta]$$

B2: The pairs $(\Theta_1, (Y_{1,1}, \ldots, Y_{1,m+n})), \ldots, (\Theta_r, (Y_{r,1}, \ldots, Y_{r,m+n}))$ are independent and identically distributed.

4.1.3 The model

We want to estimate the pure premium at the beginning of year i with data from years 1 to $i-1$ for each risk k $(k = 1, \cdots, r)$ and for the portfolio using Bühlmann's linear credibility estimator (see (4.24)). However it involves some parameters that need to be estimated. This way we use its empirical Bayes version (see (4.26)) and we will call it P_{ki}^C .

Let us define:

$\mu_0 = \mathrm{E}[\mu(\Theta_k)]$	(the expected value of the hypothetical means also referred to as the collective premium)
$\sigma^2 = \mathrm{E}[\sigma^2(\Theta_k)]$	(the expected value of the process variance)
$\tau^2 = \mathrm{Var}[\mu(\Theta_k)]$	(the variance of the hypothetical means)

$\mu(\Theta_k)$ is referred to as the hypothetical mean and $\sigma^2(\Theta_k)$ is called the process variance.

Bühlmann´s (1967) credibility pure premium estimator per risk k and year i is given by:

$$\widehat{\mu}(\Theta_k) = z\overline{Y}_k + (1-z)\mu_0 \tag{4.24}$$

where: $\overline{Y}_k = \frac{1}{i-1}\sum_{l=1}^{i-1} Y_{kl}$ and $z = \frac{i-1}{i-1+\frac{\sigma^2}{\tau^2}}$, $i > 1$. z is referred to as the Bühlmann credibility factor.

Bühlmann and Gisler (2005), Theorem 3.7 give the homogeneous credibility pure premium estimator;

$$\widehat{\widehat{\mu}}(\Theta_k) = z\overline{Y}_k + (1-z)\overline{Y} \tag{4.25}$$

and it is obtained from (4.24) by replacing μ_0 by the "classical" estimator, the observed collective mean in the portfolio, $\overline{Y} = \frac{1}{r(i-1)}\sum_{k=1}^{r}\sum_{l=1}^{i-1} Y_{kl}$.

We will use instead the empirical credibility estimator obtained from (4.25) by replacing, in z, the structural parameters σ^2 and τ^2 by their estimators, derived for instance in Section 4.8 of Bühlmann and Gisler (2005). We will have:

$$P_{ki}^C = \widehat{z}\overline{Y}_k + (1-\widehat{z})\overline{Y} \tag{4.26}$$

where:

$$\widehat{\sigma^2} = \frac{1}{r}\sum_{k=1}^{r}\frac{1}{(i-2)}\sum_{l=1}^{i-1}(Y_{kl}-\overline{Y}_k)^2 \quad \text{and,} \quad \widehat{\tau^2} = \max\left(\frac{1}{r-1}\sum_{k=1}^{r}(\overline{Y}_k-\overline{Y})^2 - \frac{\widehat{\sigma^2}}{r}, 0\right)$$

for $i > 2$ and $r > 1$.

The discussion of the assumptions and the properties of the estimators are well studied in the literature. See for instance Bühlmann and Gisler (2005).

Note that the pure credibility premium for the portfolio is the number of risks times the observed collective mean in the portfolio (i.e. $\sum_{k=1}^{r} P_{ki}^C = r\overline{Y}$).

4.2 Methodology

Our goal is to evaluate the ruin probability of a portfolio over n years starting from an initial surplus u. We are also interested in the ruin probabilities of each one of the risks of the portfolio. We have some extra knowledge about each risk that the actuary does not have. We know the individual risk parameter. We can produce results for each risk when the actuary does not have enough information to assign the correct initial surplus for each risk and decides to divide the surplus of the portfolio by the number of risks. To achieve it consider that:

(a) for simplicity, m, the number of years of past observed claim amounts, is assumed to be the same for each risk. The model also works with different observed exposure years for each risk,

(b) the actuary knows the distribution of the risk parameter, but does not know the individual outcome for each risk,

(c) the distribution of the number of claims for each risk does not depend on the risk parameter and is known to the actuary. We assume the number of claims follows a Poisson distribution with parameter λ. In this chapter we will use the same two models for the Poisson parameter, for each risk, defined in Section 3.2:

  N1 the Poisson parameter, denoted λ, is constant and equal to $1\,000$ each year, for each risk.

  N2 the Poisson parameter for risk k in year i, denoted λ_{ki}, is a random variable and $\{\{\lambda_{ki}\}_{k=1}^{r}\}_{i=1}^{n}$ is a set of *i.i.d.* random variables, each with a $U(800, 1\,200)$ distribution,

(d) the actuary also knows the distribution of claim amount (as function of the unknown risk parameter),

(e) the initial surplus is allocated at the end of time period m. As the actuary cannot initially distinguish between the risks, we obtain the initial surplus for each risk in the portfolio by dividing the initial surplus for the portfolio by the number of risks,

(f) the safety loading depends on the surplus. First we assume that it depends on the initial surplus and, secondly, that it depends on the surplus at the end of the preceding year. We will use formula (3.19).

(g) The premiums P1 and P2 are the pure collective premiums. The premiums P4 and P5 are going to be updated using the Bühlmann credibility model defined in Sections 4.1.2 and 4.1.3.

We will use the simulation procedure defined in Section 2.4 with some adjustments:

(i) For each run we need to generate the aggregate claim amounts and calculate $\psi_{TG}(u(i-1), 1, u(i))$ for each risk k and each year i. For this, we need the parameters for the translated gamma approximation. These parameters will depend on the risk parameter θ_k, because we are approximating the conditional distribution of Y_k given θ_k. Let the translated gamma parameters be: $\alpha(\theta_k)$, $\beta(\theta_k)$ and $\kappa(\theta_k)$.

$$\begin{aligned} \alpha(\theta_k) &= \tfrac{4\lambda\mu_2'^3}{\mu_3'^2}, \\ \beta(\theta_k) &= \tfrac{2\mu_2'}{\mu_3'}, \\ \kappa(\theta_k) &= \lambda(\mu - \tfrac{2\mu_2'^2}{\mu_3'}) \end{aligned} \tag{4.27}$$

where $\mu_r' = \mathrm{E}[Z_{ki}^r|\Theta_k = \theta]$ and the random variable $Z_{ki}|\Theta$ is the individual claim amount conditional on $\Theta_k = \theta_k$.

(ii) We also need the translated gamma approximation parameters α, β and γ for the portfolio. These parameters are given by:

$$\alpha = \frac{4}{\gamma(Y_{1i} + \ldots + Y_{ri})},$$

$$\beta = \frac{2}{\gamma(Y_{1i} + \ldots + Y_{ri})\sigma(Y_{1i} + \ldots + Y_{ri})}, \tag{4.28}$$

$$\kappa = \mathrm{E}(Y_{1i} + \ldots + Y_{ri}) - \frac{2\sigma(Y_{1i} + \ldots + Y_{ri})}{\gamma(Y_{1i} + \ldots + Y_{ri})}$$

with $\mathrm{E}(Y_{1i}+\ldots+Y_{ri})=\sum_{k=1}^{r}\mathrm{E}(Y_{ki})$, $\sigma^2(Y_{1i}+\ldots+Y_{ri})=\sum_{k=1}^{r}\sigma^2(Y_{ki})$,

$\mu_3(Y_{1i}+\ldots+Y_{ri})=\sum_{k=1}^{r}\mu_3(Y_{ki})$, and $\gamma(Y_{1i}+\ldots+Y_{ri})=\dfrac{\mu_3(Y_{1i}+\ldots+Y_{ri})}{\sigma(Y_{1i}+\ldots+Y_{ri})^3}$

4.3 Numerical examples

In our applications in this section we will use a lognormal distribution for the individual claim size with parameters θ and $\sigma^{\bullet 2}=0.97411$. The risk parameter Θ, has the discrete distribution shown in Table 4.41:

θ	$p_i^{\bullet}$
0.1	0.4
0.2	0.4
0.4	0.2

Table 4.41: Bühlmann: Distribution of Θ.

We are going to use throughout this chapter only one target for the ultimate ruin probability (T2, $\omega=0.01$).

Table 4.42 shows for target T2 the values of the parameters A and B to be used in formula (3.19) in Section 3.1.

target	A	B
T2	43.13933	-1.21074

Table 4.42: Bühlmann: Parameters for the power function for formula (3.19).

For the chosen initial surplus, Table 4.43 shows, in columns 2 and 3, values of the safety loading obtained using De Vylder's approximation ($(\zeta(u_{\tau_i},\omega))$) and given by the fitted power function ($Au_{\tau_i}^{B}$) and in the last two columns the results for the (approximate) probability of ultimate ruin using formula (3.18) and the safety loading indicated.

We will use the four cases for the premium defined previously in Section 4.1.1. The future time period is 10 years ($n=10$). As we are working with an existing

	safety loading		$\psi(u)$	
u	$\zeta(u_{\tau_i},\omega)$	$Au^B_{\tau_i}$	$\zeta(u_{\tau_i},\omega)$	$Au^B_{\tau_i}$
250	0.0531	0.0539	0.01	0.0095
300	0.0437	0.0432	0.01	0.0096
350	0.0371	0.0359	0.01	0.0098
400	0.0323	0.0305	0.01	0.0100
450	0.0286	0.0265	0.01	0.0103

Table 4.43: Bühlmann: Safety loading obtained by De Vylder's formula *vs* fitted power function and respective values for $\psi(u)$.

portfolio we assume that there is a previous record for the aggregate claim amounts of 5 years ($m = 5$) as illustrated in Figure 4.15.

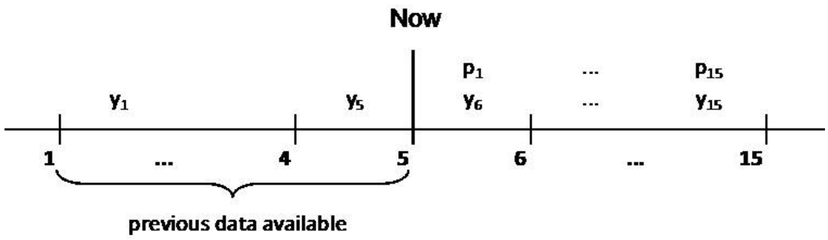


Figure 4.15: Bühlmann: Time axis.

In this chapter the results for our estimates of ruin probabilities are obtained with 50 000 simulations.

For each Poisson parameter case (N1 and N2) the simulated set of aggregate claims are the same in each simulation for the four premium types and for the different surpluses. We present for selected cases our estimate of the ruin probability, $\hat{\psi}(u,n)$, and the standard errors of the estimates for each initial surplus. Let us consider our portfolio with 5 risks ($r = 5$). This number was chosen in order to have the distribution of Θ represented exactly in the portfolio. This way, risks 1 and 2 have $\theta = 0.1$, risks 3 and 4 have $\theta = 0.2$ and risk 5 has $\theta = 0.4$. We assume that the actuary knows Table 4.41 but does not know which risk has which θ. In other words, he knows the structural distribution of Θ but not the outcome, since the risk parameter is not observable.

We choose a simple distribution of the structural parameter to illustrate our methodology. Our main goal is to study the impact of credibility updated premiums

Risk	$\alpha(\theta_k)$	$\beta(\theta_k)$	$\kappa(\theta_k)$
1	215.233	0.15848	440.576
2	215.233	0.15848	440.576
3	215.233	0.143398	486.912
4	215.233	0.143398	486.912
5	215.233	0.117405	594.715
Port.	**1 032.55**	**0.138681**	**2 555.54**

Table 4.44: One run: Translated gamma parameters.

on the ruin probability in continuous and finite time. That can be achieved using a simple discrete structural distribution and assuming that the actuary knows it. Our model can also be used with continuous structural distributions and using Poisson parameters affected by the risk parameter.

4.3.1 Illustration of one run

For a better understanding of the methodology, in this section we are going to illustrate it by considering in detail one simulation for $u = 300$, case N1. First of all we need the translated gamma parameters, obtained using formula (4.28) and shown in Table 4.44 to generate the aggregate claims for each risk. If the Poisson parameter is not constant in each year we will have a different set of parameters $\alpha(\theta_k)$, $\beta(\theta_k)$ and $\kappa(\theta_k)$ for each year.

Table 4.45, shows the generated aggregate claims for each year and risk and for the portfolio in each year ($= \sum_{k=1}^{r} Y_{ki}$). It also shows the estimate of the credibility factor for each year, $\widehat{z}$, calculated using the aggregate claim amounts of the years 1 to $i-1$, $i > 1$, of each risk and of the portfolio (formulas in Section 4.1.3). The credibility factor is used to calculate the pure credibility premium, $P_{ki}^{C} = \widehat{z}\overline{Y}_k + (1-\widehat{z})\overline{Y}$, shown in Table 4.46. Table 4.46 also shows the pure collective premium, $P_{ki}^{E} = \mathrm{E}[\mathrm{E}[Y_{ki}|\Theta_k = \theta]]$, for each year i and risk k. Table 4.47 shows the premiums calculated using the four different approaches defined previously (P1, P2, P4, P5), for each risk and year and the premium for the portfolio ($= \sum_{k=1}^{r} P_{k,i}$) per year and premium type. For instance, the safety loading for premium P1 is constant throughout the periods and is given by $0.043224 = 43.13933(300)^{-1.21074}$. Recall that we consider years 1 to 5 as previous information and we are at the beginning of year 6 with a surplus of 60 for each risk (300 for the portfolio).

Table 4.48 shows values of the surplus for each combination of P and k at the end of year i. $u(i) = u(i-1) + P_{ki} - Y_{ki}$, for instance for case (P1, $k = 4$, $i = 6$), $u(6) = u + \mathrm{P1}_{4,6} - Y_{4,6} = 30.0 + 2086.7 - 1903.73 = 242.9$, $u(7) = 242.9 + 2086.7 - 2053.54 = 276.1$, ... If for a given combination P/k $u(i) < 0$, ruin occurred. We do not need to calculate the surplus at the end of the year for the years to come.

Table 4.49 shows the within year ruin probabilities and the probability of ruin for a 10 year period. The within ruin probability for each premium type for each risk and for the portfolio is calculated using formula (2.9) replacing p_i by $P_{k,i}$ or $\sum_{k=1}^{r} P_{k,i}$ for the portfolio, and α, β and κ by the respective values of Table 4.44. If for a given combination P/k $u(i) < 0$, ruin occurred. We set the estimate of the within ruin probability to 1 and do not need to calculate it for the years to come. The $\psi_{TG}(u, 10)$ is calculated using (2.10).

	$y_{k,i}$							
$k\backslash i$	1	2	3	4	5	6	7	8
1	1770.79	1748.91	1792.61	1770.35	1932.10	1875.70	1714.93	1836.97
2	1831.42	1843.29	1751.94	1854.29	1827.36	1825.74	1688.30	1720.20
3	1986.23	1943.47	2072.31	2097.22	1904.95	2079.53	2159.25	1849.84
4	1949.52	1870.13	2034.46	2169.49	1889.20	1903.73	2053.54	1933.17
5	2491.10	2389.90	2406.73	2432.50	2639.19	2491.51	2515.65	2317.15
Port.	**10029.06**	**9795.70**	**10058.05**	**10323.85**	**10192.80**	**10176.21**	**10131.67**	**9657.33**
$\widehat{z}$	-	1.00000	0.98672	0.98359	0.98071	0.97860	0.98428	0.98639

$k\backslash i$	9	10	11	12	13	14	15
1	1810.37	1732.46	1806.65	1701.77	1752.13	1617.28	1787.32
2	1857.15	1695.56	1818.71	1936.27	1858.32	1797.42	1582.67
3	2137.96	1933.64	1966.02	1937.08	1953.32	2076.88	1899.78
4	2040.49	1967.37	1875.51	2132.34	1992.28	2041.06	2011.15
5	2418.44	2406.94	2318.34	2768.22	2541.40	2324.60	2513.46
Port.	**10264.41**	**9735.97**	**9785.23**	**10475.68**	**10097.45**	**9857.24**	**9794.38**
$\widehat{z}$	0.98618	0.98806	0.98979	0.99053	0.98953	0.99108	0.99144

Table 4.45: One run: Aggregate claims and credibility factor for year i.

P	$k \setminus i$	6	7	8	9	10
P^E_{ki}	1...5	2000.2	2000.2	2000.2	2000.2	2000.2
	Port.	**10001.1**	**10001.1**	**10001.1**	**10001.1**	**10001.1**
P^C_{ki}	1	1807.5	1818.3	1803.8	1808.1	1808.3
	2	1825.8	1825.4	1806.2	1795.8	1802.5
	3	2001.2	2014.0	2034.5	2011.6	2025.5
	4	1983.3	1970.2	1982.0	1975.9	1983.0
	5	2462.1	2468.0	2474.7	2454.2	2450.5
	Port.	**10079.9**	**10096.0**	**10101.1**	**10045.6**	**10069.9**
P	$k \setminus i$	11	12	13	14	15
P^E_{ki}	1...5	2000.2	2000.2	2000.2	2000.2	2000.2
	Port.	**10001.1**	**10001.1**	**10001.1**	**10001.1**	**10001.1**
P^C_{ki}	1	1800.7	1801.2	1793.4	1790.1	1777.9
	2	1791.8	1794.2	1806.4	1810.2	1809.3
	3	2016.4	2011.8	2005.7	2001.7	2007.0
	4	1981.4	1971.8	1985.2	1985.7	1989.6
	5	2446.4	2434.7	2461.5	2468.0	2457.7
	Port.	**10036.5**	**10013.7**	**10052.2**	**10055.7**	**10041.5**

Table 4.46: One run: Pure premiums in year i.

		P_{ki}									
P	$k\backslash i$	6	7	8	9	10	11	12	13	14	15
1	1...5	2086.7	2086.7	2086.7	2086.7	2086.7	2086.7	2086.7	2086.7	2086.7	2086.7
	Port.	**10433.3**	**10433.3**	**10433.3**	**10433.3**	**10433.3**	**10433.3**	**10433.3**	**10433.3**	**10433.3**	**10433.3**
2	1...5	2086.7	2041.1	2035.4	2017.2	2021.0	2014.3	2011.4	2016.2	2016.5	2013.4
	Port.	**10433.3**	**10205.4**	**10176.9**	**10086.0**	**10105.2**	**10071.6**	**10056.9**	**10081.1**	**10082.4**	**10067.0**
	1	1885.6	1896.9	1881.7	1886.3	1886.5	1878.5	1879.0	1871.0	1867.5	1854.8
	2	1904.7	1904.3	1884.2	1873.4	1880.4	1869.2	1871.7	1884.4	1888.4	1887.5
4	3	2087.7	2101.1	2122.5	2098.5	2113.1	2103.5	2098.7	2092.4	2088.2	2093.7
	4	2069.0	2055.4	2067.6	2061.3	2068.7	2067.0	2057.0	2071.0	2071.5	2075.6
	5	2568.6	2574.7	2581.6	2560.3	2556.4	2552.1	2540.0	2567.9	2574.7	2563.9
	Port.	**10515.6**	**10532.3**	**10537.6**	**10479.8**	**10505.2**	**10470.3**	**10446.5**	**10486.7**	**10490.3**	**10475.5**
	1	1885.6	1849.7	1828.3	1820.7	1822.8	1810.7	1809.4	1804.3	1800.9	1786.8
	2	1904.7	1857.0	1830.8	1808.3	1816.9	1801.8	1802.3	1817.3	1821.0	1818.3
5	3	2087.7	2048.9	2062.2	2025.5	2041.7	2027.6	2020.9	2017.9	2013.7	2017.0
	4	2069.0	2004.3	2009.0	1989.6	1998.8	1992.5	1980.7	1997.3	1997.6	1999.5
	5	2568.6	2510.7	2508.4	2471.3	2470.1	2460.1	2445.8	2476.5	2482.8	2469.9
	Port.	**10515.6**	**10270.5**	**10238.7**	**10115.3**	**10150.3**	**10092.6**	**10059.0**	**10113.3**	**10116.0**	**10091.4**

Table 4.47: One run: Premiums in year i.

		$u(i)$									
P	$k\backslash i$	6	7	8	9	10	11	12	13	14	15
1	1	271.0	642.7	892.4	1168.7	1522.9	1802.9	2187.8	2522.3	2991.7	3291.1
	2	320.9	719.3	1085.8	1315.3	1706.4	1974.3	2124.7	2353.1	2642.3	3146.3
	3	67.1	-5.5								
	4	242.9	276.1	429.6	475.7	595.0	806.2	760.5	854.9	900.5	976.0
	5	-344.8									
	Port.	**557.1**	**858.8**	**1634.8**	**1803.7**	**2501.0**	**3149.1**	**3106.8**	**3442.7**	**4018.7**	**4657.7**
2	1	271.0	597.1	795.5	1002.3	1290.9	1498.6	1808.2	2072.3	2471.4	2697.5
	2	320.9	673.7	988.9	1148.9	1474.4	167	1745.1	1903.0	2122.0	2552.8
	3	67.1	-51.0								
	4	242.9	230.5	332.7	309.4	363.0	501.8	380.9	404.8	380.2	382.5
	5	-344.8									
	Port.	**557.1**	**630.8**	**1150.3**	**971.9**	**1341.1**	**1627.4**	**1208.6**	**1192.2**	**1417.3**	**1689.9**
4	1	69.9	251.9	296.6	372.5	526.6	598.4	775.7	894.5	1144.7	1212.2
	2	139.0	355.0	519.0	535.3	720.2	770.7	706.1	732.2	823.2	1128.0
	3	68.1	1	282.6	243.1	422.5	56	721.7	860.7	872.0	1066.0
	4	225.3	227.1	361.6	382.3	483.7	675.2	599.9	678.6	709.1	773.5
	5	137.0	196.1	460.5	602.4	751.9	985.7	757.4	784.0	1034.0	1084.5
	Port.	**639.4**	**104**	**1920.4**	**2135.7**	**2904.9**	**359**	**3560.8**	**395**	**4583.1**	**5264.2**
5	1	69.9	204.7	196.1	206.4	296.7	300.8	408.4	460.6	644.2	643.6
	2	139.0	307.7	418.3	369.4	490.7	473.8	339.8	298.8	322.4	558.0
	3	68.1	-42.3								
	4	225.3	176.0	251.8	200.9	232.4	349.3	197.7	202.7	159.3	147.6
	5	137.0	132.1	323.3	376.1	439.3	581.0	258.5	193.6	351.8	308.2
	Port.	**639.4**	**778.2**	**1359.6**	**1210.5**	**1624.8**	**1932.2**	**1515.6**	**1531.4**	**1790.2**	**2087.2**

Table 4.48: One run: Surplus at the end of year i.

P	$k\backslash i$	$\psi_{TG}(u(i-1),1,u(i))$ 6	7	8	9	10	11	12	13	14	15	$\psi_{TG}(u,10)$
1	1	0.046064	1.47E-15	2.79E-48	6.66E-95	4.3E-223	0	0	0	0	0	4.61E-02
	2	0.023898	9.14E-21	3.13E-72	6.9E-136	0	0	0	0	0	0	2.39E-02
	3	0.47879	ruin									ruin
	4	0.055893	7.88E-06	2.81E-10	1.96E-16	2.2E-23	8.55E-42	1.28E-47	3.81E-54	2.55E-63	2.51E-74	5.59E-02
	5	ruin										ruin
	Port.	**0.003068**	**6.68E-08**	**1.00E-22**	**1.39E-44**	**1.35E-71**	**1.29E-125**	**3.04E-147**	**2.71E-167**	**1.19E-224**	**0.00E+00**	**3.07E-03**
2	1	0.046064	5.21E-15	2.27E-41	2.85E-74	1.8E-140	5.4E-275	0	0	0	0	4.61E-02
	2	0.023898	3.54E-20	2.45E-63	1.2E-108	1.6E-234	0	0	0	0	0	2.39E-02
	3	0.47879	ruin									ruin
	4	0.055893	3.79E-05	3.62E-07	4.61E-09	4.09E-10	8.03E-17	9.59E-16	1.53E-13	3.14E-13	9.94E-13	5.59E-02
	5	ruin										ruin
	Port.	**0.003068**	**3.33E-06**	**1.34E-12**	**2.77E-18**	**3.70E-22**	**9.51E-37**	**2.80E-31**	**9.76E-24**	**2.49E-28**	**2.77E-40**	**3.07E-03**
4	1	0.392613	0.015413	8.82E-08	2.86E-11	2.29E-20	1.15E-31	4.52E-50	1.29E-75	1.3E-133	3.9E-179	4.02E-01
	2	0.153811	1.04E-05	2.55E-19	2.6E-27	2.51E-41	3.94E-57	1.84E-52	4.02E-51	4.99E-62	1.1E-118	1.54E-01
	3	0.473878	0.888862	0.463808	1.04E-05	7.9E-09	2.39E-19	1.29E-33	2.28E-52	1.85E-60	9.58E-84	9.69E-01
	4	0.066411	0.000101	1.91E-07	1.56E-11	1.23E-15	1.12E-28	7.11E-32	9.8E-34	4.14E-39	2.8E-45	6.65E-02
	5	0.357286	0.042763	1.31E-05	2.68E-15	6.6E-25	9.92E-43	1.09E-36	7.53E-31	4.43E-46	1.51E-60	3.85E-01
	Port.	**0.001428**	**1.57E-10**	**1.59E-31**	**1.72E-61**	**1.11E-97**	**6.12E-167**	**3.98E-194**	**4.24E-221**	**3.00E-297**	**0.00E+00**	**1.43E-03**
5	1	0.392613	0.029152	0.000104	7.9E-05	3.52E-07	7.29E-10	4.4E-14	7.73E-21	1.15E-35	2.57E-46	4.10E-01
	2	0.153811	3.15E-05	2.64E-14	2.96E-16	1.79E-20	6.62E-25	2.13E-16	8.52E-11	1.51E-10	1.13E-21	1.54E-01
	3	0.473878	ruin									ruin
	4	0.066411	0.000587	0.000152	6.38E-05	0.000104	5.28E-08	2.92E-06	0.000412	0.002165	0.010637	7.96E-02
	5	0.357286	0.10926	0.003534	1.75E-07	5.86E-10	1.17E-15	3.15E-08	0.002231	0.00012	1.53E-06	4.31E-01
	Port.	**0.001428**	**2.17E-08**	**7.46E-18**	**1.77E-26**	**7.19E-33**	**1.58E-52**	**2.42E-46**	**1.11E-37**	**2.11E-45**	**1.75E-62**	**1.43E-03**

Table 4.49: One run: Ruin probabilities $\psi_{TG}(u(i-1),1,u(i))$ and $\psi_{TG}(u,10)$.

4.3.2 Results

Tables 4.50 and 4.51 show numerical results for N1 and N2 respectively. Estimated values of $\psi_{TG}(u, 10)$, together with the standard error of each estimate, are shown for the portfolio and for each one of the risks, k, for various values of the initial surplus, u, and four cases for the premium.

The most interesting aspects of these tables, are the effect of adjusting the premium at the start of each year by the safety loading (P2 and P5) and by credibility (P4 and P5) and the effect of the variability of a key parameter, λ (N2). We are now able to compare the aggregate effect of:

(a) using the collective premium (μ_0) as pure premium (P^E_{ki}) with a fixed safety loading (P1) and a varying one (P2). This item has the same comments as the ones made in the previous chapter Section 3.4;

(b) using credibility (P^C_{ki}) with a fixed safety loading (P4) and with a varying one (P5);

(c) for a fixed safety loading the impact of updating the premium using credibility (P4) *vs* fixed premium (P1);

(d) and finally for a varying safety loading, comparing the non-credibility premium (P2) with the credibility premium (P5). Our comments on this must be cautious because there are two variables; the safety loading that depends on the surplus at the beginning of the year and the way of calculating the pure premium.

		P1		P2		P4		P5	
k	u	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$
1	50	0.048	7.94E-08	0.049	9.88E-08	0.322	2.17E-06	0.508	3.35E-06
2	50	0.048	7.77E-08	0.049	9.73E-08	0.325	2.18E-06	0.509	3.35E-06
3	50	0.321	1.83E-06	0.463	2.95E-06	0.378	2.30E-06	0.573	3.21E-06
4	50	0.321	1.82E-06	0.464	2.95E-06	0.377	2.28E-06	0.574	3.21E-06
5	50	1.000	1.60E-08	1.000	1.60E-08	0.477	2.34E-06	0.679	2.72E-06
Pt.	**250**	**0.009**	**3.81E-08**	**0.011**	**5.57E-08**	**0.013**	**7.66E-08**	**0.016**	**1.08E-07**
1	60	0.034	6.66E-08	0.035	9.70E-08	0.334	2.61E-06	0.502	3.55E-06
2	60	0.034	6.52E-08	0.035	9.55E-08	0.337	2.63E-06	0.503	3.56E-06
3	60	0.326	2.25E-06	0.457	3.18E-06	0.394	2.76E-06	0.570	3.41E-06
4	60	0.325	2.22E-06	0.458	3.19E-06	0.393	2.75E-06	0.571	3.42E-06
5	60	1.000	1.60E-08	1.000	1.60E-08	0.501	2.77E-06	0.680	2.90E-06
Pt.	**300**	**0.010**	**7.18E-08**	**0.012**	**7.61E-08**	**0.015**	**1.40E-07**	**0.018**	**1.46E-07**
1	70	0.024	5.37E-08	0.026	9.23E-08	0.343	2.95E-06	0.491	3.71E-06
2	70	0.023	5.26E-08	0.025	9.06E-08	0.345	2.97E-06	0.492	3.72E-06
3	70	0.327	2.57E-06	0.445	3.36E-06	0.407	3.12E-06	0.561	3.59E-06
4	70	0.326	2.55E-06	0.446	3.36E-06	0.406	3.11E-06	0.562	3.59E-06
5	70	1.000	1.60E-08	1.000	1.60E-08	0.519	3.10E-06	0.676	3.06E-06
Pt.	**350**	**0.011**	**1.06E-07**	**0.011**	**8.53E-08**	**0.018**	**2.02E-07**	**0.017**	**1.62E-07**
1	80	0.016	4.21E-08	0.018	8.37E-08	0.349	3.21E-06	0.478	3.83E-06
2	80	0.016	4.14E-08	0.018	8.24E-08	0.351	3.23E-06	0.478	3.84E-06
3	80	0.327	2.82E-06	0.431	3.48E-06	0.416	3.39E-06	0.550	3.74E-06
4	80	0.326	2.80E-06	0.432	3.49E-06	0.415	3.38E-06	0.550	3.74E-06
5	80	1.000	1.60E-08	1.000	1.60E-08	0.532	3.35E-06	0.669	3.22E-06
Pt.	**400**	**0.012**	**1.38E-07**	**0.009**	**8.18E-08**	**0.020**	**2.57E-07**	**0.015**	**1.57E-07**
1	90	0.011	3.25E-08	0.013	6.86E-08	0.352	3.40E-06	0.463	3.91E-06
2	90	0.011	3.20E-08	0.013	6.77E-08	0.354	3.41E-06	0.463	3.92E-06
3	90	0.324	3.00E-06	0.415	3.57E-06	0.422	3.60E-06	0.537	3.86E-06
4	90	0.325	2.99E-06	0.416	3.58E-06	0.421	3.58E-06	0.537	3.87E-06
5	90	1.000	1.60E-08	1.000	1.60E-08	0.542	3.53E-06	0.660	3.37E-06
Pt.	**450**	**0.013**	**1.68E-07**	**0.008**	**7.12E-08**	**0.022**	**3.04E-07**	**0.013**	**1.40E-07**

Table 4.50: Bühlmann: Estimates and standard deviations of $\psi(u, 10)$, N1.

		P1		P2		P4		P5	
k	u	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$
1	50	0.181	1.77E-06	0.350	3.69E-06	0.493	4.12E-06	0.675	3.84E-06
2	50	0.183	1.78E-06	0.351	3.70E-06	0.494	4.13E-06	0.678	3.83E-06
3	50	0.540	4.10E-06	0.709	3.61E-06	0.585	4.03E-06	0.754	3.22E-06
4	50	0.541	4.10E-06	0.709	3.60E-06	0.589	4.02E-06	0.753	3.23E-06
5	50	1.000	1.64E-08	1.000	1.60E-08	0.731	3.24E-06	0.853	2.14E-06
Pt.	**250**	**0.171**	**2.38E-06**	**0.304**	**3.52E-06**	**0.204**	**2.81E-06**	**0.340**	**3.81E-06**
1	60	0.176	1.90E-06	0.355	3.89E-06	0.520	4.28E-06	0.679	3.90E-06
2	60	0.178	1.92E-06	0.357	3.89E-06	0.521	4.29E-06	0.683	3.88E-06
3	60	0.570	4.23E-06	0.711	3.67E-06	0.616	4.08E-06	0.756	3.27E-06
4	60	0.570	4.22E-06	0.712	3.66E-06	0.621	4.07E-06	0.756	3.28E-06
5	60	1.000	1.60E-08	1.000	1.60E-08	0.760	3.12E-06	0.854	2.18E-06
Pt.	**300**	**0.217**	**3.00E-06**	**0.318**	**3.66E-06**	**0.251**	**3.38E-06**	**0.354**	**3.93E-06**
1	70	0.169	1.97E-06	0.354	3.99E-06	0.536	4.38E-06	0.678	3.96E-06
2	70	0.171	1.98E-06	0.357	4.00E-06	0.540	4.39E-06	0.682	3.95E-06
3	70	0.590	4.28E-06	0.710	3.73E-06	0.637	4.08E-06	0.755	3.33E-06
4	70	0.591	4.27E-06	0.711	3.72E-06	0.642	4.07E-06	0.756	3.33E-06
5	70	1.000	1.60E-08	1.000	1.62E-08	0.778	3.03E-06	0.853	2.23E-06
Pt.	**350**	**0.254**	**3.44E-06**	**0.322**	**3.74E-06**	**0.288**	**3.78E-06**	**0.358**	**3.99E-06**
1	80	0.162	1.99E-06	0.349	4.04E-06	0.547	4.44E-06	0.674	4.03E-06
2	80	0.164	2.00E-06	0.351	4.04E-06	0.551	4.46E-06	0.678	4.01E-06
3	80	0.604	4.31E-06	0.707	3.79E-06	0.651	4.07E-06	0.752	3.40E-06
4	80	0.606	4.29E-06	0.708	3.78E-06	0.657	4.05E-06	0.754	3.39E-06
5	80	1.000	1.60E-08	1.000	1.62E-08	0.791	2.95E-06	0.851	2.28E-06
Pt.	**400**	**0.282**	**3.73E-06**	**0.319**	**3.79E-06**	**0.316**	**4.01E-06**	**0.355**	**4.03E-06**
1	90	0.155	1.98E-06	0.339	4.04E-06	0.554	4.48E-06	0.668	4.10E-06
2	90	0.156	1.99E-06	0.341	4.05E-06	0.558	4.48E-06	0.673	4.08E-06
3	90	0.613	4.33E-06	0.703	3.86E-06	0.660	4.13E-06	0.748	3.46E-06
4	90	0.615	4.31E-06	0.703	3.85E-06	0.667	4.04E-06	0.749	3.46E-06
5	90	1.000	1.60E-08	1.000	1.60E-08	0.799	2.91E-06	0.850	2.32E-06
Pt.	**450**	**0.304**	**3.94E-06**	**0.312**	**3.71E-06**	**0.337**	**4.19E-06**	**0.349**	**3.99E-06**

Table 4.51: Bühlmann: Estimates and standard deviations of $\psi(u,10)$, N2.

We make the following comments about the results in Tables 4.50 and 4.51:

(i) The standard deviations of $\psi(u, 10)$ are, as in Chapter 3, all very small.

(ii) The effect of a varying Poisson parameter compared with a constant one is the increase of the ruin probability for the four cases of premium calculation. The extra variability of the Poisson parameter is not absorbed by the updated credibility premium.

(iii) Risks 1 and 2 have approximately the same results as they have the same risk parameter. A similar behavior occurs with risks 3 and 4. With P_{ki}^{E} there is a major difference between risks 1 and 2 and 5, ruin occurs with probability 1 for risk 5. Using credibility premiums (P_{ki}^{C}) the results for each risk increase from risks 1 and 2 to 5 but are all very similar. Having this in mind from now on we will comment only on the results for the portfolio.

(iv) The order between the ruin probabilities for the four cases of premiums start as P1<P4<P2<P5 for low values of initial surplus and end as P2<P5<P1<P4 for high values of initial surplus. This may be seen better in Figure 4.16 later in this chapter. This order does not depend on the Poisson parameter, but the value of u where the order changes does depend on the Poisson parameter.

(v) P4 (P5) increases slightly the ruin probability when compared with P1 (P2). The premiums calculated using P4 (P5) have an extra degree of variability because they depend on previous data.

(vi) Comparing P1 with P2 and P4 with P5 we can see that the effect of adjusting the safety loading is the same as commented in Section 3.4.

As in Chapter 3, to help us to understand the results, we have produced statistical information, and some figures for several initial surpluses. Tables 4.52 to 4.55 show some statistical information for the paths where at the end of the year we have a surplus below zero ("end of year" ruin). We recorded, as in Chapter 3, for each simulation where $u(i) < 0$ for some $i \geq 6$ the usual information for scenarios T2/N1 and T2/N2 and for a low and a high value of the initial surplus. This information was obtained for different values of u, for the different cases of premium (P1, P2, P4, P5) and for the portfolio as for each risk and correspond to the results of Tables 4.50 and 4.51 in the sense that they were produced with a subset of the same simulations, 10 000 for each case of the Poisson parameter. We were not able to produce this information using all 50 000 simulations because of the size of the resulting files. In this example we will have many more records in the file because we are recording the information also for each risk. Since the tables are so large we present all values only for $u = 250$ and $u = 450$. Tables 4.56 and 4.57 present results only for the portfolio. These last tables were produced with a different set of 50 000 simulations, where we recorded the information for the portfolio only. For them we also present the standard deviation (SD). This is due to the lack of end of year ruin cases for the portfolio in the first file.

u	P	k	NRuins	Prop	Avg i	Avg $u(i)$	Avg $u(i-1)$	Avg p_i	Avg y_i
50	1	1	4	0.992	6.00	- 23.27	50.00	2 108.02	2 181.29
50	1	2	1	0.998	6.00	- 5.53	50.00	2 108.02	2 163.55
50	1	3	737	0.771	6.37	- 47.32	53.16	2 108.02	2 208.50
50	1	4	708	0.779	6.36	- 47.93	55.46	2 108.02	2 211.41
50	1	5	10 000	0.000	6.01	- 273.52	49.84	2 108.02	2 431.38
250	**1**	**Pt.**	**4**	**0.957**	**6.00**	**- 81.18**	**250.00**	**10 540.10**	**10 871.28**
50	2	1	4	0.992	6.00	- 23.27	50.00	2 108.02	2 181.29
50	2	2	2	0.996	6.50	- 16.00	69.59	2 064.05	2 149.64
50	2	3	2 330	0.497	8.68	- 52.97	74.78	2 044.83	2 172.58
50	2	4	2 319	0.500	8.82	- 52.71	78.35	2 043.99	2 175.05
50	2	5	10 000	0.000	6.01	- 274.62	49.83	2 106.93	2 431.38
250	**2**	**Pt.**	**10**	**0.911**	**6.90**	**- 89.90**	**438.96**	**10 350.88**	**10 879.75**
50	4	1	1 050	0.674	6.50	- 47.76	56.05	1 872.02	1 975.84
50	4	2	1 014	0.688	6.49	- 49.22	53.91	1 871.45	1 974.59
50	4	3	1 224	0.676	6.52	- 53.96	57.09	2 064.46	2 175.51
50	4	4	1 248	0.669	6.52	- 51.64	57.82	2 064.69	2 174.15
50	4	5	1 687	0.646	6.55	- 69.68	58.80	2 506.73	2 635.21
250	**4**	**Pt.**	**9**	**0.930**	**6.11**	**- 97.64**	**270.21**	**10 427.36**	**10 795.20**
50	5	1	3 101	0.389	8.80	- 54.00	72.23	1 822.00	1 948.24
50	5	2	3 181	0.375	8.85	- 53.10	72.63	1 821.21	1 946.93
50	5	3	3 598	0.372	8.81	- 59.61	79.06	2 010.00	2 148.68
50	5	4	3 612	0.371	8.80	- 59.38	77.67	2 010.22	2 147.27
50	5	5	4 529	0.333	8.67	- 76.60	88.71	2 445.21	2 610.51
250	**5**	**Pt.**	**20**	**0.878**	**7.35**	**- 95.93**	**448.65**	**10 232.27**	**10 776.85**

Table 4.52: Bühlmann: Statistical information for ruin cases, N1 $u = 250$.

u	P	k	NRuins	Prop	Avg i	Avg $u(i)$	Avg $u(i-1)$	Avg p_i	Avg y_i
90	1	1	4	0.965	6.00	- 38.17	90.00	2 053.13	2 181.29
90	1	2	2	0.982	6.50	- 14.37	82.15	2 053.13	2 149.64
90	1	3	1 630	0.498	7.34	- 49.97	81.56	2 053.13	2 184.66
90	1	4	1 551	0.522	7.35	- 51.73	81.14	2 053.13	2 185.99
90	1	5	10 000	0.000	6.01	- 287.98	89.48	2 053.13	2 430.58
450	**1**	**Pt.**	**33**	**0.755**	**7.18**	**- 90.66**	**312.85**	**10 265.60**	**10 669.15**
90	2	1	4	0.969	6.00	- 38.17	90.00	2 053.13	2 181.29
90	2	2	2	0.985	6.50	- 29.93	82.15	2 037.57	2 149.64
90	2	3	2 495	0.399	8.43	- 53.04	84.60	2 034.28	2 171.92
90	2	4	2 468	0.407	8.60	- 53.00	87.16	2 033.68	2 173.83
90	2	5	10 000	0.000	6.01	- 288.23	89.48	2 052.85	2 430.56
450	**2**	**Pt.**	**18**	**0.766**	**6.61**	**- 84.53**	**485.89**	**10 251.12**	**10 821.56**
90	4	1	2 076	0.411	7.56	- 51.38	79.78	1 827.75	1 958.91
90	4	2	2 093	0.409	7.56	- 50.39	80.12	1 827.38	1 957.89
90	4	3	2 463	0.416	7.57	- 57.91	83.12	2 015.75	2 156.78
90	4	4	2 547	0.395	7.58	- 57.09	84.13	2 015.86	2 157.08
90	4	5	3 408	0.371	7.46	- 74.39	91.92	2 449.12	2 615.43
450	**4**	**Pt.**	**67**	**0.702**	**7.51**	**- 109.61**	**285.03**	**10 182.19**	**10 576.83**
90	5	1	3 160	0.317	8.66	- 54.47	81.54	1 812.01	1 948.02
90	5	2	3 229	0.303	8.71	- 53.41	81.29	1 812.16	1 946.86
90	5	3	3 763	0.299	8.63	- 59.86	87.58	1 999.55	2 147.00
90	5	4	3 783	0.296	8.63	- 59.81	86.34	1 999.42	2 145.57
90	5	5	4 865	0.263	8.38	- 77.18	96.94	2 431.57	2 605.69
450	**5**	**Pt.**	**35**	**0.732**	**6.80**	**- 85.19**	**488.23**	**10 142.07**	**10 715.48**

Table 4.53: Bühlmann: Statistical information for ruin cases, N1 $u = 450$.

u	P	k	NRuins	Prop	Avg i	Avg $u(i)$	Avg $u(i-1)$	Avg λ	Avg p_i	Avg y_i
50	1	1	622	0.656	6.18	- 64.22	54.13	1 001.12	2 108.02	2 226.37
50	1	2	642	0.649	6.16	- 66.49	54.13	997.39	2 108.02	2 228.64
50	1	3	4 351	0.194	6.86	- 127.48	78.62	1 000.08	2 108.02	2 314.13
50	1	4	4 422	0.183	6.93	- 131.18	80.54	1 001.23	2 108.02	2 319.75
50	1	5	10 000	0.000	6.35	- 380.40	69.09	999.14	2 108.02	2 557.51
250	**1**	**Pt.**	**1 197**	**0.301**	**6.52**	**- 245.43**	**280.41**	**5 005.91**	**10 540.10**	**11 065.94**
50	2	1	711	0.797	6.41	- 69.32	61.14	1 002.12	2 087.94	2 218.40
50	2	2	730	0.792	6.32	- 68.57	62.19	995.48	2 090.40	2 221.16
50	2	3	5 601	0.210	7.54	- 132.98	103.86	999.31	2 067.63	2 304.47
50	2	4	5 616	0.208	7.57	- 137.05	106.16	1 000.25	2 066.81	2 310.02
50	2	5	9 988	0.001	6.30	- 390.24	66.99	999.21	2 088.19	2 545.41
250	**2**	**Pt.**	**2 319**	**0.237**	**8.28**	**- 237.69**	**501.04**	**5 014.01**	**10 343.78**	**11 082.51**
50	4	1	3 999	0.189	6.94	- 126.92	81.00	1 000.04	1 878.16	2 086.08
50	4	2	3 899	0.210	6.90	- 129.84	82.73	1 000.40	1 879.16	2 091.74
50	4	3	4 850	0.171	6.99	- 146.98	90.72	1 000.59	2 052.91	2 290.61
50	4	4	4 914	0.166	7.03	- 148.14	91.68	999.69	2 053.30	2 293.12
50	4	5	6 520	0.108	6.99	- 218.79	108.09	1 000.44	2 418.98	2 745.86
250	**4**	**Pt.**	**1 577**	**0.228**	**6.63**	**- 284.30**	**285.27**	**4 999.55**	**10 355.42**	**10 924.98**
50	5	1	4 912	0.272	7.51	- 131.60	102.44	999.12	1 845.71	2 079.75
50	5	2	4 865	0.283	7.52	- 130.24	104.59	1 000.82	1 846.61	2 081.44
50	5	3	6 013	0.202	7.59	- 153.06	114.15	999.15	2 015.94	2 283.15
50	5	4	5 982	0.206	7.55	- 155.16	113.94	998.93	2 017.05	2 286.14
50	5	5	7 585	0.111	7.36	- 223.72	131.14	1 000.82	2 381.71	2 736.57
250	**5**	**Pt.**	**2 755**	**0.189**	**8.14**	**- 272.82**	**501.84**	**5 008.05**	**10 191.54**	**10 966.20**

Table 4.54: Bühlmann: Statistical information for ruin cases, N2 $u = 250$.

u	P	k	NRuins	Prop	Avg i	Avg $u(i)$	Avg $u(i-1)$	Avg λ	Avg p_i	Avg y_i
90	1	1	849	0.451	6.37	- 69.10	88.79	1 000.16	2 053.13	2 211.01
90	1	2	856	0.453	6.35	- 69.96	91.75	995.55	2 053.13	2 214.83
90	1	3	5 582	0.089	7.32	- 133.96	117.07	1 000.33	2 053.13	2 304.16
90	1	4	5 629	0.085	7.35	- 137.59	117.93	1 000.77	2 053.13	2 308.65
90	1	5	10 000	0.000	6.29	- 392.75	96.83	999.14	2 053.13	2 542.70
450	**1**	**Pt.**	**2 623**	**0.136**	**7.62**	**- 279.04**	**405.65**	**5 006.04**	**10 265.60**	**10 950.32**
90	2	1	809	0.762	6.35	- 70.65	91.98	999.50	2 048.50	2 211.12
90	2	2	822	0.759	6.28	- 72.19	91.95	996.87	2 048.90	2 213.03
90	2	3	5 647	0.196	7.45	- 137.18	121.25	999.91	2 040.98	2 299.41
90	2	4	5 680	0.192	7.47	- 140.17	123.19	999.63	2 040.56	2 303.93
90	2	5	9 981	0.001	6.28	- 395.37	96.48	999.24	2 046.74	2 538.59
450	**2**	**Pt.**	**2 544**	**0.185**	**8.06**	**- 244.78**	**562.32**	**5 011.20**	**10 248.57**	**11 055.68**
90	4	1	5 042	0.089	7.40	- 131.37	115.83	1 000.47	1 833.09	2 080.30
90	4	2	4 986	0.106	7.40	- 132.53	117.26	1 000.95	1 834.11	2 083.90
90	4	3	6 059	0.082	7.41	- 152.98	126.23	1 000.25	2 002.76	2 281.97
90	4	4	6 149	0.078	7.45	- 154.59	126.85	998.72	2 003.78	2 285.22
90	4	5	7 585	0.051	7.22	- 229.32	141.69	1 000.51	2 363.93	2 734.93
450	**4**	**Pt.**	**2 989**	**0.114**	**7.58**	**- 303.06**	**424.56**	**5 002.52**	**10 118.20**	**10 845.83**
90	5	1	4 937	0.261	7.44	- 132.88	121.94	999.36	1 822.44	2 077.26
90	5	2	4 880	0.275	7.46	- 133.15	122.89	1 001.25	1 823.84	2 079.89
90	5	3	6 050	0.191	7.50	- 155.95	131.49	998.88	1 991.37	2 278.82
90	5	4	6 051	0.192	7.46	- 157.89	130.91	999.34	1 992.85	2 281.66
90	5	5	7 669	0.098	7.28	- 229.90	148.74	1 000.76	2 350.81	2 729.46
450	**5**	**Pt.**	**2 958**	**0.152**	**7.96**	**- 280.25**	**569.09**	**5 007.31**	**10 093.63**	**10 942.97**

Table 4.55: Bühlmann: Statistical information for ruin cases, N2 $u = 450$.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	SD $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg p_i	SD p_i	Avg y_i	SD y_i
250	1	34	0.926	6.00	0.00	- 70.88	65.54	250.00	0.00	10 540.10	0.00	10 860.98	65.55
250	2	73	0.871	7.67	2.43	- 67.05	62.47	417.43	196.98	10 372.41	169.00	10 856.88	91.61
250	4	80	0.876	6.08	0.27	- 72.30	61.06	245.34	46.26	10 404.08	116.53	10 721.72	147.45
250	5	158	0.807	7.44	2.09	- 74.16	67.56	405.54	177.82	10 252.90	182.03	10 732.60	124.08
300	1	62	0.879	6.11	0.32	- 87.02	69.98	288.50	63.90	10 433.30	0.00	10 808.84	98.17
300	2	100	0.828	7.27	2.10	- 79.67	67.39	426.47	168.46	10 332.69	118.52	10 838.84	86.91
300	4	150	0.806	6.25	0.57	- 88.61	71.43	280.11	75.51	10 314.70	104.55	10 683.41	146.87
300	5	217	0.754	7.14	1.90	- 84.06	71.43	405.50	154.98	10 228.00	144.82	10 717.56	124.61
350	1	109	0.806	6.36	0.60	- 79.88	71.94	315.04	93.16	10 359.70	0.00	10 754.64	117.36
350	2	123	0.773	7.04	1.95	- 79.77	70.21	432.99	137.88	10 305.65	80.35	10 818.42	90.78
350	4	230	0.745	6.46	0.72	- 93.24	80.43	297.03	114.43	10 247.75	100.45	10 638.02	156.19
350	5	249	0.708	7.01	1.81	- 87.30	73.94	421.46	127.53	10 193.24	120.67	10 702.00	128.46
400	1	150	0.757	6.67	0.86	- 83.81	73.63	310.15	135.98	10 306.20	0.00	10 700.14	156.50
400	2	126	0.732	7.03	1.93	- 80.37	70.53	466.52	121.83	10 271.14	59.96	10 818.02	90.80
400	4	311	0.695	6.83	1.06	- 95.92	82.79	306.67	143.16	10 205.31	101.04	10 607.90	162.98
400	5	255	0.667	7.00	1.80	- 87.18	74.33	450.09	111.07	10 164.27	112.82	10 701.54	127.80
450	1	190	0.718	7.18	1.38	- 90.34	76.60	306.51	160.32	10 265.60	0.00	10 662.47	170.51
450	2	117	0.696	7.09	1.99	- 79.50	69.63	499.23	107.32	10 245.06	46.84	10 823.81	89.81
450	4	368	0.672	7.16	1.26	- 103.93	82.57	309.31	162.93	10 170.77	97.92	10 584.01	170.40
450	5	236	0.638	7.07	1.84	- 87.16	73.04	484.50	104.08	10 136.05	109.03	10 707.70	127.24

Table 4.56: Bühlmann: Statistical information for ruin cases for the portfolio, N1.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	SD $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg λ	SD λ	Avg p_i	SD p_i	Avg y_i	SD y_i
250	1	6 071	0.291	6.53	0.66	- 248.66	196.10	275.14	115.34	4 998.30	262.26	10 540.10	0.00	11 063.90	218.90
250	2	11 590	0.237	8.29	2.67	- 241.56	203.75	502.98	320.51	5 013.70	268.76	10 343.67	250.75	11 088.21	243.13
250	4	7 782	0.238	6.57	0.74	- 288.38	233.34	288.09	142.89	4 997.31	261.85	10 344.41	242.17	10 920.88	317.74
250	5	13 506	0.205	8.11	2.61	- 275.55	231.88	497.98	334.03	5 008.95	265.76	10 191.04	309.62	10 964.57	312.30
300	1	8 419	0.223	6.84	1.02	- 259.16	212.70	321.17	147.41	4 997.22	259.95	10 433.30	0.00	11 013.65	244.14
300	2	12 397	0.220	8.11	2.62	- 246.44	207.72	504.40	298.37	5 011.94	266.44	10 314.45	194.81	11 065.30	244.54
300	4	10 217	0.186	6.83	1.10	- 299.61	247.25	330.08	167.48	4 997.41	259.21	10 256.17	238.54	10 885.86	329.61
300	5	14 345	0.190	7.98	2.54	- 282.11	239.86	502.85	309.51	5 008.16	264.22	10 159.30	279.53	10 944.26	320.35
350	1	10 453	0.177	7.13	1.42	- 267.19	221.39	357.62	169.48	4 998.64	259.16	10 359.70	0.01	10 984.53	261.03
350	2	12 786	0.205	8.05	2.58	- 248.87	211.51	518.57	278.60	5 012.00	265.85	10 287.39	155.62	11 054.84	248.78
350	4	12 224	0.151	7.08	1.44	- 304.20	256.79	368.03	181.02	4 999.44	259.31	10 195.23	234.34	10 867.46	338.25
350	5	14 741	0.176	7.92	2.52	- 284.52	244.05	519.32	288.40	5 007.36	263.89	10 132.21	262.22	10 936.05	326.09
400	1	11 992	0.149	7.37	1.71	- 273.23	229.99	384.91	191.01	4 999.21	259.58	10 306.20	0.01	10 964.32	275.58
400	2	12 841	0.195	8.03	2.56	- 249.33	213.18	539.03	261.47	5 011.29	265.58	10 264.84	135.13	11 053.19	251.70
400	4	13 792	0.127	7.31	1.69	- 305.34	261.43	398.94	201.42	5 000.30	259.43	10 149.67	231.03	10 853.95	342.10
400	5	14 786	0.168	7.91	2.49	- 284.80	246.06	540.68	271.06	5 007.29	264.27	10 109.45	254.03	10 934.93	327.68
450	1	13 198	0.131	7.62	1.89	- 277.22	234.21	407.24	204.85	5 000.37	259.51	10 265.60	0.01	10 950.09	283.30
450	2	12 686	0.187	8.07	2.56	- 248.31	213.45	561.19	248.06	5 011.56	265.66	10 247.95	124.36	11 057.47	251.93
450	4	14 962	0.113	7.54	1.87	- 308.62	263.14	425.06	216.19	5 000.83	258.67	10 115.03	227.84	10 848.71	346.80
450	5	14 636	0.161	7.94	2.49	- 283.40	246.16	565.13	257.43	5 007.61	264.31	10 091.21	251.47	10 939.74	327.67

Table 4.57: Bühlmann: Statistical information for ruin cases for the portfolio, N2.

From these tables we can say:

(i) Ruin occurs mainly in the first two or three years (recall that we start assessing ruin from the start of year 6).

(ii) In cases P1 and P2 ruin occurs for risk 5 in all simulations (100% in case N1) or almost all (in case N2). This does not happen in cases P4 and P5. Although in these cases it is the risk with the highest probability of end of year ruin.

(iii) The mean of the aggregate claim in the year of ruin increases from risk 1 to 5 as expected, having in mind Table 4.41.

The following comments relate to Tables 4.56 and 4.57 (the portfolio):

(iv) The proportion of the probability of ruin due to within year ruin decreases in case N1 from P1 to P5 and with the surplus. In case N2 it also decreases with the surplus but the order in terms of premiums at some point changes to P2>P5>P1>P4.

(v) The average surplus at the start of the year of ruin is comparable for (P1, P4) and (P2, P5). It is higher for (P2, P5) than (P1, P4), almost double in all cases. Case N2 is slightly higher than N1.

(vi) We can see from Table 4.57 that the average value of the Poisson parameter in the year of ruin is around 5 000, for all types of premiums.

(vii) The average premium in the end of year ruin cases has the following order P1>P2>P4>P5 (except for $u = 250$, N1/P4) in both cases of the Poisson parameter and decreases with the surplus as expected.

(viii) In cases P4 and P5 we have lower premiums than P1 and P2 in both cases of the Poisson parameter. This is due to the fact that the premiums are being adjusted by credibility. In case N2 the fact that the Poisson parameter is not constant may influence the result for the average premium because in this case P4 and P5 are lower than P4 and P5 in case N1. Premiums with a variable safety loading (P2, P5) are higher than premiums with a constant safety loading (P1, P4). This relation decreases as the surplus increases.

(ix) On average, the aggregate claims in the year that ruin occurred (from 10 600 to 10 900 for N1 and from 10 900 to 11 100 for N2) is higher than the premium (from 10 100 to 10 550 for each case, having low average premium for P4 and P5 in case N2) and is higher than the expected value of aggregate claims for the portfolio, 10 001 (the expected value of the individual claim amount given the risk parameter is 2.00021).

(x) The severity of ruin has similar values for all cases of the premium for each case N1 and N2. For case N2 it is much higher than case N1. The average number of claims is very similar and close to $\mathrm{E}[N] = 5\,000$. This indicates that ruin in these cases was not due to the number of claims but due to the high aggregate claim amount and low premium.

(xi) The ruin probabilities by risk in Tables 4.52 to 4.55 are a little inflated. As the safety loading is a function of the surplus of the portfolio in a few cases the portfolio at the end of year is ruined but some of the risks were not. In this case we assume that all the individual risks are ruined if the portfolio is ruined. In our simulations only P2 and P5 were affected. Table 4.58 shows the percentage of these cases which occurred in our set of 50 000 simulations for u=300.

N	P	Total
N1	2	0.10%
	5	0.15%
N2	2	12.20%
	5	10.74%

Table 4.58: Bühlmann: Percentage of ruined risks due to ruined portfolio.

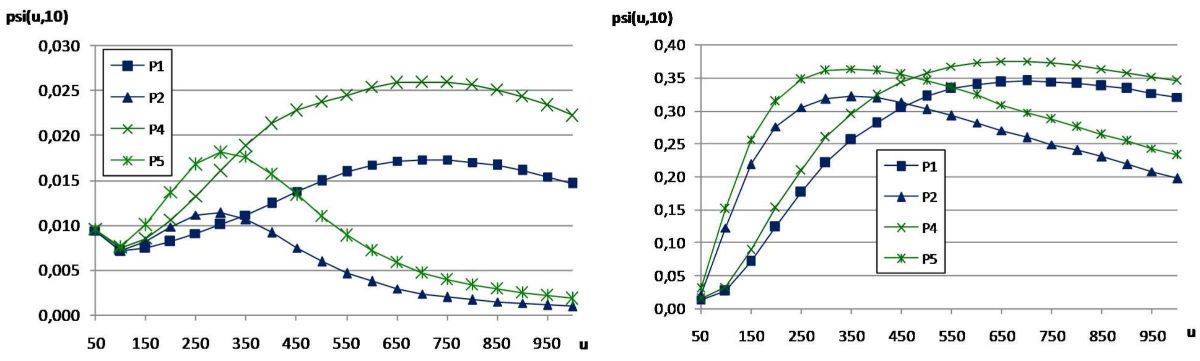
Table 4.59 shows the Pearson's correlation coefficient between the aggregate claim amounts y_{i-1} and y_i in the cases where $u(i) < 0$. For each initial surplus and type of premium we calculate for each Poisson parameter, the number of records used to obtain the correlation (Nr) and the estimate of the correlation coefficient (r_{y_{i-1},y_i}) for the portfolio. These results were obtained with 100 000 simulations. Recall that

P	u	N1 Nr	N1 r_{y_{i-1},y_i}	N2 Nr	N2 r_{y_{i-1},y_i}
1	250	50	*- 0.23*	12 208	- 0.02
	300	126	- 0.45	16 843	- 0.07
	350	209	- 0.53	20 821	- 0.12
	400	287	- 0.64	23 941	- 0.17
	450	375	- 0.68	26 368	- 0.21
2	250	110	- 0.38	23 410	- 0.10
	300	184	*- 0.08*	25 054	- 0.05
	350	218	*0.04*	25 786	- 0.03
	400	222	*0.04*	25 903	- 0.03
	450	201	*0.02*	25 592	- 0.04
4	250	130	*0.16*	15 553	0.22
	300	304	*0.09*	20 565	0.19
	350	473	- 0.11	24 501	0.16
	400	650	- 0.25	27 567	0.11
	450	813	- 0.33	29 895	0.08
5	250	257	0.17	27 036	0.20
	300	406	0.39	28 810	0.22
	350	492	0.39	29 584	0.23
	400	503	0.40	29 687	0.23
	450	465	0.40	29 373	0.22

Table 4.59: Bühlmann: Correlation between y_{i-1} and y_i.

the italic case numbers are the cases where the hypotheses $H_0 : \rho = 0$ is accepted for $\alpha = 5\%$.

From this table we can observe that there is a negative correlation for P1, P2/N2 and positive correlation for P4/N2 and P5. Recall that P1 and P2 are the same type of premiums used in Chapter 3 and we saw that there was a negative correlation in the examples in that chapter. P4 and P5 are the credibility premiums. Tables 4.56 and 4.57 show that for the credibility premiums the Avg y_i is around 10 700 for N1 and 10 900 for N2. With a positive correlation factor we may expect that two years of heavy claims and low premiums lead to end of year ruin. There are no clear patterns in cases P2/N1 and P4/N1. It depends on particular cases and may also vary because we obtained r_{y_{i-1},y_i} with a low Nr. For instance the confidence interval of (u=350, P4, N1) with r_{y_{i-1},y_i} =-0.11 goes from -0.2 to -0.02. The significance level used is $\alpha = 5\%$.



(a) N1. (b) N2.

Figure 4.16: Bühlmann: $\psi(u, 10)$ for several values of initial surplus.

	N1				N2			
u	P1 P2	P4 P5	P1 P4	P2 P5	P1 P2	P4 P5	P1 P4	P2 P5
250	40.0%	45.0%	30.0%	25.0%	44.7%	50.1%	45.1%	40.9%
300	47.1%	51.4%	36.0%	23.7%	53.5%	58.0%	46.7%	42.7%
350	56.0%	51.0%	25.5%	26.7%	58.9%	62.5%	47.6%	43.5%
400	53.1%	50.8%	26.2%	28.3%	61.7%	65.4%	47.8%	43.7%
450	37.8%	37.8%	20.5%	26.2%	63.0%	66.8%	47.6%	43.5%

Table 4.60: Bühlmann: Common ruin scenarios.

Figure 4.16 shows, for different combinations of N and P, the ruin probabilities for the portfolio over a 10 year period for a range of initial surpluses. The results for this figure were obtained with 10 000 runs. We can see the same shape as in Chapter 3 between the pairs (P1, P2) and (P4, P5). These figures reinforce the comments (ii), (iv) and (v) made in this section, page 93.

Table 4.60 shows what proportion of end of year ruins are common for P_{ki}^E and P_{ki}^C for the portfolio. For instance for ($u = 400$, N1) case, premiums types P4 and P5 have around 50% of common paths that lead to ruin at the end of some year during the 10 year period. These results came from the same file that produced Tables 4.52 to 4.55. We may add that from around 40% to 65% of cases, the pairs (P1, P2) and (P4, P5) have common ruin scenarios and increase slightly with the variability of the Poisson parameter. Case N1 and pairs (P1, P4) and (P2, P5) have around 20% to 30% and the same pairs in case N2 have around 40% to 50% common ruin scenarios.

Table 4.61 and Figure 4.17 show statistics for the credibility factor ($\hat{z}$) over time: first quartile, Median, Mean and third quartile for two cases of the Poisson parameter. These results were obtained from 50 000 simulations independently of the value of $u(i)$. We can see that the credibility factor for case N2 is lower and has more variability than in case N1 as we expected.

	Year	3	4	5	6	7	8	9
N1	1st Qu.:	0.8916	0.9307	0.9495	0.9604	0.9675	0.9725	0.9761
	Median:	0.9331	0.9512	0.9623	0.9693	0.9741	0.9776	0.9803
	Mean:	0.9174	0.9452	0.9590	0.9673	0.9727	0.9767	0.9796
	3rd Qu:	0.9606	0.9662	0.9720	0.9764	0.9794	0.9819	0.9839
N2	1st Qu.:	0.3834	0.5837	0.6866	0.7519	0.7956	0.8261	0.8496
	Median:	0.6628	0.7383	0.7919	0.8276	0.8530	0.8720	0.8868
	Mean:	0.5756	0.6748	0.7472	0.7971	0.8312	0.8557	0.8742
	3rd Qu:	0.8204	0.8373	0.8603	0.8794	0.8937	0.9052	0.9143

	Year	10	11	12	13	14	15
N1	1st Qu.:	0.9789	0.9812	0.9830	0.9845	0.9858	0.9868
	Median:	0.9824	0.9841	0.9855	0.9867	0.9877	0.9886
	Mean:	0.9819	0.9837	0.9852	0.9864	0.9875	0.9884
	3rd Qu:	0.9854	0.9867	0.9878	0.9887	0.9895	0.9901
N2	1st Qu.:	0.8673	0.8811	0.8925	0.9018	0.9099	0.9168
	Median:	0.8986	0.9081	0.9159	0.9225	0.9282	0.9332
	Mean:	0.8885	0.8999	0.9091	0.9168	0.9233	0.9289
	3rd Qu:	0.9218	0.9282	0.9337	0.9384	0.9424	0.9459

Table 4.61: Bühlmann: Statistics for the credibility factor.

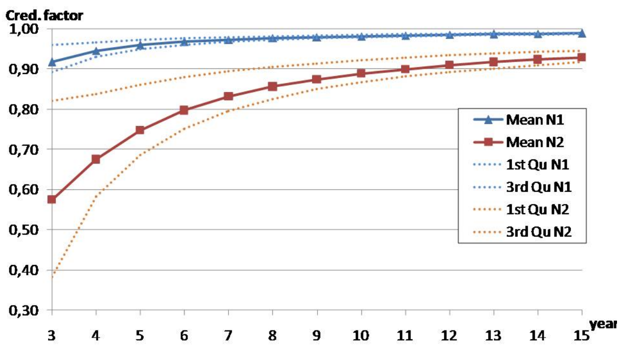
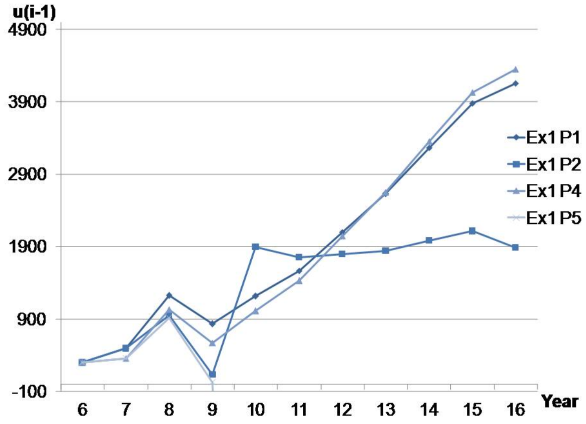
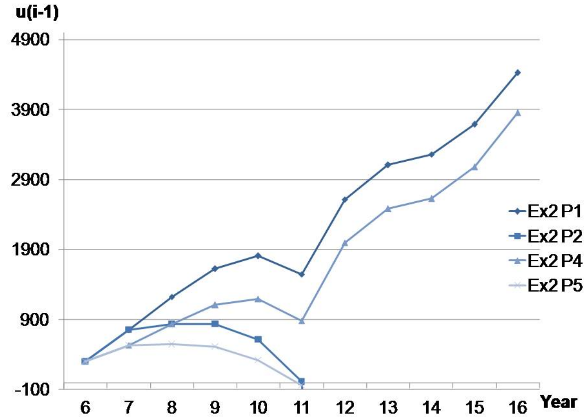


Figure 4.17: Bühlmann: Statistics for the credibility factor.

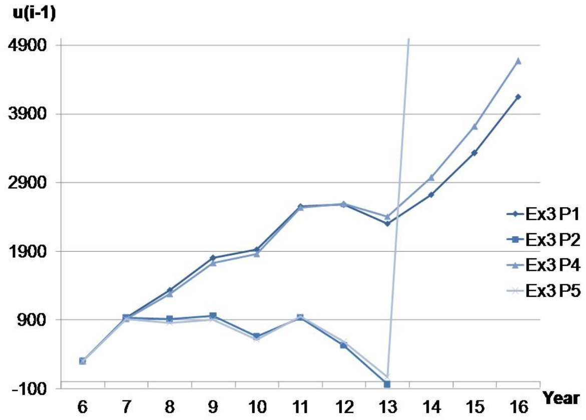
Figure 4.18 shows four examples of paths of the surplus process leading to ruin in at least one case. They are all for case ($u(5) = 300$, N1).



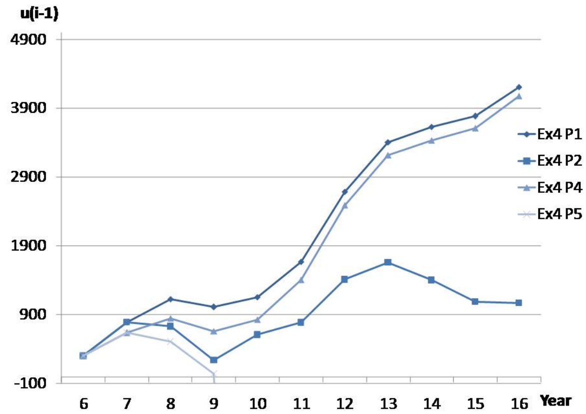
(a) Path 1.



(b) Path 2.



(c) Path 3.



(d) Path 4.

Figure 4.18: Bühlmann: Simulated paths, case N1, $u = 300$.

Figure 4.18(a) shows how P2 can recover and push the surplus upwards. In Figure 4.18(c) we have the same behavior for P5. In this last case, as the surplus at the beginning of year 13 is very low (65.7) the safety loading is going to be very high in order to avoid ruin.

The simulation for Figure 4.18(b) has a lower average aggregate claim. During year 10 the aggregate claim amount is high (10 804.3) and cases P2 and P5 are ruined at the end.

In Figure 4.18(d) we have one high value for aggregate claims during year 8, in this case only P5 leads to ruin.

4.4 Comments on results

In this chapter we gave a step forward in the calculation of the ruin probabilities of the portfolio. Using the Bühlmann (1967, 1969) model, every risk k in the collective is characterised by its individual (unknown) risk profile θ_k. Although the specific value of θ_k is unknown to the insurer, there is information about the structure of the collective.

Throughout this chapter we saw the results of applying our methodology when premiums change from year to year using credibility, and combined it with a safety loading depending on the surplus at the end of previous year. To achieve that we used four different ways of calculating the premium and two approaches to the Poisson parameter (constant and variable), in a 10 year period (with a previous record of 5 year of aggregate claims), for a portfolio with 5 risks.

We make the following general comments about the results:

(i) The distribution of Θ is a very simple one but our methodology is expected to continue working very well with other structural distributions.

(ii) Varying the Poisson parameter, scenario N2, increases the probability of ruin in a significant way, as we would expect.

(iii) With credibility premiums we have more similar results for the ruin probability for the risks.

With the insight on the causes of end of year ruin we may comment:

(iv) The average aggregate claims in the year of ruin is almost 10% higher than the expected aggregate claims.

(v) The average surplus at the start of the year of ruin is comparable for P1 and P4, but higher for P2 and P5.

(vi) The average premium in the year of ruin, has the following order P1>P2>P4>P5.

We plotted $\psi(u, 10)$ for a large range of values for the initial surplus, combining P and N.

(vii) We have the same shape as in Chapter 3 for both credibility and non-credibility premiums. Credibility premiums lead to high values of $\psi(u, 10)$ and case N2 has much higher ruin probabilities than case N1.

Finally we can also add:

(viii) The standard deviations of our estimates $\psi_{TG}(u, 10)$ are still all very small.

(ix) The correlation factor between y_{i-1} and y_i is negative for the non-credibility updated premium and positive for the credibility updated premium (except P4/N1).

(x) The Bühlmann credibility factor is lower and has more variability in case N2.

(xi) The running times of the results of this chapter are higher than the previous one, as expected. If the actuary is not interested in having the ruin probabilities of each risk the speed is increased, in our examples, by a factor of five.

Chapter 5

Premium using the Bühlmann-Straub credibility model

We can read in page 560 of Klugman et al. (2004) that *the Bühlmann model is the simplest of credibility models because it effectively requires that past claims experience of a policyholder comprise independent and identically components with respect to each past year. An important practical difficulty with this assumption is that it does not allow for variations in exposure or size.*

In this chapter we will consider the case where the premium is updated using the Bühlmann and Straub (1970) model which allows for variation in exposure or size. We are going to study the behavior of ruin probabilities within n years when the premium is updated using the Bühlmann-Straub credibility model. We are going to compare these results with the ruin probabilities within n years when the premium is calculated using the collective premium (μ_0). We will use the classical approach to the safety loading: we will consider a fixed (non dependent on the surplus) safety loading throughout the n-years period. In Section 5.1 we will define the Bühlmann-Straub model briefly. The reader may find more information in Bühlmann and Straub (1970), the original paper or in the same references cited in the previous chapter, for instance, Bühlmann and Gisler (2005) or Klugman et al. (2004). We will continue to apply our method to estimate ruin probabilities in continuous and finite time using only the translated gamma approximation. The methodology will be described in Section 5.2. In Section 5.3 we will illustrate it with a numerical example. Some considerations and comments are set out in Section 5.4. This chapter has a similar structure to Chapter 4.

5.1 The Bühlmann-Straub model

Consider that we have a portfolio of risks. We also have observed m years of past claim amounts for each risk and by now we have a known weight associated to the claim ratio. It can be number of years at risk, total amount of annual wages, sum insured, the number of separate policies,... It measures the "amount of business" in year i and we are going to call it the risk volume and denote it w_{ki}.

To deal with the risk volume variations we will consider the known generalisation of the Bühlmann model, the Bühlmann and Straub (1970) credibility model, to rate each risk.

Before defining the assumptions and the model we need to settle the notation. This notation is general for a single risk and for the portfolio and complements the notation defined in Section 4.1.1.

5.1.1 Notation

In this chapter the definitions of Port., r, k, m, n, i, j, θ_k, $Y_{ki}(\Theta)$ and P^E_{ki} are as in Section 4.1.1. Let us define the extra notation that will be used throughout this chapter.

ζ is the safety loading. In this chapter the safety loading is fixed,

w_{ki} is the risk volume of risk k in year i; if there are non-observed years $w_{ki} = 0$,

We are going to consider four different approaches to the risk volume:

W1 w_{ki} varies between the risks but it is constant for each year, one of the risks with a lower risk parameter is the dominant risk in the portfolio, W2 w_{ki} varies between the risks but it is constant for each year, one of the risks with a higher risk parameter is the dominant risk in the portfolio,

W3 w_{ki} varies among both the risks and the years. w_{ki} is a random variable and $\{\{w_{ki}\}_{k=1}^{r}\}_{i=1}^{n}$ is a set of $i.i.d$ random variables, each with a $U(a, b)$ distribution,

W4 w_{ki} varies among both the risks and the years. w_{ki} varies under a Markovian framework.

$X_{ki}(\theta_k)$ is the aggregate claims standardised in year i for risk k (other possible interpretations, claims ratio, average claim size), so that $X_{ki}(\theta_k) = \frac{Y_{ki}(\theta_k)}{w_{ki}}$. Having in mind that, the aggregate claims depend always on a risk parameter, we will use the notation X_{ki} for simplicity,

P_{ki} is the premium for risk k in year i. $P_{ki} = (1 + \zeta)P_{ki}^{\bullet}$, with $i = m + 1, \ldots, n$,

We are going to consider two different approaches to the premium:

P6 $P_{ki} = (1 + \zeta)P_{ki}^{E}$,

P7 $P_{ki} = (1 + \zeta)P_{ki}^{C}$, with P_{ki}^{C} now being the Bühlmann-Straub credibility premium for risk k in year i.

5.1.2 Assumptions

Following Model Assumptions 4.1 of Bühlmann and Gisler (2005) we have for the Bühlmann-Straub model the following assumptions:

BS1: Conditional on a given θ_k, the $X_{ki} : i = 1, 2, ..., m + n$ are independent with:

$$\mathrm{E}[X_{ki}|\Theta_k] = \mu(\Theta_k)$$
$$\mathrm{Var}[X_{ki}|\Theta_k] = \frac{\sigma^2(\Theta)}{w_{ki}}$$

BS2: The pairs $(\Theta_1, (X_{1,1}, \ldots, X_{1,m+n})), \ldots, (\Theta_r, (X_{r,1}, \ldots, X_{r,m+n}))$ are independent and the risk profiles $\Theta_1, \ldots, \Theta_r$ are $i.i.d.$ with structural distribution $U(\theta)$.

5.1.3 The model

We want to estimate P_{ki}^{C}. Let us define:

$$\mu_0 = \mathrm{E}[\mu(\Theta_k)]$$
$$\sigma^2 = \mathrm{E}[\sigma^2(\Theta_k)]$$
$$\tau^2 = \mathrm{Var}[\mu(\Theta_k)]$$

with the same interpretation as in 4.1.3.

The Bühlmann-Straub inhomogeneous credibility pure premium for a risk k is given by (see Bühlmann and Gisler (2005) Theorem 4.2):

$$z_k X_k + (1 - z_k)\mu_0 \tag{5.29}$$

where:

$X_k = \sum_{l=1}^{i-1} \frac{w_{kl}}{w_{k\bullet}} X_{kl}$, with $w_{k\bullet} = \sum_{l=1}^{i-1} w_{kl}$ and, $z_k = \frac{\sum_{l=1}^{i-1} w_{kl}}{\sum_{l=1}^{i-1} w_{kl} + \frac{\sigma^2}{\tau^2}}$ where $\frac{\sigma^2}{\tau^2}$ is called the credibility coefficient.

The homogeneous credibility estimator is obtained from (5.29) by replacing μ_0 by the credibility weighted average (not by the observed average as intuition would lead us):

$$z_k X_k + (1 - z_k)\widehat{\widehat{\mu}}_0 \tag{5.30}$$

where:
$\widehat{\widehat{\mu}}_0 = \sum_{k=1}^{r} \frac{z_k}{z_\bullet} X_k$ and, $z_\bullet = \sum_{k=1}^{r} z_k$.

We will use the empirical credibility estimator obtained from (5.30) by replacing the structural parameters σ^2 and τ^2 by their estimators, derived for instance in Section 4.8 of Bühlmann and Gisler (2005). We will have:

$$P_{ki}^{C} = \widehat{z}_k X_k + (1 - \widehat{z}_k)\widehat{\widehat{\mu}}_0 \tag{5.31}$$

where:

$$\widehat{\sigma^2} = \frac{1}{r}\sum_{k=1}^{r}\frac{1}{(i-2)}\sum_{l=1}^{i-1} w_{kl}(X_{kl}-\overline{X}_k)^2, \text{ and } \quad \widehat{\tau^2} = \max(\widehat{\widehat{\tau^2}}, 0)$$

$$\widehat{\widehat{\tau^2}} = c\left\{\frac{r}{r-1}\sum_{k=1}^{r}\frac{w_{k\bullet}}{w_{\bullet\bullet}}(\overline{X}_k-\overline{X})^2 - \frac{r\widehat{\sigma^2}}{w_{\bullet\bullet}}\right\}, \; i>2, \; r>1 \text{ with}$$

$$c = \frac{r-1}{r}\left\{\sum_{k=1}^{r}\frac{w_{k\bullet}}{w_{\bullet\bullet}}\left(1-\frac{w_{k\bullet}}{w_{\bullet\bullet}}\right)\right\}^{-1}, \; \overline{X} = \sum_{k=1}^{r}\frac{w_{k\bullet}}{w_{\bullet\bullet}}X_k \text{ and, } w_{\bullet\bullet} = \sum_{k=1}^{r} w_{k\bullet}.$$

The discussion of the assumptions and the properties of the estimators are well studied in the literature, see for instance Bühlmann and Gisler (2005).

5.2 Methodology

We want to evaluate the ruin probability of a portfolio over n years starting from an initial surplus u. Assumptions (a), (b) and (d) in Section 4.2 are unchanged for this chapter. Assumptions (c) and (e) to (h) become:

(c) the distribution of the number of claims for each risk does not depend on the risk parameter and is known to the actuary. We assume that for each risk this follows a Poisson distribution with parameter λ. In this chapter we will use the same two models for the Poisson parameter, for each risk, defined in Section 3.2:

N1 The Poisson parameter, denoted λ, is constant and equal to 10 each year.

N2 The Poisson parameter for risk k in year i, denoted λ_{ki}, is a random variable and $\{\{\lambda_{ki}\}_{k=1}^{r}\}_{i=1}^{n}$ is a set of *i.i.d.* random variables, each with a $U(8,12)$ distribution;

(e) the initial surplus is allocated at the end of time period m. As the actuary cannot initially distinguish between the risks, first we obtain the initial surplus for each risk in the portfolio by dividing the initial global surplus by the number of risks, and secondly we rate the initial surplus by the risk volume. This is done just for one example because in this chapter we will focus only on the results of the ruin probabilities for the portfolio as will be explained in Section 5.3.1;

(f) the safety loading, ζ, is fixed. In this chapter the safety loading is not going to depend on the surplus. It needs further research to estimate the expected value of the aggregate claim amounts not knowing the individual risk parameters. By now we have an idea of the effect of having the safety loading depending on the surplus of the portfolio (Chapters 3 and 4). We now focus only on the effect of the Bühlmann-Straub credibility premium.

(g) the premium P6 is the pure collective premium. The premium P7 is going to be updated using the Bühlmann-Straub credibility model defined in Sections 5.1.2 and 5.1.3, and,

(h) we assume that the actuary knows a prediction of the risk volume by risk for the coming year.

We will use the simulation procedure defined on Section 2.4 with the adjustments (i) and (ii) defined in Section 4.2.

5.3 Numerical examples

In our applications in this section we will use the same distribution as in Chapter 4, a lognormal distribution for the individual claim size with parameters θ and $\sigma^{\bullet 2} = 0.97411$. Θ is the risk parameter and has a discrete distribution given by Table 4.41. The safety loading is $\zeta = 0.1$

We will consider four cases for the risk volume, see Table 5.62 for values or Figure 5.19 for a better view of the differences. Note that in Table 5.62 the lines in bold have corresponding equal values.

W1 $w_{1,i} = 30, w_{2,i} = 250, w_{3,i} = 60, w_{4,i} = 120, w_{5,i} = 40$. The risk volume vary between the risks but are constant for each year.

W2 $w_{1,i} = 30, w_{2,i} = 40, w_{3,i} = 60, w_{4,i} = 120, w_{5,i} = 250$. The risk volume vary between the risks but are constant for each year.

W3 $w_{k,i} = U(w^1_{k,i}/2, w^1_{k,i} \times 1.5)$ where $w^1_{k,i}$ is the risk volume of case W1,

W4 $w_{k,i} = w_{k,i-1} \times g(rnd), i = 7, \ldots, 15,\ w_{k,i} = w_{k,i+1} \times g(rnd), i = 1, \ldots, 5$ and $w_{k,6} = w^1_{k,6}$. rnd is a random number between 0 and 1.

$$g(rnd) = \begin{cases} 1.10, & rnd \leq 0.1; \\ 1.05, & 0.1 < rnd \leq 0.5; \\ 1.00, & 0.5 < rnd \leq 0.8; \\ 0.90, & rnd > 0.8. \end{cases} \tag{5.32}$$

and two cases for the premium:

P6 $P_{ki} = (1 + 0.1)P^E_{ki}$,

P7 $P_{ki} = (1 + 0.1)P^C_{ki}$,

The time period is 10 years ($n = 10$). As we are working with an existing portfolio, similar to the one in Chapter 4, we assume that there is a previous record for the aggregate claim amount of 5 years ($m = 5$) as illustrated in Figure 4.15.

For each Poisson parameter and each risk volume case the simulated set of aggregate claims are the same in each simulation for the two cases of premium and for the different surpluses. We present for the selected cases our estimate of the ruin probability, $\hat{\psi}(u, n)$, and the standard errors of the estimates for each initial surplus. Let us consider our portfolio with 5 risks ($r = 5$). The reason for choosing this number was mentioned in Section 4.3. Risks 1 and 2 have $\theta = 0.1$, risks 3 and 4 have $\theta = 0.2$ and risk 5 has $\theta = 0.4$. The actuary does not know this prior information. The results for our estimates of ruin probabilities are obtained with 50 000 simulations.

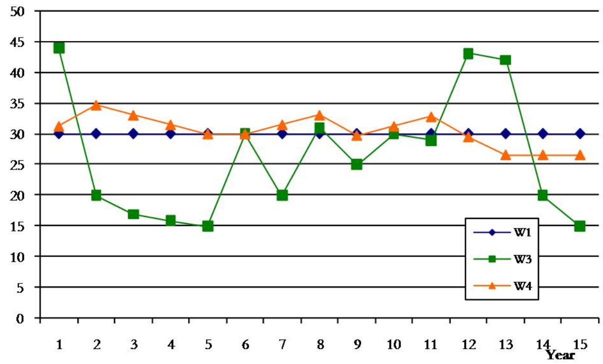
		Risk					
Risk Volume	Year	1	2	3	4	5	Total
W1	$1, \cdots, 15$	**30.00**	**250.00**	**60.00**	**120.00**	**40.00**	**500.00**
W2	$1, \cdots, 15$	30.00	40.00	60.00	120.00	250.00	500.00
W3	1	44.00	277.00	34.00	66.00	33.00	454.00
	2	20.00	298.00	67.00	170.00	24.00	579.00
	3	17.00	217.00	90.00	171.00	45.00	540.00
	4	16.00	287.00	62.00	157.00	22.00	544.00
	5	15.00	153.00	77.00	168.00	50.00	463.00
	6	**30.00**	**250.00**	**60.00**	**120.00**	**40.00**	**500.00**
	7	20.00	337.00	71.00	76.00	28.00	532.00
	8	31.00	354.00	71.00	117.00	38.00	611.00
	9	25.00	288.00	52.00	105.00	38.00	508.00
	10	30.00	203.00	62.00	144.00	35.00	474.00
	11	29.00	313.00	88.00	104.00	42.00	576.00
	12	43.00	148.00	37.00	101.00	41.00	370.00
	13	42.00	289.00	56.00	91.00	31.00	509.00
	14	20.00	318.00	87.00	157.00	43.00	625.00
	15	15.00	360.00	69.00	105.00	59.00	608.00
W4	1	31.26	236.25	65.49	152.81	34.02	519.82
	2	34.73	262.50	62.37	145.53	37.80	542.93
	3	33.08	262.50	69.30	138.60	42.00	545.48
	4	31.50	250.00	69.30	138.60	42.00	531.40
	5	30.00	250.00	63.00	132.00	42.00	517.00
	6	**30.00**	**250.00**	**60.00**	**120.00**	**40.00**	**500.00**
	7	31.50	262.50	63.00	108.00	40.00	505.00
	8	33.08	288.75	63.00	113.40	36.00	534.23
	9	29.77	259.88	56.70	119.07	36.00	501.41
	10	31.26	259.88	51.03	119.07	39.60	500.83
	11	32.82	272.87	53.58	119.07	41.58	519.92
	12	29.54	286.51	48.22	119.07	43.66	527.00
	13	26.58	300.84	48.22	119.07	39.29	534.01
	14	26.58	315.88	50.63	130.98	41.26	565.33
	15	26.58	331.67	45.57	117.88	41.26	562.96

Table 5.62: Bühlmann-Straub: Risk volume by case W, risk and year.

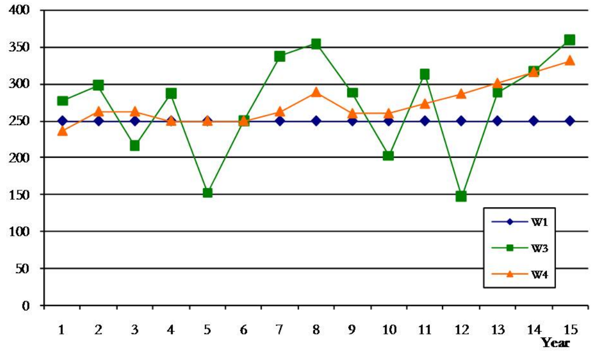
5.3.1 Results

In this chapter we are going to present results only for the portfolio. It is always difficult to have a reasonable assignment of the surplus by risk because the actuary does not know the behavior of each risk and does not know the correct amount of surplus to assign to each risk. We have now a new variable, the risk volume, that is different from risk to risk. The actuary may rate the surplus of the portfolio by the risk volume and consider a high surplus for a high risk volume, and a low surplus for a low risk volume but the main difficulty remains, he does not know which are the riskier classes. To illustrate how the ruin probabilities change among the risks with a different assignment of surplus we have Tables 5.63 and 5.64 that show numerical results for combinations of N and two different ways of assigning the surplus to each risk. First the surplus of the portfolio is divided by the number of risks, secondly the surplus is rated by the risk volume. This pattern of ruin probabilities for each risk is the same in the several combinations of W, N and P. The results will produce big tables and we will be lost in numbers to understand the results. This way we will focus only on the portfolio.

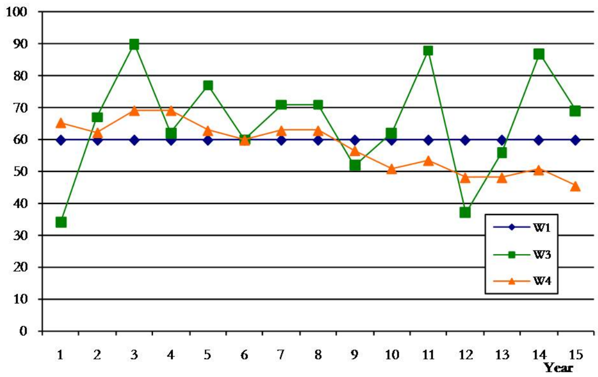
The estimated values of $\psi_{TG}(u, 10)$, together with the standard error of each estimate, are shown for the portfolio and for each one of the risks, k, for various values of the initial surplus, u, and two cases for the premium for the W1 case in Tables 5.63 and 5.64. These results were obtained with 10 000 simulations.



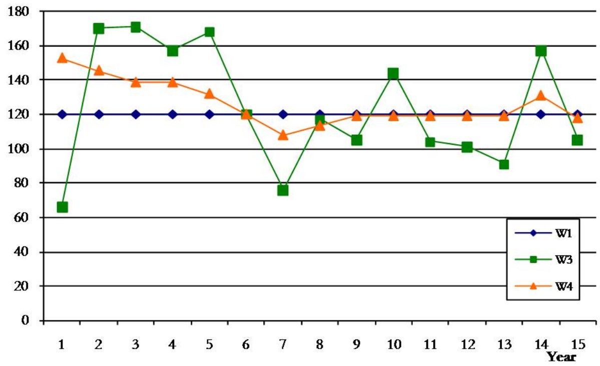
(a) k=1.



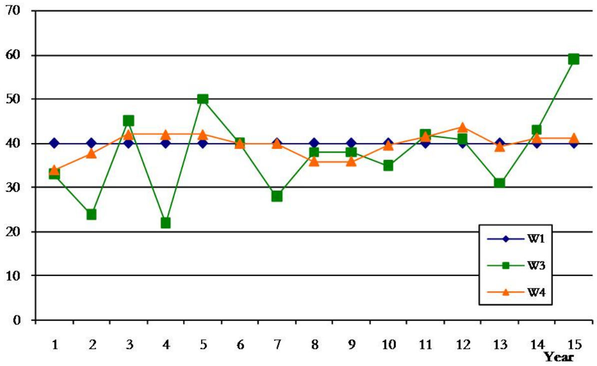
(b) k=2.



(c) k=3.



(d) k=4.



(e) k=5.

Figure 5.19: Bühlmann-Straub: Evolution of the risk volume along the years by risk.

		P6 N1		P7 N1		P6 N2		P7 N2	
k	u	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$
1	18	0.215	3.24E-06	0.460	1.09E-05	0.271	7.67E-06	0.546	1.55E-05
2	18	0.211	3.30E-07	0.448	1.95E-06	0.289	5.12E-06	0.566	1.30E-05
3	18	0.450	5.53E-06	0.481	6.74E-06	0.533	1.28E-05	0.569	1.36E-05
4	18	0.455	3.15E-06	0.482	3.90E-06	0.537	1.22E-05	0.570	1.28E-05
5	18	1.000	8.00E-08	0.554	8.15E-06	0.998	1.90E-07	0.602	1.27E-05
Port.	**90**	**0.013**	**1.30E-08**	**0.046**	**1.99E-07**	**0.053**	**1.69E-06**	**0.184**	**9.58E-06**
1	22	0.166	2.81E-06	0.403	1.15E-05	0.225	7.34E-06	0.508	1.68E-05
2	22	0.161	2.68E-07	0.389	2.06E-06	0.244	4.97E-06	0.533	1.44E-05
3	22	0.392	5.87E-06	0.426	7.31E-06	0.494	1.40E-05	0.534	1.50E-05
4	22	0.398	3.36E-06	0.426	4.25E-06	0.500	1.35E-05	0.536	1.42E-05
5	22	1.000	7.99E-08	0.504	9.06E-06	0.997	2.18E-07	0.566	1.41E-05
Pt.	**110**	**0.005**	**3.48E-09**	**0.024**	**9.66E-08**	**0.037**	**1.32E-06**	**0.157**	**9.17E-06**
1	26	0.128	2.37E-06	0.355	1.17E-05	0.188	6.86E-06	0.474	1.76E-05
2	26	0.124	2.09E-07	0.338	2.09E-06	0.209	4.72E-06	0.506	1.56E-05
3	26	0.342	6.00E-06	0.377	7.64E-06	0.461	1.50E-05	0.504	1.61E-05
4	26	0.348	3.44E-06	0.377	4.45E-06	0.470	1.45E-05	0.508	1.53E-05
5	26	1.000	8.00E-08	0.458	9.71E-06	0.997	2.47E-07	0.535	1.52E-05
Port.	**130**	**0.002**	**8.93E-10**	**0.013**	**4.66E-08**	**0.027**	**1.05E-06**	**0.138**	**8.73E-06**

Table 5.63: Bühlmann-Straub: Estimates and standard deviations of $\psi(u,10)$, W1 per risk with equal surplus.

k	u	P6 N1		P7 N1		P6 N2		P7 N2	
		$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$
1	5	0.497	3.07E-06	0.700	5.30E-06	0.523	6.01E-06	0.723	7.87E-06
2	45	0.035	4.56E-08	0.173	1.58E-06	0.107	3.17E-06	0.419	1.84E-05
3	11	0.580	4.32E-06	0.604	4.98E-06	0.626	9.61E-06	0.648	1.01E-05
4	22	0.404	3.32E-06	0.433	4.27E-06	0.506	1.32E-05	0.542	1.40E-05
5	7	1.000	8.00E-08	0.716	4.50E-06	0.998	1.42E-07	0.726	7.35E-06
Pt.	**90**	**0.013**	**1.37E-08**	**0.045**	**2.24E-07**	**0.054**	**1.82E-06**	**0.185**	**9.76E-06**
1	7	0.457	3.28E-06	0.671	6.07E-06	0.488	6.49E-06	0.699	8.89E-06
2	55	0.018	1.83E-08	0.122	1.25E-06	0.080	2.60E-06	0.394	1.90E-05
3	13	0.534	4.87E-06	0.561	5.67E-06	0.593	1.08E-05	0.618	1.14E-05
4	26	0.344	3.41E-06	0.373	4.51E-06	0.469	1.44E-05	0.507	1.53E-05
5	9	1.000	8.00E-08	0.687	5.18E-06	0.998	1.54E-07	0.703	8.33E-06
Pt.	**110**	**0.005**	**3.85E-09**	**0.024**	**1.14E-07**	**0.038**	**1.48E-06**	**0.159**	**9.33E-06**
1	8	0.421	3.43E-06	0.643	6.79E-06	0.456	6.87E-06	0.678	9.85E-06
2	65	0.009	7.03E-09	0.087	9.52E-07	0.062	2.15E-06	0.374	1.94E-05
3	16	0.492	5.32E-06	0.521	6.26E-06	0.564	1.19E-05	0.590	1.26E-05
4	31	0.293	3.37E-06	0.323	4.58E-06	0.438	1.52E-05	0.479	1.63E-05
5	10	1.000	8.00E-08	0.661	5.83E-06	0.998	1.66E-07	0.682	9.25E-06
Pt.	**130**	**0.002**	**1.06E-09**	**0.013**	**5.71E-08**	**0.028**	**1.23E-06**	**0.140**	**8.89E-06**

Table 5.64: Bühlmann-Straub: Estimates and standard deviations of $\psi(u, 10)$, W1 per risk with rated surplus.

We make the following comments about Tables 5.63 and 5.64:

(i) The results for the portfolio case N1 are equal in both tables as they should be. The residual differences arise from the low number of simulations and the use of a different set of simulated aggregate claims, because of the different portfolios. Although case N2 has an extra variability factor, the Poisson parameter, the results for the portfolio are very similar.

(ii) Risk 2 has a low risk parameter, but has more risk volume in the portfolio. Assigning the surplus rating by the risk volume, this risk will have the lowest ruin probabilities in all combinations of P and N,

(iii) Risk 5 and P6 in both cases of allocation of surplus has estimated ruin probabilities 1 or close to 1. As the risk volume is low and the actuary does not know that this risk has a high risk parameter he does not assign the correct amount of surplus,

(iv) The credibility premium (P7) has more similar results (Table 5.63) when each risk has the same amount of surplus. The results by risk for the ruin probability in case P6 vary from 0.1 to 1.0. In case P7 the results are around 0.4 for N1 and 0.5 for N2 for all risks.

From now on we only present results for the portfolio.

Tables 5.65 to 5.72 show numerical results for different combinations of N and W obtained with 50 000 simulations. Estimated values of $\psi_{TG}(u, 10)$, together with the standard error of each estimate, are shown for various values of the initial surplus, u, and for two cases for the premium, P6 and P7.

The more interesting aspects of these tables, are the effect of adjusting the premium at the start of each year by credibility (P7), the effect of the variability of a key parameter, λ (N2), and the effect of a different structure of portfolio (W1, W2, W3 and W4). We are now able to compare the effect of:

(i) using the Bühlmann-Straub credibility premium, P_{ki}^{C} (P7) compared with a non-credibility premium, P_{ki}^{E} (P6);

(ii) the variability of the Poisson parameter, N1 compared with N2;

(iii) having a constant portfolio with low risk (W1), or having a constant portfolio with the same risk volume but distributed over the risks in a way that produces a more risky portfolio (W2), or a non-constant portfolio (W3 and W4).

	P6		P7	
u	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$
80	0.02076	5.02E-09	0.06277	6.06E-08
90	0.01319	2.67E-09	0.04558	4.30E-08
100	0.00838	1.40E-09	0.03314	3.02E-08
110	0.00533	7.29E-10	0.02411	2.11E-08
120	0.00339	3.75E-10	0.01757	1.47E-08
130	0.00215	1.92E-10	0.01281	1.02E-08

Table 5.65: Bühlmann-Straub: Estimates and standard deviations of $\psi(u,10)$, N1 W1.

	P6		P7	
u	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$	$\psi_{TG}(u,10)$	SD$[\psi_{TG}(u,10)]$
80	0.06499	3.86E-07	0.19870	1.94E-06
90	0.05286	3.40E-07	0.18130	1.90E-06
100	0.04367	3.01E-07	0.16705	1.86E-06
110	0.03657	2.68E-07	0.15518	1.82E-06
120	0.03099	2.40E-07	0.14511	1.77E-06
130	0.02653	2.16E-07	0.13645	1.73E-06

Table 5.66: Bühlmann-Straub: Estimates and standard deviations of $\psi(u,10)$, N2 W1.

u	P6 $\psi_{TG}(u, 10)$	P6 SD[$\psi_{TG}(u, 10)$]	P7 $\psi_{TG}(u, 10)$	P7 SD[$\psi_{TG}(u, 10)$]
80	0.77204	1.69E-06	0.08674	8.44E-08
90	0.74871	1.93E-06	0.06551	6.34E-08
100	0.72604	2.15E-06	0.04952	4.69E-08
110	0.70398	2.36E-06	0.03747	3.44E-08
120	0.68254	2.56E-06	0.02837	2.51E-08
130	0.66170	2.74E-06	0.02149	1.82E-08

Table 5.67: Bühlmann-Straub: Estimates and standard deviations of $\psi(u, 10)$, N1 W2.

u	P6 $\psi_{TG}(u, 10)$	P6 SD[$\psi_{TG}(u, 10)$]	P7 $\psi_{TG}(u, 10)$	P7 SD[$\psi_{TG}(u, 10)$]
80	0.79550	2.66E-06	0.24552	2.30E-06
90	0.78726	2.81E-06	0.22723	2.29E-06
100	0.78000	2.94E-06	0.21211	2.27E-06
110	0.77354	3.05E-06	0.19936	2.24E-06
120	0.76769	3.15E-06	0.18847	2.21E-06
130	0.76232	3.24E-06	0.17901	2.18E-06

Table 5.68: Bühlmann-Straub: Estimates and standard deviations of $\psi(u, 10)$, N2 W2.

u	P6 $\psi_{TG}(u, 10)$	P6 SD[$\psi_{TG}(u, 10)$]	P7 $\psi_{TG}(u, 10)$	P7 SD[$\psi_{TG}(u, 10)$]
80	0.02080	5.01E-09	0.06269	5.93E-08
90	0.01322	2.67E-09	0.04550	4.22E-08
100	0.00840	1.40E-09	0.03305	2.97E-08
110	0.00534	7.27E-10	0.02403	2.09E-08
120	0.00339	3.74E-10	0.01749	1.46E-08
130	0.00216	1.92E-10	0.01274	1.03E-08

Table 5.69: Bühlmann-Straub: Estimates and standard deviations of $\psi(u, 10)$, N1 W3.

u	P6 $\psi_{TG}(u,10)$	P6 SD$[\psi_{TG}(u,10)]$	P7 $\psi_{TG}(u,10)$	P7 SD$[\psi_{TG}(u,10)]$
80	0.06427	3.73E-07	0.20648	2.06E-06
90	0.05213	3.27E-07	0.18932	2.03E-06
100	0.04295	2.89E-07	0.17527	1.99E-06
110	0.03587	2.57E-07	0.16353	1.95E-06
120	0.03032	2.30E-07	0.15353	1.90E-06
130	0.02589	2.07E-07	0.14490	1.86E-06

Table 5.70: Bühlmann-Straub: Estimates and standard deviations of $\psi(u,10)$, N2 W3.

u	P6 $\psi_{TG}(u,10)$	P6 SD$[\psi_{TG}(u,10)]$	P7 $\psi_{TG}(u,10)$	P7 SD$[\psi_{TG}(u,10)]$
80	0.02081	5.11E-09	0.06290	6.04E-08
90	0.01323	2.73E-09	0.04569	4.28E-08
100	0.00842	1.44E-09	0.03322	3.00E-08
110	0.00535	7.50E-10	0.02417	2.09E-08
120	0.00340	3.88E-10	0.01761	1.45E-08
130	0.00217	2.00E-10	0.01284	1.00E-08

Table 5.71: Bühlmann-Straub: Estimates and standard deviations of $\psi(u,10)$, N1 W4.

We make the following comments about the results in Tables 5.65 and 5.72:

(i) The standard deviations of $\psi(u,10)$ are, as in Chapters 3 and 4, all very small.

(ii) The effect of the variability of λ is considerable: $\psi_{TG}(u,10)$ increases by a factor from 2 to 11. This increment is slightly bigger in the case P6.

(iii) In this chapter we do not choose the value of the target ruin probability to settle the safety loading. The safety loading is fixed so the ruin probability decreases with the initial surplus in all combinations of N, W and P, as expected.

(iv) The case W2 is a more risky portfolio. The ruin probability increases, comparing with W1, by a factor of 36 ($u = 80$) to 306 ($u = 130$) for P6 and 0.38 to 0.68 for P7 for case N1. For case N2 it increases by a factor of 37 to 353 for P6 and 3 to 13 for P7. Adjusting the premium using the Bühlmann-Straub credibility model can (even increasing) keep the ruin probability in an acceptable

u	P6 $\psi_{TG}(u,10)$	P6 SD$[\psi_{TG}(u,10)]$	P7 $\psi_{TG}(u,10)$	P7 SD$[\psi_{TG}(u,10)]$
80	0.06508	3.83E-07	0.20249	1.98E-06
90	0.05288	3.37E-07	0.18500	1.94E-06
100	0.04364	2.98E-07	0.17064	1.90E-06
110	0.03651	2.65E-07	0.15861	1.85E-06
120	0.03091	2.37E-07	0.14836	1.81E-06
130	0.02644	2.14E-07	0.13947	1.76E-06

Table 5.72: Bühlmann-Straub: Estimates and standard deviations of $\psi(u, 10)$, N2 W4.

range of values. In case W2/P7 the ruin probabilities are higher than W1/P7 but are much smaller than W2/P6. Credibility premiums can control, in some sense, the ruin probabilities.

(v) The W1 and W2 cases have constant risk volumes per risk and year. If we compare W1 with W3 and W4 we can see that the results are quite similar. We may see in Table 5.62 that for year 6 the risk volume is equal for all W, except W2.

To help us to understand the results we have, as in Chapters 3 and 4, produced some statistical information concerning end of year ruin, and some figures for several initial surpluses.

Tables 5.73 to 5.79 show some statistical information for the paths where at the end of the year we have a surplus below zero (end of year ruin). We recorded, similarly to Chapters 3 and 4, for each simulation where $u(i) < 0$ for some $i \geq 6$ the information for different combinations of W, N, P and initial surplus. These results are for the portfolio only. We also present the standard deviation of each average (SD).

These results were produced with a different set of 100 000 simulations. There are no results for W3/N1 since in this case the 100 000 simulations did not produced a single end of year ruin. Cases W1/N1, W2/N1/P7 and W4/N1 do not have enough information to set out conclusions about the mean and the standard deviation, but allow us to comment that the results of Table 5.65, Table 5.67 case P7, and Table 5.71 are obtained with a great proportion of “within year ruin” probability.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	SD $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg p_i	SD p_i	Avg y_i	SD y_i
80	7	2	0.9997	6.00	0	- 63.04	88.72	80	0	10 365.95	-	10 509.00	65.48
90	7	1	0.9998	6.00	-	- 115.77	-	90	-	10 256.90	-	10 462.70	-
100	7	1	0.9997	6.00	-	- 105.77	-	100	-	10 256.90	-	10 462.70	-
110	7	1	0.9996	6.00	-	- 95.77	-	110	-	10 256.90	-	10 462.70	-
120	7	1	0.9994	6.00	-	- 85.77	-	120	-	10 256.90	-	10 462.70	-
130	7	1	0.9992	6.00	-	- 75.77	-	130	-	10 256.90	-	10 462.70	-

Table 5.73: Bühlmann-Straub: Statistical information for ruin cases, W1 N1.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	sd $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg λ	SD λ	Avg p_i	SD p_i	Avg y_i	SD y_i
80	6	685	0.894605	6.01	0.11	-152.93	133.77	81.18	15.74	5690.73	91.73	11001.10	-	11235.25	134.56
80	7	9267	0.533611	6.16	0.48	-284.62	243.07	110.75	122.54	5494.52	160.84	10263.93	274.06	10659.30	321.46
90	6	643	0.878359	6.01	0.12	-152.57	132.46	91.25	16.24	5692.43	92.01	11001.10	-	11244.97	133.21
90	7	9032	0.501814	6.16	0.48	-282.62	241.69	120.10	120.62	5496.85	160.05	10261.87	273.41	10664.59	319.62
100	6	616	0.858935	6.02	0.13	-149.07	131.56	101.15	17.08	5694.40	91.83	11001.10	-	11251.37	132.50
100	7	8808	0.47274	6.16	0.48	-280.45	240.52	129.68	120.36	5498.55	159.95	10259.45	272.78	10669.58	319.03
110	6	581	0.841131	6.02	0.13	-148.18	130.46	111.06	18.13	5695.80	92.34	11001.10	-	11260.39	131.26
110	7	8584	0.446845	6.17	0.49	-278.66	239.18	139.27	120.75	5500.48	159.62	10257.63	272.81	10675.56	318.47
120	6	548	0.82318	6.02	0.13	-146.80	129.37	121.12	18.66	5700.75	90.12	11001.10	-	11269.06	130.15
120	7	8390	0.421837	6.17	0.49	-276.40	238.05	148.16	119.44	5502.47	159.17	10255.77	272.44	10680.33	317.16
130	6	520	0.804008	6.02	0.13	-144.42	128.44	131.37	18.64	5703.26	89.09	11001.10	-	11276.95	128.91
130	7	8167	0.401478	6.17	0.50	-274.75	236.71	157.25	118.42	5504.71	158.42	10253.66	271.70	10685.66	315.99

Table 5.74: Bühlmann-Straub: Statistical information for ruin cases, W1 N2.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	sd $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg p_i	SD p_i	Avg y_i	SD y_i
80	6	45589	0.409499	7.26	1.96	-146.67	124.70	119.17	98.08	11001.10	-	11266.99	148.29
80	7	2	0.999769	6.00	0.00	-92.50	114.26	80	0	11870.25	41.22	12042.75	155.49
90	6	44473	0.406004	7.29	1.98	-144.67	123.73	126.00	97.51	11001.10	-	11271.82	147.51
90	7	2	0.999695	6.00	0	-82.50	114.26	90	0	11870.25	41.22	12042.75	155.49
100	6	43312	0.403446	7.34	2.00	-143.32	122.97	132.61	97.19	11001.10	-	11277.07	146.74
100	7	1	0.999798	6.00	-	-153.29	-	100	-	11899.40	-	12152.70	-
110	6	42156	0.40118	7.38	2.03	-142.23	122.32	138.86	97.23	11001.10	-	11282.23	146.18
110	7	1	0.999733	6.00	-	-143.29	-	110	-	11899.40	-	12152.70	-
120	6	41013	0.399116	7.42	2.05	-141.05	121.51	144.53	96.91	11001.10	-	11286.73	145.62
120	7	1	0.999647	6.00	-	-133.29	-	120	-	11899.40	-	12152.70	-
130	6	39939	0.396416	7.47	2.07	-139.45	120.73	150.83	97.61	11001.10	-	11291.42	145.20
130	7	1	0.999535	6.00	-	-123.29	-	130	-	11899.40	-	12152.70	-

Table 5.75: Bühlmann-Straub: Statistical information for ruin cases, W2 N1.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	sd $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg λ	SD λ	Avg p_i	SD p_i	Avg y_i	SD y_i
80	6	72976	0.082642	7.22	2.04	-543.72	399.57	251.20	309.56	5344.99	190.43	11001.10	-	11796.07	436.89
80	7	12084	0.507829	6.18	0.53	-361.49	300.05	121.47	153.74	5468.86	166.11	11663.51	-	12146.47	378.70
90	6	72715	0.076349	7.23	2.05	-540.64	397.76	258.77	307.31	5346.70	189.72	11001.10	-	11800.56	434.85
90	7	11876	0.47736	6.18	0.54	-359.22	298.77	130.49	151.72	5470.68	165.34	11661.64	-	12151.34	376.67
100	6	72459	0.071044	7.25	2.06	-537.23	395.81	266.39	305.58	5348.28	188.97	11001.10	-	11804.76	432.90
100	7	11651	0.450702	6.19	0.54	-357.24	297.54	139.85	150.85	5472.60	164.59	11659.89	-	12156.98	374.65
110	6	72172	0.066985	7.26	2.07	-534.14	393.87	273.78	303.67	5349.97	188.18	11001.10	-	11809.06	430.94
110	7	11418	0.427278	6.19	0.54	-355.52	296.29	148.63	148.97	5474.16	164.26	11657.37	-	12161.53	373.66
120	6	71883	0.063648	7.27	2.07	-531.17	392.07	281.50	302.13	5351.78	187.41	11001.10	-	11813.81	428.87
120	7	11195	0.406	6.19	0.54	-354.14	294.85	157.31	147.19	5475.86	163.59	11655.34	-	12166.79	372.52
130	6	71598	0.060786	7.28	2.08	-528.14	390.28	289.03	300.58	5353.47	186.79	11001.10	-	11818.31	427.09
130	7	10986	0.386283	6.19	0.55	-352.44	293.39	166.25	145.49	5477.63	163.13	11653.50	-	12172.19	370.14

Table 5.76: Bühlmann-Straub: Statistical information for ruin cases, W2 N2.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	sd $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg λ	SD λ	Avg p_i	SD p_i	Avg y_i	SD y_i
80	6	660	0.897305	6.02	0.13	-140.80	128.27	81.41	16.97	5691.37	122.20	11013.33	122.15	11235.59	187.50
80	7	10064	0.512596	6.25	0.57	-295.33	250.30	140.54	179.34	5601.18	328.81	10391.53	502.81	10827.39	594.91
90	6	626	0.879923	6.02	0.14	-138.28	127.53	91.35	17.77	5695.03	122.01	11014.00	125.39	11243.67	189.05
90	7	9846	0.479934	6.25	0.57	-293.01	248.96	149.67	178.56	5604.43	329.07	10391.32	504.98	10834.00	595.12
100	6	581	0.864737	6.02	0.14	-138.61	126.64	101.17	17.10	5697.78	122.05	11013.79	126.91	11253.61	189.39
100	7	9621	0.451071	6.26	0.58	-291.17	247.87	158.42	177.11	5608.20	331.73	10392.45	510.89	10842.05	598.95
110	6	549	0.846961	6.02	0.14	-136.40	125.98	111.24	17.59	5699.10	124.02	11014.53	130.53	11262.21	191.36
110	7	9406	0.424808	6.26	0.58	-289.32	246.74	167.17	175.96	5611.29	331.85	10392.48	512.37	10848.97	598.56
120	6	501	0.834748	6.02	0.13	-139.13	124.69	120.70	16.70	5700.31	113.66	11012.34	88.35	11272.21	161.17
120	7	9179	0.402135	6.26	0.58	-287.66	245.82	176.26	174.91	5614.39	332.56	10392.14	514.40	10856.05	599.82
130	6	472	0.817669	6.02	0.13	-137.42	123.75	130.74	17.21	5705.15	113.38	11013.03	90.98	11281.24	161.75
130	7	8974	0.380669	6.27	0.59	-286.15	244.58	185.02	173.95	5619.34	334.05	10394.50	519.35	10865.67	601.16

Table 5.77: Bühlmann-Straub: Statistical information for ruin cases, W3 N2.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	sd $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg p_i	SD p_i	Avg y_i	SD y_i
80	7	2	0.9997	6.00	0	- 57.21	42.25	80	0	10 286.30	160.51	10 423.50	118.23
90	7	2	0.9996	6.00	0	- 47.21	42.25	90	0	10 286.30	160.51	10 423.50	118.23
100	7	2	0.9994	6.00	0	- 37.21	42.25	100	0	10 286.30	160.51	10 423.50	118.23
110	7	1	0.9996	6.00	-	- 57.09	-	110	-	10 172.80	-	10 339.90	-
120	7	1	0.9994	6.00	-	- 47.09	-	120	-	10 172.80	-	10 339.90	-
130	7	1	0.9992	6.00	-	- 37.09	-	130	-	10 172.80	-	10 339.90	-

Table 5.78: Bühlmann-Straub: Statistical information for ruin cases, W4 N1.

u	P	NRuins	Prop	Avg i	SD i	Avg $u(i)$	sd $u(i)$	Avg $u(i-1)$	SD $u(i-1)$	Avg λ	SD λ	Avg p_i	SD p_i	Avg y_i	SD y_i
80	6	699	0.892596	6.01	0.11	-135.79	121.30	80.76	15.00	5683.27	91.51	11002.36	11.72	11218.96	123.13
80	7	9171	0.547091	6.18	0.52	-287.41	239.99	111.89	122.59	5511.71	177.06	10290.37	291.56	10689.67	346.20
90	6	647	0.877644	6.01	0.11	-136.31	120.04	90.82	15.59	5687.28	90.35	11002.46	12.18	11229.63	121.85
90	7	8947	0.516366	6.18	0.52	-285.20	238.56	121.07	120.71	5513.60	176.92	10287.84	291.17	10694.10	345.74
100	6	609	0.860446	6.01	0.11	-134.51	119.01	100.87	16.07	5688.48	90.59	11002.55	12.55	11237.97	120.78
100	7	8722	0.488864	6.18	0.52	-283.44	237.03	129.94	118.33	5515.87	176.30	10286.85	291.39	10700.23	344.40
110	6	564	0.845529	6.01	0.11	-134.84	117.68	110.61	14.73	5689.56	91.52	11002.47	12.20	11247.96	119.36
110	7	8498	0.46422	6.18	0.53	-281.36	235.71	139.17	116.94	5517.99	175.75	10284.97	291.53	10705.50	343.99
120	6	537	0.826273	6.01	0.11	-131.41	116.80	120.09	8.31	5690.29	90.54	11002.33	11.58	11253.87	118.10
120	7	8271	0.442494	6.19	0.53	-279.88	234.11	148.13	115.30	5520.79	175.17	10283.52	292.20	10711.52	343.00
130	6	497	0.812029	6.01	0.11	-131.58	115.54	130.10	8.64	5694.07	89.69	11002.43	12.04	11264.16	116.83
130	7	8063	0.421898	6.19	0.54	-277.66	232.69	157.57	114.68	5522.93	174.76	10281.07	292.54	10716.30	342.98

Table 5.79: Bühlmann-Straub: Statistical information for ruin cases, W4 N2.

From these tables we can add:

(i) Ruin occurs mainly in the first two years (recall that year 6 is the first year in which ruin can occur).

(ii) As we already noticed, the proportion of the ruin probability due to within ruin is almost 1 in cases W1/N1, W3/N1 and W4/N1 for both P6 and P7 and W2/N1/P7. Case W2/N1/P6 has values around 50%. In case W2/N2 we have a low within year ruin probability for P6 and for P7 around 50%. The remaining cases have a high within year ruin probability for P6 and still around 50% for P7.

(iii) The severity of ruin in case N2 has lower values for P6 and is comparable for cases W1, W3 and W4. It also decreases with the surplus. In case W2 we have P6 $>$ P7 and N2 $>$ N1. This has to do with the mean of the Poisson parameter being higher than $\mathrm{E}[N] = 5\,000$ in case N2.

(iv) The average premium in the cases that lead to ruin has the following order P6>P7 (except W2/N2). For P6 it is constant (by definition) and for P7 it decreases slightly with the surplus.

(v) On average the aggregate claims in the year ruin occurred is around 11 250 for N2/P6, except W2 where it is around 11 800, and 10 700 for N2/P7, except W2 where it is 12 160. It is higher than the premium, 11 001 for P6 (except W3 where it is 11 013) and from 10 300 to 11 900 for P7.

(vi) The average surplus at the start of the year of ruin is comparable for N2/P6 except in case W2, where, as we expected it is much higher. The case P7 has high values of average surplus at the start of the year compared with P6. For case N2 we have the following order W1<W4<W2<W3. W2/N1/P6 is lower than W2/N2/P7. For the case W2/N1/P7 we have a low average surplus at the start of the year compared with W2/N2/P7.

(vii) The average value of the Poisson parameter in the year of ruin for P6 in case N2 is greater than for P7 except in case W2. The cases W1, W3 and W4 have comparable values of around 5 700 for P6 and 5 500 for P7.

W/P	u	N1		N2	
		Nr	r_{y_{i-1},y_i}	Nr	r_{y_{i-1},y_i}
W1/P6	80	-	-	685	*0.05*
	90	-	-	643	*0.04*
	100	-	-	616	*0.02*
	110	-	-	581	*0.03*
	120	-	-	548	*0.04*
	130	-	-	520	*0.06*
W1/P7	80	2	–	9 267	0.30
	90	1	-	9 032	0.30
	100	1	-	8 808	0.30
	110	1	-	8 584	0.30
	120	1	-	8 390	0.30
	130	1	-	8 167	0.29
W2/P6	80	45 589	- 0.04	72 976	- 0.04
	90	44 473	- 0.05	72 715	- 0.04
	100	43 312	- 0.06	72 459	- 0.04
	110	42 156	- 0.07	72 172	- 0.04
	120	41 013	- 0.09	71 883	- 0.04
	130	39 939	- 0.10	71 598	- 0.04
W2/P7	80	2	–	12 084	0.28
	90	2	–	11 876	0.28
	100	1	-	11 651	0.28
	110	1	-	11 418	0.28
	120	1	-	11 195	0.28
	130	1	-	10 986	0.28

Table 5.80: Bühlmann-Straub: Correlation between y_{i-1} and y_i, for combinations of W1, W2, N and P.

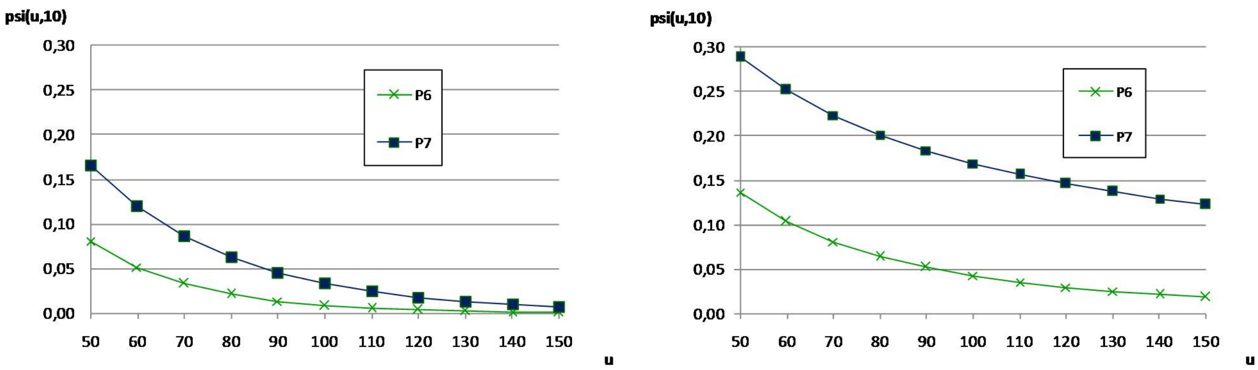
Tables 5.80 and 5.81 show the Pearson's correlation coefficient between the claim amounts y_{i-1} and y_i in the cases where $u(i) < 0$. For each initial surplus and type of premium we calculate for each Poisson parameter, the number of records used to obtain the correlation (Nr) and the estimate of the correlation coefficient (r_{y_{i-1},y_i}) for the portfolio. These results were obtained with the same set of 100 000 simulations that produced the statistical information for the end year ruin cases. Recall that the italic case numbers are the cases where the hypotheses $H_0 : \rho = 0$ is accepted for $\alpha = 5\%$.

		N1		N2	
W/P	u	Nr	r_{y_{i-1},y_i}	Nr	r_{y_{i-1},y_i}
W3/P6	80	-	-	660	0.23
	90	-	-	626	0.22
	100	-	-	581	0.22
	110	-	-	549	0.23
	120	-	-	501	0.21
	130	-	-	472	0.22
W3/P7	80	-	-	10 064	0.55
	90	-	-	9 846	0.55
	100	-	-	9 621	0.56
	110	-	-	9 406	0.56
	120	-	-	9 179	0.56
	130	-	-	8 974	0.56
W4/P6	80	-	-	699	*0.03*
	90	-	-	647	*0.04*
	100	-	-	609	*0.04*
	110	-	-	564	*0.02*
	120	-	-	537	*0.01*
	130	-	-	497	*0.00*
W4/P7	80	2	–	9 171	0.30
	90	2	–	8 947	0.30
	100	2	–	8 722	0.31
	110	1	-	8 498	0.31
	120	1	-	8 271	0.31
	130	1	-	8 063	0.31

Table 5.81: Bühlmann-Straub: Correlation between y_{i-1} and y_i, for combinations of W3, W4, N and P.

From these tables we can observe that for P6, the constant premium, we have negative or no correlation except for the case W3 where the risk volume varies in a uniform way. For P7, the credibility premium, we have a positive correlation as in the previous chapter. We also may expect that two years of heavy claims and low premiums lead to end of year ruin.

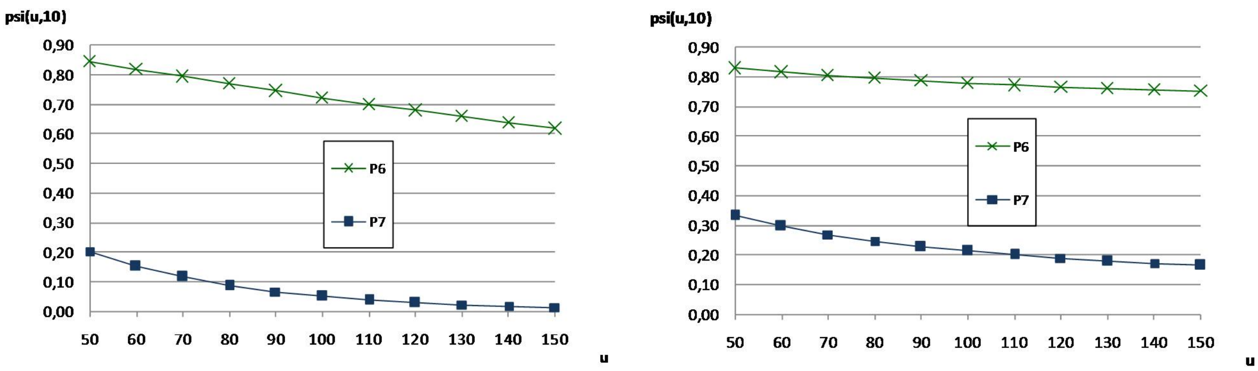
The graphs in Figures 5.20 to 5.23 show for different combinations of N, W and P the ruin probabilities for the portfolio over a 10 year period for a range of initial surpluses. The results for these figures were obtained with 10 000 runs. We can see that we do not have anymore the same shape as in Chapter 3 and 4. That is due to the fixed safety loading. These figures reinforce comments (ii) to (v) made in this section, page 124.



(a) N1.

(b) N2.

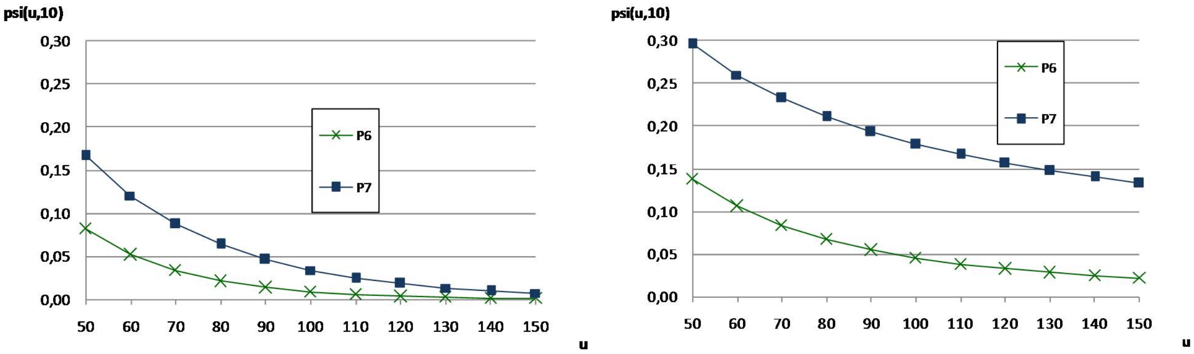
Figure 5.20: Bühlmann-Straub: $\psi(u, 10)$ for several values of initial surplus, W1.



(a) N1.

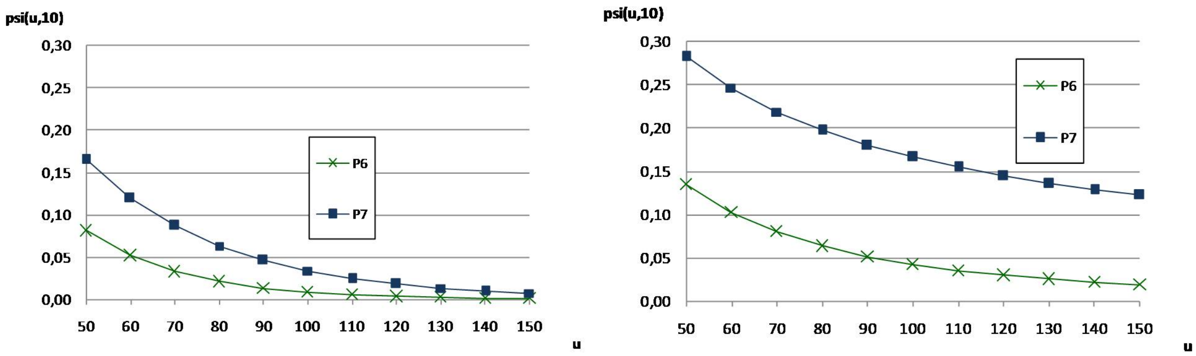
(b) N2.

Figure 5.21: Bühlmann-Straub: $\psi(u, 10)$ for several values of initial surplus, W2.



(a) N1. (b) N2.

Figure 5.22: Bühlmann-Straub: $\psi(u, 10)$ for several values of initial surplus, W3.



(a) N1. (b) N2.

Figure 5.23: Bühlmann-Straub: $\psi(u, 10)$ for several values of initial surplus, W4.

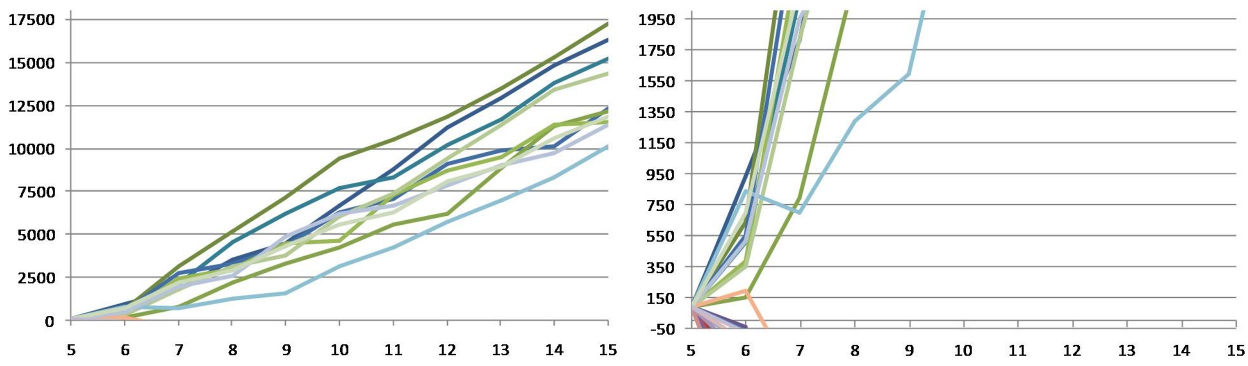
Similar to Chapter 4, Table 5.82 shows what proportion of simulations resulting in end of year ruin are identical for P6 and P7 for the portfolio. For instance for u=110 premiums types P6 and P7, case W2/N2, have around 19% of common paths that lead to ruin at the end of some year during the 10 year period. These results came from the same file that produced Tables 5.73 to 5.79. Recall that case W3/N1 does not have any end of year ruin cases in our simulations and cases W1/N1 and W4/N1 have very few results. All the remaining results have a low percentage of common ruin cases between P6 and P7.

We are going to show some figures of interest for cases W1 and W2. Figures 5.24 to 5.26 show three sets of examples of paths of the surplus process leading to ruin in at least one case for the evaluation horizon ($i = 5, \ldots, 15$). They are all for $u = 90$, the blue/green lines are for premium type P6 and the pink/violet/red lines are for

u	N1 W1	N1 W2	N1 W3	N1 W4	N2 W1	N2 W2	N2 W3	N2 W4
80	0.0%	4.1%	-	0.0%	7.8%	19.7%	6.8%	7.9%
90	0.0%	4.0%	-	0.0%	7.5%	19.5%	6.6%	7.6%
100	0.0%	3.9%	-	0.0%	7.3%	19.3%	6.4%	7.3%
110	0.0%	3.8%	-	0.0%	7.1%	19.1%	6.2%	7.0%
120	0.0%	3.7%	-	0.0%	6.9%	18.8%	5.8%	6.9%
130	0.0%	3.6%	-	0.0%	6.7%	18.6%	5.6%	6.5%

Table 5.82: Bühlmann-Straub: Common ruin scenarios

P7. In each figure, picture (b), we give detail of low values of the surplus process.

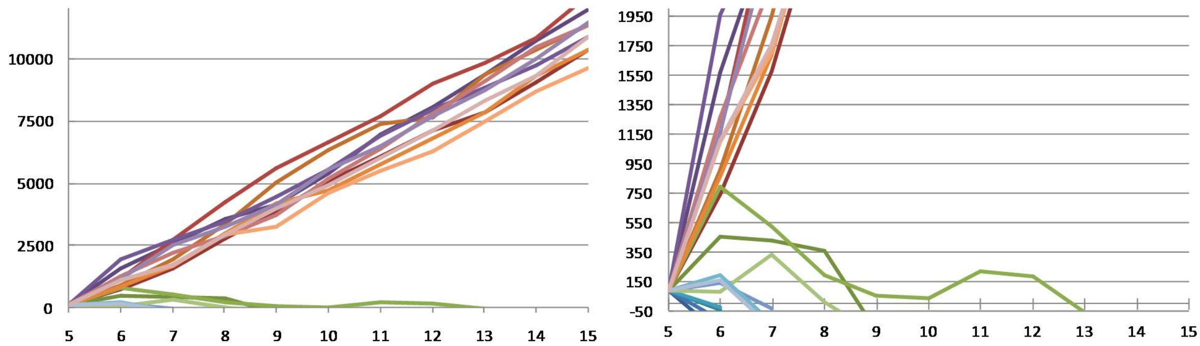


(a)

(b) Detail.

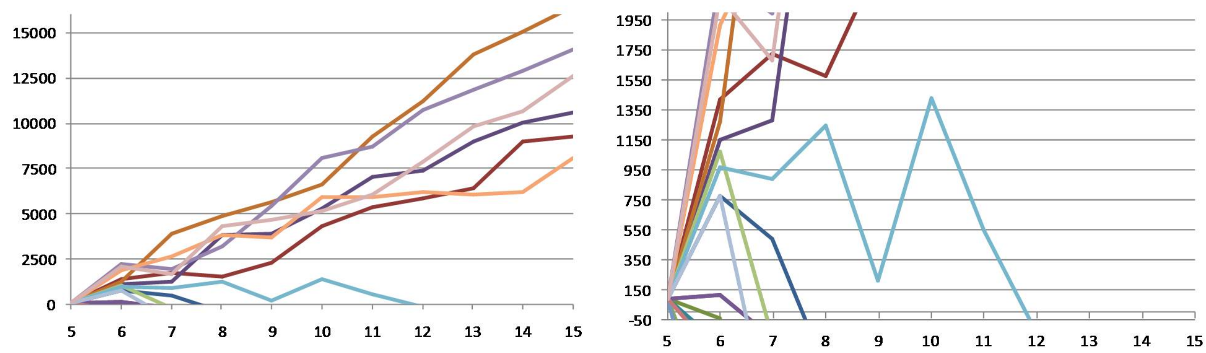
Figure 5.24: Bühlmann-Straub: Simulated paths, case W1 N2, $u = 90$.

Figure 5.24 illustrates the results of Table 5.74. End of year ruin occurs more frequently for P7 than for P6. Figure 5.25 illustrates the results of Table 5.75. End of year ruin occurs almost only for P6 and mainly in the first year. Figure 5.26 illustrates the results of Table 5.76. End of year ruin occurs more frequently for P6 than for P7.



(a) (b) Detail.

Figure 5.25: Bühlmann-Straub: Simulated paths, case W2 N1, $u = 90$.



(a) (b) Detail.

Figure 5.26: Bühlmann-Straub: Simulated paths, case W2 N2, $u = 90$.

5.4 Comments on results

In this chapter we gave another step forward in the calculation of the ruin probabilities of the portfolio. Using the Bühlmann-Straub model we allow for a portfolio with variations in exposure or size by introducing the risk volume variable.

Throughout this chapter we presented the results of applying our methodology when premiums change from year to year using credibility, and using a fixed safety loading. To achieve that we used two ways of calculating the premium combined with four approaches to the risk volume and two approaches to the Poisson parameter (constant and variable), in a 10 year period (with a previous record of 5 years of aggregate claims), for a portfolio with 5 risks.

We make the following general comments about the results:

(i) Varying the Poisson parameter, scenario N2, increases the probability of ruin significantly as we would expect.

(ii) With credibility premiums we have more similar results for the ruin probability of each risk if we divide the surplus of the portfolio by the number of risks.

(iii) With a fixed safety loading the ruin probability decreases with the surplus in all combinations of P, N and W as expected.

(iv) Even for a portfolio with a high number of risks with a high risk parameter (scenario W2) the ruin probabilities were kept at an acceptable level using the Bühlmann-Straub model to calculate the premium.

With the insight on the causes of end year ruin we may comment:

(v) Ruin occurs mainly in the first two years.

(vi) The average aggregate claims in the year of ruin is higher than the expected aggregate claims.

(vii) The average surplus at the start of the year of ruin is higher for P7/N2 in cases W1,W3,W4 than P6/N2 but also has a higher standard deviation. For W2/N2 we have P6>P7.

(vii) The average premium in the year of ruin has the following order P6>P7.

We plotted $\psi(u, 10)$ for a large range of values for the initial surplus, combining P and N.

(ix) The ruin probability decreases with the surplus, as it must.

(x) Case P7 has higher ruin probabilities than case P6.

Finally we can also add:

(xi) All the standard deviations of our estimates $\psi_{TG}(u, 10)$ are still very small.

(xii) For the constant premium, we have negative or no correlation between y_{i-1} and y_i, except for the case W3 where the risk volume varies in a uniform way. For the credibility premium we have a positive correlation.

(xiii) The running times of the results of this chapter are comparable to Chapter 4.

(xiv) The results for the ruin probabilities for cases W1, W3 and W4 are quite similar. This is presumably because the average year for end of year ruin probability is year 6 (first year for the period of calculating the ruin probability). If ruin does not occur in the first years it is not likely to occur and for year 6 the risk volume is equal for this cases of W.

Chapter 6

Conclusions

Throughout this thesis we presented a method for calculating the probability of ruin in continuous and finite time for a compound Poisson risk process where the premium rate is constant throughout the year but can change at the start of the year.

Our method involves simulating the aggregate claims for each year, calculating the premium to be charged each year given the past aggregate claim amounts, and then calculating the within year probability of ruin assuming either a Brownian motion approximation to the surplus process or a translated gamma distribution approximation for aggregate claim amounts. Both approximations work very well and they are described and tested in Chapter 2. We tested the model by comparing the results produced using our methodology with published results for compound Poisson risk processes with a fixed premium rate. Wikstad (1971) and Seal (1978a) provide values of ruin probabilities in finite and continuous time and Gerber (1979) provides an exact formula to calculate ruin probabilities in infinite time. The translated gamma distribution approximation produces closer results to $\psi(u, n)$ in the classical risk model as was expected. The standard errors of our estimates are almost identical for the two approximations to the within year probability of ruin and are very small. The ruin probabilities calculated using the translated gamma approximation are generally closer to $\psi(u, n)$ than the ones calculated using the Brownian motion approximation. So, for the remainder of the thesis we used only the translated gamma approximation. The model we propose is quite fast because we do not simulate the process claim by claim only the aggregate claims amount, and there are well established and fast algorithms for calculating gamma densities.

Our main goal was not the classical risk process with constant premium rate but the case where the premium change from year to year. First in Chapter 3 we consider the case where the premium depends, in particular, on the surplus level at the end of the previous year or at some earlier time. The premium rate is set each year so that the probability of ultimate ruin from that time is always (approximately) equal to a pre-determined value. We did this using De Vylder (1978)'s approximation. We also used two different models (N1 and N2) for the parameter of the Poisson distribution. In the first case the Poisson parameter is constant and in the second the Poisson parameter in year i is a random variable. Varying the Poisson parameter, scenario N2, increases the probability of ruin considerably, as we expected. The attempt to control the ruin probability has problems related to the fitted curve for high values of the initial surplus. The curves of the ruin probabilities as functions of the initial surplus have similar patterns for all combinations of parameters and distributions studied.

A further step was to consider in Chapter 4 the Bühlmann model to update the premium in each year combined with the safety loading depending on the surplus. In this chapter we have the opportunity to study a portfolio with inhomogeneous risks using the risk parameter. An interesting result was to notice how high is the ruin probability by risk compared with the ruin probability of the portfolio (Tables 4.50 and 4.51). We used the same scenarios for the Poisson parameter, N1 and N2. We have now a risk parameter that defines the behavior of each risk. We suppose for simplicity that the actuary knows the distribution but does not know the value of the risk parameter for each risk. The major conclusions are that adjusting the premium using the Bühlmann credibility model increases the ruin probability and the Bühlmann credibility factor decreases with a varying Poisson parameter and has more variability too.

We consider in Chapter 5 the Bühlmann-Straub model to update the premium in each year. This model allows for variations in the exposure. This way we can consider a portfolio with different risk volumes for each risk that also may vary each year. In this model we also suppose that the actuary only knows the distribution of the risk parameter and knows a predicted value for the risk volume. It's important to notice that in our risky portfolio (W2) the premium updated using the Bühlmann-Straub model can keep the ruin probability at a controlled level similar to the results produced with different risk volumes (W1, W3 and W4).

In Chapters 4 and 5 the distribution of the risk parameter, Θ, is a very simple one but our methodology is expected to work very well with other structural distributions, for instance continuous distributions. The choice of this simple discrete distribution was only to illustrate the method. Our model to estimate the ruin probabilities in continuous and finite time is very flexible concerning the way that premiums change from year to year, the distributions of the claim amount, the number of claims, risk volumes, and the structural distribution. A major consideration in the development of this methodology is that it should be easily applicable to large portfolios, where the usual numerical approximation methods take too long to run. It is our belief that this model is very interesting from a practical point of view. It can be applied to several types of portfolios. In a practical environment we will have a portfolio with more risks, which would increase the running time, but we will have only one value for the initial surplus.

There are still some questions to be answered.

The fitted safety loading function $\zeta(u_{\tau_i}, \omega) = Au_{\tau_i}^B$ needs some improvement for large values of the surplus. One approach may be fitting two curves, one for low values and the other for high values of the surplus. Also in Chapter 5, since the actuary does not know the risk parameter for each risk we can not use for instance Theorem 12.4.1 of Bowers et al. (1997) to calculate the aggregate claim distribution of the portfolio. This way we do not apply the model using the safety loading depending on the surplus and on the ruin target $\zeta(u_{\tau_i}, \omega)$ when the premium was updated using the Bühlmann-Straub model. We could estimate θ_k for each risk based on the past experience but in that case we have to do it each year for each risk (in our examples $5 \times 10 \times 50\,000$ estimates for the risk parameter). Then we would have the estimate for the expected values of aggregate claim amounts and with that we could use the method in Section 3.1 formula (3.19). We would have to fit a different curve for each year and for each run. That would be, in our examples, $10 \times 50\,000$ fitted functions. Our goal was to have a simple formula for the safety loading not to increase the running time of the simulation procedure. But if we decided to use this approach would the running times continue to be acceptable?

One of the positive points of this model is that the running times do not depend on the portfolio size (λ). But what happens if we consider some more complicated models or if we apply our method to a real portfolio, for instance a Worker's Compensation portfolio. Can we obtain results within an acceptable period of time? Do the conclusions in Chapters 4 and 5 still hold?

Varying the Poisson parameter has a great impact on the ruin probabilities. We know that in a real portfolio we don't have a constant Poisson parameter throughout the period. We may also study some other ways of varying the Poisson parameter in order to have a more realistic approach to it.

We saw in Chapters 4 and 5 that the ruin probabilities by risk vary from around 0.05 to 1.0. This is due to the allocation of the initial surplus to the risks. What is the optimal allocation of initial surplus for each risk? We need some further research to answer this question.

We considered a very simple distribution for the risk parameter to illustrate numerically our model. We even chose the number of risks in order to have the distribution of Θ represented exactly in the portfolio. What if the risk parameter does not have precisely the structural distribution? Shall we expect the conclusions to hold?

We had some insight of what may be the causes of end of year ruin. But in some cases the within year ruin proportion of the ruin probability is very high. What may be the causes of the within year ruin, are they the same ones as the end of year ruin?

In our examples the ruin horizon (n) was fixed and was ten years. What would we expect if we had a different horizon time? We saw that end of year ruin occurred mainly in the first two years. Shall we expect that the ruin probabilities don't change much with the horizon period?

It is our opinion that the model presented in this thesis can be used as a starting point to take decisions on short term and can also be used to control and/or analyse the surplus process. It is very flexible and can be easily improved to be considered as a decision tool.

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