UNIVERSIDADE TÉCNICA DE LISBOA
INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

MESTRADO EM: ECONOMETRIA APLICADA E PREVISÃO

REAL BUSINESS CYCLES IN A SMALL OPEN ECONOMY: PORTUGAL 1960-2005

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Abstract

In this work after briefly reviewing the Real Business Cycles literature we summarize the main stylized facts of business cycles in the Euro-zone and consider some international dimensions of the business cycles. We then emphasize the stylized facts for the Portuguese economy. We consider a Stationary Small Open Economy Model and obtain an approximate solution by linearizing the Euler equations of the appropriately detrended real business cycle model around the non-stochastic steady state solution of the model. We then estimate the model using the Generalized Method of Moments with annual data from Portugal, 1960-2005, and compare the volatility of the macroeconomic aggregates generated by the model and the volatility of real data.

(Neste trabalho após uma breve revisão da literatura dos ciclos económicos reais sumariamos os principais factos estilizados dos ciclos económicos na zona Euro e consideramos algumas vertentes dos ciclos económicos internacionais, com ênfase em Portugal, respeitante a factos estilizados. Consideramos um modelo estacionário para uma pequena economia aberta cuja solução aproximada é obtida através da linearização das equações de Euler do modelo em torno da solução de equilíbrio de estado estacionário. É realizada a estimação por recurso ao Método dos Momentos Generalizado do modelo estacionário, para Portugal, com dados de periodicidade anual para os anos de 1960-2005 e comparada a volatilidade das variáveis macroeconómicas geradas pelo modelo estimado e a volatilidade dos dados reais.)

Keywords: Real Business Cycles, Small Open Economy, Generalized Method of Moments, Dynamic Stochastic General Equilibrium, Hodrick-Prescott Filter, Stylized Facts.

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1. Introduction

Business cycles refer to the recurrent fluctuations of national output around a growth trend. The qualitative features of these fluctuations are common to virtually all economies, with their quantitative properties differing somewhat across countries and time periods. Modern research seeks to summarize the statistical properties of business cycles and formally model them as the outcome of purposeful decisions by individuals and firms who react to government decisions and other variables beyond their immediate control. Whereas closed economy analysis focuses on domestic shocks and policy actions and their propagation over time through responses by consumers and firms, open economy analysis adds to this international policy interaction, foreign shocks and their propagation across national borders.

The United States has long been the focus of closed economy business cycle research, while major industrialized countries, such as the G7 - the U.S. plus Canada, France, Germany, Italy, Japan and the United Kingdom - have traditionally been the focus of open economy business cycle research.

This paper is organized as follows:

In the first sections of part one, 2.1-2.3, we summarize Real Business Cycles (RBC) and International Real Business Cycles (IRBC) theories.

In the last sections of part one, 3.1-3.2, we summarize the main features of business cycles in the Euro-zone. Then we use GDPs of the Euro-zone countries and consider some international dimensions of the business cycle. Finally we emphasize the stylized facts for the Portuguese economy.

In the second part of this paper we consider the neoclassical growth model with adjustment costs for a small open economy. We obtain approximate solutions by linearizing the Euler equations of the appropriately detrended real business cycle model around the nonstochastic steady state solution of the model. This solution method is the method suggested by King, Plosser and Rebelo (1988) which is useful when solving models which have no closed form solutions for the decision variables. We estimate the model using a variant of Hansen's (1982) Generalized Method of Moments as in Christiano and Eichenbaum (1992).

The second part of the paper is organized as follows. From section 4.1-4.4 we describe our basic model. In section 4.5 we solve the model for the steady state and linearize the Euler equations that characterize the competitive equilibrium around the steady state allowing us to write them as the solution of a linear quadratic stochastic optimization and solving it numerically as suggested in King, Plosser and Rebelo (1988). Section 4.6 describes our econometric methodology. In section 4.7 we present our empirical results. In section 4.8 we present some impulse response functions to explore the properties of the model. In section 4.9 we present some conclusions and hints for further research.

Part I

Business Cycles

2. A Survey of Business Cycles

2.1. Real Business Cycles

Real Business Cycles (RBC) are recurrent fluctuations in an economy's incomes, products and factors inputs that are due to nonmonetary sources. The term business cycles was used by Long and Plosser (1983) to describe cycles generated by random changes in technology. But RBC models also became a point of departure for many theories in which technology shocks do not play a central rule. Others real sources of fluctuations that have been studied include fiscal shocks, labor markets, tastes and terms of trade among others.

The breakdown of macroeconometric models in 1970s and the associated rational expectations revolution pioneered by Lucas set the stage for the interest in equilibrium RBC models.

Kydland and Prescott (1982) and Plosser (1983), suggested that one could build a successful business cycle model that involved market clearing, no monetary factors and no rationale for macroeconomic management. Kydland and Prescott (1982), who studied the quantitative predictions of a stochastic growth model with shocks to technology, found that covariances between model series where consistent with correspondent statistics for U.S. data.

According to Rebelo (2005), Kydland and Prescott introduced three revolutionary ideas:

The first idea is that business cycles can be studied using dynamic general equilibrium models; The second idea is that it is possible to unify business cycle and growth theory by insisting that business cycle models must be consistent with the empirical regularities of long-run growth; The third idea is that we can go way beyond the qualitative comparison of model properties with stylized facts that dominated theoretical work on macroeconomics until 1982.

During the 1980s and 1990s business cycle research was exploratory but methodological rooted.

Since the seminal work by Kydland and Prescott (1982), a number of papers have tested the neoclassical general-equilibrium models to account for economic fluctuations. The original work of Kydland and Prescott has been extended to include labor market rigidities, Hansen (1985), taxes and government expenditures, McGrattan (1994), money and inflation, Cooley and Hansen (1989), open economies, Backus, Kehoe and Kydland (1992), among others.

Kydland and Prescott (1982) and Plosser (1983) emphasize technology shocks as an important source of fluctuations. Greenwood, Hercovitz and Huffman (1988) also explore the rule of technology shocks for the business cycle but restrict attention to technology changes affecting the productivity of new capital goods and allow for acelareted depreciation of old capital. Christiano and Eichenbaum (1992), and McGrattan (1994) add fiscal shocks which are important for movement in hours and labor productivity.

The original technology driven business cycle models underpredicted fluctuations in observed hours and overpredicted the correlation between hours and productivity, leading to further investigations of the model of labor market and alternative mechanisms for propagation shocks. The most widely used mechanism of this kind was proposed by Rogerson (1988) and implemented by Hansen (1985) in a RBC model. Hansen (1985) finds a significant increase in hours fluctuations relative to Kydland and Prescott (1982). When labor-market search frictions are introduced as in Andolfato (1996), the empirical performance of RBC model is also improved.

Cooley and Hansen (1989) included monetary shocks and a cash in advance constraint and show negligible effects of these additions on business cycle predictions.

Benhabib, Rogerson and Wright (1991) and Greenwood and Hercovitz (1991) introduced home production. They show that business cycle predictions depend on the willingness and opportunity of households to substitute time in homework and market work.

A summary of more recent work is in King and Rebelo (1999) and Rebelo (2005).

Many theoretical and empirical articles have been used the RBC approach. The methods of RBC research are now commonly applied, being used in work in monetary economics, public finance; labor economics asset pricing international macroeconomics and trade. The dynamic stochastic general equilibrium model is established as the laboratory in which modern macroeconomic analysis is conducted.

2.2. International Real Business Cycles

The stylized facts on international business cycles are largely based on the seminal contributions of Backus, Kehoe and Kydland (1992). They build a two country dynamic general equilibrium model and compare the model's predictions concerning the cross correlations of macroeconomic aggregates with the data. They found that the one sector two-country international business cycle model generates negligible output comovement and consumption correlations near unity. In fact international consumption correlations are low, lower even than international income correlations. Productivity driven models of the business cycle tend to predict low or negative correlations of output and factor inputs across countries, whereas output and factor inputs tend to move together across countries. This is known as the international comovement puzzle. Baxter and Crucini (1995), Ambler, Cardia and Zimmermann (2002), Kehoe and Perri (2002) are a few examples of recent research that tackle empirical and/or theoretical dimensions of the puzzle.

The most part of general equilibrium open economy business cycle literature has considered models in which asset markets may trade any contingent claims they wish, Backus, Kehoe and Kydland (1992), and Baxter and Crucini (1993). In equilibrium individuals attain the optimal degree of consumption smoothing, and pool all idiosyncratic risks. Some studies modify the constraints on trading among agents and find that incomplete markets help to reduce the cross country correlation of consumption, but the cross correlations of output, investment and hours worked remain couterfactual negative, see Ambler, Cardia and Zimmermann (2004) for a survey on this puzzle.

Tesar (1991) and Baxter and Crucini (1993) address the Feldstein-Horioka Puzle. Feldstein and Horioka documented log run correlations between national savings and investment rates that were very close to unity. These correlations suggest an apparent low rate of international capital mobility despite the liberalization of capital controls in most countries.

2.3. Small Open Economies

Mendoza (1991) was one of the very first international RBC papers, and it deals with a small open economy (SOE); he includes shocks of the terms of trade in an international business cycle model and shows that responses of real exchange rates to productivity shocks and terms of trade shocks are different, both qualitatively and quantitatively. The standard small open economy model with incomplete markets features a steady state that depend on initial conditions. In addition, equilibrium dynamics posses a random walk component, Correia, Neves and Rebelo (1995). The random walk property of the dynamics implies that the unconditional variance of variables such asset holdings and consumption is infinite. To resolve this problem a number of modifications to removing the built in random walk of the canonical model was done. Smith-Grohé and Uribe (2003) compare the business cycle properties of five variations of the small open economy model: endogenous discount factor, debt-contingent interest rate premium, portfolio adjustment costs, complete asset markets and the non stationary case. They conclude that the models are similar in predicting volatility of main economic aggregates.

The small open economy literature, for example, Cardia (1991), Mendoza (1991), Correia, Neves and Rebelo (1995) and Smith-Grohé and Uribe (2003) typically assume that the only financial asset available to individuals is a risk free real bond. Individuals can engage in consumption smoothing, but not risk pooling. In line with previously obtained in a two country real business cycle models, Baxter and Crucini (1995) whether asset markets are complete or incomplete make no significant quantitative difference. Smith-Grohé and Uribe (2003) also conclude the same for SOE model.

2.4. Measurement of Business Cycles.

The basic element in dating business cycles are the calendar dates of peaks and troughs. A business cycle peak is the date at which the economy is in transition from an expansionary or boom phase to a contractionary or recession phase. The procedure by which business cycle peaks and troughs are determined is subjective and in the United States it is decided by the Business Cycle Dating Committee at National Bureau of Economic Research. In Europe the dating committee is affiliated with the Center for Economic Policy Studies. The basis of the methods used by both committees dates back to the classic contribution of Burns and Mitchell (1946) who identified peaks and troughs in the U.S. business cycle using industrial production data (which is available over a much longer historical period than Gross Domestic Product data).

The subjective dating-of-turning-points approach contrasts with the method used in most academic research which boils down to a statistical decomposition of output into a growth trend and a business cycle component. While the statistical method is easier to replicate, it is not entirely without subjectivity, since, as Quah (1990) notes there are an infinite set of alternatives.

Most macroeconomists adhere to the view that a business cycle decomposition should separate frequencies of the data that are typically attributed to business cycles from those associated with secular or growth trends. The most widely used method is the Hodrick-Prescott (1997), (HP) though the band-pass filters developed more recently by Baxter and King (1999), (BK) are becoming more common. One advantage of the latter approach is that is allows one to focus the cyclical variation in a range of frequencies that typify business cycles, 6 to 32 quarters in the U.S. historical context. Below we use the more common Hodrick-Prescott filter since it remains more widely applied.

Taking the logarithm of real gross domestic product as a reference variable for the cycle, the growth trend and business cycle components add up to the original series:

$$y_t = y_{g,t} + y_{c,t}$$

where $y_{g,t}$ is an estimate of the growth component, which is integrated of order 1 in most cases (i.e. for most macroeconomic aggregates in most countries grow over time) and $y_{c,t}$ is the stationary, cyclical, component, integrated of order 0.

Figure 2.1 displays the output (GDP) along with the Hodrick-Prescott growth trend for Portugal. The difference between the original series and the growth component is the business cycle component.

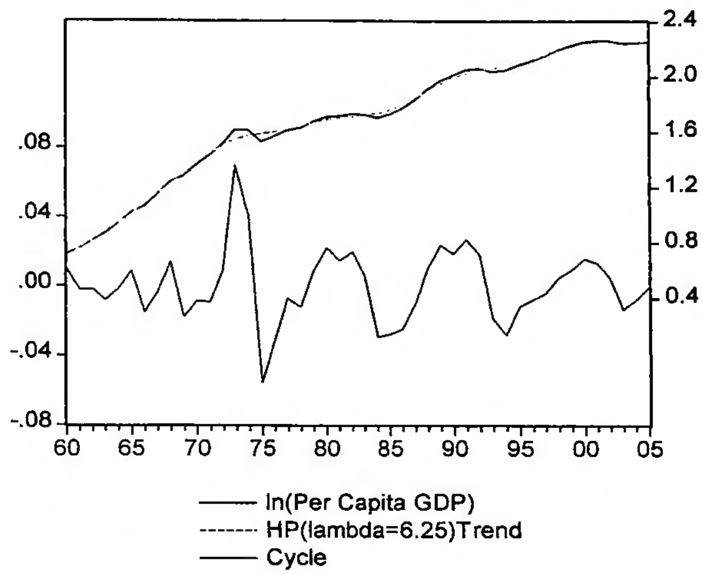
Most real quantities, such GDP in figure 2.1, grow through time. Hence the statistical measurement (computation of moments) of business cycles necessary involves same way of making the series stationary. Figure 2.2 shows the cyclical and non-cyclical components of Portuguese GDP based on $BK_3(2,8)$ filter. The $BK_3(2,8)$ passes frequencies between 2 and 8 years. We see in Figure 2.3 how the cyclical output measure produced by HP(λ=6.25) filter, compares with the $BK_3(2,8)$ filter. Both filters HP(6.25) and $BK_3(2,8)$ give identical detrended output. Figure 2.3 show some important shocks to the Portuguese economy and a regular pattern of the cycle on the last 30 years.[1] Figure 2.3 shows, that there is a very close correspondence between the cycles isolated by those filters when applied to Portuguese GDP. The HP(6.25) filter resembles an approximate high-pass filter designed to eliminate stochastic components with periodicity greater than eight years.

For annual data, the $BK_3(2,8)$ filter and the HP(6.25) filter generally yield similar results.[2] With annual data currently practice is to set $\lambda = 100$. However the value of smoothing parameter for annual data required to approximate

[1]On Appendix C we provide sampling statistics for HP(λ =6.25), $BK_3(2,8)$, $BK_3(2,10)$ and HP(λ =100) of Portuguese agreggates.

[2]See Ravn and Uhlig (2002)

Figure 2.1: Output, HP(6.25), Detrended and Trend



the HP($\lambda = 1600$) for quarterly data is $\lambda = 6.25$, Ravn and Uhlig (2002).

Higher values of lambda for annual data pass of the low-frequencies and variation of the series, and do not produce a filtered time series for GDP that resembles the produced by the $BK_3(2,8)$, rising the volatility, and persistence of filtered series, (see Appendix C) ≪ $\lambda = 100$ and $\lambda = 400$ filters pass trough nearly all components of the data which cycles between 9 and 16 years - components that most researchers would not identify as "business cycle" components≫ Baxter and King (1995, p.27).

Figure 2.2: Output, $BK_3(2,8)$, Non-Cyclical and Cyclical Components

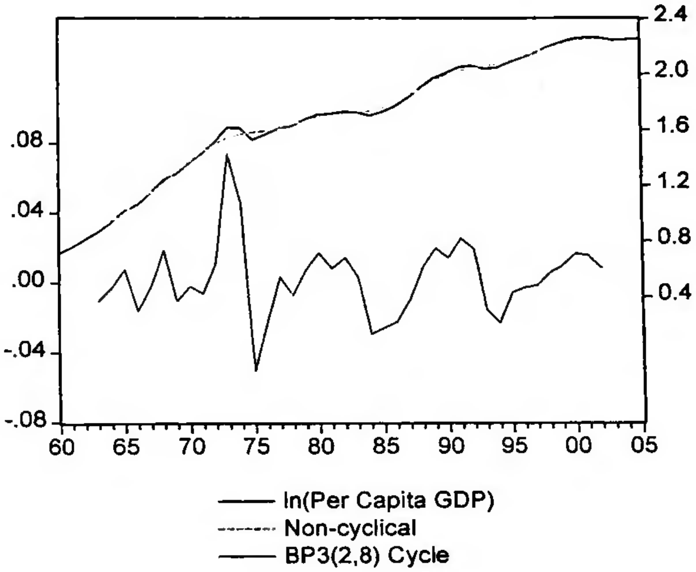
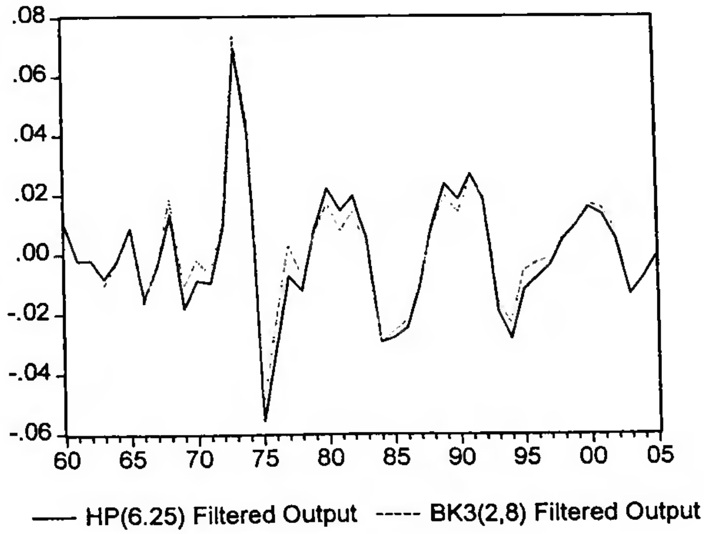


Figure 2.3: $HP(\lambda = 6.25)$ and $BK_3(2,8)$ Output Cyclical Components



3. Euro-zone Business Cycles: Stylized Facts, 1960-2005

Looking at literature on statistical properties of international business cycles we find that fluctuations across countries and across time periods display consistency in the key stylized facts.[3] Within-countries, consumption and investment tend to be strongly procyclical, with consumption less volatile then output(when durables are not included) and investment more volatile. Net exports are contercyclical. Across countries, cyclical movements in output tend to be positively correlated, as are cyclical movements in consumption, investment and labor input. Backus, Kehoe and Kydland (1992), Baxter (1995), Newmeyer and Perry (2003) provide a detailed discussion of these stylized facts and Correia, Neves and Rebelo (1995) for stylized facts within Portugal.

We organize our discussion of business cycle moments around two identities. The first identity links the business cycle reference variable (output) to it's expenditure components according to the conventions of national income accounting: The amount of output produced in the home country equals the sum of its uses in domestic private consumption and investment and government spending and exports. Imports from abroad are subtracted because they are not part of domestic production and imports account for some of the expenditure in each of the consumption, investment and government spending categories. The variables

[3]See Ambler, Cardia and Zimmermann (2002) for a diferente conclusion about these stylized facts.

are ordered in terms of the fraction of output accounted for on average (across countries and time) by each component. These ratios do not differ much across industrialized countries when long time averages are taken.

The second identity is the supply side and relates output with factor inputs.

We present results for the following macroeconomic aggregates: output, consumption purchases (including durables), investment (Gross Fixed Capital Formation), government purchases, exports, imports, net exports as a fraction of output, capital and employment for the Euro-zone countries. We also present results for capital and employment (hours, when data is available).

3.1. Data and Summary Statistics

Our analysis is performed on annual data from AMECO.[4] We applied the HP (λ=6.25) filter,[5] Hodrick and Prescott (1997), to the natural logarithm of all the variables (these variables are in real terms and converted to per capita quantities using the population series) with the exception of the ratio of net exports to output, which is measured in level form. List of variables are provided in Appendix A. The sample period goes from 1960 to 2005. The calculations were performed by posing the estimation problem as a generalized method of moments problem.[6]

[4]Anual MacroECOnomic data.

[5]Ravn and Uhlig (2002) sugested λ=6.25 for anual data.

[6]We used the Hansen-Heaton-Ogaki's (1992) GMM software for GAUSS. We also thank Wouter Denhaan and Andrew T. Levin for making available an alternative rats code to compute these statistics and alternative standard errors at http://ideas.repec.org/c/dge/qmrbcd/66.html

All the statistics were estimated by GMM and standard errors are reported in parentheses. On the within countries statistics we use a data set for each country. On the cross country correlations on each correlation we use all country panel for each aggregate.

3.2. Euro-zone Within-Countries Business Cycles

Tables 3.1, 3.2 and 3.3 summarize the point estimates of the most important second order moments of different macroeconomic variables.

We divided the Euro-zone countries in two groups, the G7 group and the Small Open Economies (SOE) group. Table 3.1 presents volatility statistics, defined as the standard deviation of each aggregate, measured as percent per year (except for net exports) and their standard errors are in parentheses for each country.

The volatility of output ranges between about one and two percent at business cycle frequencies. The G7 group shows an average volatility lower than the small open economies for all the aggregates. Portugal has the most variable business cycle, 2.7 per year while France has the least, 0.77.

Investment and trade flows are more volatile than output. Consumption is slightly less variable than output (on average) and government spending is least variable of expenditure (on average).

Table 3.1: Volatility in Euro-zone countries (percentage per year)

	$\sigma(y)$	$\sigma(c)$	$\sigma(i)$	$\sigma(g)$	$\sigma(x)$	$\sigma(m)$	$\sigma(\frac{NX}{Y})$	$\sigma(k)$	$\sigma(h)$
Austria	0.95	1.03	2.65	0.64	2.60	3.04	0.52	0.20	0.74
	(0.08)	(0.11)	(0.25)	(0.07)	(0.21)	(0.30)	(0.05)	(0.02)	(0.08)
Belgium	0.98	0.87	3.10	0.98	2.34	2.70	0.46	0.30	0.89
	(0.10)	(0.10)	(0.40)	(0.14)	(0.34)	(0.37)	(0.06)	(0.03)	(0.10)
Finland	1.82	1.80	5.36	1.12	3.75	4.56	1.02	0.52	1.50
	(0.22)	(0.26)	(0.70)	(0.15)	(0.60)	(0.54)	(0.12)	(0.06)	(0.22)
Greece	1.88	1.39	6.60	1.97	5.92	4.54	0.68	0.54	0.85
	(0.25)	(0.16)	(0.92)	(0.22)	(0.56)	(0.90)	(0.07)	(0.10)	(0.09)
Ireland	1.44	1.91	5.11	1.66	2.92	4.14	1.24	0.41	1.13
	(0.16)	(0.17)	(0.54)	(0.18)	(0.24)	(0.57)	(0.19)	(0.04)	(0.15)
Luxembourg	2.04	1.21	6.45	1.26	4.01	3.61	1.83	1.00	0.82
	(0.22)	(0.16)	(0.87)	(0.12)	(0.45)	(0.32)	(0.19)	(0.16)	(0.08)
Nederland	0.96	1.17	2.85	0.90	2.08	2.57	0.55	0.25	1.34
	(0.07)	(0.10)	(0.27)	(0.10)	(0.20)	(0.28)	(0.06)	(0.02)	(0.16)
Portugal	2.07	1.90	5.79	2.24	6.08	5.93	1.10	0.82	2.05
	(0.30)	(0.21)	(0.53)	(0.31)	(0.75)	(0.64)	(0.11)	(0.08)	(0.29)
Spain	1.04	1.07	3.34	1.08	3.17	4.55	0.62	0.43	1.29
	(0.10)	(0.11)	(0.39)	(0.14)	(0.38)	(0.40)	(0.06)	(0.04)	(0.16)
Mean	1.46	1.37	4.58	1.32	3.65	3.96	0.89	0.50	1.18
Median	1.44	1.21	5.11	1.12	3.17	4.14	0.68	0.43	1.13
Germany	1.45	1.23	3.09	0.95	3.92	3.48	0.79	1.21	0.76
	(0.23)	(0.12)	(0.27)	(0.11)	(0.90)	(0.44)	(0.09)	(0.32)	(0.10)
France	0.77	0.66	2.36	0.69	2.16	3.19	0.38	0.23	0.55
	(0.08)	(0.07)	(0.21)	(0.08)	(0.17)	(0.40)	(0.04)	(0.02)	(0.05)
Italy	1.12	1.21	3.41	0.68	2.68	4.07	0.73	0.36	1.02
	(0.11)	(0.12)	(0.41)	(0.06)	(0.29)	(0.51)	(0.08)	(0.04)	(0.12)
Mean	1.11	1.03	2.95	0.77	2.92	3.58	0.63	0.60	0.78
Median	1.12	1.21	3.09	0.69	2.68	3.48	0.73	0.36	0.76

Table 3.2: Volatility relative to own country output in Euro-zone countries

	$\sigma(c)/\sigma(y)$	$\sigma(i)/\sigma(y)$	$\sigma(g)/\sigma(y)$	$\sigma(x)/\sigma(y)$	$\sigma(m)/\sigma(y)$	$\sigma(\frac{NX}{Y})/\sigma(y)$	$\sigma(k)/\sigma(y)$	$\sigma(h)/\sigma(y)$
Austria	1.08	2.79	0.68	2.73	3.20	0.54	0.21	0.78
	(0.12)	(0.28)	(0.09)	(0.22)	(0.28)	(0.07)	(0.03)	(0.11)
Belgium	0.89	3.17	1.01	2.39	2.77	0.47	0.31	0.91
	(0.10)	(0.40)	(0.19)	(0.26)	(0.31)	(0.08)	(0.04)	(0.12)
Finland	0.98	2.94	0.61	2.06	2.50	0.56	0.28	0.82
	(0.07)	(0.22)	(0.10)	(0.41)	(0.24)	(0.07)	(0.04)	(0.07)
Greece	0.74	3.52	1.05	3.16	2.42	0.36	0.29	0.46
	(0.10)	(0.31)	(0.18)	(0.46)	(0.37)	(0.06)	(0.04)	(0.08)
Ireland	1.33	3.55	1.15	1.96	2.87	0.86	0.28	0.79
	(0.17)	(0.40)	(0.19)	(0.27)	(0.51)	(0.17)	(0.03)	(0.08)
Luxembourg	0.59	3.17	0.62	1.97	1.77	0.90	0.49	0.40
	(0.09)	(0.53)	(0.09)	(0.16)	(0.18)	(0.07)	(0.09)	(0.06)
Nederland	1.22	2.96	0.93	2.16	2.67	0.57	0.26	1.39
	(0.11)	(0.27)	(0.13)	(0.21)	(0.26)	(0.08)	(0.02)	(0.20)
Portugal	0.92	2.79	1.08	2.93	2.86	0.53	0.40	0.99
	(0.15)	(0.32)	(0.21)	(0.32)	(0.31)	(0.09)	(0.05)	(0.20)
Spain	1.03	3.20	1.03	3.04	4.36	0.60	0.41	1.23
	(0.09)	(0.28)	(0.14)	(0.46)	(0.42)	(0.08)	(0.04)	(0.16)
Mean	0.98	3.12	0.91	2.49	2.82	0.60	0.33	0.86
Median	0.98	3.17	1.01	2.39	2.77	0.56	0.28	0.82
Germany	0.85	2.13	0.65	2.70	2.40	0.54	0.83	0.53
	(0.09)	(0.32)	(0.12)	(0.31)	(0.24)	(0.07)	(0.17)	(0.09)
France	0.86	3.05	0.89	2.78	4.12	0.49	0.29	0.71
	(0.08)	(0.25)	(0.13)	(0.20)	(0.35)	(0.05)	(0.04)	(0.07)
Italy	1.07	3.04	0.60	2.38	3.62	0.65	0.32	0.91
	(0.11)	(0.36)	(0.09)	(0.30)	(0.37)	(0.09)	(0.05)	(0.13)
Mean	0.93	2.74	0.71	2.62	3.38	0.56	0.48	0.72
Median	0.86	3.04	0.65	2.70	3.62	0.54	0.32	0.71

There are some differences in the relative volatility of each component across countries, but the ranking across them are robust. On the factor inputs side capital is less cyclically variable than labor input (exceptions are Luxembourg and Germany). Table 3.3 shows the correlations of output and other variables and indicates the cyclicality of a variable. If the correlation is positive, the variable is said to be procyclical: on average it rises when the economy is in a boom and falls when the economy moves into a recession. All expenditure components except government spending are strongly procyclical, consumption investment and imports particularly so. In a statistical sense government spending is acyclical (except for Portugal and Spain). Imports are consistently more highly correlated with domestic output than are exports. Import demand is strongly influenced by domestic income, thus the strong correlation between the two. Exports of a country are influenced by the demands of various countries for the goods a country exports and since international business cycles are far from perfectly correlated across countries,[7] exports tend to be less correlated with the domestic cycle, consequently net exports are countercyclical.

[7]See next section for some evidence on this point.

Table 3.3: Correlation with own country output in Euro-zone countries

	y	c	i	g	x	m	$\sigma(\frac{NX}{Y})$	k	h
Austria	1	0.66	0.67	0.17	0.58	0.66	-0.23	0.41	0.19
		(0.08)	(0.08)	(0.14)	(0.11)	(0.09)	(0.14)	(0.13)	(0.11)
Belgium	1	0.70	0.68	0.07	0.78	0.73	-0.12	0.30	0.41
		(0.07)	(0.07)	(0.11)	(0.08)	(0.08)	(0.14)	(0.14)	(0.14)
Finland	1	0.93	0.84	0.22	0.31	0.86	-0.57	0.27	0.75
		(0.02)	(0.05)	(0.17)	(0.15)	(0.03)	(0.10)	(0.17)	(0.08)
Greece	1	0.64	0.71	-0.18	0.33	0.54	-0.11	0.41	-0.02
		(0.09)	(0.11)	(0.13)	(0.14)	(0.14)	(0.16)	(0.22)	(0.13)
Ireland	1	0.64	0.67	0.06	0.39	0.47	0.21	0.43	0.58
		(0.08)	(0.09)	(0.13)	(0.11)	(0.10)	(0.10)	(0.13)	(0.12)
Luxembourg	1	0.50	0.48	0.12	0.82	0.67	0.44	0.39	0.31
		(0.10)	(0.09)	(0.15)	(0.06)	(0.11)	(0.16)	(0.12)	(0.13)
Nederland	1	0.76	0.72	-0.05	0.70	0.78	-0.36	0.59	0.27
		(0.07)	(0.07)	(0.13)	(0.07)	(0.05)	(0.11)	(0.11)	(0.13)
Portugal	1	0.49	0.72	0.30	0.62	0.80	-0.29	0.54	0.33
		(0.13)	(0.08)	(0.15)	(0.11)	(0.06)	(0.15)	(0.11)	(0.11)
Spain	1	0.86	0.78	0.46	0.09	0.72	-0.53	0.23	0.70
		(0.03)	(0.07)	(0.12)	(0.15)	(0.07)	(0.08)	(0.18)	(0.06)
Mean		0.69	0.70	0.13	0.51	0.69	-0.17	0.40	0.39
Median		0.66	0.71	0.12	0.58	0.72	-0.23	0.41	0.33
Germany	1	0.81	0.72	0.07	0.69	0.83	0.12	0.65	0.67
		(0.05)	(0.06)	(0.18)	(0.13)	(0.05)	(0.24)	(0.11)	(0.10)
France	1	0.71	0.83	-0.24	0.73	0.75	-0.36	0.32	0.69
		(0.09)	(0.05)	(0.14)	(0.07)	(0.07)	(0.14)	(0.12)	(0.08)
Italy	1	0.73	0.82	-0.00	0.21	0.77	-0.45	0.31	0.54
		(0.06)	(0.04)	(0.13)	(0.16)	(0.06)	(0.11)	(0.13)	(0.09)
Mean		0.75	0.79	-0.06	0.54	0.78	-0.23	0.43	0.63
Median		0.73	0.82	-0.00	0.69	0.77	-0.36	0.32	0.67

3.3. International Dimensions of Business Cycles

Two key aspects of international business cycle research distinguish it from closed economy counterpart. The first is the international business cycle comovment and the second is relative price determination. International business cycle comovement refers to the relation of output cycle in one country to that of another country or group of countries. The relative prices that are most studied in the literature are the real exchange rate, defined as the ratio of foreign to domestic price levels in a common currency and the terms of trade, the exchange rate between a nation's imports and its exports. Questions ranging from the structure of international asset markets, transmission of technology coordination of fiscal and monetary policy are subjects of analysis of business cycle comovment and relative price determination.

3.3.1. Euro-zone Cross-Country Business Cycles

To measure international comovement we use the correlation of a foreign variable with it's Portugal counterpart.[8]

Table 3.4 show the pattern of international comovment in macro aggregates in Euro-zone. It gives the correlation of each Portuguese aggregate with the corresponding aggregate in other Euro-zone countries at business cycle frequencies.

[8]See Kose and Witeman (2004) for alternative forms of mesuring international business comovement.

Stock and Watson (2003) analyzing the G7 countries GDPs concerning to synchronization of business cycles refer that there appears to have been an emergence of one cyclical coherent group, the G7 countries in the Euro-zone, after the 1970s.

Table 3.4: Cross correlation with same Portuguese variable

	$y_t,,y_t^*$	$c_t,,c_t^*$	$i_t,,i_t^*$	$g_t,,g_t^*$	$x_t,,x_t^*$	$m_t,,m_t^*$	$ne_t,,ne_t^*$	$k_t,,k_t^*$	n,n_t^*
Austria	0.57	0.25	0.40	-0.08	0.35	0.31	-0.16	0.27	0.12
	(0.09)	(0.11)	(0.12)	(0.13)	(0.12)	(0.17)	(0.14)	(0.15)	(0.14)
Belgium	0.62	0.39	0.28	-0.35	0.46	0.53	0.05	0.33	0.40
	(0.10)	(0.13)	(0.10)	(0.17)	(0.14)	(0.13)	0.17()	(0.13)	(0.15)
Finland	0.36	0.03	0.38	0.24	0.40	0.32	-0.08	0.25	0.48
	(0.13)	(0.12)	(0.10)	(0.14)	(0.13)	(0.11)	(0.10)	(0.11)	(0.14)
Greece	0.29	-0.17	0.19	-0.03	0.31	0.55	-0.02	0.42	0.24
	(0.22)	(0.18)	(0.17)	(0.09)	(0.14)	(0.10)	(0.12)	(0.16)	(0.17)
Irland	0.36	0.12	0.34	0.10	0.16	0.47	-0.07	0.59	0.62
	(0.11)	(0.14)	(0.14)	(0.09)	(0.14)	(0.13)	(0.16)	(0.09)	(0.11)
Luxembourg	0.51	0.33	0.50	0.07	0.51	0.45	0.10	0.17	0.38
	(0.13)	(0.08)	(0.14)	(0.12)	(0.09)	(0.11)	(0.18)	(0.16)	(0.13)
Nederlands	0.40	-0.17	0.11	0.02	0.59	0.34	0.08	0.49	0.28
	(0.12)	(0.14)	(0.16)	(0.14)	(0.12)	(0.15)	(0.15)	(0.08)	(0.12)
Spain	0.52	0.45	0.54	0.13	0.51	0.43	0.36	0.49	0.57
	(0.10)	(0.11)	(0.09)	(0.16)	(0.14)	(0.12)	(0.12)	(0.11)	(0.09)
Average	0.47	0.15	0.34	0.01	0.41	0.43	0.03	0.38	0.39
Germany	0.33	-0.26	0.34	-0.02	0.16	0.19	-0.03	-0.06	0.48
	(0.14)	(0.16)	(0.14)	(0.20)	(0.19)	(0.12)	(0.16)	(0.07)	(0.13)
France	0.70	0.14	0.63	-0.13	0.47	0.59	0.24	0.55	0.55
	(0.08)	(0.18)	(0.07)	(0.16)	(0.09)	(0.11)	(0.14)	(0.08)	(0.10)
Italy	0.49	0.24	0.13	0.17	0.20	0.25	0.21	0.59	0.28
	(0.12)	(0.11)	(0.12)	(0.09)	(0.17)	(0.20)	(0.12)	(0.09)	(0.11)
Average	0.51	0.04	0.37	0.02	0.28	0.34	0.14	0.36	0.44

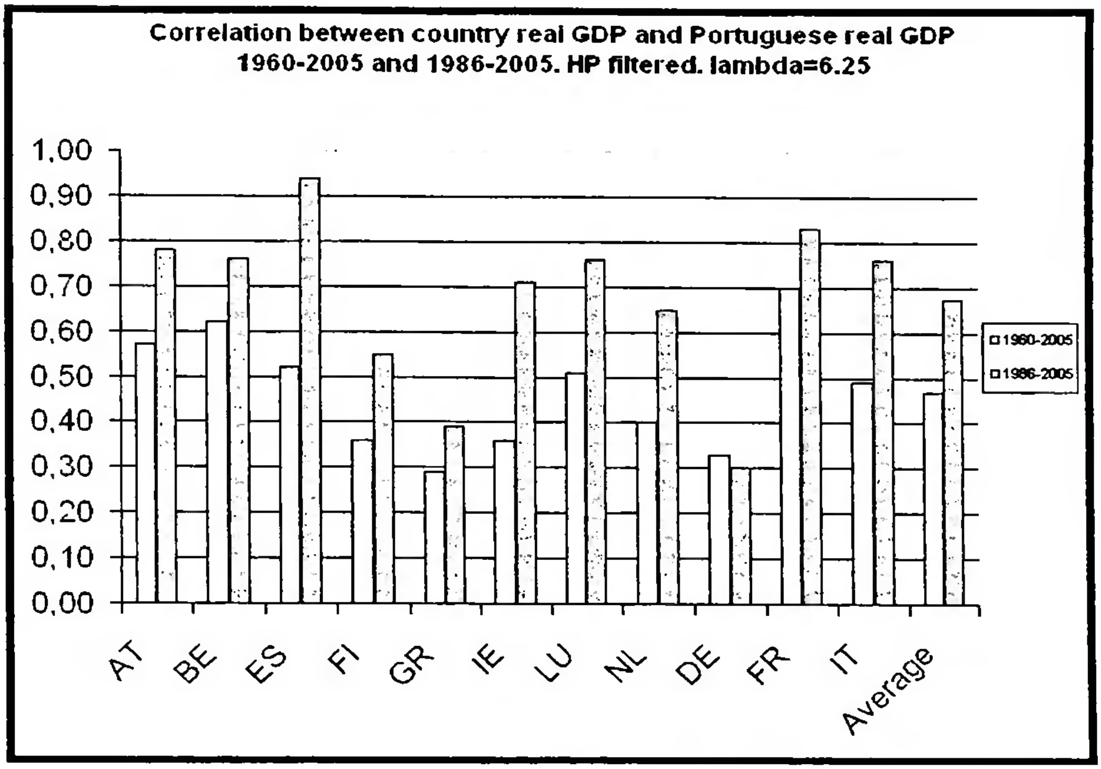
Table 3.4 show that outputs move together, the correlation of every country's output with the Portuguese output is positive and quite substantial for several countries. Consumption (on average) also tend to be slightly positive correlated across countries, but the correlation is smaller. There is no country in our sample for which consumption correlation exceeds the output correlation. Investment also tend to be positively correlated across countries as do employment. Government spending and net exports have low international correlation.[9] Even though exports and imports are positively correlated net exports exhibit much lower correlation across countries, which some countries showing a negative correlation between their net exports and those of Portugal.

On the supply side the capital correlations ranges from 0.17 to 0.59 and employment ranges from 0.12 to 0.57 for SOE.

Figure 3.1 shows the correlation between detrended GDP in Portugal and the remaining countries in the Euro-zone for 1960-2005 and 1986-2005 periods. The average correlation for 1960-2005 is 47 percent. There is a significant comovement, above the average, between Portugal and France, Belgium, Austria, Luxembourg and Spain. All countries exhibit a positive correlation. The figure also shows information on correlations between detrended GDP in Portugal and the remaining

[9]The ranking is more ambigous in a statistical sense and for a broadrer range of countries table 3.4 suggestes. See Ambler, Cardia and Zimmermann (2002) for a more complete review of international business cycles.

Figure 3.1: Euro-zone Detrended Output Correlations



countries in the Euro-zone for the period 1986-2005. Still the comovement between Portugal and France, Belgium, Austria, Luxembourg and Spain are above the average as well as Italy and Ireland. There is an increase in all the correlations for this period except for Germany (Germany was reunified in 1991). The correlation of Portugal and Spain for this period increased a lot (0.91) as well as with Ireland. Portugal and Spain became a member of EEC on 1986. All the tree countries also benefit from Coesion's Fund. The creation of the Eurozone and the Stability and Growth Pact also contributed for a reduction of the volatility and a higher correlation between those countries. A more detailed analysis of the correlation of the components of expenditure and of production would provide insight on this comovement. During the period 1986-2005 the volatility of the detrended output also decreased shown by the standard deviations of detrended output (not reported) and there was a tendency towards increasing international synchronization of cyclical fluctuations on G7 Euro-zone countries as mentioned before.

3.4. The Portuguese Economy: 1960-2005

We apply the HP(λ =6.25), HP(λ =100), $BK_3(2,8)$, and $BK_3(2,10)$ filters to produce the cyclical components for the main Portuguese macroeconomic variables. The results are in appendix C.[10] The results of $BK_3(2,8)$, $BK_3(2,10)$ and HP(λ =6.25) filters are similar.

[10]Standard errors computed by alternative methos are also provided, see Appendix C.

Table 3.5: Moments of macroeconomic variables HP(λ=6.25) detrended. Portuguese data: 1960-2005

	$\sigma(x)$	$\sigma(x)/\sigma(y)$	$\rho(x_t, x_{t-1})$	$\rho(y_t, x_{t-1})$	$\rho(y_t, x_t)$	$\rho(y_t, x_{t+1})$
y	2.08 (0.30)		0.41 (0.17)	0.41 (0.17)	1.00	0.41 (0.17)
c	1.90 (0.21)	0.92	0.30 (0.15)	-0.19 (0.24)	0.49 (0.13)	0.63 (0.17)
i	5.79 (0.53)	2.79	0.47 (0.13)	0.52 (0.16)	0.72 (0.08)	0.38 (0.14)
g	2.24 (0.31)	1.08	-0.07 (0.23)	-0.06 (0.16)	0.30 (0.15)	0.33 (0.15)
x	6.08 (0.75)	2.93	0.36 (0.16)	0.47 (0.18)	0.62 (0.11)	0.17 (0.15)
m	5.93 (0.64)	2.86	0.27 (0.23)	0.35 (0.22)	0.80 (0.06)	0.39 (0.21)
NX/Y	1.10 (0.11)	0.53	0.11 (0.14)	0.10 (0.18)	-0.29 (0.15)	-0.29 (0.16)
k	0.82 (0.09)	0.40	0.56 (0.16)	-0.10 (0.21)	0.54 (0.11)	0.68 (0.16)
h	2.05 (0.29)	0.99	0.30 (0.15)	0.15 (0.15)	0.33 (0.11)	0.37 (0.18)

Our results show lower volatility and persistence that the ones reported in Correia, Neves and Rebelo (1995). The differences are due basically to the filtering, since when we use the HP(λ=100) our statistics are identical to theirs even though there is some evidence for lower volatility on last decade. Table 3.5 show that the Portuguese cyclical fluctuations conform with cyclical fluctuations showed by the other countries. Consumption investment and exports show some degree of persistence. Investment is tree times more volatile than output, consumption is as volatile as output , while the ratio of net exports to output is less volatile than output. The most macroeconomic variables are procyclical. They exhibit a positive contemporaneous correlation with output with exception of net exports which is countercyclical. Private consumption, capital and hours cycles lag output cycle. Investment, exports, and imports cycles are coincident with output cycle.

Figures 3.2 to 3.7 provide graphs of the HP(λ =6.25) business cycle components of Portuguese aggregates. We use the cyclical component of output as a reference variable, placing it in each panel of each figure.[11]

[11] Summary statistics for selected series are provided in Table 3.5 for volatility, relative volatility, persistence, correlation of each variable with output and comovements of output with other aggregates.

Figure 3.2: Consumption and Output

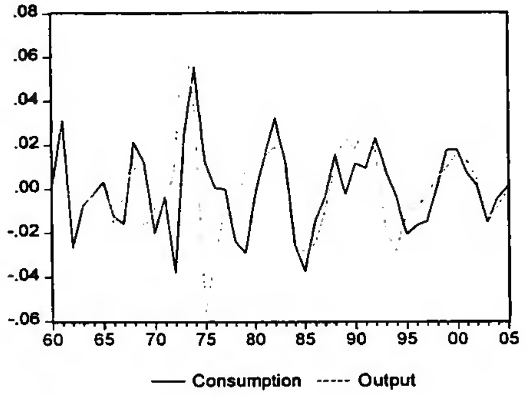


Figure 3.3: Investment and Output

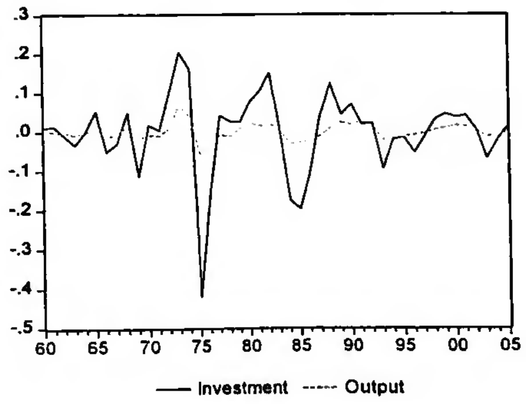


Figure 3.4: Government Spending and Output

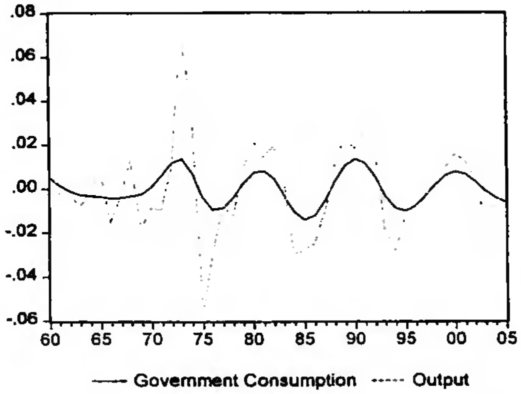


Figure 3.5: Net Exports and Output

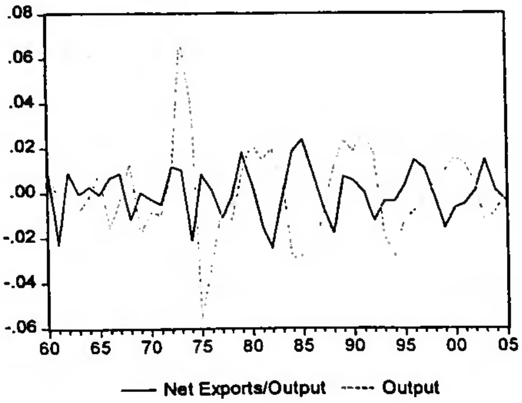


Figure 3.6: Total Hours and Output

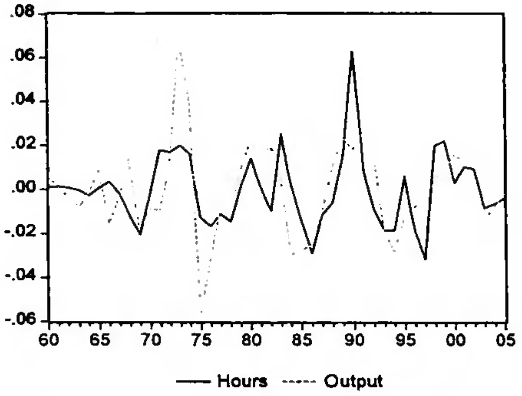


Figure 3.7: Capital and Output

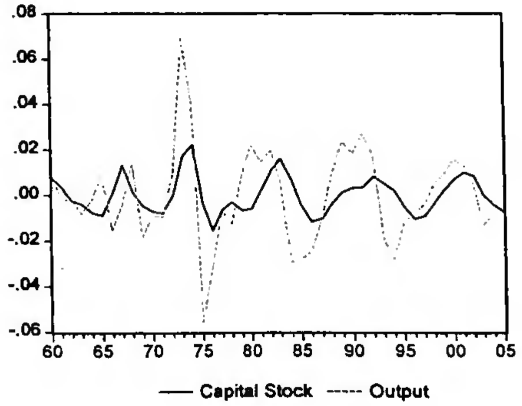


Table 3.6: Volatility of Portuguese Terms of Trade

$\sigma(y)$	$\sigma(NE/Y)$	$\sigma(TOT)$
0.021	0.011	0.023
(0.003)	(0.001)	(0.003)

Table 3.7: Cross - correlations of Portuguese Terms of Trade

	$\rho(y_t, NE_{t-1}/Y_{t-1})$	$\rho(y_t, NE_t/Y_t)$	$\rho(y_t, NE_{t+1}/Y_{t+1})$
Output and	0.10	-0.29	-0.29
Net Exports	(0.18)	(0.15)	(0.16)
	$\rho(y_t, TOT_{t-1})$	$\rho(y_t, TOT_t)$	$\rho(y_t, TOT_{t+1})$
Output and	0.20	0.33	-0.04
Terms of Trade	(0.17)	0.16	(0.23)
	$\rho(NE_t/Y_t, TOT_{t-1})$	$\rho(NE_t/Y_t, TOT_t)$	$\rho(NE_t/Y_t, TOT_{t+1})$
Terms of Trade	-0.02	0.09	0.19
and Net Exports	(0.16)	(0.15)	(0.16)

3.4.1. Relative prices

On tables 3.6 and 3.7 we present some statistics for Terms of Trade (price of exports over price of imports) for the Portuguese economy. While we se a positive correlation between output and terms of trade, net exports are not correlated to terms of trade at business cycle frequencies.

3.5. Some Stylized Facts of Economic Growth

While the Portuguese time series for many aggregates grow over time, there are many "great ratios " that appear to be relatively constant. Figures 3.8 to 3.12 illustrate that the process of sustained growth appears to leave many of the shares of income components and output unaffected. Stability of the great ratios implies that most series have a similar rate of growth, so there is no deterministic trend in the ratios, and the factors causing permanent changes in the level of economic activity do so that makes their effects proportional across series.

The ratios of investment to output, trade balance to output capital to output appear to fluctuate around constant means. The ratio of consumption to output does decrease from 1960 onwards and there is nothing the large trend that we saw in output in figure 2.1. Government consumption is the exception.

Over the period 1986-2005 there is no evidence of a trend in hours worked per person.

Figure 3.8: Consumption-Output Ratio

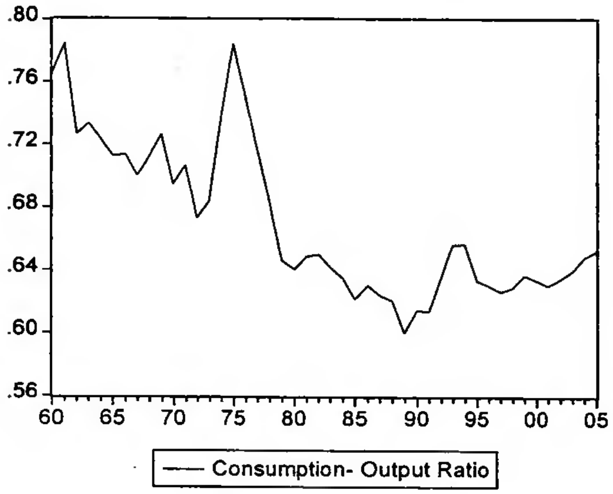


Figure 3.9: Investment-Output Ratio

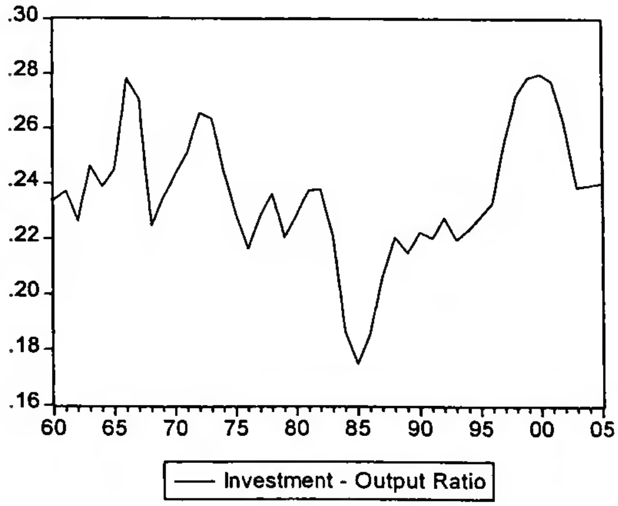


Figure 3.10: Government Consumption-Output Ratio

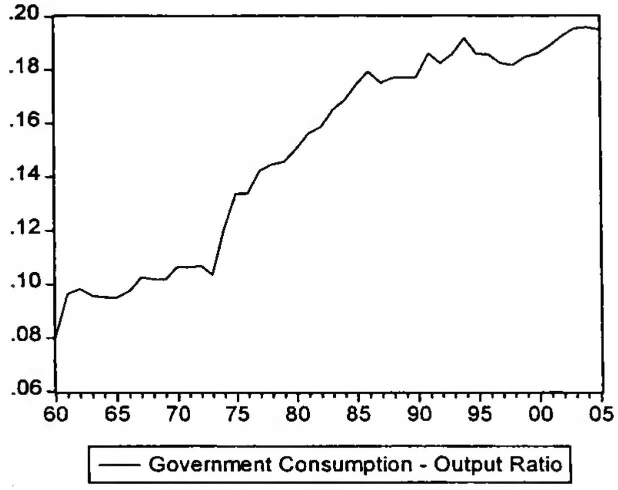


Figure 3.11: Trade Balance-Output Ratio

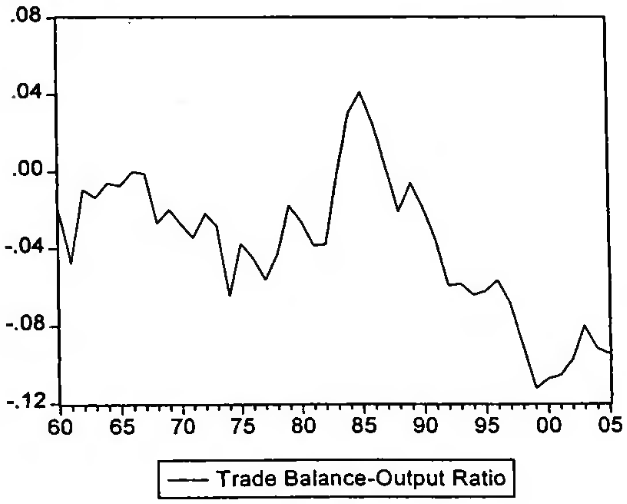


Figure 3.12: Capital-Output Ratio

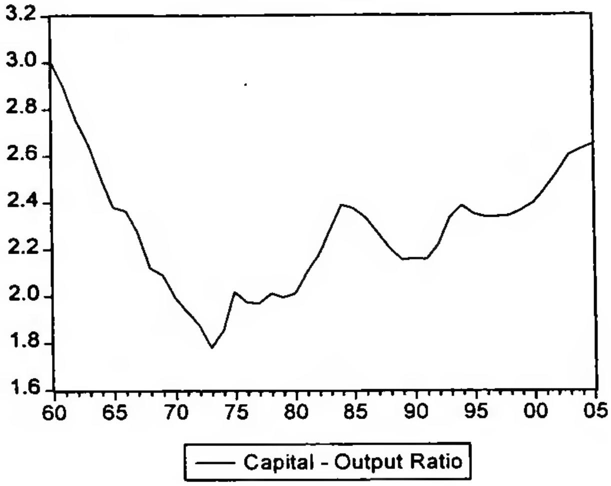
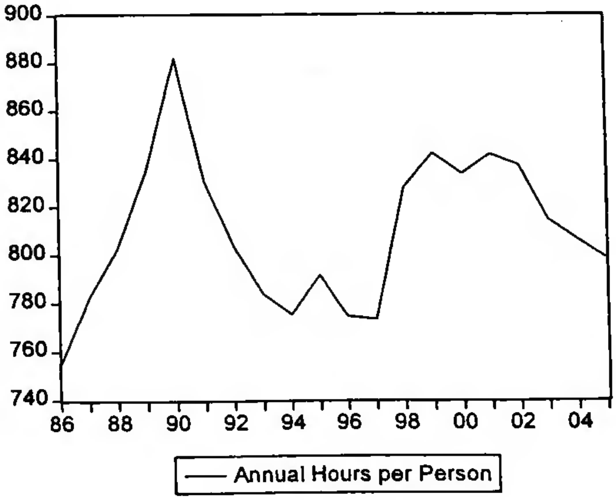


Figure 3.13: Hours per Person



Part II

A dynamic small open economy model

4. A Stationary Small Open Economy Model

The standard Small Open Economy (SOE) model with incomplete markets features a steady state that depend on initial conditions. In addition, equilibrium dynamics posses a random walk component, Correia, Neves and Rebelo (1995). The random walk property of the dynamics implies that the unconditional variance of variables such asset holdings and consumption is infinite. These endogenous variables in general wonder around an infinitely large region in response to bounded shocks. This introduces serious computational difficulties because all the available techniques are valid locally around a stationary path. To resolve this problem a number of modifications to removing the built in random walk of the canonical model was done. Smith-Grohé and Uribe (2003) compare the business cycle properties of five variations of the small open economy model: endogenous discount factor, debt-contingent interest rate premium, portfolio adjustment costs, complete asset markets and the non stationary case. They conclude once the five models share the same calibration, their quantitative predictions regarding the behavior of key macroeconomic variables, as measured by unconditional second moments and impulse response functions, is virtually identical.

Our model is based on the neoclassical model of optimal capital accumulation extended to an open economy with capital and portfolio adjustment costs and augmented by technology and government shocks. The economy is populated by a large number of identical agents who act as price takers in the various markets in which they interact. All the variables are in per capita terms. There are several alternative ways of describing the decentralized economy in such a way that there is an equivalence between the social planner problem and the decentralized economy. The structure of the economy is the following: Firms own the capital stock and choose their investment plans and the quantity of labor that they employ in order to maximize their value. Households choose optimally their supply of labor and their savings, which are allocated between foreign bonds which yield a rate of return r_t^*, and equity shares in the representative firm. This structure involves a spot labor market and a stock market.

4.1. Preferences

Each agent seeks to maximize his expected utility defined over random sequences of consumption (C_t) and leisure ($1 - N_t$):

$$U = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} [(C_t - \psi X_t N_t^{\nu})^{1-\sigma} - 1] \right\}, \qquad \sigma>0,\ \beta>0,\ \nu>1,\ \psi>0.$$

E_0 denotes the expectation based on the information set available at time zero (which includes current and lagged values of all variables), β is a constant discount factor and X_t represents the level of technical progress and σ, ν and ψ are parameters. This utility function was proposed by Greenwood, Hercowitz and Huffman (1988), has the property that the elasticity of intertemporal substitution associated with leisure is zero, and have been used in open economy models by Mendoza (1991), Correia, Neves and Rebelo (1995) among others. According to Correia, Neves and Rebelo (1995) the adoption of this utility function is fundamental to the good performance of the model of Mendoza (1991) and their model as well.

4.2. Technology

Output (Y_t) is produced by combining labor (N_t) and capital (K_t) according to a Cobb-Douglas production function:

$$Y_t = A_t K_t^{1-\alpha} (X_t N_t)^{\alpha}, \qquad 0<\alpha<1.$$

The quantity $X_t N_t$ is usually referred to as effective labor units. The level of output is influenced by productivity disturbances (A_t) and by the level of technological progress (X_t). We assume that $\ln(A_t)$ follows an AR(1) process and that X_t grows at a constant rate:

$$\ln(A_t) = (1 - \rho_a)\ \ln(A) + \rho_a\ \ln(A_{t-1}) + \varepsilon_{at},$$

$$X_{t+1} = \gamma X_t \qquad \gamma > 1.$$

The unconditional mean of $\ln(A_t)$ equals $\ln(A)$, $|\rho_a| < 1$, and ε_{at} is the innovation to $\ln(A_t)$ with a standard deviation $\sigma_{\varepsilon a}$. Output can be used for private consumption (C_t), for investment (I_t), and for government consumption (G_t). The difference between output and domestic absorption ($C_t + I_t + G_t$) is the trade balance (TB_t):

$$Y_t = C_t + I_t + G_t + TB_t.$$

Domestic investment increases the stock of capital (K_t) which evolves according to:

$$K_{t+1} = (1 - \delta)K_t + (\tfrac{I_t}{K_t})^\tau K_t,$$

where δ is the rate of depreciation. SOE models typically include capital adjustment costs to avoid excessive investment volatility in response to variations in the domestic-foreign interest rate differential. $(\frac{I_t}{K_t})^\tau K_t$ is an adjusting cost of capital function and τ is a parameter.

4.3. Government Policy

Government expenditures (G_t) are viewed as exogenous from the standpoint of the private sector and are financed with lump sum taxes. We assume that

$$G_t = \gamma^t g_t, \qquad \text{where } g_t \text{ has the following law of motion:}$$

$$\ln(g_t) = (1-\rho_g)\ \ln(g) + \rho_g\ \ln(g_{t-1}) + \varepsilon_{gt}.$$

Here $\ln(g)$ is the unconditional mean of $\ln(g_t)$, $|\rho_g|<1$, and ε_{gt} is the innovation to $\ln(g_t)$ with a standard deviation $\sigma_{\varepsilon g}$.

4.4. International Borrowing and Lending

Agents in this economy can buy and sell foreign bonds in the international capital market. The level of net holdings of bonds (B_t) evolves according to:

$$B_{t+1} = R_t^* B_t - \frac{\xi}{2}\frac{(B_{t+1}-\bar{B})}{Y}^2 + TB_t,$$

following Neumeyer and Perri (2001), we introduce portfolio adjustment costs to induce stationarity.[12] $\frac{\xi}{2}\frac{(B_{t+1}-\bar{B})}{Y}^2$ represent a quadratic cost of holding a quantity of bonds different from a long run level $\bar{B}$ (that will determine the steady

[12]This is needed because otherwise the model has multiple steady states and bond holdings are not a stationary variable. Dividing the cost by steady state GDP implies the long run cost grows at the same rate of others variables in the economy.

state debt) and ξ and $\bar{B}$ are constant parameters defining the portfolio adjustment cost function. The interest rate, $r^*_{t,}$ at which households can borrow from the rest of the world is assumed constant, $R^*_t = 1 + r^*_t = 1 + r^*$.

To rule out the possibility of the economy playing a Ponzi game in the international capital markets we also assume:

$$\lim_{t \to \infty} \mathrm{E}(B_{t+1}/R^*_t) = 0$$

and we also assume that the following restriction involving the international real rate of return, hold: $\beta R^* = \gamma^\sigma$, since without this assumption a deterministic version of the model has unappealing asymptotic properties.

4.5. The Competitive Equilibrium: Incomplete markets

Considering the social planner problem which solution is equivalent to the equilibrium of our decentralized competitive economy:

$$\max U = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} [(C_t - \psi X_t N_t^{\nu})^{1-\sigma} - 1] \right\}$$

s.t.

$$C_t + I_t + G_t + B_{t+1} + \frac{\xi}{2} \frac{(B_{t+1} - \bar{B})^2}{Y} - R^* B_t = A_t K_t^{1-\alpha} (X_t N_t)^{\alpha}$$

$$K_{t+1} = (1-\delta) K_t + (\frac{I_t}{K_t})^{\tau} K_t$$

Setting up the Lagrangean for this problem we get:

$$\mathcal{L} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \frac{1}{1-\sigma} [(C_t - \psi X_t N_t^{\nu})^{1-\sigma} - 1] + \Lambda_{1t} [A_t K_t^{1-\alpha} (X_t N_t)^{\alpha} - C_t - I_t - G_t - B_{t+1} - \frac{\xi}{2} \frac{(B_{t+1} - \bar{B})^2}{Y} + R^* B_t] + \Lambda_{2t} [-K_{t+1} + (1-\delta) K_t + (\frac{I_t}{K_t})^{\tau} K_t] \}$$

The efficiency conditions for this problem are:

$$\frac{\partial \mathcal{L}}{\partial C_t} = (C_t - \psi X_t N_t^{\nu})^{-\sigma} - \Lambda_{1t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = \nu \psi X_t N_t^{\nu - 1} (C_t - \psi X_t N_t^{\nu})^{-\sigma} + \Lambda_{1t} \alpha A_t K_t^{1-\alpha} (X_t N_t)^{\alpha - 1} X_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial I_t} = -\Lambda_{1t} + \Lambda_{2t} \tau (\frac{I_t}{K_t})^{\tau - 1} = 0$$

$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = -\Lambda_{2t} + \beta\, \mathrm{E}_t \left\{ \Lambda_{1t+1}[(1-\alpha)A_{t+1}K_{t+1}^{-\alpha}(X_{t+1}N_{t+1})^{\alpha}] + \Lambda_{2t+1}[(1-\delta)+(1-\tau)(\frac{I_{t+1}}{K_{t+1}})^{\tau}] \right\} = 0$

$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = -\Lambda_{1t}\left[1 + \xi \frac{\mathrm{E}_t(B_{t+1}-\bar{B})}{Y}\right] + \beta R^{*}\mathrm{E}_t\{\Lambda_{1t+1}\} = 0$

$\frac{\partial \mathcal{L}}{\partial \Lambda_{1t}} = A_t K_t^{1-\alpha}(X_t N_t)^{\alpha} - C_t - I_t - G_t - B_{t+1} - \frac{\xi}{2}\frac{(B_{t+1}-\bar{B})^2}{Y} + R^{*}B_t = 0$

$\frac{\partial \mathcal{L}}{\partial \Lambda_{2t}} = -K_{t+1} + (1-\delta)K_t + (\frac{I_t}{K_t})^{\tau}K_t = 0$

$\lim_{t\to\infty} \beta^t \Lambda_{1t} B_{t+1} = 0$

$\lim_{t\to\infty} \beta^t \Lambda_{2t} K_{t+1} = 0$

Transforming the economy into a stationary one and defining the transformed variables $c_t = \frac{Ct}{X_t}, i_t = \frac{It}{X_t}, k_t = \frac{K_t}{X_t}, b_t = \frac{B_t}{X_t}, g_t = \frac{G_t}{X_t}, \lambda_{1t} = \Lambda_{1t}X_t^{\sigma}$ and $\lambda_{2t} = \Lambda_{2t}X_t^{\sigma}$ we get:

$$(c_t - \psi N_t^{\nu})^{-\sigma} - \lambda_{1t} = 0 \tag{4.1}$$

$$-\nu\psi N_t^{\nu-1}(c_t - \psi N_t^{\nu})^{-\sigma} + \lambda_{1t}(\alpha A_t k_t^{1-\alpha} N_t^{\alpha-1}) = 0 \tag{4.2}$$

$$-\lambda_{1t} + \lambda_{2t}\tau(\frac{i_t}{k_t})^{\tau-1} = 0 \tag{4.3}$$

$$-\gamma^{\sigma}\lambda_{2t}+\beta E_t\{\lambda_{1t+1}[(1-\alpha)A_{t+1}k_{t+1}^{-\alpha}N_{t+1}^{\alpha}]+\lambda_{2t+1}[(1-\delta)+(1-\tau)(\frac{i_{t+1}}{k_{t+1}})^{\tau}]\}=0 \quad (4.4)$$

$$-\gamma^{\sigma}\lambda_{1t}\left[1+\xi\frac{E_t(\gamma b_{t+1}-\bar{b})}{y}\right]+\beta R^{*}E_t\{\lambda_{1t+1}\}=0 \quad (4.5)$$

$$A_t k_t^{1-\alpha}N_t^{\alpha}-c_t-i_t-g_t-\gamma b_{t+1}-\frac{\xi}{2}\frac{(\gamma b_{t+1}-\bar{b})^2}{y}+R^{*}b_t=0 \quad (4.6)$$

$$-\gamma k_{t+1}+(1-\delta)k_t+(\frac{i_t}{k_t})^{\tau}k_t=0 \quad (4.7)$$

A competitive equilibrium is a set of processes $\{b_{t+1}, c_t, N_t, y_t, i_t, k_{t+1}, \lambda_{1t}, \lambda_{2t}\}$ satisfying equations 4.1 - 4.7 given the law of motion of the shocks and A_0, g_0, B_0 and K_0.

Writing the Lagrangean in terms of transformed variables:

$$\mathcal{L}=\mathrm{E}_0\sum_{t=0}^{\infty}(\beta^{*})^{t}\{\frac{1}{1-\sigma}[(c_t-\psi N_t^{\nu})^{1-\sigma}-X_t^{\sigma-1}]+\lambda_{1t}[A_t k_t^{1-\alpha}N_t^{\alpha}-c_t-i_t-g_t-\gamma b_{t+1}- \frac{\xi}{2}\frac{\left(\gamma b_{t+1}-\bar{b}\right)^2}{y}+R_t^{*}b_t+\lambda_{2t}[-\gamma k_{t+1}+(1-\delta)k_t+(\frac{i_t}{k_t})^{\tau}k_t]\}$$

$$\frac{\partial\mathcal{L}}{\partial c_t}=(c_t-\psi N_t^{\nu})^{-\sigma}-\lambda_{1t}=0$$

$$\frac{\partial\mathcal{L}}{\partial N_t}=-\nu\psi N_t^{\nu-1}(c_t-\psi N_t^{\nu})^{-\sigma}+\lambda_{1t}(\alpha A_t k_t^{1-\alpha}N_t^{\alpha-1})=0$$

$\frac{\partial \mathcal{L}}{\partial i_t} = -\lambda_{1t} + \lambda_{2t}\tau(\frac{i_t}{k_t})^{\tau-1} = 0$

$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = -\gamma^{\sigma}\lambda_{2t} + \beta\, \mathrm{E}_t\{\lambda_{1t+1}[(1-\alpha)A_{t+1}k_{t+1}^{-\alpha}N_{t+1}^{\alpha}] + \lambda_{2t+1}[(1-\delta)+(1-\tau)(\frac{i_{t+1}}{k_{t+1}})^{\tau}]\} = 0$

$\frac{\partial \mathcal{L}}{\partial b_{t+1}} = -\gamma^{\sigma}\lambda_{1t}\left[1+\xi\frac{E_t(\gamma b_{t+1}-\bar{b})}{y}\right] + \beta R^{*}\mathrm{E}_t\{\lambda_{1t+1}\} = 0$

$\frac{\partial \mathcal{L}}{\partial \lambda_{1t}} = A_t k_t^{1-\alpha}(N_t)^{\alpha} - c_t - i_t - g_t - \gamma b_{t+1} - \frac{\xi}{2}\frac{\left(\gamma b_{t+1}-\bar{b}\right)^2}{y} + R^{*}b_t = 0$

$\frac{\partial \mathcal{L}}{\partial \lambda_{2t}} = -\gamma k_{t+1} + (1-\delta)k_t + (\frac{i_t}{k_t})^{\tau}k_t = 0$

where $\beta^{*} = \beta(\gamma)^{1-\sigma}$ and $\beta^{*} < 1$ is required to guarantee finiteness of lifetime utility and we also used the assumption $\beta^{*} = \frac{\gamma}{R^{*}}$.

A nonstochastic steady state in terms of transformed variables will exist since they all have been divided by their common growth component X_t. To find that steady state we simply remove time subscripts from the Euler equations,

$$(c - \psi N^{\nu})^{-\sigma} - \lambda_1 = 0 \tag{4.8}$$

$$-\nu\psi N^{\nu-1}(c - \psi N^{\nu})^{-\sigma} + \lambda_1 \alpha A k^{1-\alpha}N^{\alpha-1} = 0 \tag{4.9}$$

$$-\lambda_1 + \tau\lambda_2(\frac{i}{k})^{\tau-1} = 0 \tag{4.10}$$

$$-\gamma^{\sigma}\lambda_2 + \beta\lambda_1[(1-\alpha)Ak^{-\alpha}N^{\alpha}] + \lambda_2[(1-\delta) + (1-\tau)(\frac{i}{k})^{\tau}] = 0 \tag{4.11}$$

$$-\gamma^{\sigma}\lambda_1\left[1 + \xi\frac{(\gamma b - \bar{b})}{y}\right] + \beta R^{*}\lambda_1 = 0 \tag{4.12}$$

$$Ak^{1-\alpha}N^{\alpha} - c - i - g - \gamma b - \frac{\xi}{2}\frac{\left(\gamma b - \bar{b}\right)^2}{y} + R^{*}b = 0 \tag{4.13}$$

$$\gamma - (1-\delta) - (\frac{i}{k})^{\tau} = 0 \tag{4.14}$$

In the steady state all the components of the national income identity grow at rate γ and the steady state values of N, c, i, k, and b are characterized by the next equations:

$$\beta R^{*} = \gamma^{\sigma}[1 + \xi\frac{(\gamma b - \bar{b})}{y}] \tag{4.15}$$

$$(\frac{i}{k})^{\tau} = [\gamma - (1-\delta)] \tag{4.16}$$

$$+\tau(\frac{i}{k})^{\tau-1} = \frac{\lambda_1}{\lambda_2} \quad (4.17)$$

$$\frac{\lambda_1}{\lambda_2}[(1-\alpha)Ak^{-\alpha}N^{\alpha}] + [(1-\delta) + (1-\tau)(\frac{i}{k})^{\tau}] = \gamma^{\sigma}\beta^{-1} \quad (4.18)$$

$$\alpha\frac{y}{N} = \nu\psi N^{\nu-1} \quad (4.19)$$

$$Ak^{1-\alpha}N^{\alpha} - c - i - g - \gamma b - \frac{\xi}{2}\frac{\left(\gamma b - \bar{b}\right)^2}{y} + R^{*}b = 0 \quad (4.20)$$

The equation (4.15) gives the steady state value of b.

Using $\beta R^{*} = \gamma^{\sigma}$ we get $\left[1 + \xi\frac{(\gamma b - \bar{b})}{y}\right] = 1$ implying that the parameter $\bar{b}$ determines the steady state of foreign debt:

$$b = \frac{\bar{b}}{\gamma} \quad (4.21)$$

From equation (4.16) we have:

$$(\frac{i}{k}) = [\gamma - (1-\delta)]^{\frac{1}{\tau}} \quad (4.22)$$

$(\frac{i}{k}) = [\gamma - (1-\delta)]^{\frac{1}{\tau}}$ then $\phi(\frac{i}{k}) = \gamma - (1-\delta) > (\frac{i}{k})$ as suggested by Correia, Neves and Rebelo (1995),[13] so at steady state we have some adjustment cost as long as τ is different from 1. We considered adjustment costs so the model could match the volatility of the trade balance.

Equations (4.17) and (4.18) determine the capital output ratio:

$$\frac{k}{y} = \frac{(1-\alpha)\tau[\gamma+\delta-1]^{\frac{\tau-1}{\tau}}}{\gamma^{\sigma}\beta^{-1} - \gamma + \tau[\delta+\gamma-1]} \tag{4.23}$$

From equation (4.14) we get:

$$N = [(\frac{\alpha}{\nu\psi})(\frac{y}{N})]^{\frac{1}{(\upsilon-1)}} \tag{4.24}$$

where

$$\frac{y}{N} = A(\frac{k}{N})^{(1-\alpha)} \tag{4.25}$$

and

$$\frac{k}{N} = \left(\frac{A(1-\alpha)\tau[\gamma+\delta-1]^{\frac{\tau-1}{\tau}}}{\gamma^{\sigma}\beta^{-1} - \gamma + \tau(\gamma+\delta-1)}\right)^{1/\alpha} \tag{4.26}$$

Equation 4.27 determines the steady state value of $\frac{c}{y}$.

[13]Note 13 in Correia, Neves and Rebelo (1995).

$$\frac{c}{y} = 1 - [\gamma - (1-\delta)]^{\frac{1}{\tau}}\frac{k}{y} - \frac{g}{y} + (1 - \gamma^{\sigma-1}\beta^{-1})\frac{\overline{b}}{y} \tag{4.27}$$

So we have a full set of steady state values for N, c, y, i, k and b.

We now proceed computing an approximate solution to the competitive equilibrium using the method proposed by KPR(1988) which approximates the Euler equations by a set of linear equations in the unknowns. By totally differentiating the Euler equations at the steady state we get a linear approximation to the Euler equations in the neighborhood of the steady state.

We define the variables $\hat{z}_t = \left(\frac{dz_t}{z}\right)$. These variables represent first-order approximations to percentage deviations from the steady state values, $\frac{dz_t}{z} \approx \ln\left(\frac{z_t}{z}\right)$. Totally differentiating the Euler equations above, (4.1) to (4.7), and rewriting them in terms of hatted variables we get:

$$\left(\frac{-\sigma c}{c-\psi N^{\nu}}\right)\hat{c}_t + \left(\frac{\sigma\psi\nu N^{\nu}}{c-\psi N^{\nu}}\right)\hat{N}_t - \hat{\lambda}_{1t} = 0$$

$$(\nu - \alpha)\,\hat{N}_t - (1-\alpha)\,\hat{k}_t - \hat{A}_t = 0$$

$$-\hat{\lambda}_{1t} + \hat{\lambda}_{2t} + (\tau - 1)\,\hat{\imath}_t - (\tau - 1)\hat{k}_t = 0$$

$$-\frac{\gamma^{\sigma}}{\beta\tau}\left(\frac{k}{i}\right)^{\tau}\hat{\lambda}_{2t} + \mu E_t\hat{\lambda}_{t+1} + \mu E_t\hat{A}_{t+1} - (\alpha\mu + 1 - \tau)E_t\hat{k}_{t+1} + \alpha\mu E_t\hat{N}_{t+1} + (1 - \delta)\frac{(1-\tau)}{\tau}E_t\hat{\lambda}_{2t+1} + (1-\tau)E_t\hat{\imath}_{t+1} = 0$$

where $\mu = (1-\alpha)\frac{y}{i}$

$$-\hat{\lambda}_{1t} + \gamma\xi\frac{b}{y}E_t\hat{b}_{t+1} + E_t\hat{\lambda}_{1t+1} = 0$$

$$\hat{A}_t + (1-\alpha)\,\hat{k}_t + \alpha\hat{N}_t - \frac{i}{y}\hat{\imath}_t - \frac{c}{y}\hat{c}_t - \frac{g}{y}\hat{g}_t + R^*\frac{b}{y}\hat{b}_t - \gamma\frac{b}{y}\hat{b}_{t+1} = 0$$

$$-\gamma E_t\hat{k}_{t+1} + (1-\delta)\hat{k}_t + \tau(\tfrac{i}{k})\,\hat{\imath}_t + (\tfrac{i}{k})(1-\tau)\hat{k}_t = 0$$

This system of equations can be written in the form:

$$M_{cc}\begin{bmatrix}\hat{c}_t\\ \hat{N}_t\\ \hat{\imath}_t\end{bmatrix} = M_{cs}\begin{bmatrix}\hat{k}_t\\ \hat{b}_t\\ \hat{\lambda}_{1t}\\ \hat{\lambda}_{2t}\end{bmatrix} + M_{ce}\begin{bmatrix}\hat{A}_t\\ \hat{g}_t\end{bmatrix}$$

$$M_{ss}^0E_t\begin{bmatrix}\hat{k}_{t+1}\\ \hat{b}_{t+1}\\ \hat{\lambda}_{1t+1}\\ \hat{\lambda}_{2t+1}\end{bmatrix} + M_{ss}^1\begin{bmatrix}\hat{k}_t\\ \hat{b}_t\\ \hat{\lambda}_{1t}\\ \hat{\lambda}_{2t}\end{bmatrix} = M_{sc}^0E_t\begin{bmatrix}\hat{c}_{t+1}\\ \hat{N}_{t+1}\\ \hat{\imath}_{t+1}\end{bmatrix} + M_{sc}^1\begin{bmatrix}\hat{c}_t\\ \hat{N}_t\\ \hat{\imath}_t\end{bmatrix}$$

$$+M_{se}^0E_t\begin{bmatrix}\hat{A}_{t+1}\\ \hat{g}_{t+1}\end{bmatrix} + M_{se}^1\begin{bmatrix}\hat{A}_t\\ \hat{g}_t\end{bmatrix}$$

$$M_{cc} = \begin{bmatrix}\left(\frac{-\sigma c}{c-\psi N^\nu}\right) & \left(\frac{\sigma\psi\nu N^\nu}{c-\psi N^\nu}\right) & 0\\ 0 & (\nu-\alpha) & 0\\ 0 & 0 & (\tau-1)\end{bmatrix} M_{cs} = \begin{bmatrix}0 & 0 & 1 & 0\\ (1-\alpha) & 0 & 0 & 0\\ (\tau-1) & 0 & 1 & -1\end{bmatrix}$$

$$M_{ce} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M_{ss}^{0} = \begin{bmatrix} -\alpha\mu - (1-\tau) & 0 & \mu & (1-\delta)\frac{(1-\tau)}{\tau} \\ 0 & \gamma\xi\frac{b}{y} & 1 & 0 \\ 0 & -\gamma\frac{b}{y} & 0 & 0 \\ -\gamma & 0 & 0 & 0 \end{bmatrix}$$

$$M_{ss}^{1} = \begin{bmatrix} 0 & 0 & 0 & \frac{-\gamma^{\sigma}}{\beta\tau}(\frac{k}{i})^{\tau} \\ 0 & 0 & -1 & 0 \\ (1-\alpha) & \frac{\gamma^{\sigma}}{\beta}\frac{b}{y} & 0 & 0 \\ (1-\delta) + (\frac{i}{k})(1-\tau) & 0 & 0 & 0 \end{bmatrix}$$

$$M_{sc}^{0} = \begin{bmatrix} 0 & -\alpha\mu & -(1-\tau) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad M_{sc}^{1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{c}{y} & -\alpha & \frac{i}{y} \\ 0 & 0 & -\tau(\frac{i}{k}) \end{bmatrix}$$

$$M_{se}^{0} = \begin{bmatrix} -\mu & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad M_{se}^{1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & \frac{g}{y} \\ 0 & 0 \end{bmatrix}$$

We solve the system of equations above numerically[14] assuming the following laws of motion for A_t and g_t :

$$\ln(A_t) = (1-\rho_a)\ln(A) + \rho_a \ln(A_{t-1}) + \varepsilon_{at}$$

$$\ln(g_t) = (1-\rho_g)\ln(g) + \rho_g \ln(g_{t-1}) + \varepsilon_{gt}$$

Where $\varepsilon_{at}, \varepsilon_{gt}$ and are i.i.d. with mean zero and variances σ_a^2,and σ_g^2. These suggest an approximation to the laws of motion of the form

$$\hat{A}_t = \rho_a \hat{A}_{t-1} + \varepsilon_{at}$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{gt}$$

Often, we are interested in variables other than states, controls and exogenous variables. For example we are also interested in properties of output, y_t, and the trade balance. These variables are all determined in terms of the others,

$$y_t = A_t k_t^{1-\alpha} N_t^{\alpha}$$

$$tb_t = A_t k_t^{1-\alpha} N_t^{\alpha} - c_t - i_t - g_t$$

These expressions can also be linearized:

$$\hat{y} = \hat{A}_t + (1-\alpha)\hat{k}_t + \alpha \hat{N}_t$$

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[14] See appendix 1 for the solution of the system of equations. For alternative methods see Uhlig, H. (1999).

$$\hat{tb} = \left(\frac{y}{tb}\right)\hat{A}_t + (1-\alpha)\left(\frac{y}{tb}\right)\hat{k}_t + \alpha\left(\frac{y}{tb}\right)\hat{N}_t - \left(\frac{c}{y}\right)\left(\frac{y}{tb}\right)\hat{c} - \left(\frac{i}{y}\right)\left(\frac{y}{tb}\right)\hat{\imath}_t - \left(\frac{g}{y}\right)\left(\frac{y}{tb}\right)\hat{g}$$

where the steady state of $\frac{tb}{y} = (\gamma^{\sigma-1}\beta^{-1} - 1)\frac{\bar{b}}{y}$.

4.6. The Econometric Methodology

This paper uses a variant of generalized method of moments (GMM) procedure discussed in Christiano and Eichenbaum (1992) to estimate and access the empirical performance of the model. Our estimation criterion is set up so that the estimated model exactly matches the sample analog of certain unconditional moments of the data generating process. We use the estimated model to calculate selected second moments of the data. These same second moments can be estimated in a way that does not involve the model. When one abstracts from sampling uncertainty, the two sets of second-moment estimates ought to coincide if the model has been specified correctly. To test this hypothesis, we employed a Wald type test statistic.

4.6.1. Estimation

Let Ψ_1 denote the 12 × 1 vector of structural parameters to be estimated:

$$\Psi_1 = \{\alpha, \psi, \delta, \ln\gamma, \ln\gamma_G, \rho_a, \sigma_{\varepsilon a}, \rho_g, \sigma_{\varepsilon g}, \ln Y, \ln G, \ln A\}.$$

Given estimated values of Ψ_1, $\hat{\Psi}_{1T}$, and distribution assumptions on $\varepsilon's$, our model provides a complete description of the data generating process. This can be used to compute the second moments of all the variables of the model. So we first calculate selected second moments of the data using our model evaluated at $\hat{\Psi}_{1T}$.Then we estimate the same second moments from the data without using the model. Then our test compares these two sets of second moments and determines whether the differences between them can be accounted for by sampling variation under the null hypothesis that the model is correctly specified.

To implement our diagnostic procedures we must estimate various moments of the data generating process. Let Ψ_2 denote the set of second moments to be estimated:

$$\Psi_2 = \{\sigma_y, \sigma_c, \sigma_i, \sigma_{tb}, \sigma_h, \sigma_k, corr(tb, y)\} .$$

Here σ_x denotes the standard deviation of the variable x, x=$\{y, c, i, tb, h, k\}$ and corr(tb, y) denotes the correlation between tb and y. Since the data displayed marked time trends, we used the Hodrick and Prescott filter($\lambda = 6.25$) to ensure that the moments in Ψ_2 exist.

The following moment conditions were used to estimate $\Psi_1$and Ψ_2:

$$E\left[\ln(Y_t) - \ln Y - \ln(\gamma)t\right] = 0; \tag{4.28}$$

$$\frac{\mathrm{E}\left[\ln(Y_t) - \ln Y - \ln(\gamma)t\right] t}{T} = 0; \tag{4.29}$$

$$E\left[\ln(G_t) - \ln G - \ln(\gamma_g)t\right] = 0; \tag{4.30}$$

$$\frac{\mathrm{E}\left[\ln(G_t) - \ln G - \ln(\gamma_g)t\right] t}{T} = 0; \tag{4.31}$$

$$E\left[\left[\ln(g_t) - (1-\rho_g)\ln(g) - \rho_g \ln(g_{t-1})\right]\ln(g_{t-1})\right] = 0; \tag{4.32}$$

$$E\left[\left[\ln(g_t) - (1-\rho_g)\ln(g) - \rho_g \ln(g_{t-1})\right]^2 - \sigma_{\varepsilon g}^2\right] = 0; \tag{4.33}$$

$$E\left[\ln(A_t) - (1-\rho_a)\ln(A) - \rho_a \ln(A_{t-1})\right] = 0; \tag{4.34}$$

$$E\left[\left[\ln(A_t) - (1-\rho_a)\ln(A) - \rho_a \ln(A_{t-1})\right]\ln(A_{t-1})\right] = 0; \tag{4.35}$$

$$E\left[\left[\ln(A_t) - (1-\rho_a)\ln(A) - \rho_a \ln(A_{t-1})\right]^2 - \sigma_{\varepsilon a}^2\right] = 0; \tag{4.36}$$

$$E[\frac{\gamma^{\frac{\ln \beta R^*}{\ln \gamma}}}{\tau}(\frac{i_t}{k_t})^{1-\tau} - \beta(\frac{C_t - \psi\gamma^t N_t^v}{(C_{t+1}/\gamma) - \psi\gamma^t N_{t+1}^v})^{\frac{\ln \beta R^*}{\ln \gamma}}[(1-\alpha)\frac{Y_{t+1}}{K_{t+1}} +$$

$$\frac{(1-\delta)}{\tau}(\frac{i_{t+1}}{k_{t+1}})^{1-\tau} + \frac{(1-\tau)}{\tau}(\frac{i_{t+1}}{k_{t+1}})] = 0; \tag{4.37}$$

$$E\left[\psi - [(\frac{\alpha}{\nu N_t^{\nu}})(\frac{Y_t}{\gamma^t})]\right] = 0; \tag{4.38}$$

$$E\left[-\gamma k_{t+1} + (1-\delta)k_t + (\frac{i_t}{k_t})^{\tau} k_t\right] = 0; \tag{4.39}$$

$$E\left[y_{hp,t}^2 - \sigma_y^2\right] = 0; \tag{4.40}$$

$$E\left[c_{hp.t}^2 - \sigma_c^2\right] = 0; \tag{4.41}$$

$$E\left[i_{hp,t}^2 - \sigma_i^2\right] = 0; \tag{4.42}$$

$$E\left[tb_{hp,t}^2 - \sigma_{tb}^2\right] = 0; \tag{4.43}$$

$$E\left[\left(\frac{y_{hp,t}}{\sigma_y}\frac{tb_{hp,t}}{\sigma_{tb}}\right) - corr\,(tb, y)\right] = 0; \tag{4.44}$$

$$E\left[n_{hp,t}^2 - \sigma_n^2\right] = 0; \tag{4.45}$$

$$E\left[k_{hp,t}^2 - \sigma_k^2\right] = 0; \tag{4.46}$$

The hp subscript denotes the HP cyclical component of the time series in question.

The first 12 equations, (4.28)-(4.39), consist of 12 unconditional moment restrictions involving the 12 elements of Ψ_1. These restrictions can be summarized as:

$$\mathrm{EH}_{1,t}\left(\Psi_1^0\right) = 0 \qquad \text{for all } t \geq 0,$$

where Ψ_1^0 is the true value of $\Psi_1$and $\mathrm{H}_{1,t}\left(\Psi_1\right)$ is the 12×1 random vector which has as its elements the left sides of the first 12 equations before expectations are taken.

The last 7 equations, (4.40)-(4.46), consist of 7 unconditional moment restrictions involving the 7 elements of Ψ_2. This restrictions can be summarized as:

$$\mathrm{EH}_{2,t}\left(\Psi_2^0\right) = 0 \qquad \text{for all } t \geq 0,$$

where Ψ_2^0 is the true value of $\Psi_2$and $\mathrm{H}_{2,t}\left(\Psi_2\right)$ is the 7×1 random vector valued function which has as its elements the left sides of the last 7 equations before expectations are taken.

Defining

$$\mathrm{H}_t\left(\Psi^0\right) = \begin{bmatrix} \mathrm{H}_{1,t}\left(\Psi_1^0\right) \\ \mathrm{H}_{2,t}\left(\Psi_2^0\right) \end{bmatrix}$$

we can represent all moment conditions as

$$\mathrm{EH}_t\left(\Psi^0\right) = 0 \qquad \text{for all } t \geq 0.$$

Let g_T denote the vector valued function

$$g_T\left(\Psi\right) = \tfrac{1}{T}\sum_{t=1}^{T}\mathrm{H}_t\left(\Psi\right)$$

To define our estimator we choose Ψ_T to minimize

$$J_T\left(\Psi\right) = g_T\left(\Psi\right)^T W_T g_T\left(\Psi\right)$$

where W_T is a 19×19 symmetric positive definite weighting matrix.

Defining

$$D_0 = \mathrm{E}\left[\frac{\partial H_t\left(\Psi^0\right)}{\partial \Psi^T}\right]$$

and let W_T converge almost surely to the symmetric positive matrix W_0. Then Hansen (1982) shows that $\hat{\Psi}_T$ is consistent and that

$$\sqrt{T}\left(\hat{\Psi}_T - \Psi^0\right) \xrightarrow{d} N\left[0, (D_0^T W_0 D_0)^{-1} D_0^T W_0 S_0 W_0 D_0 (D_0^T W_0 D_0)^{-1}\right]$$

where

$$S_0 = \mathrm{E}\left[\sum_{j=-\infty}^{\infty} H_t\left(\Psi^0\right)\, H_{t+j}\left(\Psi^0\right)^T\right]$$

Hansen (1982) also shows that across all GMM estimators which exploit the same moment restrictions, the asymptotic variance covariance matrix is smallest when W_T converges almost surely to S_0^{-1}. Then we use the following two step procedure:

We estimate Ψ using the identity matrix as the weighting matrix. Since this estimated parameter vector, $\hat{\Psi}_T$, is consistent it is used to construct a consistent estimator of S_0. To estimate S_0 we use the Newey-West estimator, $\hat{S}_T$ [15],

$$\hat{S}_T = \hat{\Gamma}_{0,T} + \sum_{v=1}^{q} \left\{1 - \left[\frac{v}{(q+1)}\right]\right\} \left(\hat{\Gamma}_{v,T} + \hat{\Gamma}_{v,T}^T\right)$$

where

$$\hat{\Gamma}_{v,T} = \frac{1}{T} \sum_{t=v+1}^{T} \left[H_t\left(\hat{\Psi}_T\right)\right]\left[H_{t-v}\left(\hat{\Psi}_T\right)\right]^T.$$

Setting $W_T = \hat{S}_T^{-1}$ we reestimate Ψ. The estimator $\hat{\Psi}_T$ has the following asymptotic distribution

$$\sqrt{T}\left(\hat{\Psi}_T - \Psi^0\right) \xrightarrow{d} N\left[0, (D_0^T S_0^{-1} D_0)^{-1}\right].$$

The variance covariance matrix of $\hat{\Psi}_T$ was consistently estimated by computing

$$V(\hat{\Psi}_T) = \frac{\left[D_T^T \hat{S}_T^{-1} D_T\right]^{-1}}{T} \text{ where } D_T = \frac{1}{T} \sum_{t=1}^{T} \frac{\partial g_t(\hat{\Psi})}{\partial \Psi^T}.$$

[15]The order of lags on our estimation was set equal to 5

4.6.2. Hypothesis Testing

Exploiting the moments conditions

$$\mathrm{EH}_t\left(\Psi^0\right) = 0,$$

we estimated the 19×1 parameter vector $\Psi = [\Psi_1, \Psi_2]'$ by GMM, denoting his variance covariance matrix by $V(\hat{\Psi}_T)$. Given a set of values for Ψ_1, we define $m(\Psi_1)$ as a function that maps the parameter vector to the model moments. The function m can be given different definitions depending on how many model moments we are interested in calculating. This function is highly non linear in Ψ_1 and is computed using numerical methods. The distance between the model moments and the data moments is given by

$$h\left(\hat{\Psi}_T\right) = m\left(\hat{\Psi}_{1T}\right) - \hat{\Psi}_{2T} .$$

Under the null hypothesis that the model is correctly specified, $h\left(\Psi^0\right) = 0$.

If our data sample were large , then $\hat{\Psi}_T = \Psi^0$ and we could perform our test by just comparing $h\left(\hat{\Psi}_T\right)$ with a vector of zeros. However $h\left(\hat{\Psi}_T\right)$ need not be zero in a small sample, because of sample uncertainty in $\hat{\Psi}_T$. So to perform our test we need the distribution of $h\left(\hat{\Psi}_T\right)$ under the null hypothesis. Christiano and Eichenbaum (1992) show that

$$V\left[h\left(\hat{\Psi}_T\right)\right] = \frac{\partial h(\hat{\Psi}_T)}{\partial \Psi'} V(\hat{\Psi}_T) \frac{\partial h(\hat{\Psi}_T)}{\partial \Psi'}^T.$$

and the test statistic

$$W_T = h\left(\hat{\Psi}_T\right)^T V\left[h\left(\hat{\Psi}_T\right)\right]^{-1} h\left(\hat{\Psi}_T\right),$$

is asymptotically distributed as a χ^2random variable with q degrees of freedom, where q is the number of moments to be tested.

By computing W_T for any pair of model and data moments we have a way of deciding whether the model is consistent with that aspect of the data.

We also perform an over-identification test, $\mathrm{H}_0 : \gamma = \gamma_C = \gamma_I = \gamma_G$ considering eight moments related to the growth rate of output and expenditure components. In our model we assumed the growth rate of all the components of expenditure should grow at rate γ.

$$E\left[\ln(Y_t) - \ln(Y) - \ln(\gamma)t\right] = 0; \tag{4.47}$$

$$\frac{\mathrm{E}\left[\ln(Y_t) - \ln(Y) - \ln(\gamma)t\right]t}{T} = 0; \tag{4.48}$$

$$\mathrm{E}\left[\ln(C_t) - \ln(C) - \ln(\gamma)t\right] = 0; \tag{4.49}$$

$$\frac{\mathrm{E}\left[\ln(C_t) - \ln(C) - \ln(\gamma)t\right]t}{T} = 0; \tag{4.50}$$

$$E\left[\ln(I_t) - \ln(I) - \ln(\gamma)t\right] = 0; \tag{4.51}$$

$$\frac{\mathrm{E}\left[\ln(I_t) - \ln(I) - \ln(\gamma)t\right]t}{T} = 0; \tag{4.52}$$

$$\mathrm{E}\left[\ln(G_t) - \ln(G) - \ln(\gamma)t\right] = 0; \tag{4.53}$$

$$\frac{\mathrm{E}\left[\ln(G_t) - \ln(G) - \ln(\gamma)t\right]t}{T} = 0; \tag{4.54}$$

4.6.3. The Data

Our data source is AMECO.[16]

All the variables are expressed in per capita terms and at 1995 prices when appropriate. The data that we used has annual frequency and covers the period from 1960-2005.

We performed two transformations to characterize the cyclical behavior of the different variables. First, we computed the logarithm of all variables with the exception of the trade balance and than filtered the data with the Hodrick-Prescott filter ($\lambda = 6.25$). Since the trade balance takes on negative values we expressed it as percentage deviations from the mean using the following approximation to $\ln(tb_t)$: $\frac{tb_t}{|mean(tb)|} - 1$. We then detrended the variable with the Hodrick-Prescott filter ($\lambda = 6.25$).

Hours for Portugal were only available for 1986-2005. We computed the per capita number of hours for 1986-2005 and extrapolated linearly the per capita number of hours for 1960-1985. Then we computed the total number of hours for 1960-1985 and than the per capita number of hours per annum and divide it for a measure of annual time endowment (5476 hours).

We computed the Solow residual and used it to estimate the stochastic process associated with technology shocks.

[16]See Appendix 6.1 for a description of the variables.

4.7. Empirical Results

The main purpose of this paper is to check if the model is able to mimic actual business cycle fluctuations when instead of choosing a baseline parameterization we estimate the model using a variant of Hansen's (1982) GMM as in Christiano and Eichenbaum (1992).

In this section we report our empirical results.

The parameters β, ν and r^* were not estimated and their values are standard in RBC literature. We set β equal to 0.971, implying a 3-percent annual subjective discount rate, $r^* = 0.0375$ and $v = 1.7$ (an elastic labor supply) or $v = 2.3$ (an inelastic labor supply).The adjustment cost parameters, τ and ε, were set equal to 0.80 and 0.01. The value assigned to $\bar{d}$ is set to match the average Portuguese Trade Balance to GDP ratio.

Table 4.1 reports our estimates of Ψ_1 along with standard errors based on Newey and West method (between parentheses) for the two different values of labor supply. Table 4.2 reports the implications of our estimates of Ψ_1 for various second moments of the data. We compare the second moments implications of the model with those of the data for each variable using the test describe in section 4.6.2. We report the results of testing only one moment at a time.

Table 4.1: Parameter Estimates

Model Parameter	$\nu = 1.7$	$\nu = 2.3$
β	0.971	0.971
R^*	1.0375	1.0375
τ	0.80	0.80
ε	0.01	0.01
$\bar{b}$	-300000	-15000
α	0.75516 (0.01150)	0.75581 (0.00950)
δ	0.13314 (0.00275)	0.13314 (0.00275)
ψ	27.9866 (1.37999)	63.8489 (3.45353)
$\ln Y$	0.95991 (0.04296)	0.95991 (0.04296)
$\ln\gamma$	0.03287 (0.00145)	0.03287 (0.00145)
$\ln G$	-1.41052 (0.05026)	-1.41052 (0.05026)
$\ln\gamma_G$	0.05253 (0.00197)	0.05253 (0.00197)
ρ_g	0.91139 (0.06056)	0.91139 (0.06056)
$\sigma^2_{\varepsilon g}$	0.00182 (0.00053)	0.00182 (0.00053)
$(1-\rho_a) \ln A$	0.05492 (0.00695)	0.05498 (0.00697)
ρ_a	0.97126 (0.00720)	0.97129 (0.00714)
$\sigma^2_{\varepsilon a}$	0.00201 (0.00054)	0.00201 (0.00054)

Table 4.2: Selected Second Moments:
$\nu = 1.7$ and $\nu = 2.3$

$\nu = 1.7$

	Model Moments	s.e	Data Moments	s.e.	Test	P-Value
σ_y	0.0490	0.0067	0.0212	0.0030	21.060	0.0000
σ_c	0.0602	0.0196	0.0194	0.0021	4.3584	0.0368
σ_i	0.0875	0.0125	0.0591	0.0054	4.7916	0.0286
σ_{tb}	1.4817	2.9660	0.2374	0.0253	0.1757	0.6751
$\text{corr}(y, tb)$	-0.6330	0.0070	-0.3443	0.1216	5.6336	0.0176
σ_h	0.0288	0.0039	0.0209	0.0029	3.0799	0.0793
σ_k	0.0079	0.0012	0.0082	0.0009	0.0572	0.8110

$\nu = 2.3$

	Model Moments	s.e	Data Moments	s.e.	Test	P-Value
σ_y	0.0408	0.0056	0.0212	0.0030	15.179	0.0001
σ_c	0.0376	0.0123	0.0194	0.0021	2.1345	0.1440
σ_i	0.0724	0.0104	0.0591	0.0054	1.4492	0.2287
σ_{tb}	0.8171	1.3226	0.2374	0.0253	0.1912	0.6619
$\text{corr}(y, tb)$	-0.6030	0.0148	-0.3443	0.1216	4.4755	0.0344
σ_h	0.0178	0.0024	0.0209	0.0029	0.8322	0.3616
σ_k	0.0065	0.0010	0.0082	0.0009	1.7997	0.1797

Looking at table 4.2, $\nu = 2.3$, we can see that the model moments match the data moments except for output. As in the data the model predicts the following ranking volatilities, in ascending order, consumption output and investment. However the expenditure components show a higher volatility than what we observe in data. The model also predicts a negative correlation between output and trade balance. Volatility of capital and hours are also correctly predicted.

We also performed an overidentification test. In our model we assumed private consumption , investment and government consumption all grow at same rate γ. We consider eight moments, equations (4.47)-(4.54), to test this hypothesis. We reject the hypothesis $\gamma = \gamma_C = \gamma_I = \gamma_G$[17].

[17]See apendix D: Gauss code and outputs for this test: P-Value=0.0001

4.8. Properties of the Model

In this section we present Impulse Response Functions, so we can see the properties of our model without filtering the data generated by the model, that are generated recursively as follows:

We derive expressions for all variables of interest in terms of current and lagged innovations in the exogenous shock vector $z_t = \begin{bmatrix} \hat{A}_t & \hat{g}_t \end{bmatrix}^T$. As showed in Appendix 1 the solutions for the state vector, $x_{t+1} = \begin{bmatrix} \hat{k}_{t+1} & \hat{b}_{t+1} \end{bmatrix}^T$, costate variable, $\lambda_t = \begin{bmatrix} \hat{\lambda}_{1t} & \hat{\lambda}_{2t} \end{bmatrix}^T$, and the vector of controls, $u_t = \begin{bmatrix} \hat{c}_t & \hat{N}_t & \hat{\imath}_t \end{bmatrix}^T$, are given by

$$x_{t+1} = \Upsilon_{xx}\, x_t + \Upsilon_{xz}\, z_t$$

$$\lambda_t = \Upsilon_{\lambda x}\, x_t + \Upsilon_{\lambda z}\, z_t$$

$$u_t = \Upsilon_{ux}\, x_t + \Upsilon_{uz}\, z_t.$$

This means we can write

$$s_{t+1} = M\, s_t + \epsilon_{t+1}$$

where

$$s_t = \begin{bmatrix} x_t \\ z_t \end{bmatrix} \quad \epsilon_t = \begin{bmatrix} 0 \\ \varepsilon_t \end{bmatrix} \quad M = \begin{bmatrix} \Upsilon_{xx} & \Upsilon_{xz} \\ 0 & \Pi \end{bmatrix}$$

Figure 4.1: Impulse Response Functions to a Unit Productivity Shock

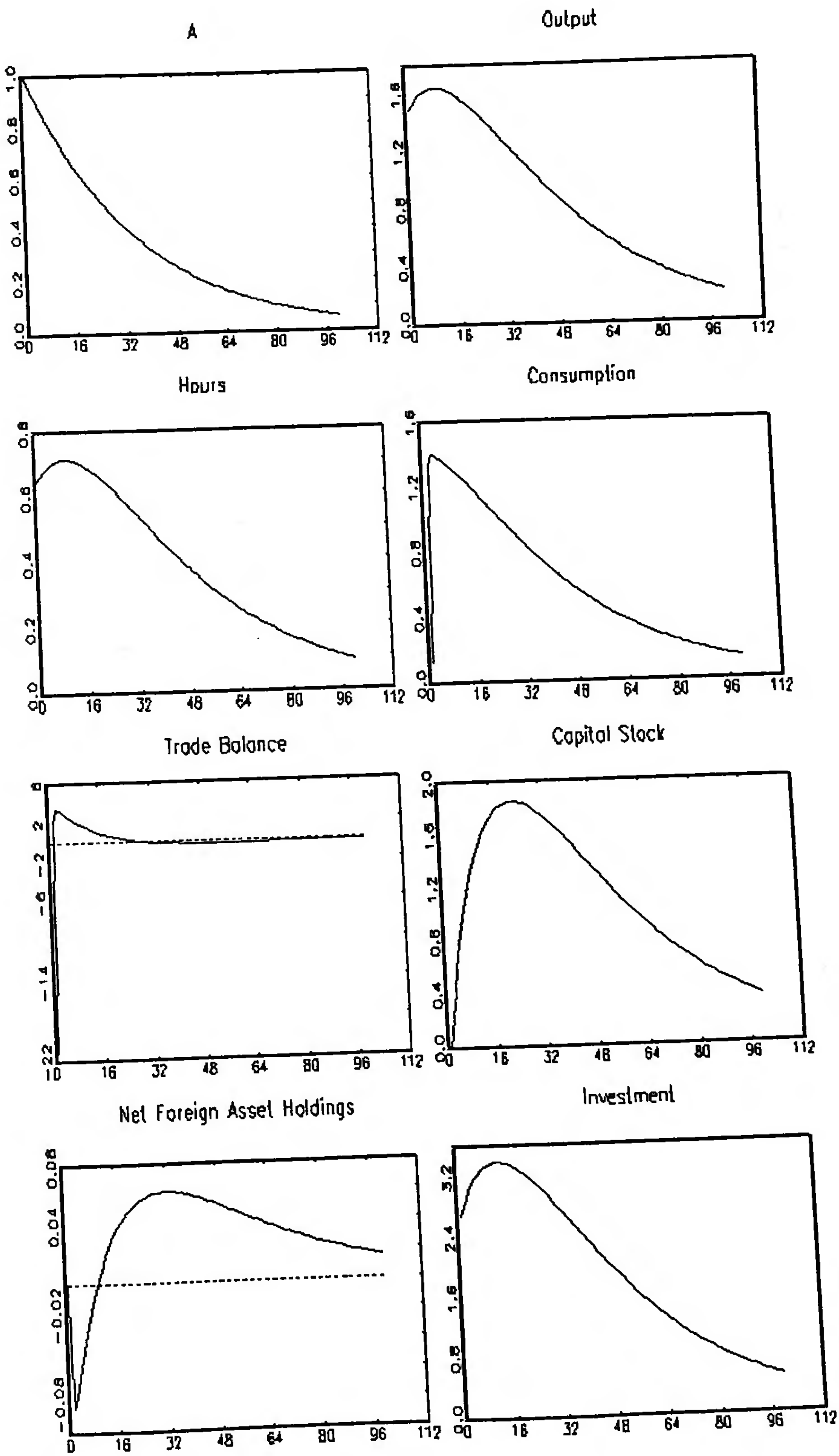
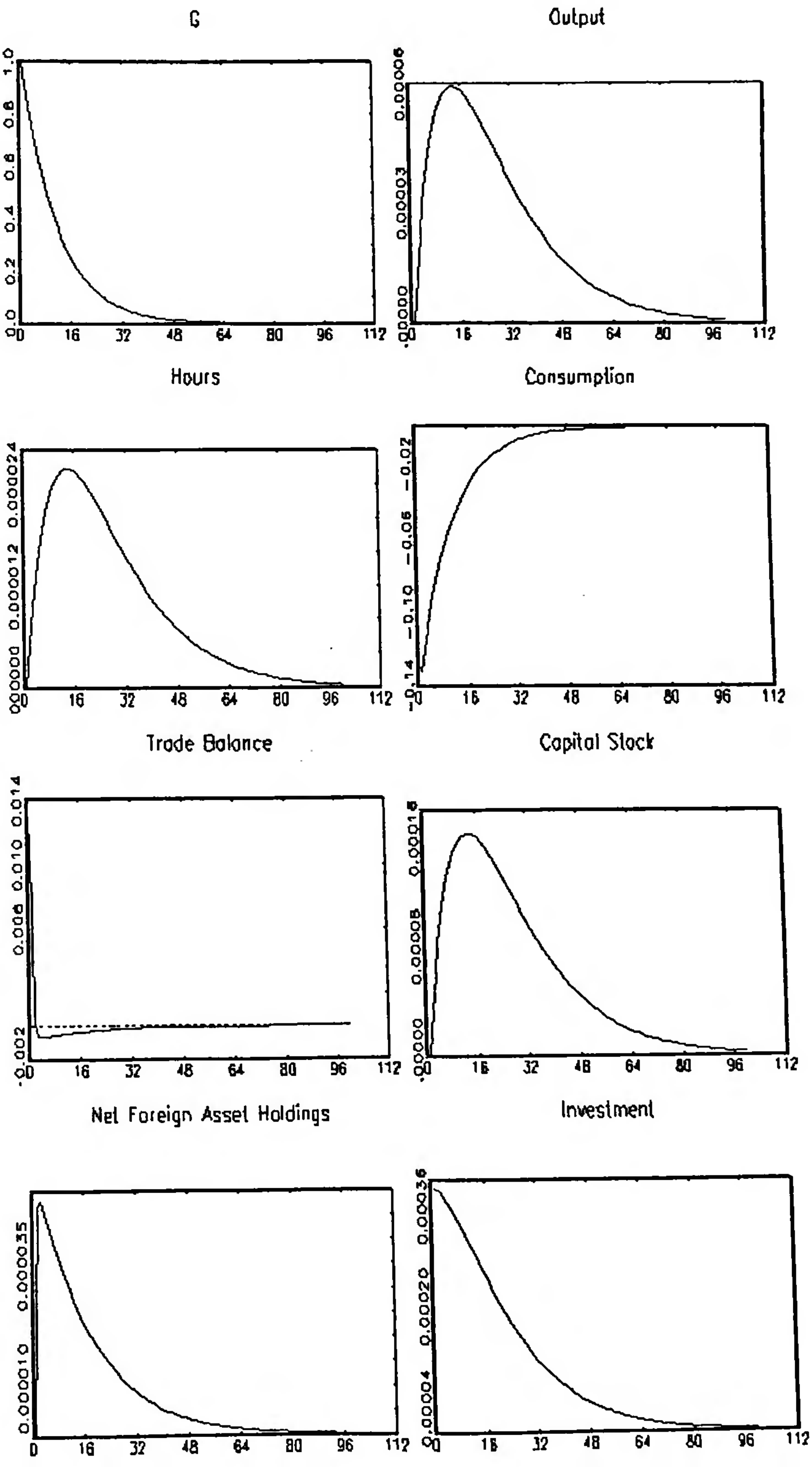


Figure 4.2: Impulse Response Functions to a Unit Government Shock



4.9. Conclusions

We conclude that even though our model generates more volatility on macroeconomic aggregates that we observe in data, the order and signs of the selected moments of our predictions look like what we observe in data.

We consider some alternatives that might improve the predictions of the model and be hints of future research:

In our model we assumed a deterministic trend and that all components of expenditure should grow at same rate. We may consider a stochastic trend in the analysis.

We may also consider the alternative SOE economy models mentioned before, a tradable - nontradable goods model or even analyze some other shocks as for instance the interest rate.

5. References

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6. Appendix

6.1. Appendix A : Data Sources and Definitions

Our data source for the Euro-zone countries is AMECO (Annual Macro-ECOnomic database of the European Comission' Directorate General for Economic and Financial Affairs (DG ECFIN).

The data is annual and goes from 1960-2005.

Definitions:

Population - NPTD: Total Population (National Accounts)

GDP - OVGD: Gross Domestic Product at 1995 market prices.

Consumption - OCPH: Private final consumption expenditure at 1995 prices.

Government spending - OCTG: Final consumption expenditure of general government at 1995 prices.

Investment - OIGT: Gross fixed capital formation at 1995 prices.

Exports - OXGS: Exports of goods and services at 1995 prices.

Imports - OMGS: Imports of goods and services at 1995 prices.

Capital - OKND: Net capital stock at 1995 prices.

Employment - NLTDLMS: Employment, persons.

Hours - NLHT: Total annual hours worked,(when available).

6.2. Appendix B : Dynamic General Equilibrium Model Solution

Defining

$$u_t = \begin{bmatrix} \hat{c}_t \\ \hat{N}_t \\ \hat{\imath}_t \end{bmatrix}, \quad x_t = \begin{bmatrix} \hat{k}_t \\ \hat{b}_t \end{bmatrix}, \quad \lambda_t = \begin{bmatrix} \hat{\lambda}_{1t} \\ \hat{\lambda}_{2t} \end{bmatrix} \quad \textit{and } z_t = \begin{bmatrix} \hat{A}_t \\ \hat{g}_t \end{bmatrix}$$

we can rewrite the following system of equations from section 3.2:

$$M_{cc} \begin{bmatrix} \hat{c}_t \\ \hat{N}_t \\ \hat{\imath}_t \end{bmatrix} = M_{cs} \begin{bmatrix} \hat{k}_t \\ \hat{b}_t \\ \hat{\lambda}_{1t} \\ \hat{\lambda}_{2t} \end{bmatrix} + M_{ce} \begin{bmatrix} \hat{A}_t \\ \hat{g}_t \end{bmatrix}$$

$$M_{ss}^0 E_t \begin{bmatrix} \hat{k}_{t+1} \\ \hat{b}_{t+1} \\ \hat{\lambda}_{1t+1} \\ \hat{\lambda}_{2t+1} \end{bmatrix} + M_{ss}^1 \begin{bmatrix} \hat{k}_t \\ \hat{b}_t \\ \hat{\lambda}_{1t} \\ \hat{\lambda}_{2t} \end{bmatrix} = M_{sc}^0 E_t \begin{bmatrix} \hat{c}_{t+1} \\ \hat{N}_{t+1} \\ \hat{\imath}_{t+1} \end{bmatrix} + M_{sc}^1 \begin{bmatrix} \hat{c}_t \\ \hat{N}_t \\ \hat{\imath}_t \end{bmatrix}$$

$$+ M_{se}^0 E_t \begin{bmatrix} \hat{A}_{t+1} \\ \hat{g}_{t+1} \end{bmatrix} + M_{se}^1 \begin{bmatrix} \hat{A}_t \\ \hat{g}_t \end{bmatrix} \text{ as}$$

$$u_t = M_{cc}^{-1} M_{cs} \begin{bmatrix} x_t \\ \lambda_t \end{bmatrix} + M_{cc}^{-1} M_{ce} z_t \tag{A1}$$

$$E\begin{bmatrix} x_{t+1} \\ \lambda_{t+1} \end{bmatrix} = W\begin{bmatrix} x_t \\ \lambda_t \end{bmatrix} + R\,E_t z_{t+1} + Q\,z_t \tag{A2}$$

where

$$W = -(M^0_{ss} - M^0_{sc}M^{-1}_{cc}M_{cs})^{-1}(M^1_{ss} - M^1_{sc}M^{-1}_{cc}M_{cs})$$

$$R = (M^0_{ss} - M^0_{sc}M^{-1}_{cc}M_{cs})^{-1}(M^0_{se} + M^0_{sc}M^{-1}_{cc}M_{ce})$$

$$Q = (M^0_{ss} - M^0_{sc}M^{-1}_{cc}M_{cs})^{-1}(M^1_{se} + M^1_{sc}M^{-1}_{cc}M_{ce})$$

Assuming that n_s+n_{cs}[18] linearly independent eigenvectors exist for W, let $P\Lambda P^{-1} = W$, where the eigenvectors are arranged in these matrices so that eigenvector in the ith column of P is an eigenvector corresponding to the ith eigenvalue of the diagonal of Λ.

Then multiplying (A2) trough P^{-1} we get

$$P^{-1}E\begin{bmatrix} x_{t+1} \\ \lambda_{t+1} \end{bmatrix} = \Lambda P^{-1}\begin{bmatrix} x_t \\ \lambda_t \end{bmatrix} + P^{-1}R\,E_t z_{t+1} + P^{-1}Q\,z_t$$

$$E\begin{bmatrix} \bar{x}_{t+1} \\ \bar{\lambda}_{t+1} \end{bmatrix} = \Lambda\begin{bmatrix} \bar{x}_t \\ \bar{\lambda}_t \end{bmatrix} + P^{-1}R\,E_t z_{t+1} + P^{-1}Q\,z_t$$

[18] n_s = number of state variables; n_{cs} = number of costate variables.

Then, Λ is constructed with the eigenvalues in increasing order of modulus and is decomposed into

$$\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}$$

with all elements of $\Lambda_1$less than one, and all values of Λ_2 greater than one. As a result the equation for $\tilde{x}_{t+1}$should be solved backward, while the equation for should be solved forward.

Now we can partition the matrices W, R, Q, P and P^{-1}as follows:

$$W = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \; R = \begin{bmatrix} R_x \\ R_\lambda \end{bmatrix} \; Q = \begin{bmatrix} Q_x \\ Q_\lambda \end{bmatrix} \; P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} P^{11} & P^{12} \\ P^{21} & P^{22} \end{bmatrix}$$

Since $W = P\Lambda P^{-1}$ we also have

$$\begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} = \begin{bmatrix} P_{11}\Lambda_1 P^{11} + P_{12}\Lambda_2 P^{21} & P_{11}\Lambda_1 P^{12} + P_{12}\Lambda_2 P^{22} \\ P_{21}\Lambda_1 P^{11} + P_{22}\Lambda_2 P^{21} & P_{21}\Lambda_1 P^{12} + P_{22}\Lambda_2 P^{22} \end{bmatrix}$$

Solving the second equation forward we get

$$E_t\, \tilde{\lambda}_{t+1} = \Lambda_2\, \tilde{\lambda}_t + (P^{21}R_x + P^{22}R_\lambda)\, E_t\, z_{t+1} + (P^{21}Q_x + P^{22}Q_\lambda)\, z_t$$

$$\tilde{\lambda}_t = \Lambda_2^{-1} E_t\, \tilde{\lambda}_{t+1} - \Lambda_2^{-1}(P^{21}R_x + P^{22}R_\lambda)\, E_t\, z_{t+1} + (P^{21}Q_x + P^{22}Q_\lambda)\, z_t$$

$$\bar{\lambda}_t = -\sum_{j=0}^{\infty} \Lambda_2^{-(j+1)} \left[(P^{21} R_x + P^{22} R_\lambda) E_t z_{t+1+j} + (P^{21} Q_x + P^{22} Q_\lambda) E_t z_{t+j} \right].$$

Going back to the original difference equation we have

$$x_{t+1} = W_{11} x_t + W_{12} \lambda_t + R_x E_t z_{t+1} + Q_x z_t \tag{A3}$$

and noticing that

$$\begin{bmatrix} \bar{x}_t \\ \bar{\lambda}_t \end{bmatrix} = \begin{bmatrix} P^{11} & P^{12} \\ P^{21} & P^{22} \end{bmatrix} \begin{bmatrix} x_t \\ \lambda_t \end{bmatrix}$$

we have $\lambda_t = -(P^{22})^{-1} P^{21} x_t + (P^{22})^{-1} \bar{\lambda}_t$. Substituting this into (A3) and using the solution for partitioned W given above

$$x_{t+1} = (P_{11} \Lambda_1 P^{11} + P_{12} \Lambda_2 P^{21}) x_t - (P_{11} \Lambda_1 P^{12} + P_{12} \Lambda_2 P^{22})(P^{22})^{-1} P^{21} x_t$$

$$+ (P_{11} \Lambda_1 P^{12} + P_{12} \Lambda_2 P^{22})(P^{22})^{-1} \bar{\lambda}_t + R_x E_t z_{t+1} + Q_x z_t$$

$$x_{t+1} = [P_{11} \Lambda_1 (P^{11} - P^{12} (P^{22})^{-1} P^{21})] x_t + (P_{11} \Lambda_1 P^{12} + P_{12} \Lambda_2 P^{22})(P^{22})^{-1} \bar{\lambda}_t$$

$$+ R_x E_t z_{t+1} + Q_x z_t$$

Recalling the partitioned inverse formula it will be ease to show that the term $(P^{11} - P^{12}(P^{22})^{-1} P^{21})$ is equal to P_{11}^{-1}so that

$$x_{t+1} = (P_{11} \Lambda_1 P_{11}^{-1}) x_t + (P_{11} \Lambda_1 P^{12} + P_{12} \Lambda_2 P^{22})(P^{22})^{-1} \bar{\lambda}_t + R_x E_t z_{t+1} + Q_x z_t$$

This expresses the solution for x_{t+1} as a function of the past state, and, given the solution for $\bar{\lambda}_t$, as a function of current and expected future values of exogenous variables, z_t.

The solution of λ_t is given by $\lambda_t = -(P^{22})^{-1}P^{21}x_t + (P^{22})^{-1}\ \bar{\lambda}_t$ while the decision rule for the control is given by (A1) above.

Assuming a AR(1) representation for z_t, so that $z_{t+1} = \Pi\, z_t + \varepsilon_{t+1}$, we have $E_t\, z_{t+j} = \Pi^j\, z_t$. To make sure this converges as $j \to \infty$ we need to assume that the eigenvalues of Π are less than one in modulus. Going back to the solution for $\bar{\lambda}_t$ given above we have

$$\bar{\lambda}_t = -\sum_{j=0}^{\infty} \Lambda_2^{-(j+1)} \left[(P^{21}R_x + P^{22}R_\lambda)\, E_t\, z_{t+1+j} + (P^{21}Q_x + P^{22}Q_\lambda)E_t\ z_{t+j}\right]$$

$$\bar{\lambda}_t = -\sum_{j=0}^{\infty} \Lambda_2^{-(j+1)}(\Phi_0 E_t\, z_{t+1+j} + \Phi_1 E_t\ z_{t+j})$$

$$\bar{\lambda}_t = -\left[\sum_{j=0}^{\infty} \Lambda_2^{-(j+1)}\, (\Phi_0\Pi + \Phi_1)\, \Pi^j\right]\ z_t$$

$$\bar{\lambda}_t = \Psi\, z_t.$$

Then the solution for $x_{t+1}$is just

$$x_{t+1} = (P_{11}\Lambda_1 P_{11}^{-1})\, x_t + (P_{11}\Lambda_1 P^{12} + P_{12}\Lambda_2 P^{22})(P^{22})^{-1}\Psi\, z_t + R_x\Pi + Q_x\, z_t$$

$$x_{t+1} = (P_{11}\Lambda_1 P_{11}^{-1})\, x_t + [(P_{11}\Lambda_1 P^{12} + P_{12}\Lambda_2 P^{22})(P^{22})^{-1}\Psi + R_x\Pi + Q_x]\ z_t$$

$$x_{t+1} = \Upsilon_{xx}\, x_t + \Upsilon_{xz}\, z_t \qquad \text{(A4)}$$

The solution for λ_t is

$$\lambda_t = -(P^{22})^{-1}P^{21}x_t + (P^{22})^{-1}\Psi\, z_t$$

$$\lambda_t = \Upsilon_{\lambda x}\, x_t + \Upsilon_{\lambda z}\, z_t \tag{A5}$$

and the solution for the controls is obtained from (A1)

$$u_t = M_{cc}^{-1}M_{cs}\begin{bmatrix} I \\ -(P^{22})^{-1}P^{21} \end{bmatrix} x_t + \left[M_{cc}^{-1}M_{ce}\begin{bmatrix} 0 \\ (P^{22})^{-1}\Psi \end{bmatrix} + M_{cc}^{-1}M_{ce} \right] z_t$$

$$u_t = \Upsilon_{ux}\, x_t + \Upsilon_{uz}\, z_t. \tag{A6}$$

6.4. Appendix D : Gauss Code and Outputs

```
THE PROGRAMS TO ESTIMATE THE MODEL@

statements for graphics @

ary pgraph ;
graphset ;

opening procedure to solve the model @

parameters which matter for the linearized solution@

ac divisib(bbar,crast,ti,epson,ssa,fi,v,lngamy,sig,beta,alpha,delta,sg,rho) ;
cal ynratio,knratio,byratio,tbyratio,gyratio,ncs,
ns,nex,nf,gammax,kyratio,iyratio,cyratio,ssn,ssy,ssc,ssb,ssk,ssi,mu,muk,mcc,mcs,
ce,mss0,mss1,msc0,msc1,mse0,mse1,fc,fx,fe,w,r,q,p0,lamb0,lambr,pr,pim,
amb,lambz,lambs,lambda,p,lamb1,lamb2,p11,p12,p21,p22,ps,ps11,ps12,ps21,
22,rxe,rle,qxe,qle,phi0,phi1,psi,i,xx,xe,solx,lx,lex,soll,cxl,ce,solc,
olf,m,h,ikratio ;

=3 ;  @number of control variables: c, N and i@
=2 ;  @number of state variables: k and b @
s=2;   @number of costate variables: lambda1 and lambda2 @
x=2 ;  @number of exogenous variables: A and g @
f=2;   @number of aditional variables: y and tb@

ratio=sg;
mmax=exp(lngamy) ;
=ln(crast*beta)/lngamy;
ratio=((ssa*(1-alpha)*ti*(gammax-1+delta)^((ti-1)/ti))/(crast-gammax+ti*(gammax+delta-
))^(1/alpha);
ratio=ssa*(knratio)^(1-alpha);
n=((alpha/v*fi)*ssa*knratio^(1-alpha))^(1/(v-1));
y=ssn*ynratio;
ratio=((1-alpha)*ti*(gammax+delta-1)^((ti-1)/ti))/(crast-gammax+ti*(gammax+delta-1));
ratio=(gammax-(1-delta))^(1/ti)*kyratio ;
ratio=iyratio/kyratio;
ratio=bbar/(gammax*ssy);
ratio=(crast-gammax)*byratio;
ratio=1-iyratio-gyratio-tbyratio ;
c=cyratio*ssy;
i=iyratio*ssy;
k=kyratio*ssy;
b=bbar/gammax;


u =(1-alpha)*(1/iyratio);
uk=-alpha*mu ;


cc = (-sig*ssc)/(ssc-(fi*ssn^v))~(sig*fi*v*ssn^v)/(ssc-(fi*ssn^v))~0|
  0~(v-alpha)~0|0~0~(ti-1);

cs = (0~0~1~0)|
    ((1-alpha)~0~0~0)|((ti-1)~0~1~(-1)) ;

mce = (0~0)|(1~0)|(0~0);

mss0 = (muk~(1-ti)~0~mu~(1-delta)*(1-ti)/ti)|
     (0~(gammax*epson*byratio)~1~0)|
      0~-gammax*byratio~0~0|
     ((-gammax)~0~0~0) ;

mss1 = (0~0~0~(-crast/ti)*(kyratio/iyratio)^ti)|
     (0~0~-1~0)|
     ((1-alpha)~crast*byratio~0~0)|
      (1-delta)+(iyratio/kyratio)*(1-ti)~0~0~0 ;

msc0 = (0~muk~-(1-ti))|
      (0~0~0)|(0~0~0)|(0~0~0) ;

msc1 = (0~0~0)|
     (0~0~0)|cyratio~(-alpha)~iyratio|
     (0~0~(-ti*(iyratio/kyratio))) ;

mse0 = (-mu)~0|
     0~0|
     0~0|0~0 ;

mse1 = 0~0|
     0~0|
     (-1~gyratio)|0~0 ;

fc = (0~alpha~0)|
   ((-cyratio/tbyratio)~(alpha/tbyratio)~(-iyratio/tbyratio));


fx = ((1-alpha)~0)|
   ((1-alpha)/tbyratio~0);

fe = (1~0)|
   ((1/tbyratio)~(-gyratio/tbyratio));

w = -inv(mss0 - msc0*inv(mcc)*mcs)*(mss1 - msc1*inv(mcc)*mcs);
r = inv(mss0 - msc0*inv(mcc)*mcs)*(mse0 + msc0*inv(mcc)*mce);
q = inv(mss0 - msc0*inv(mcc)*mcs)*(mse1 + msc1*inv(mcc)*mce);

{lamb0,p0} = eigv(w);
alamb=abs(real(lamb0)) ;
if sumc(abs(imag(lamb0))) gt 1e-10 ;
 print "Error - some eigenvalues of W are complex numbers" ;
 print "Results will be misleading" ;
endif ;
lambr=real(lamb0) ;
pr=real(p0) ;
pr=pr./(sqrt(sumc(pr^2))') ;

lambz=sortind(alamb) ;
lambs=lambr[lambz,1] ;

lambda=diagrv(eye(ns+ncs),lambs) ;
p=pr[.,lambz] ;

lamb1 = lambda[1:ns,1:ns] ;
lamb2 = lambda[ns+1:ns+ncs,ns+1:ns+ncs] ;

p11 = p[1:ns,1:ns] ;
p12 = p[1:ns,ns+1:ns+ncs] ;
p21 = p[ns+1:ns+ncs,1:ns] ;
p22 = p[ns+1:ns+ncs,ns+1:ns+ncs] ;

ps=inv(p) ;
ps11 = ps[1:ns,1:ns] ;
ps12 = ps[1:ns,ns+1:ns+ncs] ;
ps21 = ps[ns+1:ns+ncs,1:ns] ;
ps22 = ps[ns+1:ns+ncs,ns+1:ns+ncs] ;

rxe = r[1:ns,1:nex] ;
rle = r[ns+1:ns+ncs,1:nex] ;
qxe = q[1:ns,1:nex] ;
qle = q[ns+1:ns+ncs,1:nex] ;

phi0=ps21*rxe+ps22*rle ;
phi1=ps21*qxe+ps22*qle ;

psi=zeros(ncs,nex) ;
i=1 ;
do while i le ncs ;
psi[i,.] = -(phi0[i,.]*rho+phi1[i,.])*inv(eye(nex)-rho./lamb2[i,i])/
        lamb2[i,i] ;
i=i+1 ;
endo ;

xx = p11*lamb1*inv(p11) ;
xe = (p11*lamb1*ps12+p12*lamb2*ps22)*inv(ps22)*psi+rxe*rho+qxe ;
solx = xx~xe ;


lx = -inv(ps22)*ps21 ;
lex = inv(ps22)*psi ;
soll = lx~lex ;

cxl = inv(mcc)*mcs ;
ce = inv(mcc)*mce ;
solc = (cxl[.,1:ns]~ce)+(cxl[.,ns+1:ns+ncs]*soll) ;


solf = (fx~fe)+fc*solc ;

m = solx|(zeros(nex,ns)~rho) ;
h = soll|solc|solf ;
retp(m|h) ;
endp ;

proc hpmom(sigma,drules,ncorr) ;
local nex,ns,m,h,sigh,dr1,vr1,sigt,gammt0,gamm0,hbig,base,maxf,jp,jm,
 hpap,hpam,hpa,ny,gammf,facv,i,j,gammj,tcorr,facr,sd,vr,dr,gamm01 ;
nex=rows(sigma) ;
ns=cols(drules)-nex ;
m=drules[1:nex+ns,.] ;
h=drules[nex+ns+1:rows(drules),.] ;
sigh=zeros(ns+nex,ns+nex) ;
sigh[ns+1:ns+nex,ns+1:ns+nex]=sigma ;
{dr1,vr1}=eigv(m) ;
if sumc(abs(imag(dr1))) gt 1e-10 ;
 print "Error - some eigenvalues of M are complex numbers" ;
 print "Results will be misleading" ;
endif ;

vr=real(vr1)./(sqrt(sumc(real(vr1)^2))') ;
dr=real(dr1) ;

sigt=inv(vr)*sigh*inv(vr') ;
gammt0=(ones(ns+nex,ns+nex)./(ones(ns+nex,ns+nex)-dr.*(dr'))).*sigt ;
gamm0=vr*gammt0*(vr') ;
hbig=eye(ns+nex)|h ;

base=181;
maxf=101 ;

jp=seqa(1,1,maxf) ;
jm=seqa(maxf,-1,maxf) ;
hpap=-(.635^jp).*(.234*cos(.439*jp)+.212*sin(.439*jp)) ;
hpam=-(.635^jm).*(.234*cos(.439*jm)+.212*sin(.439*jm)) ;
hpa=hpam|(1-(.234*cos(0)+.212*sin(0)))|hpap ;

ny=rows(drules) ;
gammf=zeros(1,ncorr+1).*.zeros(ns+nex,ns+nex) ;
facv=zeros(ny,(ncorr+1)*ny) ;
i=0 ;
do while i le ncorr ;
 j=0 ;
 do while j le base ;
  if j eq 0 ;
   gammj=gamm0 ;
   gammf[.,i*(ns+nex)+1:(i+1)*(ns+nex)]=gammf[.,i*(ns+nex)+1:(i+1)*(ns+nex)]+
    gammj*(hpa[i+1:2*maxf+1,1]'*hpa[1:2*maxf+1-i,1]) ;
  else ;
   gammj=m*gammj ;
   if j le i ;
    gammf[.,i*(ns+nex)+1:(i+1)*(ns+nex)]=gammf[.,i*(ns+nex)+1:(i+1)*(ns+nex)]+
       gammj*(hpa[i+1-j:2*maxf+1,1]'*hpa[1:2*maxf+1-i+j,1])+
       (gammj')*(hpa[i+j+1:2*maxf+1,1]'*hpa[1:2*maxf+1-i-j,1]) ;
   else ;
    gammf[.,i*(ns+nex)+1:(i+1)*(ns+nex)]=gammf[.,i*(ns+nex)+1:(i+1)*(ns+nex)]+
       gammj*(hpa[1:2*maxf+1+i-j,1]'*hpa[1-i+j:2*maxf+1,1])+
       (gammj')*(hpa[i+j+1:2*maxf+1,1]'*hpa[1:2*maxf+1-i-j,1]) ;
   endif ;
  endif ;
```

```
+1 ;
o ;
{.,i*ny+1:(i+1)*ny]=hbig*gammf[.,i*(ns+nex)+1:(i+1)*(ns+nex)]*(hbig') ;
+1 ;
o ;
sqrt(diag(facv[1:ny,1:ny])) ;
=sd*(sd') ;
=facv./(ones(1,ncorr+1).*.tcorr) ;
(facv|facr) ;
p ;

c hpfilter(y,lamb) ;
 d,a,i ;
ows(y) ;
eros(d,d) ;

while i le d-2 ;
,i]=6*lamb+1;
,i+1]=-4*lamb;
,i+2]=lamb;
,i-2]=lamb;
,i-1]=-4*lamb;
=+1 ;
o ;
,2]=1+5*lamb;
,3]=-4*lamb;
,4]=lamb;
,1]=-2*lamb;
,1]=1+lamb;
,2]=-2*lamb;
,3]=lamb ;
-1,d-1]=5*lamb+1;
-1,d]=-2*lamb;
-1,d-2]=-4*lamb;
-1,d-3]=lamb;
,d]=1+lamb;
,d-1]=-2*lamb;
,d-2]=lamb;
((eye(d)-inv(a))*y) ;
p ;

oc moments(b) ;
al bbar,crast,ti,epson,ssa,fi,sg,v,sig,beta,alpha,lngamy,
lta,rho,sigma,drules,ny,mms,sd,facr,mm,facv;
=b[12,1];
=2.3;
=0.8;
son=0.01;
ar=-15000;
ast=1.0375;
ta=0.971 ;
=(ln(beta*crast))/b[2,1];
pha=b[7,1] ;
gamy=b[2,1] ;
lta=b[11,1] ;
```

```
rho=(b[9,1]~0)|(0~b[5,1]);
sg=exp(b[3,1]-b[1,1]);
ssa=exp(b[8,1]/(1-b[9,1]));
sigma=b[10,1]~0|0~b[6,1];


drules=divisib(bbar,crast,ti,epson,ssa,fi,v,lngamy,sig,beta,alpha,delta,sg,rho) ;
ny=rows(drules) ;
mms=hpmom(sigma,drules,0) ;
sd=sqrt(diag(mms[1:ny,.])) ;
facv=mms[1:ny,.] ;
facr=mms[ny+1:2*ny,.] ;
mm=sd[10,1]|sd[7,1]|sd[9,1]|sd[11,1]
|facr[10,11]|sd[8,1]|sd[1,1];
retp(mm) ;
endp ;

proc impulse(b,nimp) ;
local crast,epson,bbar,ti,fi,ssa,sg,v,sig,beta,theta,alpha,lngamy,delta,rho,sigma,drules,
 nex,ns,m,h,irf,i,irfs,irff,t,zers ;

v=2.3;
ti=0.8;
epson=0.01;
bbar=-15000;
crast=1.0375;
beta=0.971;
fi=b[12,1];
sig=(ln(beta*crast))/b[2,1];
alpha=b[7,1] ;
lngamy=b[2,1] ;
delta=b[11,1] ;
rho=(b[9,1]~0)|(0~b[5,1]);
sg=exp(b[3,1]-b[1,1]);
ssa=exp(b[8,1]/(1-b[9,1]));
sigma=b[10,1]~0|0~b[6,1];



drules=divisib(bbar,crast,ti,epson,ssa,fi,v,lngamy,sig,beta,alpha,delta,sg,rho) ;
nex=rows(sigma) ;
ns=cols(drules)-nex ;
m=drules[1:nex+ns,.] ;
h=drules[nex+ns+1:rows(drules),.] ;
irf=zeros(nimp,8) ;
i=1 ;
do while i le nimp ;
 if i eq 1 ;
  irfs=eye(ns+nex) ;
 else ;
  irfs=m*irfs ;
 endif ;
 irff=h*irfs ;
```

```
[i,1]=irfs[ns+1,ns+1] ;
[i,2]=irfs[1,ns+1] ;
[i,3]=irfs[2,ns+1] ;
[i,4]=irff[3,ns+1] ;
[i,5]=irff[4,ns+1] ;
[i,6]=irff[5,ns+1] ;
[i,7]=irff[6,ns+1] ;
[i,8]=irff[7,ns+1] ;

=i+1 ;
do ;
=seqa(1,1,nimp) ;
s=zeros(nimp,1) ;
gwind ;
ndow(4,2,0) ;
wind(1) ;
aphset ;
protate=1 ; _plwidth=2 ;
ptitlht=0.40 ; _pnumht=0.30 ; _pltype=6|6 ;
tle("A") ;
tics(0,nimp+1,int((nimp+1)/6),1) ;
y(t,irf[.,1]~zers) ;
extwind ;
raphset ;
protate=1 ; _plwidth=2 ;
ptitlht=0.40 ; _pnumht=0.30 ; _pltype=6|3|2|6 ;
tle("Output ") ;
tics(0,nimp+1,int((nimp+1)/6),1) ;
y(t,irf[.,7]~zers) ;
extwind ;
raphset ;
protate=1 ; _plwidth=2 ;
ptitlht=0.40 ; _pnumht=0.30 ; _pltype=6|3|6 ;
tle("Hours ") ;
tics(0,nimp+1,int((nimp+1)/6),1) ;
y(t,irf[.,5]~zers) ;
extwind ;
raphset ;
protate=1 ; _plwidth=2 ;
ptitlht=0.40 ; _pnumht=0.30 ; _pltype=6|3|6 ;
tle("Consumption  ") ;
tics(0,nimp+1,int((nimp+1)/6),1) ;
y(t,irf[.,4]~zers) ;
extwind ;
raphset ;
protate=1 ; _plwidth=2 ;
ptitlht=0.40 ; _pnumht=0.30 ; _pltype=6|3|6 ;
tle(" Trade Balance") ;
tics(0,nimp+1,int((nimp+1)/6),1) ;
y(t,irf[.,8]~zers) ;
extwind ;
raphset ;
protate=1 ; _plwidth=2 ;
ptitlht=0.40 ; _pnumht=0.30 ; _pltype=6|3|2|6 ;
tle("Capital Stock") ;
```

```
 xtics(0,nimp+1,int((nimp+1)/6),1) ;
 xy(t,irf[.,2]~zers) ;
nextwind ;
 graphset ;
 _protate=1 ; _plwidth=2 ;
 _ptitlht=0.40 ; _pnumht=0.30 ; _pltype=6|3|2|6 ;
 title("Net Foreign Asset Holdings") ;
 xtics(0,nimp+1,int((nimp+1)/6),1) ;
 xy(t,irf[.,3]~zers) ;
nextwind ;
 graphset ;
 _protate=1 ; _plwidth=2 ;
 _ptitlht=0.40 ; _pnumht=0.30 ; _pltype=6|3|2|6 ;
 title("Investment") ;
 xtics(0,nimp+1,int((nimp+1)/6),1) ;
 xy(t,irf[.,6]~zers) ;
nextwind;
nextwind;
endwind;
retp(irf) ;
endp ;


proc datamom(b) ;
local dm ;
dm=b[13:19,1];
retp(dm) ;
endp ;

varb=0 ;

proc tests(b) ;
local nm,mm,gm,dm,gd,vm,vd,testn,testd,test,pv,i ;
mm=moments(b) ;
nm=rows(mm) ;
gm=grad2(&moments,b,mm,0) ;
dm=datamom(b) ;
gd=grad2(&datamom,b,dm,0) ;
load varb ;
vm=zeros(nm,1) ;
vd=zeros(nm,1) ;
test=zeros(nm,1) ;
pv=zeros(nm,1) ;
i=1 ;
do while i le nm ;
 vm[i,1]=gm[i,.]*varb*(gm[i,.]') ;
 vd[i,1]=gd[i,.]*varb*(gd[i,.]') ;
 testn=mm[i,1]-dm[i,1] ;
 testd=(gm[i,.]-gd[i,.])*varb*((gm[i,.]-gd[i,.])') ;
 test[i,1]=(testn^2)/testd ;
 pv[i,1]=cdfchic(test[i,1],1) ;
 i=i+1 ;
endo ;
format /rd /m1 11,4 ;
print "Model Moments  s.e.     Data Moments     s.e     Test     P-Value";
```

```
am~sqrt(vm)~dm~sqrt(vd)~test~pv ;
);
;

***************************************************************** @
------- Step 1: Prepare Output File ----------------------@
***************************************************************** @
u file=tesemestrado.out reset; @ Specify the name of the output file. @

*********************** GMM Results *******************************";
mestrado.out";

***************************************************************** @
------- Step 2:  Define Global Variables ---------------- @
***************************************************************** @

-----------------@
Section A @
-----------------@

=46;  @ # of observations=T @

=1|0.03|-1.5|0.05|0.9|0.002|0.53|0.13|0.9
01|0 1|20|0.02|0.02|0.05|0.23|-0.02|0.02|0.01;

        @ bgm=b(1)|..|b(kgm); kgm by 1 vector
          Initial values of the coefficients@

=19;       @ # of disturbance terms in w(t); scalar @

=ones(nw,1) ;
        @ nzv=nz(1)|..|nz(nw); nw by 1 vector
          For each i, nz(i) is # of elements in zi(t) i=1,...,nw.
          Let L=sumc(nzv), then L is # of orthogonal condition.@

=ones(nw,1) ;
          @ rwv=rw(1)|..|rw(nw); nw by 1 vector
           rw(i) is defined so that
           wi(t) is in I(t+rw(i)) i=1,...,nw @

-----------------@
 @ Section B@
-----------------@

dname=&GRAD2; @ Specify the name of proc that calculates
          the gradient dgT(b)/db. @

nst=1/sqrt(tend);   @ Scaling Multiplier when W0=eye(L) @
nst2=1/sqrt(tend);  @ Scaing Multiplier when W0 is not eye(L) @
            @ These scaling multipliers should be set so that
             the value of function (vof) in nonlinear search of
             MINQUAD.SET is near 1.  If vof is too close to 0,
             the search will not work properly.
```

```
            const2=1/sqrt(tend) will generally be good. @
w0flag=0;   @ scalar,
         If w0flag=0, W0=I is used as the initial weighing matrix W0.
         If w0flag=1, initial bgm is used to calculate initial W0.
         If w0flag=2, W0 in the memory is used as initial W0.
         If w0flag=3, W0 and bgm in the memory are used to give
             the first GMM result.              @

maxitegm=2; @ Sets maximum # of iteration over weighting matrix, W0.  Set
         w0flag=0 and maxitegm=2 to execute usual 2-Stage GMM. @

zero=1E-2;   @ Iteration over W0 continues until the maximum
         difference of the current and the previous W0 in
         absolute value becomes less than 'zero', or
         the # of iteration exceeds maxitegm. @

calwflag=1;  @ This variable is used to choose the method to calculate
        the distance matrics, W0.

        If calwflag=0, Durbin's method will be used when W0
        If calwflag=1, Newey and West Method will be used when
           W0 is singular.
        If calwflag=2, Parzen's lag window will be used when
          W0 is singular.
        If calwflag=3, Durbin's method will be used.
        If calwflag=4, Newey and West Method will be used.
        If calwflag=5, Parzen's lag window will be used.
      Durbin's method imposes zero restrictions while Newey and West
      method does not.  @

ordard=floor(sqrt(8190/sumc(nzv)^2)); @ Order of AR representation for
          Durbin's method when W0 is singular @

lend=5;  @ Order of lags used for Newey and West method @

  @ See MINQUAD.SET for the following globals @
hflag=2;
dfpflag=1;
sstol=1e-25;
  @ See MAXMUM.DOC on MODULE9 of GAUSS for the following globals @
gradtol=1e-5;
btol=1e-5;
typf=1;
typb=1;

@ ******************************************************** @
@ ------- Step 3: READING IN DATA -------------------- @
@ ******************************************************** @
load x[46,13]=PT.dat;

y=(1000*x[.,2]./x[.,1]);
c=(1000*x[.,3]./x[.,1]);
i=(1000*x[.,4]./x[.,1]);
dk=(1000*x[.,5]./x[.,1]);
g=(1000*x[.,6]./x[.,1]);
```

```
=(1000*x[.,7]./x[.,1]);
=(1000*x[.,8]./x[.,1]);
=1000*(x[.,7]-x[.,8]);

=(1000*x[.,9]./x[.,1]);
=(1000*x[.,11]./(5476*x[.,1]));

=((2.876663562*1000)^(-1).*nx)-1;

=hpfilter(ln(y),6.25) ;
=hpfilter(ln(c),6.25) ;
=hpfilter(ln(dk),6.25) ;
=hpfilter(ln(g),6.25) ;
ex=hpfilter(ln(ex),6.25) ;
=hpfilter(ln(m),6.25) ;
=hpfilter(lnx,6.25) ;
=hpfilter(ln(k),6.25) ;
=hpfilter(ln(h),6.25) ;

clear x;

@ ***************************************************************** @
* --------------------------------------------------
    Step 4: Define the proc hu(b) that returns tend by L matrix
              |[z(1)w(1)]'     |
              |   |            |
              |[z(tend)w(tend)]'|

          where [z(t)w(t)]'=[w1(t)z1(t)',...,wnw(t)znw(t)']
------------------------------------------------------------ */
@ ***************************************************************** @

proc hu(b);
local epson,bbar,sig,ti,crast,fi,ssa,sg,rho,delta,v,alpha,t,t0,w31,beta,a,w1,w2,w3,
w4,w5,w6,w7,w8,w9,w10,w11,w12,w13,w14,w15,w16,w17,w18,w19;

=(b[9,1]~0)|(0~b[5,1]);
=exp(b[3,1]-b[1,1]);
a=exp(b[8,1]/(1-b[9,1]));

=seqa(1,1,44) ;
=0|t|45 ;

epson=0.01;
=2.1;
ssa=0.971 ;
crast=1.0375;
=0.8,
bar=-15000;

=y[1:46,1] ./(k[1:46,1]^(1-b[7,1])).*(h[1:46,1].*exp(b[2,1]*t0))^(b[7,1]);
= 1/(v*h[2:45,1]^v).*(b[7,1]*(y[2:45,1]./exp(b[2,1]*t)));

sig=(ln(beta*crast))/b[2,1];
```

```
w1=ln(y[2:45,1])-b[1,1]-b[2,1].*t ;
w2= w1 .*t/tend ;
w3=ln(g[2:45,1])-b[3,1]-b[4,1].*t;
w4=w3 .*t/tend;
w31=ln(g[1:44,1])-b[3,1]-b[4,1].*(t-1);
w5=(w3-b[5,1]*w31).*w31;
w6=(w3-b[5,1]*w31)^2-b[6,1];

w7=-((exp(b[2,1])^sig)/ti).*(dk[2:45,1]./k[2:45,1])^(1-ti)
+beta.*((c[2:45,1]-exp(b[2,1]*t).*b[12,1].*h[2:45,1]^v)./(c[3:46,1]/exp(b[2,1])-
b[12,1].*exp(b[2,1]*t).*h[3:46,1]^v))^sig
.*((1-b[11,1])/ti.*(dk[3:46,1]./k[3:46,1])^(1-ti)+((1-ti)/ti).*(dk[3:46,1]./k[3:46,1])+(1-
b[7,1]).*(y[3:46,1]./k[3:46,1]));

w8=ln(a[2:45,1])-b[8,1]-b[9,1].*ln(a[1:44,1]) ;
w9=(w8).*ln(a[1:44,1]) ;
w10=w8^2-b[10,1] ;
w11=-(k[3:46,1]./exp(b[2,1]*t))+(k[2:45,1]./exp(b[2,1]*t)).*(dk[2:45,1]./k[2:45,1])^ti+(1-
b[11,1]).*(k[2:45,1]./exp(b[2,1]*t)) ;
w12=(fi[1:44]-b[12,1]);
w13=hy[2:45,1]^2-b[13,1]^2;
w14=hc[2:45,1]^2-b[14,1]^2;
w15=hi[2:45,1]^2-b[15,1]^2 ;
w16=htb[2:45,1]^2-b[16,1]^2 ;
w17=((hy[2:45,1].*htb[2:45,1])./(b[13,1]*b[16,1]))-b[17,1];
w18=hh[2:45,1]^2-b[18,1]^2 ;
w19=hk[2:45,1]^2-b[19,1]^2 ;


retp(w1~w2~w3~w4~w5~w6~w7~w8~w9~w10~w11~w12~w13~w14~w15~w16~w17~w18~w19
);

endp;

proc (0)=prntrslt(x,s);

? "
------------ Minimization Results ----------------
Step size: " s "
Value of the objective function: " vof "
Minimiser is"  x' "
--------------------------------------------";
endp;

@ ------------------------------------------------------- @
@ ******************************************************************* @
@ THE PROGRAM STARTS @

chi=gmmq(gradname,tend,nzv,rwv,nw,rows(bgm),zero,maxitegm);
load bgm ;
dmmy=tests(bgm) ;
dmmy=impulse(bgm,100) ;
output off;
@ ------------------ END OF PROGRAM -------------------- @
end ;
```

************************** GMM Results **********************************
tesemestrado.out
Initial values of the coefficients= 1.0000 0.0300 -1.5000 0.0500 0.9000
0.0020 0.5300 0.1300 0.9000 0.0010 0.1000 20.0000 0.0200 0.0200
0.0500 0.2300 -0.0200 0.0200 0.0100
lend= 5.0000
rwv= 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
1.0000 1.0000
Initial W0=I
Scaling Constant used for first iteration= 0.14744196

initial function value is 32945.15579000
------------- Iteration number 1.00000000 ---------------
Step Size 1.00000000
Value of objective function 65.86960776
Current parameter values: 0.95991417 0.03286967 -1.41052353 0.05253068
0.84730409 0.00177437 0.75990127 0.05606415 0.82406447 -0.00729155
0.13093224 65.44352317 0.02121144 0.01945364 0.05988586 0.23753845
-0.34120161 0.02095170 0.00839640
Current relative gradients: 1.47436552 1361882.04892072 7.77190883
186.65781303 0.00435601 0.00171705 105878.40775886 4.88414457
0.89952535 0.54917744 58.17195671 1169.73680672 280.39694239
0.00003655 0.00413374 24.44661611 14.90000161 0.00006367 0.00002682
Outer product used for Hessian: DFP=1, Outer Prodct=2
------------- Iteration number 2.00000000 ---------------
Step Size 1.00000000
Value of objective function 0.00337420
Current parameter values: 0.95991417 0.03286967 -1.41052353 0.05253068
0.91138557 0.00173115 0.75581587 0.05455556 0.96935715 -0.00686428
0.13313712 63.84937505 0.02117685 0.01944597 0.05906989 0.23741883
-0.34431328 0.02093008 0.00824326
Current relative gradients: 0.00000012 5142777.37236157 0.00663541
0.73760719 0.07638127 0.34582928 283475.22492197 108.54647974
88.76482107 49.22233737 203.39125132 3356.41187282 103.59555102
0.00003121 0.03145482 9.16949037 6.38956601 0.00010199 0.00011604
Outer product used for Hessian: DFP=1, Outer Prodct=2
------------- Iteration number 3.00000000 ---------------
Step Size 1.00000000
Value of objective function 0.00000000
Current parameter values: 0.95991417 0.03286967 -1.41052353 0.05253068
0.91138557 0.00181710 0.75581104 0.05497716 0.97129397 0.00200788
0.13313712 63.84896621 0.02117682 0.01944597 0.05906425 0.23741880
-0.34431453 0.02093007 0.00824184
Current relative gradients: 0.00000001 3.79459369 0.00009546 0.00942366
0.00000000 0.23098714 0.37846223 2.58230619 1.39803886 23.85223780
0.00441067 0.00000017 0.03280369 0.00000001 0.00021137 0.00290508
0.00201575 0.00000005 0.00000104
Outer product used for Hessian: DFP=1, Outer Prodct=2
------------- Iteration number 4.00000000 ---------------
Step Size 1.00000000
Value of objective function 0.00000000
Current parameter values: 0.95991417 0.03286967 -1.41052353 0.05253068
0.91138557 0.00181710 0.75581104 0.05497717 0.97129401 0.00200990

0.13313712 63.84896621 0.02117682 0.01944597 0.05906425 0.23741880
-0.34431453 0.02093007 0.00824184
Current relative gradients: 0.00000000 0.00006473 0.00000000 0.00000000
0.00000000 0.00000000 0.00001974 0.00004149 0.00002429 0.00543567
0.00000000 0.00000000 0.00001826 0.00000000 0.00000001 0.00000163
0.00000112 0.00000000 0.00000000
Outer product used for Hessian: DFP=1, Outer Prodct=2

-------------- Minimization Results ---------------------
Step size: 1.00000000
Value of the objective function: 0.00000000
Minimiser is 0.95991417 0.03286967 -1.41052353 0.05253068 0.91138557
0.00181710 0.75581104 0.05497717 0.97129401 0.00200990 0.13313712
63.84896621 0.02117682 0.01944597 0.05906425 0.23741880 -
0.34431453 0.02093007 0.00824184

------- for next GMM iteration
All the zero restirictions are successfully imposed on W0 -------
max(|difference|)=323536538.88751417

initial function value is 0.00000000

-------------- Minimization Results ---------------------
Step size: 1.00000000
Value of the objective function: 0.00000000
Minimiser is 0.95991417 0.03286967 -1.41052353 0.05253068 0.91138557
0.00181710 0.75581104 0.05497717 0.97129401 0.00200990 0.13313712
63.84896621 0.02117682 0.01944597 0.05906425 0.23741880 -
0.34431453 0.02093007 0.00824184

================ GMM ITERATION 2.00000000 ================
b= 0.95991417 0.03286967 -1.41052353 0.05253068 0.91138557
0.00181710 0.75581104 0.05497717 0.97129401 0.00200990
0.13313712 63.84896621 0.02117682 0.01944597 0.05906425 0.23741880
-0.34431453 0.02093007 0.00824184
s.e.= 0.04295951 0.00145458 0.05025457 0.00196990 0.06055147
0.00052895 0.00950363 0.00697471 0.00714367 0.00054457 0.00275159
3.45352780 0.00304961 0.00214060 0.00539702 0.02531140 0.12156809
0.00291037 0.00088220
Just Identified
scaling const used is 0.14744196

Model Moments	s.e.	Data Moments	s.e	Test	P-Value
0.0408	0.0056	0.0212	0.0030	15.1795	0.0001
0.0376	0.0123	0.0194	0.0021	2.1345	0.1440
0.0724	0.0104	0.0591	0.0054	1.4492	0.2287
0.8171	1.3226	0.2374	0.0253	0.1912	0.6619
-0.6030	0.0148	-0.3443	0.1216	4.4755	0.0344
0.0178	0.0024	0.0209	0.0029	0.8322	0.3616
0.0065	0.0010	0.0082	0.0009	1.7997	0.1797

```
tend=46;  @ # of observations=T; scalar @

bgm=0.03|0.03|0.03|0.03|0.03;
 @ Initial values of the coefficients@
nw=8;          @ # of disturbance terms in w(t); scalar @
nzv=ones(nw,1);        @ nzv=nz(1)|..|nz(nw); nw by 1 vector
             For each i, nz(i) is # of elements in zi(t) i=1,..,nw.
             Let L=sumc(nzv), then L is # of orthogonal condition.@
rwv=ones(nw,1);        @ rwv=rw(1)|..|rw(nw); nw by 1 vector
             rw(i) is defined so that
              wi(t) is in I(t+rw(i)) i=1,..,nw @
gradname=&GRAD2; @ Specify the name of proc that calculates
             the gradient dgT(b)/db. @
const=1/sqrt(tend);   @ Scaling Multiplier when W0=eye(L) @
const2=1/sqrt(tend);  @ Scaing Multiplier when W0 is not eye(L) @
              @ These scaling multipliers should be set so that
               the value of function (vof) in nonlinear search of
               MINQUAD.SET is near 1.  If vof is too close to 0,
               the search will not work properly.
               const2=1/sqrt(tend) will generally be good. @
w0flag=0;   @ scalar;
            If w0flag=0, W0=I is used as the initial weighing matrix W0.
            If w0flag=1, initial bgm is used to calculate initial W0.
            If w0flag=2, W0 in the memory is used as initial W0.
            If w0flag=3, W0 and bgm in the memory are used to give
               the first GMM result.              @
maxitegm=2; @ Sets maximum # of iteration over weighting matrix, W0.  Set
             w0flag=0 and maxitegm=2 to execute usual 2-Stage GMM. @
zero=1E-2;   @ Iteration over W0 continues until the maximum
             difference of the current and the previous W0 in
             absolute value becomes less than 'zero', or
             the # of iteration exceeds maxitegm. @
calwflag=0;  @ This variable is used to choose the method to calculate
            the distance matrics, W0.
            If calwflag=0, Durbin's method will be used when W0
              is singular.
            If calwflag=1, Newey and West Method will be used when
              W0 is singular.
            If calwflag=2, Parzen's lag window will be used when
              W0 is singular.
            If calwflag=3, Durbin's method will be used.
            If calwflag=4, Newey and West Method will be used.
            If calwflag=5, Parzen's lag window will be used.
          Durbin's method imposes zero restrictions while Newey and West
          method dose not.  @
ordard=floor(sqrt(8190/sumc(nzv)^2)); @ Order of AR representation for
             Durbin's method when W0 is singular @
lend=5;  @ Order of lags used for Newey and West method @
  @ See MINQUAD.SET for the following globals @
hflag=2;
dfpflag=1;
sstol=1e-25;
  @ See MAXMUM.DOC on MODULE9 of GAUSS for the following globals @
btol=1e-5;
typf=1;
typb=1;

@ ***************************************************************** @
@ ---------- Step 3: READING IN DATA ---------------------- @
@ ***************************************************************** @
load x[46,13]=PT.dat;

Y=(1000*x[.,2]./x[.,1]);
c=(1000*x[.,3]./x[.,1]);
dk=(1000*x[.,5]./x[.,1]);
G=(1000*x[.,6]./x[.,1]);
k=(1000*x[.,9]./x[.,1]);
N=(1000*x[.,10]./x[.,1]);
h=(1000*x[.,11]./(5476*x[.,1]));
clear x;

proc hu(b);
local w1,w2,w3,w4,w5,w6,w7,w8,t,t0;

t=seqa(1,1,46) ;

w1=ln(Y[1:46,1])-b[1,1]-b[2,1].*t ;
w2= w1 .*t/tend ;
w3=ln(dk[1:46,1])-b[3,1]-b[2,1].*t ;
w4= w3 .*t/tend ;
w5=ln(c[1:46,1])-b[4,1]-b[2,1].*t ;
w6= w5 .*t/tend ;
w7=ln(g[1:46,1])-b[5,1]-b[2,1].*t;
w8=w7 .*t/tend;
retp(w1~w2~w3~w4~w5~w6~w7~w8);

endp;

@=========== The User does not have to change the code below. ============ @

@------------------- Print results of estimation-------------------@
proc (0)=prntrslt(x,s);

? "
-------------- Minimization Results --------------------
Step size: " s "
Value of the objective function: " vof "
Minimiser is"  x' "
-----------------------------------------------------";
endp;

@ THE PROGRAM STARTS @

chi=gmmq(gradname,tend,nzv,rwv,nw,rows(bgm),zero,maxitegm);
output off;

@ --------------------- END OF PROGRAM ---------------------------- @
```

Initial values of the coefficients= 0.030000000 0.030000000 0.030000000
0.030000000 0.030000000
lend= 5.0000000
rwv= 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
1.0000000 1.0000000 1.0000000
Initial W0=I
Scaling Constant used for first iteration= 0.14744196

initial function value is 136.85716
------------- Iteration number 1.0000000 ---------------
Step Size 1.0000000
Value of objective function 0.18447855
Current parameter values: 0.81675972 0.036913277 -0.63029306 0.40838908
-1.0873441
Current relative gradients: 60.546031 142.96073 30.722288 34.789333
59.549348
Outer product used for Hessian: DFP=1, Outer Prodct=2

--------------- Minimization Results ----------------------
Step size: 0.015625000
Value of the objective function: 0.18447855
Minimiser is 0.81675972 0.036913277 -0.63029306 0.40838908 -1.0873441

------- for next GMM iteration
All the zero restirictions are successfully imposed on W0 -------
max(|difference|)= 49082.694

initial function value is 42.679445
------------- Iteration number 1.0000000 ---------------
Step Size 1.0000000
Value of objective function 25.747373
Current parameter values: 0.91443938 0.034005182 -0.55138076 0.48861318
-0.91983855
Current relative gradients: 8.6275163 214.47225 3.4232757 0.066303459
4.2628671
Outer product used for Hessian: DFP=1, Outer Prodct=2

--------------- Minimization Results ----------------------
Step size: 1.0000000
Value of the objective function: 25.747373
Minimiser is 0.91443938 0.034005182 -0.55138076 0.48861318 -0.91983855

==================== GMM ITERATION 2.0000000 ====================
b= 0.91443938 0.034005182 -0.55138076 0.48861318 -0.91983855
s.e.= 0.036233348 0.0012366456 0.040254831 0.037753901 0.049904385
chi square= 25.747373 (1.0772460e-005)
d.f.= 3.0000000
difference in prob. value from last GMM iteration= 99.999989
(1/sqrt(T))g(b)= -0.15806605 -0.10619204 -0.031809621 -0.039149048
0.0047878974 -0.13274760 -0.83877728 0.057313707
s.e.= 0.055053032 0.047474257 0.010674124 0.023768652
0.072077609 0.058655847 0.18148280 0.031571616
scaling const used is 0.14744196