

RESERVADO



Universidade Técnica de Lisboa

Instituto Superior de Economia e Gestão

MASTER COURSE IN FINANCE

## Calculating Rates of Return on Futures

Pedro Miguel da Silva Gonçalves

Supervisor: Professor Dr. João Duque

Presidente do Júri: Doutor João Luís Correia Duque

Vogais: Doutor Paulo Fernando de Sousa Pereira Alves

Mestre João Augusto Cantiga Esteves

May 2008

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## CALCULATING RATES OF RETURN ON FUTURES

**Author:** Pedro Miguel da Silva Gonçalves

**Master Course in:** Finance

**Supervisor:** Professor Dr. João Duque

**Date:** October 2007

### ABSTRACT

The purpose of this dissertation is to address the practical problems faced when calculating futures returns. A methodology is proposed to allow extending standard procedures to calculate rates of return of futures and portfolios holding futures positions.

We generally define a derivative instrument position as the sum of the required margin and the derivative value. Since a futures contract is daily marked-to-market its value is zero and the investment value is the amount of funds tied to the investment in the margin account. The rate of return can be calculated on the margining cash flows required to maintain the investment position, but in relation to the daily adjusted tied investment.

The methodology is shown to accurately provide daily returns obtained and risk incurred in futures positions, namely through the use of Value at Risk using calculated historical return series.

**Keywords:** Future Contracts, Rates of return, Non-traditional portfolios

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## CÁLCULO DE TAXAS DE RETORNO DE FUTUROS

**Autor:** Pedro Miguel da Silva Gonçalves

**Mestrado em:** Finanças

**Orientador:** Professor Dr. João Duque

**Data:** Outubro 2007

### RESUMO

Esta dissertação propõe-se abordar os problemas práticos com que somos deparados ao calcular taxas de retorno de contractos futuros. É proposta uma metodologia que permite estender os procedimentos padrão para o cálculo de taxas de retorno de futuros e carteiras investidas em futuros.

A posição de um derivado é definida como a soma da margem exigida e do valor do derivado. Uma vez que um contracto futuro é liquidado diariamente, o seu valor é zero e o valor da posição são os fundos na conta margem necessários ao investimento. A taxa de retorno pode ser calculada utilizando os fluxos resultantes da liquidação diária do contrato face aos fundos que é necessário manter em cada dia na conta margem.

É demonstrado que a metodologia permite calcular com acuidade os retornos obtidos e o risco incorrido em posições de futuros, nomeadamente através do cálculo do *Value at Risk* utilizando series históricas calculadas desses retornos.

**Palavras Chave:** Contractos Futuro, Taxas de Retorno, Portfolios não-tradicionais



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## CONTENTS

1. Introduction .....	5
2. Literature Review .....	6
Hancock's approach .....	8
3. Alternative Approach .....	9
Why is this modification important? .....	11
4. Research Questions .....	16
5. Methodology and Data .....	16
5.1. Methodology .....	16
5.2. Data .....	18
6. Empirical Analysis .....	18
6.1. Calculating Historical Risk and Standard Deviations .....	18
6.2. Forecasting Risk .....	24
7. Summary .....	28
References .....	30

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## 1. Introduction

A portfolio return has been shown by Markowitz (1952) to be a simple weighted average of its individual securities return. However, in portfolios holding non-traditional assets, like futures positions or short options, obtaining the investment amount, proportional portfolio weight and rate of return is not straightforward. A practical difficulty exists in extending standard procedures to calculate returns on these portfolios.

Meanwhile what were then relatively exotic instruments are nowadays a common presence in investment portfolio balances, particularly futures contracts. The difficulty in identifying the rate of return can mean an inaccurate measurement of performance and misperception of the risk represented by these positions in portfolios. What may seem a tiny and irrelevant detail may in fact have important implications as eventually leading to capital inadequacies in financial institutions or their shareholders having a remuneration misadjusted from the risk incurred. Regulatory enforcement problems have become particularly relevant in a Basel II implementation context.

The purpose of this dissertation is to propose a practical solution to the problems faced when calculating rates of returns on futures positions and portfolios holding futures.

Hancock (2005) has previously addressed this issue but we believe some modifications are needed in his proposals.

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This dissertation is organized as follows: on chapter 2 we trace a general view of the theoretical and empirical work developed on the subject; on chapter 3 our alternative approach is presented; on chapter 4 we identify our research questions; on chapter 5 the methodology and empirical data used to address the problem are explained; on chapter 6 we present, analyse and discuss empirical results obtained and on chapter 7 we summarize the main conclusions and identify areas of further future research.

## **2. Literature Review**

The problem of calculating futures contract rates of return has been considered by several authors and different solutions have been proposed, but none of them obtained consensus.

Figlewski (1984), and Cheung, Kwan and Yip (1990) calculate the rate of return of futures as the change in the futures price divided by the initial level of the spot index. As if the investor posted a 100% margin account. Baxter, Conine and Tamarkin (1985) use the change in the futures settlement price. Kalavathi and Shanker (1991), and Kuo and Chen (1995) calculate the absolute return on the difference between the initial and final futures price, as if no investment is made.

The most immediate difficult is to identify the initial investment position. In practice it is not possible to enter in a futures position without the funds required by the broker (or exchange) to the margin account. Margins work as "security deposits required by brokers from their clients in certain kind of transactions, as trading futures contracts,

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and reflect their self interest in protecting themselves against the possibility of their clients defaulting" (Telser 1981).

A possibility would be to consider the margin as the investment amount in the futures contract and use it to calculate returns rates. But some authors contest it. Dusak (1973) argues that the margin account cannot be considered as an investment because there is no fund transfer from the buyer to the seller and the trader retains ownership of the funds deposited. Figlewski (1984) in addition points out that the margin account represents no real cost since interest bearing treasury securities can be posted to meet margin requirements.

Telser, on the other hand, considers interest bearing treasury securities to be "highly liquid assets that stand ready for use by the holder in case of emergency or to take advantage of profitable opportunities that may suddenly present themselves. They are part of the holders' precautionary balance. If he commits them for use as margin, they are unavailable for other uses. They are no longer part of his precautionary balance. Hence, the margin requirement surely imposes a cost on the investor".

Although different for each investor there is a liquidity cost that results from the existence of the margin account increasing the probability of the investor being caught short of liquid assets, and there is a "positive probability that an investor will not be able to convert quickly the Treasury bills in a margin account into cash" if price limit moves occur (Hartzman 1986). Hartzman empirical results also support the contention that margin requirements impose costs on traders. He finds futures open interest and margin requirements to be significantly inversely related, but not volume, and that changes in

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margins cause a significant change in the composition of traders in the market although not in the price of the contracts.

Even if liquidity is not needed the margin deposited becomes unavailable for investment in higher yielding assets. An opportunity cost seems to remain. Funds deposited cannot be used on alternative purposes and the risk free rate cannot compensate the risk inherent on a futures contract position where there is no guarantee of those funds being returned. If the trader is in a liquidity shortage or wants to invest in higher yielding assets, he needs to borrow. Kalavathi (1991) therefore defines the cost of the margin as the difference between the trader's borrowing and lending rates.

Fishe and Goldberg (1999) try to determine whether margins have an impact on trading and develop a model in which their most significant effect is to increase the cost of defaulting. They find an inverse relation between open interest and margin changes.

### **Hancock's approach**

Hancock has specifically addressed the issue of providing techniques for identifying returns on non-traditional portfolios.

Although assuming there is no consensus in considering the funds necessary to enter a futures contract as an investment, he generally defines an investment as "the deferral of consumption due to the commitment of dollars for a period of time in order to derive a payoff that is expected to compensate the investor for the time commitment, the expected rate of inflation and the risk of the medium", and concludes funds involved in

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futures margining possess some characteristics which liken them to it - opportunity costs, liquidity costs and no guarantee of its return.

He identifies the amount of the investment in a futures contract as the tied-investment or initial deposit, and calculates its realized global return on the cash flows required to maintain the position.

Hancock uses Elton and Gruber arguments<sup>1</sup> to defend the futures position return should not be influenced by “interim cash flows created by marking-to-market”, and that the way to eliminate that effect is to calculate the return on the cash-flows required to maintain the position as if there were no mark-to-market mechanism. The return is obtained as if it were of a forward contract with an initial margin requirement.

### **3. Alternative Approach**

Empirical research presenting a relationship between changes in margin requirements and changes on market activity, whether on open interest or the composition of traders in the market, is a strong indicator that margin requirements represent a cost to futures contract traders. If so, as is our opinion, that means they cannot be ignored when calculating return rates.

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<sup>1</sup> “Unless the inflow and outflows are under the control of the manager (and in most cases they are not), the manager should not be rewarded or penalized for the good or bad fortune of having extra funds available at a particular time.

We eliminate the effect of having different amounts of funds available if we calculate the rate of return in each period and then compound the return to determine it in the overall period. When the rate of return is calculated this way it is called the time weighted rate of return.” (Elton and Gruber, 1995).

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Our approach differs from Hancock in considering that the daily margining mechanism of futures contracts must also be integrated. Although in and out flows resulting from the marking-to-market of the contract are not under the investor's control, they are a result of an investment decision. Their nature is very different from in and out flows referred by Elton and Gruber, like a capital injection or a capital withdraws, and are of a particular importance for they determine the real daily tied investment amount.

They cannot be ignored because the real tied investment is daily adjusted to the initial margin whenever the interval between the maintenance and initial margin is exceeded<sup>2</sup>. It is on that tied investment that the return on the cash-flows required to maintain the position must be calculated.

We will generally define a futures investment position and return rate as:

$$\text{Futures Investment Position} = \text{Margin required}^3 + \text{Futures Value} \quad (1a)$$

Since a futures contract is daily marked-to-market its value is zero and the tied funds in the margin account represent the value of the future investment position.

$$\text{Futures Inv Position Return } t = \frac{\text{Margin in/out flow } t}{\text{Futures Inv Position } t-1} \quad (1b)$$

$$\text{Margin in/outflow} = \Delta \text{ Futures Price} * \text{No Contracts} * \text{Contract Size} \quad (1c)$$

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<sup>2</sup> For simplicity, and because it is indifferent for our purposes, through out this dissertation we will assume the futures initial and maintenance margin to be equal.

<sup>3</sup> Only funds tied to the investment; surplus eventually not withdrawn are not included.

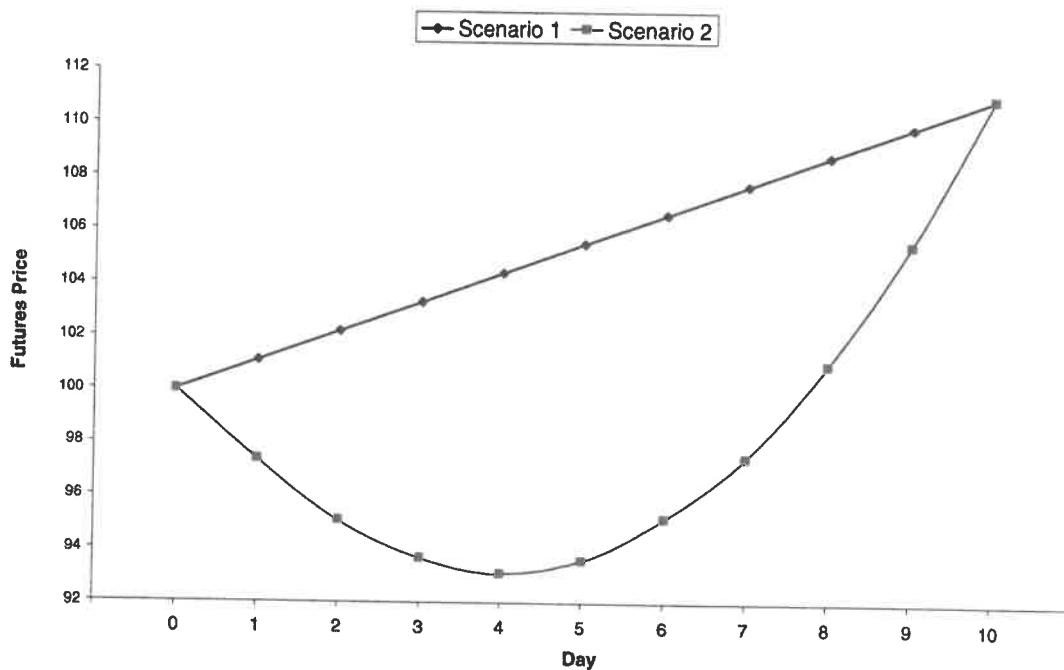
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## Why is this modification important?

In a futures contract, after the margin account initially constituted, we cannot be sure of the additional amount of funds needed (or released) to maintain a position during a period of time. Chart I presents two possible scenarios for an investor opening a long futures position between day 0 and day 10. Both scenarios coincide on the futures price in day 0 and day 10, but whereas scenario 1 is of a constant daily increase in the futures price, in scenario 2 the price decreases till day 4 and then sharply increases till day 10.

### Chart I

#### Futures Price Scenarios



The margining mechanism means that, even assuming the given daily information holds in  $t=0$  and  $t=10$ , what happens in the moments between is not neutral on the portfolio return, either by different financing costs or allocation effects created between assets on the portfolio. They can cause liquidity shortages (or surplus) with associated costs (or revenues) and impact other investment decisions.

Again, if we believe margin requirements do represent a cost to the investor, it is not neutral to the investor which of these scenarios becomes true. Whereas in scenario 1 the investor would steadily recover his initial investment, in scenario 2, although he would also profit on the 10 day balance, besides the initial deposit he would have to face additional margin calls till day 4. The amount of funds required to maintain the futures contract position is higher in scenario 2 than in scenario 1, with impact on the return rate obtained.

However, using Hancock technique the return on the futures position is the same in both scenarios (table I) because daily cash flows resulting from the margining mechanism are not considered.

**Table I**  
**Position Return and Standard Deviation**

	Scenario 1	Scenario 2
<b>Alternative</b>		
Return	630,5%	136,2%
Standard Deviation	0,0%*	57,0%
<b>Hancock</b>		
Return	220,0%	220,0%
Standard Deviation	4,8%	608,6%

\* Price increases in the same magnitude every day

Equations 3a-3d and Tables II and III show in more detail Hancock's procedure to measure global futures returns. When the future price movement is adverse funds

injected by margin calls are not considered in the investment amount, and when surpluses occur they are assumed to remain in the margin account.

$$\text{Futures Return } t1 = \frac{\Delta \text{ Margin } t1}{\text{Initial Margin } t} \quad (3a)$$

$$\text{Futures Return } t2 = \frac{\Delta \text{ Margin } t2}{\text{Initial Margin } t + \Delta \text{ Margin } t1} \quad (3b)$$

$$\text{Futures Return } t3 = \frac{\Delta \text{ Margin } t3}{\text{Initial Margin } t + \Delta \text{ Margin } t1 + \Delta \text{ Margin } t2} \quad (3c)$$

$$\Delta \text{ Margin} = \Delta \text{ Futures Price} * \text{No Contracts} * \text{Contract Size} \quad (3d)$$

**Table II**  
**Daily Return on Futures using Hancock Technique (Scenario 1)**

t	Number of contracts	Futures Price	Underlying	Initial and Maintenance Margin	Investment Position	Δ * Underlying	Return
0	100	100,0	100.000	5.000	5.000		
1	100	101,1	101.100	5.000	6.100	1.100	22,0%
2	100	102,2	102.200	5.000	7.200	1.100	18,0%
3	100	103,3	103.300	5.000	8.300	1.100	15,3%
4	100	104,4	104.400	5.000	9.400	1.100	13,3%
5	100	105,5	105.500	5.000	10.500	1.100	11,7%
6	100	106,6	106.600	5.000	11.600	1.100	10,5%
7	100	107,7	107.700	5.000	12.700	1.100	9,5%
8	100	108,8	108.800	5.000	13.800	1.100	8,7%
9	100	109,9	109.900	5.000	14.900	1.100	8,0%
10	100	111,0	111.000	5.000	16.000	1.100	7,4%

The investment amount changes from day to day, and would be set again to the initial margin only if there was

**Table III****Daily Return on Futures using Hancock Technique (Scenario 2)**

t	Number of contracts	Futures Price	Underlying	Initial and Maintenance Margin	Investment Position	$\Delta^*$ Underlying	Return
0	100	100,0	100.000	5.000	5.000		
1	100	97,4	97.400	5.000	2.400	-2.600	-52,0%
2	100	95,1	95.100	5.000	100	-2.300	-95,8%
3	100	93,7	93.700	5.000	-1.300	-1.400	-1400,0%
4	100	93,1	93.100	5.000	-1.900	-600	46,2%
5	100	93,6	93.600	5.000	-1.400	500	-26,3%
6	100	95,2	95.200	5.000	200	1.600	-114,3%
7	100	97,5	97.500	5.000	2.500	2.300	1150,0%
8	100	101,0	101.000	5.000	6.000	3.500	140,0%
9	100	105,5	105.500	5.000	10.500	4.500	75,0%
10	100	111,0	111.000	5.000	16.000	5.500	52,4%

The investment amount changes from day to day, and would be set again to the initial margin only if there was

In fact there is no reason to make such an assumption. Hancock observes that "(...)a local return on a futures position can be obtained by identifying the initial deposit as the investment". But the investment required to maintain the position during a given period of time is daily adjusted to the initial margin whenever the interval between the maintenance and initial level is exceeded, and the rate of return has to reflect it.

If at some moment a margin call is made, funds have to be added to the margin account, either from selling other assets or from debt, and both will have an associated cost or opportunity cost. On the inverse situation, when surplus funds become available from the margin account, they are no longer tied up to the investment and can be used in new investments or in liquidity, with a potential benefit.

Even if not removed margin account surpluses cannot be considered as part of the investment because they are no longer tied up to the futures position, and so do not fill out the characteristics that liken the funds in the margin account to an investment. It

follows that futures contract returns can only be calculated on relation to the part of the margin account which is tied to the investment.

**Table IV**

**Daily Return on Futures using Alternative Technique (Scenario 1)**

t	Number of contracts	Futures Price	Underlying	Initial and Maintenance Margin	Investment Position	Δ Underlying	Return
0	100	100,0	100.000	5.000	5.000		
1	100	101,1	101.100	5.000	5.000	1.100	22,0%
2	100	102,2	102.200	5.000	5.000	1.100	22,0%
3	100	103,3	103.300	5.000	5.000	1.100	22,0%
4	100	104,4	104.400	5.000	5.000	1.100	22,0%
5	100	105,5	105.500	5.000	5.000	1.100	22,0%
6	100	106,6	106.600	5.000	5.000	1.100	22,0%
7	100	107,7	107.700	5.000	5.000	1.100	22,0%
8	100	108,8	108.800	5.000	5.000	1.100	22,0%
9	100	109,9	109.900	5.000	5.000	1.100	22,0%
10	100	111,0	111.000	5.000	5.000	1.100	22,0%

The investment position is considered to be the funds tied to the investment, i.e., the initial or maintenance margin.

**Table V**

**Daily Return on Futures using Alternative Technique (Scenario 2)**

t	Number of contracts	Futures Price	Underlying	Initial and Maintenance Margin	Investment Position	Δ Underlying	Return
0	100	100,0	100.000	5.000	5.000		
1	100	97,4	97.400	5.000	5.000	-2.600	-52,0%
2	100	95,1	95.100	5.000	5.000	-2.300	-46,0%
3	100	93,7	93.700	5.000	5.000	-1.400	-28,0%
4	100	93,1	93.100	5.000	5.000	-600	-12,0%
5	100	93,6	93.600	5.000	5.000	500	10,0%
6	100	95,2	95.200	5.000	5.000	1.600	32,0%
7	100	97,5	97.500	5.000	5.000	2.300	46,0%
8	100	101,0	101.000	5.000	5.000	3.500	70,0%
9	100	105,5	105.500	5.000	5.000	4.500	90,0%
10	100	111,0	111.000	5.000	5.000	5.500	110,0%

The investment position is considered to be the funds tied to the investment, i.e., the initial or maintenance margin.

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#### **4. Research Questions**

Futures contracts return rates can be calculated on the cash flows required to maintain the position, considering the investment position as the sum of the derivative value (zero) and the daily adjusted funds tied to the investment in the margin account.

The application of the proposed methodology to historical returns allows the computation of Value at Risk to estimate the risk incurred in a futures position (or a portfolio holding futures) as if it were a traditional asset.

#### **5. Methodology and Data**

##### **5.1 Methodology**

We empirically test the proposed technique in counterpoint to Hancock's one.

The track record of a hypothetical investment fund is simulated since 2001 to September 2007, and only the equity portfolio performance is evaluated. We assume the portfolio to be proportionately invested on FT 100 Index members and also on FT 100 future contracts to increase equity exposure.

The use of surplus funds is a responsibility of the asset allocation manager. Margin calls are assumed to be made available. Any potential costs or benefits are not considered responsibility of the equity manager; he can only be made responsible for the investment amount at his charge.

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The equities and futures contracts risk and return are measured separately and then integrated on the equity portfolio as a whole. Risk is measured using standard deviation. For the reasons already exposed the amount invested in the futures contract is daily adjusted to the initial margin amount. This is the investment necessary to maintain the futures position, and therefore the one which can be considered associated to the manager being evaluated. Results from the application of Hancock' technique will also be presented.

Futures price changes are used to calculate the daily margin flow (Equation 3d), and the resulting investment return. The portfolio daily return is a weighted sum of the equities and futures contracts return. Differences in the considered investment amount on futures contracts explain different daily returns and relative weights.

The application of the proposed methodology to past returns is afterwards tested to forecast risk. Calculated returns, variances and covariances are used to compute Value at Risk and estimate the risk incurred in a futures position and a portfolio holding future positions.

The back testing period is coincident with the used data period, what can seem redundant, but we will see that it is not useless. It will allow us to check if the return rates calculated (and resulting risk measures) do in fact inform of the position performance and behaviour. Return rates inaccurately measured would lead to inaccurate risk measures. If the calculated futures contract returns are correctly measured the Value at Risk should prove a faithful measure of their daily risk. Value at

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Risk of the equity portfolio is calculated using variances and covariances of FT 100 and FT 100 futures.

## **5.2 Data**

Four different futures contracts are annually available for the FT 100 Index and daily price quotations were obtained since 2001 (Reuters), as well as for the FT 100 Index itself (Bloomberg).

## **6. Empirical Analysis**

### **6.1 Calculating Historical Risk and Standard Deviations**

Annualized return rates and standard deviations of FT 100 index are straightforward to obtain. Differences appear when dealing with futures. Using Hancock's technique futures standard deviations assume enormous proportions, and seem to have no relation with the market risk on each year. Extreme and apparently erratic return rates are obtained (see Table VI), for example 2409% in a year where the index result was -16.2% (2001), or -347.6% in a year where the index rose by 10.7% (2006).

The problem is that if the amount invested goes close to zero, any little change of the futures price will create very high returns, and risk, but with little meaning. On chart II we can check the relation between investment amount and return rates amplitude using the example of future prices path in scenario 2. This explains why futures positions risk and return measured by Hancock can amount to extreme and little intuitive values.

**Table VI**

**Portfolio invested on FT 100 Index and on FT 100 Futures with an underlying equivalent to 3 times the direct investment**

Year	FT 100 *		FT 100 Futures **		Equity Portfolio	
	Return	Stdev	Return	Stdev	Return	Stdev
<b>Hancock</b>						
2001	-16,2%	21,8%	3303,8%	43387,0%	-78,7%	94,2%
2002	-24,5%	27,5%	253,9%	10536,8%	-72,2%	121,5%
2003	13,6%	19,5%	501,4%	1414,6%	53,4%	84,1%
2004	7,5%	10,4%	224,4%	2058,6%	27,5%	34,5%
2005	16,7%	8,7%	932,3%	1627,8%	68,3%	30,4%
2006	10,7%	12,6%	-360,3%	889,3%	32,3%	45,5%
2007	1,3%	13,4%	-125,0%	1109,7%	0,1%	53,0%
<b>Alternative</b>						
2001	-16,2%	21,8%	-100,0%	442,7%	-55,0%	75,6%
2002	-24,5%	27,5%	-100,0%	528,2%	-73,6%	95,3%
2003	13,6%	19,5%	-99,3%	370,4%	34,1%	66,8%
2004	7,5%	10,4%	-47,2%	198,8%	22,8%	34,7%
2005	16,7%	8,7%	404,5%	178,0%	66,6%	30,5%
2006	10,7%	12,6%	-69,7%	250,4%	33,0%	43,6%
2007	1,3%	13,4%	-97,1%	255,3%	-2,4%	45,8%
<b>Difference: Hancock - Alternative</b>						
			<b>Δ Return</b>	<b>Δ Stdev</b>	<b>Δ Return</b>	<b>Δ Stdev</b>
2001			3403,8%	42944,3%	-23,7%	18,7%
2002			353,9%	10008,6%	1,3%	26,2%
2003			600,7%	1044,2%	19,3%	17,3%
2004			271,6%	1859,8%	4,7%	-0,1%
2005			527,8%	1449,8%	1,7%	-0,1%
2006			-290,5%	638,9%	-0,7%	2,0%
2007			-27,9%	854,3%	2,5%	7,2%

Return Rates and Standard Deviations are annualized

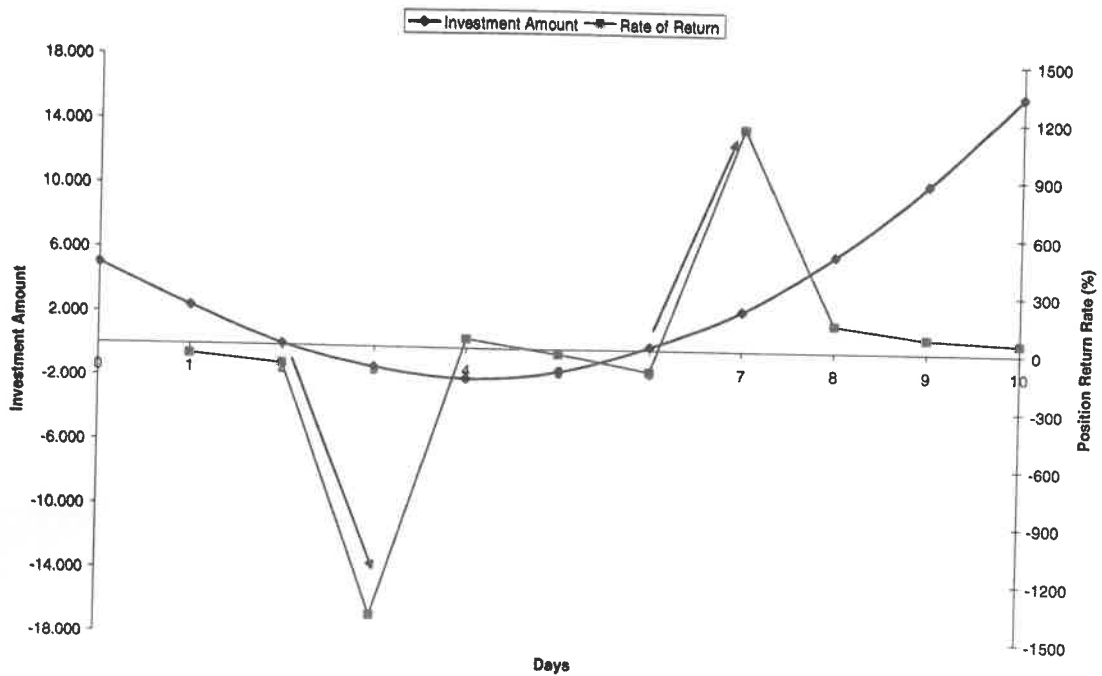
\* Dividends were not considered, but their impact is negligible

\*\*Contracts are rolled over every 3 months and their initial margin is assumed to be 5% of the underlying value

Their effect is smoothed on the total portfolio figures because the days in which they amount to these very big numbers is when the investment is close to zero, and therefore what happens to the futures contract has little weight.

## Chart II

### Investment Amount vs. Rates of Return (Hancock Technique)



In fact, if the amount invested in any day gets to zero, which is perfectly, it is not possible to calculate further return rates. The methodology can no longer be applied.

Standard deviations computed using the Alternative technique are more stable and change from year to year in proportion with those of the equity investment (the same as the underlying of the contract), as would be expected.

Return rates are also more stable, but still with little intuition. In some years the index has a positive return and future contracts have a negative one. The reason is that the investment amount is daily readjusted to the initial margin level, and therefore the compounding of daily return rates supplies us with annual return rates that do not

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transmit the real return delivered by the investment. We think futures annualized return rates presented should be ignored. However the portfolio return rate is not affected by this and individual daily return rates are correct and express the real position payoff. It makes more sense to look at these returns in daily and not in annualized rates.

The futures presence in the portfolio increases exposure to the equity market and variations of return (and therefore risk) of the total portfolio.

When looking to the equity portfolio differences between standard deviations obtained by applying Hancock' and the Alternative methodology are still significant, especially in more volatile periods. They are positively correlated with market volatility.

What can justify these differences? The reason was already pointed out and it is that one method and the other have different assumptions of the amount invested in the futures contract in each moment, which means that the daily returns obtained will be more diluted or enhanced in one case or another.

To better understand how these effects impact results, we have repeated the same example but manipulated futures prices so that in some years they would always increase and in others they would always decrease, although both in a random manner in order for risk to remain (see Table VII). In 2001 prices increase every trading day, and in 2002 they decrease. In 2003 they increase again, and in 2004 they decrease, but in both cases with stronger amplitude. In 2005 and 2006 they respectively increase and decrease with even stronger amplitude. And in 2007 they finally increase again in the strongest movement.

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**Table VII**

**Portfolio invested on FT 100 Index and on FT 100 Futures (underlying equivalent to 3 times the direct investment) with futures prices manipulated**

Year	FT 100 *		FT 100 Futures **		Equity Portfolio		
	Return	Stdev	Return	Stdev	Return	Stdev	
<b>Hancock</b>							
Bullish	2001	-16,2%	21,8%	403,2%	5,9%	16,1%	17,9%
Bearish	2002	-24,5%	27,5%	-91,6%	10,0%	-44,8%	23,3%
Bullish	2003	13,6%	19,5%	1469,6%	10,8%	120,2%	14,5%
Bearish	2004	7,5%	10,4%	-100,0%	301,5%	-51,6%	9,4%
Bullish	2005	16,7%	8,7%	3616,9%	15,5%	157,9%	7,4%
Bearish	2006	10,7%	12,6%	-100,3%	1842,6%	-55,8%	11,9%
Bullish	2007	1,3%	13,4%	1947,9%	16,2%	84,4%	10,7%
<b>Alternative</b>							
Bullish	2001	-16,2%	21,8%	618,5%	7,0%	15,3%	18,6%
Bearish	2002	-24,5%	27,5%	-84,5%	6,9%	-43,1%	22,4%
Bullish	2003	13,6%	19,5%	4442,6%	13,7%	138,4%	15,9%
Bearish	2004	7,5%	10,4%	-98,0%	12,5%	-48,7%	8,7%
Bullish	2005	16,7%	8,7%	21849,9%	20,9%	182,5%	8,1%
Bearish	2006	10,7%	12,6%	-99,5%	19,3%	-51,9%	10,7%
Bullish	2007	1,3%	13,4%	12041,0%	22,8%	95,9%	12,0%
<b>Difference: Hancock - Alternative</b>							
				<b>Δ Return</b>	<b>Δ Stdev</b>	<b>Δ Return</b>	<b>Δ Stdev</b>
Bullish	2001			-215,3%	-1,1%	0,8%	-0,7%
Bearish	2002			-7,1%	3,0%	-1,7%	0,9%
Bullish	2003			-2973,0%	-2,9%	-18,1%	-1,4%
Bearish	2004			-2,0%	289,1%	-2,9%	0,8%
Bullish	2005			-18233,0%	-5,3%	-24,6%	-0,7%
Bearish	2006			-0,8%	1823,3%	-3,9%	1,3%
Bullish	2007			-10093,1%	-6,7%	-11,5%	-1,3%

Return Rates and Standard Deviations are annualized

\* Dividends were not considered, but their impact is negligible

\*\* Contracts are rolled over every 3 months and their initial margin is assumed to be 5% of the underlying value

As explained we will ignore annualized return rates of future contracts. Standard deviations measured with Hancock technique almost explode in bearish years (the reason is again that when the investment amount is close to zero any little market movement will create enormous returns with impact on volatility), and are very low in bullish years (returns are smoothed by an increasing investment amount).

Because futures prices were manipulated, we know risk should be in line with the magnitude of the price changes simulated. This magnitude is similar in 2001, 2002,



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higher in 2003 and 2004, higher again in 2005 e 2006, and even higher in 2007. Standard deviations obtained with the alternative technique risk are consistent with these assumptions. They are similar in 2001 and 2002, increase in 2003 and 2004, again in 2005 and 2006, and finally again in 2007. This is what could be expected.

Looking at the total equity portfolio results, Hancock's method tends to decrease risk in bullish periods, and to increase it in bearish periods. When the margin account is receiving more and more money, this return is being obtained in relation to an investment amount superior to the real one, and therefore return and risk come diluted. When margin calls are constantly made, the amount invested is considered to decrease and even to become negative, and therefore the loss is in relation to a smaller investment... again the return is lower, but the risk is higher.

We see no reason to apply the methodology being proposed only to speculative futures. On Table VIII we present the results for an identical portfolio but using futures to hedge the equity direct investment.

Margin requirements are usually set at different levels for speculators and for hedgers, but that was not considered in our analysis and is harmless to our conclusions.

**Table VIII**

**Portfolio invested on FT 100 Index and on short FT 100 Futures positions to hedge equity market exposure**

Year	FT 100 *		FT 100 Futures **		Equity Portfolio	
	Return	Stdev	Return	Stdev	Return	Stdev
<b>Alternative</b>						
2001	-16,2%	21,8%	-100,1%	442,7%	-0,4%	6,4%
2002	-24,5%	27,5%	-100,0%	528,2%	0,6%	4,2%
2003	13,6%	19,5%	-100,0%	370,4%	-0,5%	3,3%
2004	7,5%	10,4%	-96,8%	198,8%	0,1%	2,4%
2005	16,7%	8,7%	-99,3%	178,0%	-0,3%	2,2%
2006	10,7%	12,6%	-99,5%	250,4%	0,0%	2,2%
2007	1,3%	13,4%	-97,8%	255,3%	-0,2%	2,2%

Return Rates and Standard Deviations are annualized

\* Dividends are excluded from the analysis, but their impact is negligible

\*\*Contracts are rolled over every 3 months and their initial margin is assumed to be 5% of the underlying value

## 6.2 Forecasting Risk

The calculation of return rates is important not only to evaluate the past performance of investments but also to allow using the information of that historical behaviour to forecast risk. We use daily FT 100 Index and FT 100 future contract returns previously obtained to compute the Value at Risk of the simulated portfolios and check if it is accurate. A simple backtesting of the portfolio's Value at Risk with 95% confidence level and a 1 day horizon was made.

**Table IX**

**Daily Variance Covariance Matrix using returns from 2001 to September 2007**

	FT 100	FT 100 Futures
FT 100	0,01%	0,24%
FT 100 Futures	0,24%	4,75%

**Table X****Daily Correlation Matrix using returns from 2001 to September 2007**

	FT 100	FT 100 Futures
FT 100	100,00%	97,46%
FT 100 Futures	97,46%	100,00%

**Table XI****Value at Risk backtesting results in a portfolio invested on FT 100 Index and on FT 100 Futures with an underlying equivalent to 3 times the direct investment**

	FT 100	FT 100 Futures	Equity Portfolio
Average Weight	87,0%	13,0%	100,0%
Standard Deviation *	1,1%	21,8%	3,8%
Average Return	0,0%	0,1%	0,0%
VaR (1 day horizon, 95% confidence level)	1,8%	36,1%	6,2%
Observations	1708	1708	1708
Losses exceeding VaR	76	74	78
Percentage of Losses exceeding VaR	4,4%	4,3%	4,6%

\* Equity Portfolio St. Dev. obtained using variances and covariances of FT 100 and FT 100 futures

Value at Risk seems to be robust in predicting not only the futures contract investment risk but the portfolio risk as well (see Table XI). The losses in a day exceeding the 1 day horizon Value at Risk with a 95% probability represent 4,5% of the daily observations on the FT 100 investment, 4,6% on the equity portfolio and about 4,3% on the FT 100 futures.

If Hancock' technique is used to calculate the inputs of the Variance – Covariance Matrix the Value at Risk of the futures contract investment is no longer robust (see Table XII). Its value is clearly blistered by the effects created when the investment amount is near zero. The variance of futures contract returns and covariance with the index returns are contaminated and Value at Risk does not reveal predictive power. We will continue testing only the Alternative technique.

**Table XII**

**Value at Risk backtesting results in a portfolio invested on FT 100 Index and on FT 100 Futures with an underlying equivalent to 3 times the direct investment, using Hancock technique**

	FT 100	FT 100 Futures	Equity Portfolio
Average Weight	87,3%	12,7%	100,0%
Standard Dev	1,1%	1083,6%	138,1%
Average Return	0,0%	-20,5%	0,0%
VaR (1 day horizon, 95% confidence level)	1,8%	1787,9%	227,2%
Observations	1708		
Losses exceeding VaR	76	2	0
Percentage of Losses exceeding VaR	4,4%	0,1%	0,0%

Equity Portfolio St. Dev. obtained using variances and covariances of FT 100 and FT 100 futures calculated with Hancock technique

**Table XIII**

**Portfolio invested on FT 100 Index and on FT 100 Futures (underlying equivalent to 3 times the direct investment) with futures prices manipulated**

	FT 100	FT 100 Futures	Equity Portfolio
Average Weight	83,1%	16,9%	100,0%
Standard Dev *	1,1%	2,0%	1,0%
Average Return	0,0%	0,3%	0,1%
VaR (1 day horizon, 95% confidence level)	1,8%	3,0%	1,6%
Observations	1708	1708	1708
Losses exceeding VaR	76	71	74
Percentage of Losses exceeding VaR	4,4%	4,2%	4,3%

Estimated using variances and covariances of FT 100 and FT 100 futures with manipulated prices

Value at Risk is also robust in the example where we manipulated futures prices. We see no reason why our Alternative methodology cannot be used on hedged portfolios (see Table XVI).

The Value at Risk accurately predicts the equity and the short future contracts individual risk but seems to overestimate the total portfolio risk. We additionally computed Value at Risk using the return rates and standard deviations of the Equity

portfolio as a whole, as if it was an individual asset, instead of using variances and covariance's of equities and futures. And Value at Risk still overestimates the total risk. This means the problem does not lie on the way futures returns are calculated, but on the particular distribution of the hedged portfolio returns.

**Table XIV**

**Daily Variance Covariance Matrix using returns from 2001 to September 2007**

	FT 100	Short FT 100 Futures	
FT 100		0,01%	-0,24%
Short FT 100 Futures		-0,24%	4,75%

**Table XV**

**Daily Correlation Matrix using returns from 2001 to September 2007**

	FT 100	Short FT 100 Futures	
FT 100		100,00%	-97,46%
Short FT 100 Futures		-97,46%	100,00%

**Table XVI**

**Portfolio invested on FT 100 Index and on short FT 100 Futures positions to hedge equity market exposure**

	FT 100	Short FT 100 Futures	Equity Portfolio
<b>Average Weight</b>	95,3%	4,7%	100,0%
<b>Standard Deviation *</b>	1,1%	21,8%	0,2%
<b>Average Return</b>	0,0%	-0,1%	0,0%
<b>VaR (1 day horizon, 95% confidence level)</b>	1,8%	36,1%	0,4%
<b>Observations</b>	1708	1708	1708
<b>Losses exceding VaR</b>	76	73	48
<b>Percentage of Losses exceding VaR</b>	4,4%	4,3%	2,8%

\* Equity Portfolio St. Dev. obtained using variances and covariances of FT 100 and Short FT 100 futures

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## 7. Summary

Hancock suggested a methodology to calculate the futures position return on the cash flows required to maintain the position, assuming the initial margin as the investment amount and ignoring the mark-to-market mechanism, as if it was a forward contract with an initial security deposit.

We proved that, on the contrary, the mark-to-market mechanism in futures contracts means the days between the beginning and end of the measurement period will determine the daily amount of funds necessary to maintain the futures contract position, with implications in its proportional weight and return rate.

Furthermore, if at a certain moment the investment amount is zero, which can easily happen, Hancock's technique can not longer be applied. Subsequent return rates become impossible to calculate.

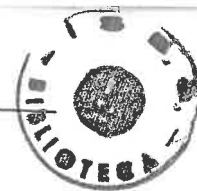
We generally defined a derivative investment position as the sum of the required margin and the derivative value. Since a futures contract is daily marked-to-market its value is zero and the investment position is the amount of funds tied to the investment in the margin account. Rates of return can be calculated on the cash flows required to maintain the futures position, but in relation to the daily adjusted tied investment. Standard procedures can therefore be extended to calculate returns on futures and on portfolios holding futures positions.

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Value at Risk computed using futures daily returns measured with our proposed methodology proved to be an accurate estimate of the risk incurred in a futures position and its contribution to the total portfolio risk. This means the methodology provides return rates and standard deviations consistent with the real returns and risk incurred in futures positions.

We believe this can be a useful and important contribute to overcome practical difficulties in accessing futures contribution to portfolios risk and return. Especially in a Basel II implementation context, a misperception of risk can lead to capital inadequacies in financial institutions or to the shareholders remuneration being misadjusted from incurred risk.

The methodology proposed should however be further tested on the particular case of hedged portfolios. We also think an extension is possible to address the short options issue.



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