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RELATIONS IN ARISTOTLE

Predicative propositions are the basic structure of a specific kind of term logic that can be traced back to Aristotle¹.

The difficulties it encompasses are evident.

An obvious one is the framing of relations, as defined in the Aristotelian doctrine of categories, within this kind of logic.

This difficulty can be summarized as follows: how can one reconcile the doctrine according to which every proposition has but two terms, a subject term and a predicate term, with Aristotle's assumption of a category of relatives and therefore with the implication that "is" can introduce relations ("master of", "in love with", "son of", etc.), whose correlates should apparently, in Aristotelian terms, be subjects (as in, for instance, "Plato is master of Aristotle" or "Socrates is son of Sophroniscus"), thus forming no longer a dyadic, but a triadic assertion?

Despite the superficially thorny aspect of the question, the answer is, I think, relatively simple and the question itself can accordingly be seen to raise no genuine difficulty at all.

In fact, as we could systematically check off if we were to analyse all of the types of predicative proposition generated according to each of the ten categories, a proposition expressing a relation, like

(1) "Socrates is son of Sophroniscus"

is not formally distinct from propositions where a quality, a location, an action or indeed the essence of a subject are ascribed, as, for instance, in:

(2) "Socrates is white",

(3) "Socrates is in Athens",

(4) "Socrates fought in the Peloponnesian War",

(5) "Socrates is a man".

¹ As is well known, the use of the term "proposition" may convey two rather different interpretations: either the identification of propositions with the content (or, in the sense introduced by Frege, the meaning) of declarative sentences, taken to be the bearer of truth and falsehood; or its identification with declarative sentences themselves, understood as the truth-bearers. Understandably enough, the interpretation here adopted is always the former, for it is the one also assumed in traditional logic.

In all these propositions — and the one expressing a relation is no exception in this regard — there are two terms and two terms only: the subject and the predicate. The only changing element in each of those propositions is the meaning of the predication, which varies according to that which the predicate states about the subject. In proposition (2), the predicate “white” states how Socrates is; in (3), the predicate “in Athens” states where Socrates is; in (4), the predicate “fought in the Peloponnesian War” states which action was performed by Socrates; in (5) the predicate “man” states what Socrates is; and, exactly in the same way, in our controversial proposition (1), the predicate “son of Sophroniscus” states with whom Socrates is related by filiation.

In the Aristotelian predicative propositions — whatever their categorical content may be, which is to say regardless of the type of predication introduced by the proposition —, there are always only two terms and relational predications (in the Aristotelian sense) constitutes no exception from this point of view. In Aristotle’s theory, propositions that introduce a relation have, exactly as any other proposition, a dyadic structure; and, through them, *one* predicate is attributed to *one* subject.

Actually, it is not hard to understand why this is so. For Aristotle, categories correspond to the several types of question one can ask of a given substance concerning its being and, therefore, exhaust the various types of predicates that, in general, any substance may receive².

Now, the question typified by the category of relatives is the question $\pi\rho\sigma$ $\tau\iota$, “in relation to what?”. In this sense, the question introduced by the category of relatives is not that which asks about the relation one given subject maintains with another, but rather that which asks about the subject *with which* another subject maintains a specific relation. The specific question regarding relatives is not thus: “what is the relation between *a* and *b*?” (where *a* and *b* are subjects); but instead: “with whom does *a* have the relation *R*?”.

Accordingly, that which is expressed by a relational predicate (in the Aristotelian sense of the term) is not a specific relation that a specific substance has. The relational predicate expresses that to which that substance is related under the specified relation. In proposition (1), for instance, “is” does not introduce a specific relation (*viz.*, that of filiation) between Socrates and Sophroniscus; it introduces that to which (to whom, in this particular case) Socrates is related by filiation, namely Sophroniscus³.

In this light, proposition (1) does not express the relation of filiation between Socrates and Sophroniscus (in which case the proposition would have had three terms), but rather states *that* Socrates is related, by filiation, to Sophronis-

² Cf. Cat. 4, 1b25-27.

³ On this issue, see Cat. 7, 8a 13 – b 24.

cus, where “being related to Sophroniscus by filiation” constitutes a single predicate that the proposition attributes to the subject. And, of course, this holds, in general, for every proposition analogous to (1).

This being the case, the problem raised in the beginning should not worry us any more. Sadly, the solution itself is not, however, altogether innocuous and have some side effects on Aristotle’s theory of the proposition.

Let us have a look at two of these effects.

The first effect I would like to highlight concerns declarative sentences, particularly those that express relational propositions (in the Aristotelian sense of the term). If our account of relational propositions is right, then names that occur within the predicative term of such sentences possess reference and contribute to the reference of the term as a whole. This typically affects sentences with relational content, but not only them: think, for example, of locative assertions.

The second effect is the following. Although every proposition must include at least one general term and therefore no proposition expresses a bare relation between individuals (a thesis rather intuitive in itself, but one that would require a great deal of time to fully justify here), relational propositions thus understood (as locative propositions, for that matter) alert us to the fact that, in a proposition, *some* relations between individuals may be implicitly involved. Such is indeed what paradigmatically happens in the case of propositions expressing relations. Let us again look at proposition (1). In spite of the fact that it only includes two terms, *viz.*, “Socrates”, the subject, which is a singular term, and “son of Sophroniscus”, the predicate, which is a general term, it seems rather plain that the predicate attributes to the individual subject a certain relation with another individual and, in so doing, notwithstanding its dyadic character, the proposition can be seen as ultimately introducing, *by mediation of the predicate*, a relation between individuals.

Be that as it may, it seems clear enough that none of these effects disqualifies the doctrine according to which every predicative proposition, regardless of its particular categorical content, has but two terms and relational predications (in the Aristotelian sense) are no exception.

However, this leads us to a further difficulty; and, this time, quite a genuine one.

In fact, if Aristotle’s doctrine survives the challenge posed by the two said effects, it is not at all clear that it can also overcome the defiance posed by relations themselves — by which I now mean not relational predications in the Aristotelian sense, but real relations between objects (the extension of predicates of arity greater than 1, if you will).

The new problem can be put as follows: subject/predicate logic, upon which the notion of predicative proposition rests, is incapable of accounting for relations, in the strong and proper sense of “relation”. To put it more incisively:

any discourse whose relations, in this sense, are ineradicable is impervious to subject / predicate logic.

We can see this at two levels.

Here is the first level: from the notion of proposition so defined it is only possible to account for an argumentative discourse, not for a narrative discourse.

In order to understand it, let us take the following lines as an example:

The car travelled from Moscow to Dmitrov. Once in Dmitrov, it travelled to Dubna. Later, it travelled back to Moscow⁴.

Following the interpretation presented, we have here four propositions, which can be expressed by the following sentences:

(6) “The car travelled from Moscow to Dmitrov” (where “the car” expresses the subject and “travelled from Moscow to Dmitrov” expresses the predicate). In notation: Fa ⁵.

(7) “The car arrived in Dmitrov” (where “the car” expresses the subject and “arrived in Dmitrov” expresses the predicate). Let it be Ga .

(8) “The car travelled to Dubna” (where “the car” expresses the subject and “travelled to Dubna” expresses the predicate). Let it be Ha .

(9) “The car travelled back to Moscow” (where “the car” expresses the subject and “travelled back to Moscow” expresses the predicate). Let it be Ja .

The problem that becomes apparent in the translation is that we are left before a succession of predications with no relation among them, except for the fact that all share the same subject. The point of the narrative, i.e., the outline of the itinerary, vanishes completely. Its various stops are subsumed, and therefore diluted, into the predicates.

Let us now consider the case of argumentative discourse.

⁴ For those who, even after the considerations already developed on the specificity of relational predications in Aristotelian-based logic, may feel inclined to dispute that the propositions here presented are predicative propositions, we would remind them that it is Aristotle himself who argues that all categorical assertions can be converted into predicative propositions (or, perhaps better, clarified as predicative propositions): cf. *Int.* 12, 21a 38 – b 10; *Analyt. Pr.* I 46, 51b 5 – 25; *Metaph.* Δ 7, 1017a 27–30.

⁵ I chose to use standard notation of predicate logic in order to render the subsequent illustration more intuitive. I have, however, made two fundamental simplifications, which naturally follow from the nature of the doctrine of the proposition under debate: first, every predicate, including those that predicate logic sees as relational predicates, are interpreted as monadic predicates; second, every name (including descriptions) are assimilated to proper nouns and therefore represented by an individual constant.

An example:

The car travelled from Moscow to Saint Petersburg. Now, to travel from Moscow to Saint Petersburg is to make a long journey. Therefore, the car made a long journey.

Here, the notion of proposition fits perfectly, as rendered clear by the formalisation⁶.

We have three propositions, namely,

(10) “The car travelled from Moscow to Saint Petersburg” (where “the car” expresses the subject and “travelled from Moscow to Saint Petersburg” expresses the predicate). Let it be Fa .

(11) “To travel from Moscow to Saint Petersburg is to make a long journey” (where “to travel from Moscow to Saint Petersburg” expresses the subject and “to make a long journey” expresses the predicate). Let it be $\forall x (Fx \rightarrow Gx)$.

(12) “The car made a long journey” (where “the car” expresses the subject and “to make a long journey” expresses the predicate). Let it be Ga .

The argument results thus:

$$Fa, \forall x (Fx \rightarrow Gx) \vdash Ga,$$

which makes indeed a valid inference.

Briefly, then, when we find ourselves before a narrative context, bringing the discourse back to the set of propositions that would allegedly constitute the meaning of its respective sentences renders the discourse unreadable and this is consequence of the fact that narrative discourses resist being analysed in terms of the proposition structure. It is only when we are dealing with an argumentative context that the discourse can be successfully analysed by looking into the proposition structure.

Explaining why this is so would perhaps not require any special effort, but it undesirably would lead us astray from our intended course here.

There is, however, no need to do it.

In fact, were this all there was to it, no major problem would arise, in that the predicative proposition model would lose explanatory power in the context of narrative discourse, but not in the context of argumentative discourse, of which it could be recognised as a basic structure — even in the case of relations.

This is not what happens, though. A few counter examples will suffice to show that relations, in the strong sense of the term, are ineradicable from argu-

⁶ According to the simplification stated in the previous note, but assuming all conventional rules concerning the translation of categorical assertions in predicate logic.

mentative discourse, and that, *because of them*, propositional structure is unable to account for the whole of the argumentative discourse.

Let us consider the following:

(13) Louis is Bernard's twin. Now, Bernard is John's twin. Therefore, Louis is John's twin.

(14) The whale destroyed the ship⁷. Now, the ship was Jerome's fortune. Therefore, the whale destroyed Jerome's fortune.

(15) The car went to Moscow. Now, Moscow is near Dmitrov. Therefore, the car went near Dmitrov.

In any of these instances, the argument only obtains if the propositions' predicate is severed into relation and object — or, equally, if the corresponding declarative sentence contains two singular terms connected by a general term. Now, this constitutes not only *another* limitation to the application range of predicative propositions, but indeed a fatal one, as well as to any term logic founded on them. And this means that (as has long been acknowledged) any such logic is necessarily limited — and limited, among other things, by its intrinsic inability to account for relations.

That said, how is such inability to be justified, and to what extent does its resulting limitation constitute a drastic restriction for the intervention of the notion of predicative proposition?

In other words: how much room does that limitation leave to such intervention? Does it leave a continuous and well-defined operating margin, within which predicative propositions may be used systematically and consistently, or, on the contrary, does that limitation outline a transversal boundary, of broken line and occasionally diffuse sections, leaving no room at all for the possibility of a consistent and systematic use?

Impossible as it is, at this juncture, to provide a full answer to the first question (and consequently to all others), we can at least advance some elements towards answering it.

Let me begin by a hypothesis of historical-philosophical tenor, a hypothesis that appeals expressly to the model in which the theory of the predicative proposition and the logic built upon it were for the first time put forth in the Western culture: the Aristotelian system.

The hypothesis is the following: within Aristotle's system, the justification for what I have called "its intrinsic inability to account for relations" lies in the system's insensitivity to propositions whose subjects are singular terms, and is rooted in the relative neglect that that system shows for arguments containing singular subject assertions. In fact, the very doctrine of proposition was devel-

⁷ The example borrows from *Strawson*. On Referring. P. 321.

oped to service a logic that acknowledges only quantified sentences and where, therefore, there are no singular subjects⁸.

It is true that Aristotle's logical texts attest to singular-subject propositions — albeit not in a systematic manner, as would be the case in the subsequent logical tradition, and, at any rate, far more modestly than one could expect⁹. However, those propositions play no role in logic proper (i.e., in the inferential systems) and it is not because of them, but because of universal propositions, that the very doctrine of proposition is built and presented as a necessary preamble to that logic.

That is why relations, as functions whose arguments are singular terms, are, and can be, overlooked by Aristotelian logic, and why such a logic does not need to overcome, break away from, or reformulate the framework set by the model of predicative propositions for purposes of accommodating atomic structures with more than two terms, that is to say with a structure other than subject / predicate.

In short: the Aristotelian system does not include a logic of relations because it does not include any singular term logic to begin with. Aristotle's logic is a logic of connections between universals; and this is why the predicative proposition structure provides it with the necessary and sufficient conditions for it to operate¹⁰.

⁸ The four propositional forms of Aristotelian syllogistic all involve universal terms and universal terms alone. In fact, they attribute, *universaliter* or *particulariter*, a universal predicate to a subject and only a subject which is itself universal can be predicated *universaliter* or *particulariter*. That is why we can speak meaningfully of “all men” or “some man”, but not of “all Socrates” or “some Socrates”. Aristotle himself explicitly testifies to this when he states: “But ‘Socrates’ is not said of many; that is why one does not say ‘every Socrates’ as one says ‘every man’” (Metaph. Δ 9, 1018a3-4). This point was first pointed out by Alexander of Aphrodisias (In Analyt. Pr. 100.11 Wallies) and was in modern times retrieved by Łukasiewicz, in *La syllogistique d’Aristote*. P. 24. That Aristotelian syllogistic does not allow for singular terms has been acknowledged by many commentators; see, especially, Aristotle's *Prior and Posterior Analytics* (Ross). P. 289; Łukasiewicz. *La syllogistique d’Aristote*. P. 21, 24–25; Patzig. *Aristotle's Theory of the Syllogism*. P. 6–8; Granger. *La théorie aristotélicienne de la science*. P. 125–126; Corcoran. *Aristotle's Natural Deduction System*. P. 100.

⁹ Examples can be found in two passages of the *Prior Analytics* (I 33, 47b 21–37, and II 27, 70a 16–38) and, in greater number, throughout the *Topics*.

¹⁰ One could surely object that, by leaving out relations, Aristotelian logic is also leaving out second-order relations, without which some such connections between universals would not even be possible. Consider, for instance, the following argument: “Mortal has greater extension than Animal; Animal has greater extension than Man; therefore, Mortal has greater extension than Man”. How, one would ask, could this argument be taken as valid inference without recognising a status of relation to “having greater extension than”? Aristotle's answer is implicit in his logical practice and consists in reabsorbing all those “relations” as alternative formulae of the copula, or its extensions (a model evidently not exportable to first-order relations).

Let us now set aside the historical-philosophical approach and move on to a more general theoretical level, from which we can draw some conclusions likely to provide an answer to the other questions left unanswered earlier — and with these conclusions I will rest.

Given the evidence presented thus far, we can easily grant, I think, that, in a framework as the one described, i.e., in the framework defined by those assertions the doctrine itself considers relevant (universal subject assertions), all assertions likely to occur within an argument can be brought back to the predicative proposition structure and, thus, that the theory of the proposition suffices to encompass all arguments generated within it¹¹.

A system founded upon the predicative proposition structure (and not the Aristotelian system specifically, but any system that adopts this basic structure), although limited, might nevertheless be complete, in the given sense.

Complete, that is within the limits it itself has defined, thus excluding singular terms; and in the given sense, that is in the sense of the system's ability to refer all assertions and arguments it deems relevant to the proposition structure.

Re-assimilation of singular assertions can then be carried out retrospectively, namely widening, by extension, the proposition's scope so as to include singular subjects, excluding only those singular terms that occur in predicative place.

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¹¹ Even though syllogistic is not: but that is not the issue here.