Caring for their parent

January 2006

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Abstract

Should someone with more adult children expect to have further chances of avoiding institutionalisation in old age than someone with fewer children? Should an aging population build more institutions for the elderly, including for the elderly with children?

We show that population aging – as the motive for the reduction of the number of children to every parent – does not imply that the elderly with adult children will have to rely more on institutions when they need long-term care.

JEL classification: D19, J14.

Keywords: living arrangements, elderly, adult children.

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0. Introduction

The aging of the developed world brings with it several problems. One of them is the need of assistance an increasingly larger part of the population - the elderly - experiments. When living alone, many elderly feel unsafe, feel too lonely, and/or are physically incapable of taking care of themselves. They need long-term care.

Additionally, this increasingly larger part of the population tends to have fewer children per capita than previous generations. This means that the accommodation of a lonely old parent is becoming more important to each surviving child.

Although we cannot establish an immediate link between the type of living arrangement and the level of well-being, it is clear that living arrangements widely influence the daily lives of the elderly and their life satisfaction.

Reasons may be pointed to justify that coresidence improves the old person's well-being: there are increasing returns in sharing a house, for instance, in domestic services (cleaning, laundry, meals), in the rent payment, in consumption (electricity, telephone, cable TV). Help with personal care, entertainment and companionship may also be more easily available when the elderly share a household than when they live alone.[2] There may be less opportunity for loneliness feelings – although that depends on the amount of contacts that the old person has during the day and the number of contacts he[3] would have if not living with children.

[2] Burch and Matthews (1987) (referred in Wolf (1994)) and also Palloni (2000) note that each household living situation is like a composite good that includes physical shelter, domestic services, personal care, privacy, power/authority, independence, recreation, companionship and consumption of economies of scale.

[3] For simplicity, when referring to someone that could be either feminine or masculine, I will use the masculine form, although it could be exactly the other way round.

In spite of the motives that justify the increase in the well-being of the elderly originated by coresidence, there are also potential negative effects. Loss of independence, loss of authority, negative personal relations or difficulty in adapting to the living style of the new coresidents may prove damaging to the elderly's well-being.

These positive and negative effects are obtained by comparison with the independent living alternative. Nevertheless, when the elderly are not capable of living alone, the comparison should be made between the alternatives of coresidence with kins and institutionalization[4]. The referred negative aspects of coresidence also apply to institutonalization. Additionally, the environment is more impersonal and sometimes less affective.

Several characteristics of the elderly and of their children have been pointed out in the literature, that help explain the living arrangement decisions.

The health status, including the number of functional disabilities is one of the main determinants of the living arrangement, especially of the probability of living alone. This is testified by Crimmins and Ingegneri (1990); Norton (2000) finds that health is the primary determinant of demand for nursing home care; Mutchler and Burr (1991) (cited by Schneider and Wolf (1997)) find health to be significant when the choice of institutionalization is considered, although insignificant when this choice is not included in the possible living arrangements. Boersch-Supan et al. (1992) find that functional ability is the primary determinant of living arrangement choices.

Empirical studies, - like Kotlikoff and Morris (1990) and Boersch-Supan, et al. (1988) - have related the level of income of the elderly, and of the children, with the probability of living in separate houses. They point to a positive influence of the income level on the probability of the old person living alone. Possible explanations are the preference for "intimacy at a distance" with the affordability of formal home care, that allows the parent to age in place, or the higher opportunity cost of restricting the supply of working hours for the children with higher income levels. A negative relation could be justified by the competition for the parent's money: the larger the parent's income or wealth, the greater the interest of children in having the parent at home.

Another factor that is often considered as a determinant of living arrangements is the number of children.

[4] We could consider instead two possible alternatives in a more general way: one relying on informal care (which includes coresidence with kins), and the other relying on formal care (which includes institutionalization). The model that we develop in the rest of this paper would still be valid, as long as the scale of the elderly's preferences were adapted accordingly.

The empirical literature is not unanimous about this subject. Several studies in the literature identify a negative and significant effect of the number of children on the odds of living alone relative to living with others. [Burr and Mutchler (1992); Crimmins and Ingegneri (1990), and Spitze and Logan (1990)] However, Wolf (1994) refers that Aquilino (1990) finds that the number of children are not significant in explaining parent-child coresidence. In this study the likelihood of coresidence is evaluated given that there is at least one unmarried child. Some other studies that analyse the probability of transitions from one living arrangement to another find that the effect of the number of children is not significant when other factors such as health, income and demographic characteristics are controlled. [Worobey and Angel (1990), Speare et al. (1991), and in part Spitze et al. (1992)]. Wolf and Soldo (1988) conclude that the composition of the kin network is more important in explaining the choice of living arrangements than its size.

Hoerger et al. (1996) find that having more children irrespectively of distance of residence has no impact on the probability of institutionalization. However, having no children increases the likelihood of nursing home entry. This is exactly in line with the results we obtain in our paper.

Theoretically, this relation between the number of children and the likelihood of each living arrangement is not entirely clear, and has not been explored. Wolf (1994) mentions that the number of available kin represents a constraint on the set of living arrangements. Engers and Stern (2002) [5] write that the number of children "determines the number of options in the choice set and it interacts with other explanatory variables". Also, in Hiedemann and Stern (1999), the estimated model points to an increase in the chance that a parent receives care from a child or lives in an institution with a larger number of children. The other alternative would be to live independently.

We agree that the number of available kin represents a constraint on the set of living arrangements. The larger the number of children, the larger the number of possible living arrangements. In the limit, an elder with no available kins, who is incapable of living alone, has institutionalization/ formal care as the single possible solution.

[5] Engers and Stern (2002) present one of a few very interesting theoretical models that incorporates strategic behaviour and consider the role of n children when modelling the decision about the living arrangements. Hiedemann and Stern (1999), Checkovich and Stern (2002), and Byrne et al. (2004) are others. Although they obtain relevant results when they estimate the models, their main purpose is not to examine the rationale for the effect of the number of children on the living arrangements of an elderly parent.

In this paper we argue that the aging of the population implies a larger demand of formal care for the frail elderly, to the extent that it is accompanied by a larger incidence of childlessness.[6] If the dominant rule was to have the elderly live with children, it should not matter much whether there were many or few children. One should be enough to guarantee the coresidence outcome, even if the costs of caregiving were concentrated in one household.[7]

We show that a parent with many children has no further chance of escaping from living all the time in an institution, when costs of effort and financial costs of caring for a parent are considered.

Then, we introduce a disappointment effect that expresses the emotional impact on the parent of having a child that does not offer the parent's preferred living arrangement. This effect is finally found to be a channel through which the number of children negatively affects the probability of full time institutionalization.

We, therefore, conclude that population aging does not necessarily imply a need for more institutions for elderly with adult children – when living with children is a cultural possibility and it is preferred by the parent.

Our paper extends the preceding literature on long-term care decisions/ living arrangements of the elderly by developing a theoretical model of family decision making – when there has been a predominance of the empirical perspective - trying to rationalize the possible relation between the number of children and the living arrangements of the elderly, which had not formally been done.

It challenges a widespread idea - although not always supported by the empirical analyses - that a smaller number of children should unequivocally imply smaller chances of parent-child coresidence and larger chances of institutionalization.

1. Hypotheses of the model

The following hypotheses are assumed.

[6] Palloni (2000, p.38) has been alone in his acknowledgement that "declines in fertility could exert immediate pressure on coresidence only if accompanied by widespread childlessness."

[7] Won and Lee (1999) for instance, refer that in Korean society elderly parents live predominantly with the only or eldest son, when there is one. Accordingly, they find that the number of children is not related to the choice of living arrangements.

H1. The parent has no conditions to live alone, either for health reasons or for psychological reasons.

H2. The parent has enough income or wealth to cover his consumption, he does not need financial help from his adult children.

H3. The parent gets along equally with all his adult children and respective families.

H4. The ranking of the parent's preferences with respect to his accommodation in old age is the following:

1st: to live with one of his children,

2nd: to live equal periods of time with each of his children,

3rd: to live in an institution.

H5. The parent's utility of living in an institution is negative.

H6. Adult children prefer to live with the narrow family: living with an old parent represents a cost of time and effort. However, children are altruistic with respect to their parent, that is, they care for their parent, integrating his utility in their utility function.[8]

2. The model with two children

2.1. Description

The adult children are the players of the game. We start by considering the existence of two adult children in a symmetric game.

N = {1, 2} children.

[8] Although we state H6 in the above terms, we could change the interpretation of costs of effort, including possible benefits in them, and allowing for negative values.

$A_i = \{V_1, V_{1/2}, V_{Ins}\}, \ \forall i \in N$

$A = \times_{i \in N} A_i$

$u_i : A \rightarrow R$

$u_i (V_i, V_{-i})$= utility of child *i* when child *i* plays $V_i \in A_i$, and the other child plays V_{-i}.

The payoffs are represented in Table 1.

(insert Table 1)

When both adult children offer to have their parent live with them full time, the parent will accept the offer of any of the children with equal probability because he is indifferent between living with one or the other. Therefore, the payoff of each child will be the utility to their parent of always living with the same child (his first option) - $U(V_1)$ - less the cost of living with the parent all the time multiplied by the probability of being the child that gets the parent.

When one child offers to have his parent live with him all the time and the other offers one of the other possibilities, the parent will live with the child that made the offer for permanent accommodation. Therefore, when one of the adult children offers to have his parent with him permanently, all the payoffs will have the utility to their parent of always living with the same child. The difference will lay in the costs. Only the child who gets the parent has positive costs. The other has zero costs, whichever offer he makes.

When both adult children offer to have their parent half the time, they will have their parent's utility of getting his second option - $U(V_{1/2})$ - but each will have half the costs compared to the full time coresidence solution.

When one of the adult children offers to have his parent half the time but the other decides to have him in an institution, several alternatives can be assumed:

a) institutional care is not constrained by the supply side, so the parent will live half the time with one child and in an institution the other half;
b) it is difficult to find institutions, it is not feasible to get in and out for periods, therefore the parent has to stay in the institution all the time;
c) it is difficult to find institutions, it is not feasible to get in and out for periods, therefore the child who refuses to have the parent in an institution during his half of the time, has to take care of the parent all the time.

We assume alternative a).

In this case, only the child who has the parent with him half the time has costs, and these are half the costs compared to the full time coresidence solution.

When both adult children decide to have their parent in an institution, they do not have costs, but they have negative utilities reflecting their parent's negative utility.

We are going to solve this simultaneous game taking into consideration the Nash equilibria in pure strategies only.

2.2. Results

We provide the solutions to the model of section 2.1. in Appendix 1.

What can we say about the determinants of each living arrangement?

What determines which living arrangement is settled is the real improvement in the well-being of the parent from passing from an outcome to the next preferred outcome and the comparison of this improvement with the increase in costs imposed on the children. It is possible to have full time coresidence as the equilibrium solution despite $U(V_1) < C(V_1)$. The reason is the negativity of $U(V_{Ins})$. What really matters to the decision is not the level of the utility it reaches but the increase in utility that it allows.

Even at a great cost of time and effort, the child may decide to have his parent with him all the time, if he observes that it is really very, very important for the parent to be with just one of the children. It may be the case that the children live very far apart and the parent has problems with travelling or that he has fragile things in his dear possessions that he hates to change from place to place, for instance. It is also decisive that the aversion the parent has with respect to living in an institution is considerable.

A larger $U(V_1) - U(V_{1/2})$ - that is, a larger A [9] - diminishes the probability of the parent going to an institution and also the probability of the parent spending part of the time with each child. With larger $U(V_1) - U(V_{1/2})$, larger costs are necessary in order for the parent to go to an institution. Furthermore, it is necessary that $U(V_1) - U(V_{1/2})$ is small for the outcome of going from one house to the other to be possible.

[9] A is defined as $U(V_1) - U(V_{1/2})$.

A larger $U(V_{1/2}) - U(V_{Ins})$ - that is, a larger B [10] - increases the probability of the parent not going to an institution. It increases the probability of the parent going from house to house and also the probability of staying all the time with the same child.

When it is possible that the parent goes from house to house, it is also possible that he stays with just one child, with the other child wanting to send him to an institution. It is not compatible with the parent staying with just one child, with the other child offering to have the parent half the time. Figure 1 illustrates the possibilities of each solution in relation to the level of costs and in relation to the structure of the preferences of the parent. We introduced the simplifying condition $A + B = 1$ that does not change the intuition and allows the graphic representation.

(Insert Figure 1)

Larger costs of having the parent at home start by increasing the probability of the shared outcome, – if $U(V_{1/2}) - U(V_{Ins})$ is sufficiently large- and then increase the probability of the institution outcome.

$C(V_1) > U(V_1) - U(V_{Ins})$ is the condition for the parent to live in an institution.

Multiple coresidence is empirically one of the less prevalent living arrangements. Checkovich and Stern (2002) assert that no papers modelled shared caregiving among children, probably because that situation is not common in the data. That is consistent with our model, since multiple coresidence only exists as a multiple equilibrium, which does not happen with full time coresidence or with full time institutionalization.

3. The extension to *n* children

3.1. Description

We have now

$N = \{1, 2, \ldots, n\}$ children,

$A_i = \{V_1, V_{1/n}, V_{Ins}\}, \ \forall i \in N$.

[10] B is defined as $U(V_{1/2}) - U(V_{Ins})$.

The extension to *n* players of a game in a discrete form, whose payoffs are not well represented by continuous, well behaved functions, is not a completely trivial exercise. To facilitate the reasoning, we introduce u_i (V_i, d, e) = utility of child *i* when child *i* plays $V_i \in A_i$, there are *d* children apart from *i* that do not play V_1 and there are *e* children apart from *i* that play $V_{1/n}$. It is now possible to generalize the payoff functions in the following way:

$$u_i(V_1, d, e) = U(V_1) - C(V_1)/(n-d), \qquad 0 \leq d,e \leq n-1 \tag{1}$$

$$u_i(V_{1/n}, d, e) = \begin{cases} U(V_1), & d < n-1 \\ \frac{(e+1)}{n}.U(V_{1/n}) + \frac{(n-e-1)}{n}.U(V_{Ins}) - \frac{C(V_1)}{n}, & d = n-1 \end{cases} \tag{2}$$

$$u_i(V_{Ins}, d, e) = \begin{cases} U(V_1), & d < n-1 \\ \frac{e}{n}.U(V_{1/n}) + \frac{n-e}{n}.U(V_{Ins}), & d = n-1 \end{cases} \tag{3}$$

If d = n-1,

and a_i (element of A_i) = V_1, then $u_i = U(V_1) - C(V_1)$; (4)

and $a_i = V_{1/n}$, then $u_i = \frac{(e+1)}{n}.U(V_{1/n}) + \frac{(n-e-1)}{n}.U(V_{Ins}) - \frac{C(V_1)}{n}$; (5)

and $a_i = V_{Ins}$, then $u_i = \frac{e}{n}.U(V_{1/n}) + \frac{n-e}{n}.U(V_{Ins})$. (6)

3.2. Results: The effect of the number of children

We provide the solutions to the model of section 3.1. in Appendix 2.

Generally, if costs are large, the parent goes to an institution, if costs are not so large, he can be all the time with one child or divide his time among children. It is only possible that he divides his time among children when the improvement in the parent's utility for not going to an institution is larger than the loss for not staying with only one child, and the children's costs are between the two values.

The main purpose of extending the model to *n* children is to allow us to investigate the effect of the number of children on the living arrangements. So, what can we say about the probability of each living arrangement as the number of children grows? For instance, is the probability of staying all the time in an institution different for a parent with many children than for a parent with few children? Of course, if the parent has any children the probability of being taken care by children rises dramatically compared to the same probability for a parent with no children. But that is a special case. Does that probability continue to rise as the number of children grows?

In our model, as long as $C(V_1) > A + B$, full time institutionalization is a unique equilibrium, and that condition is not affected by the number of children.

When $C(V_1) \leq A + B$, the full time coresidence outcome is a solution. The other outcomes apart from the full time institutionalization are never unique equilibria. When the conditions for their existence are verified, the full time coresidence outcome is also a possible equilibrium. The number of children changes the zones where multiple equilibria exist.

The number of children has an indeterminate effect on the likelihood of dividing time equally among the children. A larger *n* has two effects on the likelihood of dividing time equally among all children. The first effect is related to the fact that $\frac{d(n/(n-1))}{dn}<0$, which conveys an enlargement of the probability of dividing time equally among all children. The second effect is a result of the fact that $U(V_{1/n})$ decreases with *n*. In this situation, a larger *n* enlarges *A* and decreases *B* by the same amount. Through this channel, the probability of dividing time equally among children is smaller with a larger *n.* The global effect is, therefore, indeterminate.

The likelihood of dividing time between his children and an institution, which was already small because the corresponding condition is very restrictive, is even smaller for a parent with more children.

As we see, it is not at all obvious that an old person with more children should with more ease avoid an undesired institution outcome and reach the favourite full time coresidence arrangement.

Naturally, this model is very basic. One may argue that the large financial costs usually associated with institutions that provide long-term care to the elderly may induce a change in these results. That is what we try to find out in the next section.

4. The model with financial costs

The financial costs are important to the choice of the living arrangement and they have not yet been included in the model. The cost of having the parent in an institution was null because we assumed that the income or wealth of the parent was enough to pay for the institution. It was also enough to pay what the adult children spent with the parent, should the parent stay with them. Let us change this setup.

It is now important to the adult children what is spent with the parent. It may be because the income/wealth of the parent is not enough to pay for all the expenses, so that the children must pay for part of it, or because the children administrate their parent's income/wealth and receive what is saved, or simply because they expect a bequest that will be smaller, the more it is spent by the parent now.

This financial cost may be introduced through a second cost component that is larger when the parent is in an institution and is smaller when the parent lives with his children. Hypothesis H2. in the original model substitutes for

H2'.: The cost of the living arrangement is important to the decision of the children. The most expensive solution is the 'institutionalization'.

Table 1' is an adaptation of Table 1 to this modified setup. It shows the payoffs of the model with financial costs and two children.

b is the ratio of the financial cost of keeping the parent at home over the financial cost of having the parent in an institution.

b <1, reflecting the idea that the financial costs of living with the parent are inferior to the financial costs of having the parent in an institution.

(insert Table 1')

5. The extension of the model with financial costs to *n* children

The payoff functions are now

$$u_i(V_1, d, e) = U(V_1) - C^a(V_1)/(n\text{-}d) - b.C^b(V_{Ins}), \qquad 0 \le d,e \le n\text{-}1 \qquad (7)$$

$$u_i(V_{1/n}, d, e) = \begin{cases} \mathrm{U}(V_1), & d < n-1 \\ \frac{(e+1)}{n}.\mathrm{U}(V_{1/n}) + \frac{(n-e-1)}{n}.\mathrm{U}(V_{Ins}) - \frac{C^a(V_1)}{n} - \frac{b.C^b(V_{Ins})}{n}, & d = n-1 \end{cases} \quad (8)$$

$$u_i(V_{Ins}, d, e) = \begin{cases} \mathrm{U}(V_1), & d < n-1 \\ \frac{e}{n}.\mathrm{U}(V_{1/n}) + \frac{n-e}{n}.\mathrm{U}(V_{Ins}) - \frac{C^b(V_{Ins})}{n}, & d = n-1 \end{cases} \quad (9)$$

In the Appendix 3, we present the conditions required for every type of solution to this model with financial costs.

As before, it is not an equilibrium to have more than one sibling offering to have the parent with them all the time. Also as before, when it is an equilibrium to have a full time coresidence outcome, it is not an equilibrium to have a full time institutionalization outcome. Which outcome is preferred when financial costs are considered, depends on the sign of (1/n –b). If b>1/n, the probability that the conditions for the full time coresidence solution are verified decreases and it is not clear what happens to the probability of the full time institutionalization outcome. If b<1/n, it is the effect on the first probability that is indeterminate and the probability of the full time institutionalization outcome that clearly decreases. In an extreme case, with null financial costs of having the parent at home, and positive financial costs of institutionalization, the financial cost effect unequivocally favours the full time coresidence outcome.

The two outcomes whose probability undoubtedly rises are the division of time among all children and the division of time between some of his children and an institution.

6. The effect of the number of children in the model with financial costs

Together with the effects that were present in the model without financial costs, there are now more effects as the number of children grows.

A large *n* reduces the attractiveness of $V_{1/n}$ in comparison to V_{Ins}, since $U(V_{1/n})$ becomes closer to $U(V_{Ins})$. If b>1/n, the inclusion of financial costs makes V_{Ins} more interesting when compared to V_1. And the larger the *n*, the easier it is that such condition is verified. Therefore, the larger the number of children, the larger the probability of the verification of the conditions for a full time institutionalization outcome when financial costs are introduced.

A large *n* has conflicting or indefinite effects on the chances of verification of the conditions for the other outcomes. The effect of the inclusion of financial costs is influenced by the number of children only as V_1 becomes less interesting when compared to V_{Ins}. This acts against the full time coresidence outcome and in favour of the division of time between children and an institution. Nevertheless, as it is not clear whether a larger *n* makes V_1 more or less attractive than $V_{1/n}$, it is not possible to state safely that the outcome full time coresidence becomes less probable as the number of children grows.

7. The model with financial costs and a disappointment effect

In the previous sections, we have considered that the living arrangement is what really matters to the parent. Nevertheless, we may consider that not only the decision but also the process that leads to the decision is relevant to the parent. As the parent prefers living with his children, and with the same all the time if possible, and as the parent knows his children know his preferences, there can be some disappointment in realizing that none of the children offers to have him at home. We could also see this effect as a kind of symmetric of the "joy-of-giving".[11] [12]

We assume that for every child that wants to have him at home only part of the time, there is a reduction in the parent's utility of α_1. For every child that wants to have him in an institution, there is a reduction in the parent's utility of α_2. α_1 and α_2 are, thus, a kind of disappointment coefficients, and, obviously, $\alpha_1 < \alpha_2$.

Considering this new disappointment effect, the payoffs become the following:

[11] This expression was appropriately suggested by a discussant of the paper.

[12] A contentment coefficient expressed as an increase in the parent's utility for getting an offer of a living arrangement that pleases him more than institutionalization, instead of a disappointment coefficient for getting offers that are not the most desired ones, would have identical results.

$$u_i(V_1, d, e) = \text{U}(V_1) - C^a(V_1)/(n-d) - b.C^b(V_{Ins}) - \alpha_1.e - \alpha_2.(d - e), \qquad 0 \le d,e \le n-1 \quad (10)$$

$u_i(V_{1/n}, d, e) =$

$$\begin{cases} \text{U}(V_1) - \alpha_1.(e+1) - \alpha_2.(d-e), & d < n-1 \\ \dfrac{(e+1)}{n}.\text{U}(V_{1/n}) + \dfrac{(n-e-1)}{n}.\text{U}(V_{Ins}) - \dfrac{C^a(V_1)}{n} - \dfrac{b.C^b(V_{Ins})}{n} - \alpha_1.(e+1) - \alpha_2.(d-e), & d = n-1 \end{cases} \quad (11)$$

$$u_i(V_{Ins}, d, e) = \begin{cases} \text{U}(V_1) - \alpha_1.e - \alpha_2.(d-e+1), & d < n-1 \\ \dfrac{e}{n}.\text{U}(V_{1/n}) + \dfrac{n-e}{n}.\text{U}(V_{Ins}) - \dfrac{C^b(V_{Ins})}{n} - \alpha_1.e - \alpha_2.(d-e+1), & d = n-1 \end{cases} \quad (12)$$

One could argue that in our model each child's utility function expresses the same valuation of the parent's disappointment with him and of the parent's disappointment with a sibling, and that it should not be so. Child i should be more upset by the parent's disappointment with himself. Nevertheless, if we introduce that distinction, nothing is changed in the results. That is because all equilibrium conditions end out a result of comparisons that differ only in what child i does in different circumstances. Child i may change, but it is always a first person evaluation.

In the Appendix 4, we present the conditions required for every type of solution to this model with costs of effort, financial costs and a disappointment effect.

This version of the model displays some new features. It is now possible that there are equilibria including more than one V_1. On the other hand, when V_1 is played, no one plays V_{Ins}.

V_{Ins} and $V_{1/n}$ are seldom played simultaneously.

A large *A* acts in favour of a full time coresidence outcome, and against the part time coresidence, part time institutionalization, and full time institutionalization outcomes. A large *B* acts only against the outcomes that include institutionalization.

The disappointment effect favours the full time coresidence outcome in relation to the part time coresidence. A large coefficient α_1 acts in favour of the full time institution outcome, whereas a large coefficient α_2 acts against it. As $\alpha_1 < \alpha_2$, the disappointment effect globally makes full time institutionalization more difficult.

8. The effect of the number of children in the model with financial costs and a disappointment effect

A large number of children reinforces the negative impact of the disappointment effect on the chances of a full time institutionalization outcome. It weakens the positive impact on the chances of full time coresidence with all but one sibling offering to have the parent at home part of the time. At the same time, it reinforces the positive impact on the chances of full time coresidence with everyone playing V_1.

These results show a kind of herd behaviour: when all the siblings play $V_{1/n}$, a large number of siblings decreases the attractiveness of playing V_1; on the contrary, when all the others play V_1, a large number of siblings increases the attractiveness of playing V_1.

A large number of children may enlarge the probability of part time coresidence and shorten the probability of institutionalization if the disappointment coefficient corresponding to every child that proposed to have the parent in an institution is really large compared to the other disappointment coefficient. It cannot be firmly stated that the probability of part time coresidence grows with the number of children because of the loss of attractiveness of the part time coresidence solution when the number of children grows (*B* decreases). Nevertheless, the disappointment effect is the only channel through which the number of children negatively affects the conditions for verification of full time institutionalization.

A larger *n* unequivocally enlarges the chances of part time institutionalization. Nevertheless, they remain small.

10. Conclusions

In this paper, we propose a game-theoretic model that intends to describe, in a simple way, what may motivate the decision of long-term care for an elderly, frail parent. In particular, we address the issue of how the number of children influences such decision.

The three possible living arrangements that are considered are coresidence with only one of his children, division of time equally among children and institutionalization. The preferences of the parent are known.

The adult children would rather not have to take care of the old parent, for it represents a cost of effort and time, not easily compatible with busy, modern life. However, they care for their parent and take into account their parent's utility when they decide.

It is found that the living arrangement obtained is a result of the comparison of the cost of taking care of the parent and the improvement in the parent's utility of adopting the next preferred arrangement. It is the improvement in utility that counts and not really the level of utility.

We add two extensions to the just described model (the canonical model): an extension to financial costs – where institutionalization is assumed to be the most expensive living arrangement, and an extension to an effect of disappointment felt by the parent with every children's undesired proposal of living arrangement.

The solutions reflect strategic interaction among adult children. In the canonical model, if one of the siblings offers to take care of the parent all the time, no other sibling will do the same. It is easier that one of the adult children stays with the parent all the time when the other siblings want to have the parent in an institution, than when there are other children who offer to take care of the parent part of the time.

Nevertheless, it is easier for one child to offer to take care of the parent part of the time when the others do the same than when some others want to have the parent in an institution. This is because the temptation to offer full time coresidence is larger in the last situation. This is so much so that it is not an equilibrium to be the only sibling to offer to have the parent part of the time with the parent living in an institution the rest of the time.

In the canonical model, although multiple equilibria are possible, the full time coresidence and the full time institutionalization outcomes are mutually exclusive.

Although sometimes economists regard multiple equilibria as an annoyance, it in fact expresses a component of arbitrariness of life. Some authors consider that realistic games possibly admit multiple equilibria for at least some combinations of parameters, and that exploiting the presence of multiplicity conveys more information than imposing simplifying assumptions [Andrews et al. (2004), Myerson (2003), Tamer (2003)].

When financial costs are introduced, the likelihood of the equally divided arrangement and the likelihood of the division of time between children and an institution increase. The likelihood of the full time institutionalization decreases if the financial cost of having the parent in an institution is larger than that of having him at home even part of the time. The likelihood of the full time coresidence arrangement decreases as long as the saving from having the parent at home instead of in an institution is inferior to the saving from paying for the parent part of time instead of all of the time.

With the extension to a disappointment effect, it is no longer impossible that more than one child offers full time coresidence.

The disappointment effect undoubtedly makes full time institutionalization more difficult, and full time coresidence more appealing compared to the other possible living arrangements.

Table 2 summarises the results concerning the impact of each considered effect – cost of effort, financial cost, and disappointment effect – on each possible outcome.

In the canonical model, the number of children does not change the condition for full time institutionalization, and the effect of the number of children on the probabilities of full time coresidence and of division among all children is indeterminate. Concerning financial costs, a large number of children acts in favour of the full time institutionalization as compared to the full time coresidence arrangement. A larger number of children strengthens the negative influence of the disappointment effect on the chances of the full time institutionalization arrangement. A larger number of children may enlarge the probability of part time coresidence and shorten the likelihood of institutionalization, if the disappointment coefficient corresponding to every child that proposed to have the parent in an institution is really large compared to the other disappointment coefficient.

The model including the disappointment effect allows for the possibility of having a positive association between the number of adult children and the probability of the parent living with children instead of in an institution. It must be noted, though, that that result is just one possibility and is not at all guaranteed.

As we see, a parent with many children that would rather live with the same child all the time, is not safely closer to his aim than a parent with few children. A parent with many children is closer to avoiding institutionalization only if the disappointment effect surpasses the cost effects.

Therefore, population aging – as the motive for the reduction of the number of children to every parent – does not imply that the unmarried elderly with adult children will have to rely more on institutions when they need long-term care.

Naturally, population aging also involves more childless elderly, and our model does not consider those.

Further empirical and theoretical work is needed on this problematic. Empirically, the positive relation between the number of children and the probability of institutionalization should be checked with several different data bases, obtained in different countries. Theoretically, it would be interesting to extend the analysis to a model that considers the increased chances of having more than one elderly parent to be cared after – a parent and a parent-in-law -, as a result of population aging. A possible competition for coresidence with children should be more severe in an aging world, therefore inducing more institutionalization.

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Appendix 1

The solutions of the canonical model, with 2 children

We managed to write the necessary conditions for each solution in a simple way with interesting properties: it allows for a graph representation, it facilitates comparisons among different solutions and it is helpful in detecting what underlies the decisions. In order to have that, we put in evidence three elements in every condition: $C(V_1)$, the difference between $U(V_1)$ and $U(V_{1/2})$ – which will be called *A* - and the difference between $U(V_{1/2})$ and $U(V_{Ins})$ – which will be called *B*.

(V_1, V_1) cannot be a Nash equilibrium: if a sibling offers to have the parent all the time, the other's best response is not to offer the same. It is indifferent if this one offers $V_{1/2}$ or V_{Ins}, since the consequences will be equal: he will have no costs and the parent will be entirely satisfied all the same.

For **(V_1, $V_{1/2}$)** and **($V_{1/2}$, V_1)** to be equilibria, it must be that:

a) There is no better answer to $V_{1/2}$ than V_1. This happens when

$$\begin{cases} C(V_1) \leq 2A & (V_1 \succ V_{1/2}) \\ C(V_1) \leq A + 0{,}5.B & (V_1 \succ V_{\text{Ins}}) \end{cases} \tag{A1}$$

b) There is no better answer to V_1 than to play $V_{1/2}$. This is always true. Playing $V_{1/2}$ delivers the largest payoff, which is equal to the payoff obtained by playing V_{Ins}.

For **(V_1, V_{Ins})** and **(V_{Ins}, V_1)** to be equilibria, it must be that:

a) there is no incentive to deviate from V_1 when the other plays V_{Ins}. This happens when:

$$\begin{cases} C(V_1) \leq 2A + B & (V_1 \succ V_{1/2}) \\ C(V_1) \leq A + B & (V_1 \succ V_{\text{Ins}}) \end{cases} \tag{A2}$$

b) V_{Ins} is the best answer to V_1. This is always true. Playing V_{Ins} delivers the largest payoff, which is equal to the payoff obtained by playing $V_{1/2}$.

For **($V_{1/2}$, $V_{1/2}$)** to be an equilibrium, it must be that there is no incentive to deviate from playing $V_{1/2}$ when the other plays $V_{1/2}$. This happens if:

$$\begin{cases} C(V_1) \geq 2A & (V_{1/2} \succ V_1) \\ C(V_1) \leq B & (V_{1/2} \succ V_{\text{Ins}}) \end{cases} \tag{A3}$$

For **(V_{Ins}, V_{Ins})** to be an equilibrium, it must be that there is no incentive to deviate from playing V_{Ins} when the other plays V_{Ins}. This happens if:

$$\begin{cases} C(V_1) \geq A + B & (V_{\text{Ins}} \succ V_1) \\ C(V_1) \geq B & (V_{\text{Ins}} \succ V_{1/2}) \end{cases} \tag{A4}$$

Systems (3) and (4) are incompatible. For all the conditions to be verified, *A* would have to be non-positive, which would go against H4. Therefore, ($V_{1/2}$, $V_{1/2}$) and (V_{Ins}, V_{Ins}) cannot both be equilibria at the same time.

(V_{Ins}, $V_{1/2}$) and **($V_{1/2}$, V_{Ins})** are not equilibria. For $V_{1/2}$ to be the best answer to V_{Ins}, it must happen that:

$$\begin{cases} C(V_1) \geq 2A + B & (V_{1/2} \succ V_1) \\ C(V_1) \leq B & (V_{1/2} \succ V_{\text{Ins}}) \end{cases} \tag{A5}$$

And for V_{Ins} to be the best answer to $V_{1/2}$, it must happen that:

$$\begin{cases} C(V_1) \geq A + 0{,}5.B & (V_{\text{Ins}} \succ V_1) \\ C(V_1) \geq B & (V_{\text{Ins}} \succ V_{1/2}) \end{cases} \tag{A6}$$

For systems (5) and (6) to be compatible, it should happen that $C(V_1) = B$. However, together with the first condition of system (5), this would imply that $A < 0$, which is not true, by assumption.

Appendix 2

The solutions of the canonical model, with *n* children

In this section, *A* accounts for $\cup(V_1) - \cup(V_{1/n})$ and *B* accounts for $\cup(V_{1/n}) - \cup(V_{Ins})$.

In equilibrium there is not more than one V_1. With someone else playing V_1 (d<n-1), $u_i(V_{1/n}, d, e) = u_i(V_{Ins}, d, e) = \cup(V_1) > u_i(V_1, d, e) = \cup(V_1) - C(V_1)/(n-d)$, always. Therefore, it is not optimal to play V_1 when someone else is already doing so.

There are three types of possible solutions that include someone playing V_1:

1.1) $(V_1, V_{1/n}, \ldots, V_{1/n})$. In this case, d = n-1, and e = d, with $V_i = V_1$;

1.2) $(V_1, V_{1/n}, \ldots, V_{Ins})$. In this case, d = n-1, and 0<e<d, with $V_i = V_1$;

1.3) $(V_1, V_{Ins}, \ldots, V_{Ins})$. In this case, d = n-1, and e = 0, with $V_i = V_1$.

There are three types of possible solutions where no sibling plays V_1 and where at least one plays $V_{1/n}$:

2.1) $(V_{1/n}, V_{1/n}, \ldots, V_{1/n})$. In this case, d = n-1, e = d, and $V_i = V_{1/n}$;

2.2) $(V_{1/n}, V_{1/n}, \ldots, V_{Ins})$. In this case, d = n-1, 0<e<d, and $V_i = V_{1/n}$;

2.3) $(V_{1/n}, V_{Ins}, \ldots, V_{Ins})$. In this case, d = n-1, e = 0, and $V_i = V_{1/n}$.

There are three types of possible solutions where no sibling plays V_1 and where at least one plays V_{Ins}:

3.1) $(V_{Ins}, V_{1/n}, \ldots, V_{1/n})$. In this case, d = n-1, e = d, and $V_i = V_{Ins}$;

3.2) $(V_{Ins}, V_{1/n}, \ldots, V_{Ins})$. In this case, d = n-1, 0<e<d, and $V_i = V_{Ins}$;

3.3) $(V_{Ins}, V_{Ins}, \ldots, V_{Ins})$. In this case, d = n-1, e = 0, and $V_i = V_{Ins}$.

In order to have a type **1.1)** equilibrium,

a) it must not pay to deviate from playing V_1 when the others play $V_{1/n}$. This happens when

$$\begin{cases} C(V_1) \le \dfrac{n}{n-1} A & (V_1 \succ V_{1/n}) \\ \\ C(V_1) \le A + \frac{1}{n} B & (V_1 \succ V_{Ins}) \end{cases} \tag{A7}$$

b) it must also not pay to deviate from playing $V_{1/n}$ when someone else plays V_1 and the others play $V_{1/n}$ (d=e<n-1). This always happens.

In order to have a type **1.2)** equilibrium,

a) it must not pay to deviate from playing V_1 when the others play either $V_{1/n}$ or V_{Ins}. This happens when d= n-1 and

$$\begin{cases} C(V_1) \leq (n/(n-1)).A + (\dfrac{n-e-1}{n-1}).B, & (V_1 \succ V_{1/n}) \\ C(V_1) \leq A + \dfrac{n-e}{n}.B & (V_1 \succ V_{\text{Ins}}) \end{cases} \qquad \text{(A8)}$$

b) It must also be optimal not to deviate from playing $V_{1/n}$ when someone else plays V_1 and other(s) play V_{Ins}. In this case d<n-1, with $V_i = V_{1/n}$. The conditions for that are

$$\begin{cases} U(V_1) \geq U(V_1) - C(V_1)/(n-d), & (V_{1/n} \succ V_1) \\ U(V_1) \geq U(V_1) & (V_{1/n} \succ V_{\text{Ins}}) \end{cases} \qquad \text{(A9)}$$

and they are always verified.

c) Additionally, it must not pay to deviate from playing V_{Ins} when there is someone playing V_1 and other(s) play $V_{1/n}$. In this situation d<n-1, with $V_i = V_{Ins}$ and

$$\begin{cases} U(V_1) \geq U(V_1) - C(V_1)/(n-d), & (V_{Ins} \succ V_1) \\ U(V_1) \geq U(V_1) & (V_{Ins} \succ V_{1/\text{n}}) \end{cases}$$

This system, equal to the previous one, is always verified.

Therefore, in order to have a type 1.2) equilibrium, the only requirement is system (A2).

In order to have a type **1.3)** equilibrium,

a) it must not pay to deviate from playing V_1 when everyone else plays V_{Ins}. This is so when d = n-1, and e = 0, with $V_i = V_1$, and:

$$\begin{cases} C(V_1) \leq (n/(n-1)).A + B & (V_1 \succ V_{1/n}) \\ C(V_1) \leq A + B & (V_1 \succ V_{\text{Ins}}) \end{cases} \qquad \text{(A10)}$$

b) It must also not pay to deviate from playing V_{Ins} when someone else plays V_1 and all the rest play V_{Ins}, that is, when d<n-1, e=0, with $V_i = V_{Ins}$ and

$$\begin{cases} U(V_1) \geq U(V_1) - C(V_1)/(n-d), & (V_{Ins} \succ V_1) \\ U(V_1) \geq U(V_1) & (V_{Ins} \succ V_{1/n}) \end{cases} \tag{A11}$$

which always happens.

In order to have a type **2.1)** equilibrium, it must not pay to deviate from playing $V_{1/n}$ when the others play $V_{1/n}$. This happens when:

$$\begin{cases} C(V_1) \geq (n/(n-1)).A & (V_{1/n} \succ V_1) \\ C(V_1) \leq B & (V_{1/n} \succ V_{Ins}) \end{cases} \tag{A12}$$

In order to have a type **2.2)** equilibrium,

a) it must not pay to deviate from playing $V_{1/n}$ when no one else plays V_1. This happens when d = n-1, 0<e<d, with $V_i = V_{1/n}$, and:

$$\begin{cases} C(V_1) \geq (n/(n-1)).A + ((n-e-1)/(n-1)).B & (V_{1/n} \succ V_1) \\ C(V_1) \leq B & (V_{1/n} \succ V_{Ins}) \end{cases} \tag{A13}$$

b) Additionally, it must not pay to deviate from playing V_{Ins} when no one else plays V_1. This happens when d=n-1, with $V_i = V_{Ins}$ and:

$$\begin{cases} C(V_1) \geq A + ((n-e)/n).B, & (V_{Ins} \succ V_1) \\ C(V_1) \geq B & (V_{Ins} \succ V_{1/n}) \end{cases} \tag{A14}$$

The combination of systems (13) and (14) lead to $C(V_1) = B \geq \frac{n}{e} A$.

In order to have a type **2.3)** equilibrium,

a) it must not pay to deviate from playing $V_{1/n}$ when everyone else plays V_{Ins}. This happens when $d = n$-1, e=0, with $V_i = V_{1/n}$,:

$$\begin{cases} C(V_1) \geq (n/(n-1)).A + B & (V_{1/n} \succ V_1) \\ C(V_1) \leq B & (V_{1/n} \succ V_{Ins}) \end{cases} \tag{A15}$$

b) Additionally, it must not pay to deviate from V_{Ins} when no one is playing V_1. d=n-1. This is the same system as (14).

We can see that the two conditions in system (15) are not compatible. Therefore, ($V_{1/n}$, V_{Ins}, …, V_{Ins}) is not a possible equilibrium.

In order to have a type **3.1)** equilibrium,

a) it must not pay to deviate from playing V_{Ins} when the others play $V_{1/n}$. This happens when d=n-1= e, with $V_i = V_{Ins}$:

$$\begin{cases} C(V_1) \geq A + (1/n).B, & (V_{Ins} \succ V_1) \\ C(V_1) \geq B & (V_{Ins} \succ V_{1/n}) \end{cases} \tag{A16}$$

b) Additionally, it must not pay to deviate from playing $V_{1/n}$ when no sibling plays V_1. That is, system (13) must be satisfied.

Conditions a) and b) together resume to

$$\begin{cases} B \geq \dfrac{n}{n-1} A \\ C(V_1) = B \end{cases} \tag{A17}$$

In order to have a type **3.2)** equilibrium,

a) it must not pay to deviate from playing V_{Ins} when no sibling plays V_1. This happens when:

$$\begin{cases} C(V_1) \geq A + ((n-e)/n).B, & (V_{Ins} \succ V_1) \\ C(V_1) \geq B & (V_{Ins} \succ V_{1/n}) \end{cases} \tag{A18}$$

b) Additionally, it must not pay to deviate from playing $V_{1/n}$ when no sibling plays V_1. This is system (13) again.

Conditions a) and b) together resume to

$$\begin{cases} B \geq \dfrac{n}{n-1} A \\ C(V_1) = B \end{cases} \tag{A19}$$

In order to have a type **3.3)** equilibrium, it must not pay to deviate from playing V_{Ins} when everyone else plays V_{Ins}. This happens when:

$$\begin{cases} C(V_1) \geq A + B, & (V_{Ins} \succ V_1) \\ C(V_1) \geq B & (V_{Ins} \succ V_{1/n}) \end{cases} \tag{A20}$$

<u>Summing up</u> the results we may say that:

a) The parent stays with one child all the time when:

$$\begin{cases} C(V_1) \leq A + B + (A - e.B)/(n-1) & (V_1 \succ V_{1/n}) \\ C(V_1) \leq A + B - \frac{e}{n}.B & (V_1 \succ V_{Ins}) \end{cases} \quad \text{and } 0 \leq e \leq d. \tag{A21}$$

The system reduces to only one of the conditions. However, what condition is active depends on the relation between $(A\text{-}e.B)/(n\text{-}1)$ and $-e.B/n$. If $A>e.B/n$, the active condition is $C(V_1) \leq A + B - (e/n).B$. If $A<e.B/n$, the active condition is $C(V_1) \leq A + B + (A - e.B)/(n-1)$. Both conditions are larger, the smaller e is. This means that the largest active condition is the one with e=0. In this case, the two conditions convert to $C(V_1) \leq A + B$ and $C(V_1) \leq A + B + A/(n-1)$, respectively. The largest active condition is, therefore, $C(V_1) \leq A + B$. That is, for all $U(V_{Ins})$, $U(V_{1/n})$ and $U(V_1)$, as long as $C(V_1)$ is smaller than $(U(V_1) - U(V_{1/n})) + (U(V_{1/n}) - U(V_{Ins}))$, V_1 is an equilibrium.

b) The parent divides time equally among children when:

$$\frac{n}{(n-1)}.A \leq C(V_1) \leq B. \tag{A22}$$

c) The parent stays in an institution all the time when:

$$C(V_1) \geq A + B. \tag{A23}$$

d) The parent divides his time between his children and an institution when:

$$C(V_1) = B \geq \frac{n}{e}.A \tag{A24}$$

Appendix 3

The solutions of the model with costs of effort, financial costs, and *n* children

Considering the taxonomy of solutions that was presented in Appendix 2, a type **1.1)** equilibrium exists when it does not pay to deviate from playing V_1 when the others play $V_{1/n}$.

a) $$\begin{cases} U(V_1) - C^a(V_1) - b.C^b(V_{Ins}) \geq U(V_{1/n}) - C^a(V_1)/n - b.C^b(V_{Ins})/n, & (V_1 \succ V_{1/n}) \\ U(V_1) - C^a(V_1) - b.C^b(V_{Ins}) \geq ((n-1)/n).U(V_{1/n}) + (1/n).U(V_{Ins}) - C^b/n & (V_1 \succ V_{Ins}) \end{cases} \Leftrightarrow$$

$$\begin{cases} C^a(V_1) \leq (n/(n-1)).A - b.C^b(V_{Ins}), & (V_1 \succ V_{1/n}) \\ C^a(V_1) \leq A + \frac{1}{n}.B - (b - \frac{1}{n}).C^b(V_{Ins}) & (V_1 \succ V_{Ins}) \end{cases} \quad \text{(A25)}$$

b) it must also not pay to deviate from playing $V_{1/n}$ when someone else plays V_1 and the others play $V_{1/n}$ (d=e<n-1). This always happens.

In order to have a type **1.2)** equilibrium,

a) it must not pay to deviate from playing V_1 when the others play either $V_{1/n}$ or V_{Ins}. This happens when:

$$\begin{cases} U(V_1) - C^a(V_1) - b.C^b(V_{Ins}) \geq ((e+1)/n).U(V_{1/n}) + ((n-e-1)/n).U(V_{Ins}) - C^a(V_1)/n - b.C^b(V_{Ins})/n, & (V_1 \succ V_{1/n}) \\ U(V_1) - C^a(V_1) - b.C^b(V_{Ins}) \geq (e/n).U(V_{1/n}) + ((n-e)/n).U(V_{Ins}) - \frac{C^b(V_{Ins})}{n} & (V_1 \succ V_{Ins}) \end{cases}$$

$$\begin{cases} C^a(V_1) \leq \frac{n}{n-1}.A + \frac{n-e-1}{n-1}.B - b.C^b(V_{Ins}) & (V_1 \succ V_{1/n}) \\ C^a(V_1) \leq A + \frac{n-e}{n}.B - (b - \frac{1}{n}).C^b(V_{Ins}) & (V_1 \succ V_{Ins}) \end{cases} \quad \text{(A26)}$$

b) It must also be optimal not to deviate from playing $V_{1/n}$ when someone else plays V_1 and other(s) play V_{Ins}. In this case d<n-1 and

$$\begin{cases} \mathsf{U}(V_1) \geq \mathsf{U}(V_1) - C^a(V_1)/(n-d) - b.C^b(V_{\text{Ins}}), & (V_{1/n} \succ V_1) \\ \mathsf{U}(V_1) \geq \mathsf{U}(V_1) & (V_{1/n} \succ V_{\text{Ins}}) \end{cases} \quad \text{(A27)}$$

This system is always verified.

c) Additionally, it must not pay to deviate from playing V_{Ins} when there is someone playing V_1 and other(s) play $V_{1/n}$. In this situation d<n-1 and

$$\begin{cases} \mathsf{U}(V_1) \geq \mathsf{U}(V_1) - C^a(V_1)/(n-d) - b.C^b(V_{\text{Ins}}), & (V_{Ins} \succ V_1) \\ \mathsf{U}(V_1) \geq \mathsf{U}(V_1) & (V_{Ins} \succ V_{1/\text{n}}) \end{cases} \quad \text{(A28)}$$

This system is always verified.

In order to have a type **1.3)** equilibrium,

a) it must not pay to deviate from playing V_1 when everyone else plays V_{Ins}. This is so when:

$$\begin{cases} \mathsf{U}(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}) \geq \frac{1}{n}.\mathsf{U}(V_{1/n}) + ((n-1)/n).\mathsf{U}(V_{\text{Ins}}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}), & (V_1 \succ V_{1/n}) \\ \mathsf{U}(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}) \geq \mathsf{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) & (V_1 \succ V_{\text{Ins}}) \end{cases}$$

$$\Leftrightarrow \begin{cases} C^a(V_1) \leq (n/(n-1)).A + B - b.C^b(V_{\text{Ins}}) & (V_1 \succ V_{1/n}) \\ C^a(V_1) \leq A + B - (b - \frac{1}{n}).C^b(V_{\text{Ins}}) & (V_1 \succ V_{\text{Ins}}) \end{cases} \quad \text{(A29)}$$

b) It must also not pay to deviate from playing V_{Ins} when someone else plays V_1 and all the rest play V_{Ins}, that is, when

$$\begin{cases} \mathsf{U}(V_1) \geq \mathsf{U}(V_1) - C^a(V_1)/(n-d) - b.C^b(V_{\text{Ins}}), & (V_{Ins} \succ V_1) \\ \mathsf{U}(V_1) \geq \mathsf{U}(V_1) & (V_{Ins} \succ V_{1/\text{n}}) \end{cases} \quad \text{(A30)}$$

This system is always verified.

In order to have a type **2.1)** equilibrium, it must not pay to deviate from playing $V_{1/n}$ when the others play $V_{1/n}$. This happens when:

$$\begin{cases} \mathsf{U}(V_{1/n}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}) \geq \mathsf{U}(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}), & (V_{1/n} \succ V_1) \\ \mathsf{U}(V_{1/n}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}) \geq ((\text{n-1})/\text{n}).\mathsf{U}(V_{1/n}) + \frac{1}{n}.\mathsf{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) & (V_{1/n} \succ V_{\text{Ins}}) \end{cases}$$

$$\Leftrightarrow$$

$$\begin{cases} C^a(V_1) \geq (n/(n-1)).A - b.C^b(V_{\text{Ins}}) & (V_{1/n} \succ V_1) \\ C^a(V_1) \leq B + (1-b).C^b(V_{\text{Ins}}) & (V_{1/n} \succ V_{\text{Ins}}) \end{cases} \quad \text{(A31)}$$

In order to have a type **2.2)** equilibrium,

a) it must not pay to deviate from playing $V_{1/n}$ when no one else plays V_1. This happens when:

$$\begin{cases} \frac{\text{e+1}}{\text{n}}.\mathsf{U}(V_{1/n}) + \frac{n-e-1}{n}.\mathsf{U}(V_{\text{Ins}}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}) \geq \mathsf{U}(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}), & (V_{1/n} \succ V_1) \\ \frac{\text{e+1}}{\text{n}}.\mathsf{U}(V_{1/n}) + \frac{n-e-1}{n}.\mathsf{U}(V_{\text{Ins}}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}) \geq \frac{e}{n}.\mathsf{U}(V_{1/n}) + \frac{n-e}{n}.\mathsf{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) & (V_{1/n} \succ V_{\text{Ins}}) \end{cases}$$

$$\Leftrightarrow$$

$$\begin{cases} C^a(V_1) \geq \frac{n}{n-1}.A + \frac{n-e-1}{n-1}.B - b.C^b(V_{\text{Ins}}) & (V_{1/n} \succ V_1) \\ C^a(V_1) \leq B + (1-b).C^b(V_{\text{Ins}}) & (V_{1/n} \succ V_{\text{Ins}}) \end{cases} \quad \text{(A32)}$$

b) Additionally, it must not pay to deviate from playing V_{Ins} when no one else plays V_1. This happens when:

$$\begin{cases} \frac{\text{e}}{\text{n}}.\mathsf{U}(V_{1/n}) + \frac{n-e}{n}.\mathsf{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) \geq \mathsf{U}(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}), & (V_{\text{Ins}} \succ V_1) \\ \frac{\text{e}}{\text{n}}.\mathsf{U}(V_{1/n}) + \frac{n-e}{n}.\mathsf{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) \geq \frac{e+1}{n}.\mathsf{U}(V_{1/n}) + \frac{n-e-1}{n}.\mathsf{U}(V_{\text{Ins}}) - \frac{1}{n}C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}), & (V_{\text{Ins}} \succ V_{1/n}) \end{cases}$$

$$\begin{cases} C^a(V_1) \geq A + \frac{n-e}{n}.B - (b - \frac{1}{n}).C^b(V_{\text{Ins}}), & (V_{Ins} \succ V_1) \\ C^a(V_1) \geq B + (1-b).C^b(V_{\text{Ins}}) & (V_{Ins} \succ V_{1/\text{n}}) \end{cases} \quad \text{(A33)}$$

a) and b) together resume to:

$$\begin{cases} C^a(V_1) = B + (1-b).C^b(V_{\text{Ins}}) \\ \\ C^b(V_1) \geq \dfrac{n}{n-1}.A - \dfrac{e}{n-1}.B \end{cases} \tag{A34}$$

In order to have a type **2.3)** equilibrium,

a) it must not pay to deviate from playing $V_{1/n}$ when everyone else plays V_{Ins}. This happens when:

$$\begin{cases} \frac{1}{n}.\text{U}(V_{1/n}) + \dfrac{n-1}{n}.\text{U}(V_{\text{Ins}}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}) \geq \text{U}(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}), & (V_{1/n} \succ V_1) \\ \frac{1}{n}.\text{U}(V_{1/n}) + \dfrac{n-1}{n}.\text{U}(V_{\text{Ins}}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}) \geq \text{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) & (V_{1/n} \succ V_{\text{Ins}}) \end{cases} \Leftrightarrow$$

$$\begin{cases} C^a(V_1) \geq (n/(n-1)).A + B - b.C^b(V_{\text{Ins}}) & (V_{1/n} \succ V_1) \\ \\ C^a(V_1) \leq B - (b-1).C^b(V_{\text{Ins}}) & (V_{1/n} \succ V_{\text{Ins}}) \end{cases} \tag{A35}$$

b) Additionally, it must not pay to deviate from V_{Ins} when no one is playing V_1. d=n-1. This is the same system as (33).

The set of conditions resumes to:

$$\begin{cases} C^a(V_1) = B + (1-b).C^b(V_{\text{Ins}}) \\ \\ C^b(V_1) \geq \dfrac{n}{n-1}.A \end{cases} \tag{A36}$$

In order to have a type **3.1)** equilibrium,

a) it must not pay to deviate from playing V_{Ins} when the others play $V_{1/n}$. This happens when:

$$\begin{cases} ((n-1))/n).\text{U}(V_{1/n}) + \frac{1}{n}.\text{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) \geq \text{U}(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}), & (V_{\text{Ins}} \succ V_1) \\ ((n-1)/n).\text{U}(V_{1/n}) + \frac{1}{n}.\text{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) \geq \text{U}(V_{1/n}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}), & (V_{\text{Ins}} \succ V_{1/n}) \end{cases} \Leftrightarrow$$

$$\begin{cases} C^a(V_1) \geq A + \tfrac{1}{n}.B - (b - \tfrac{1}{n}).C^b(V_{\text{Ins}}), & (V_{\text{Ins}} \succ V_1) \\ C^a(V_1) \geq B + (1-b).C^b(V_{\text{Ins}}), & (V_{\text{Ins}} \succ V_{1/n}) \end{cases} \tag{A37}$$

b) Additionally, it must not pay to deviate from playing $V_{1/n}$ when no sibling plays V_1. That is, system (32) must be satisfied.

The set of conditions resumes to:

$$\begin{cases} C^a(V_1) = B + (1-b).C^b(V_{\text{Ins}}) \\ C^b(V_1) \geq \dfrac{n}{n-1}.A - \dfrac{e}{n-1}.B \end{cases} \tag{A38}$$

In order to have a type **3.2)** equilibrium,

a) it must not pay to deviate from playing V_{Ins} when no sibling plays V_1. This happens when system (33) is verified.

b) Additionally, it must not pay to deviate from playing $V_{1/n}$ when no sibling plays V_1. That is, system (32) must be satisfied.

The set of conditions resumes to:

$$\begin{cases} C^a(V_1) = B + (1-b).C^b(V_{\text{Ins}}) \\ C^b(V_1) \geq \dfrac{n}{n-1}.A - \dfrac{e}{n-1}.B \end{cases} \tag{A39}$$

In order to have a type **3.3)** equilibrium, it must not pay to deviate from playing V_{Ins} when everyone else plays V_{Ins}. This happens when:

$$\begin{cases} \text{U}(V_{\text{Ins}}) - \tfrac{1}{n}.C^b(V_{\text{Ins}}) \geq \text{U}(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}), & (V_{\text{Ins}} \succ V_1) \\ \text{U}(V_{\text{ns}}) - \tfrac{1}{n}.C^b(V_{\text{Ins}}) \geq \tfrac{1}{n}.\text{U}(V_{1/n}) + ((n-1)/n).\text{U}(V_{\text{Ins}}) - \tfrac{1}{n}.C^a(V_1) - \tfrac{b}{n}.C^b(V_{\text{Ins}}), & (V_{\text{Ins}} \succ V_{1/n}) \end{cases} \Leftrightarrow$$

$$\begin{cases} C^a(V_1) \geq A + B + (\tfrac{1}{n} - b).C^b(V_{\text{Ins}}), & (V_{Ins} \succ V_1) \\ C^a(V_1) \geq B + (1-b).C^b(V_{\text{Ins}}) & (V_{Ins} \succ V_{1/\text{n}}) \end{cases} \tag{A40}$$

Appendix 4

The solutions of the model with costs of effort, financial costs, disappointment effects, and *n* children

We consider again the taxonomy of solutions that was presented in Appendix 2. The conditions for a type **1.1)** equilibrium are:

a) d = n-1 and e = n-1

$$\begin{cases} U(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}) - \alpha_1.e - \alpha_2.(d-e) \geq \\ U(V_{1/n}) - C^a(V_1)/n - b.C^b(V_{\text{Ins}})/n - \alpha_1.(e+1) - \alpha_2.(d-e), & (V_1 \succ V_{1/n}) \\ U(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}) - \alpha_1.e - \alpha_2.(d-e) \geq \\ ((n-1)/n).U(V_{1/n}) + (1/n).U(V_{\text{Ins}}) - C^b/n - \alpha_1.e - \alpha_2.(d-e+1), & (V_1 \succ V_{\text{Ins}}) \end{cases} \Leftrightarrow$$

$$\begin{cases} C^a(V_1) \leq (n/(n-1)).A - b.C^b(V_{\text{Ins}}) + (n/(n-1)).\alpha_1, & (V_1 \succ V_{1/n}) \\ C^a(V_1) \leq A + \frac{1}{n}.B - (b - \frac{1}{n}).C^b(V_{\text{Ins}}) + \alpha_2 & (V_1 \succ V_{\text{Ins}}) \end{cases} \tag{A41}$$

b) d= n-2 and e = n- 2

$$\begin{cases} U(V_1) - \alpha_1.(n-1) \geq U(V_1) - C^a(V_1)/2 - b.C^b(V_{\text{Ins}}) - (n-2).\alpha_1, & (V_{1/n} \succ V_1) \\ U(V_1) - \alpha_1.(n-1) \geq U(V_1) - \alpha_1.(n-2) - \alpha_2 & (V_{1/n} \succ V_{\text{Ins}}) \end{cases} \Leftrightarrow$$

$$\begin{cases} C^a(V_1) \geq -2b.C^b(V_{\text{Ins}}) + 2.\alpha_1, & (V_{1/n} \succ V_1) \\ \alpha_1 \leq \alpha_2 & (V_{1/n} \succ V_{\text{Ins}}) \end{cases} \tag{A42}$$

It now happens that when someone else plays V_1, it is not indifferent to play $V_{1/n}$ or V_{Ins}, as it was before. As the only consequence of offering to have the parent at home part of the time, instead of offering to pay in order to have him in an institution, is an increase in the parent's utility, V_{Ins} is dominated by $V_{1/n}$. Therefore, there will be no equilibrium including V_{Ins} and V_1 simultaneously. That is to say the **1.2)** and the **1.3)** equilibria of section 2.3 will not be equilibria here.

The conditions for a type **2.1)** equilibrium are the following:

d = n-1 and e = n-1

$$\begin{cases} \mathrm{U}(V_{1/n}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}) - n.\alpha_1 \geq \mathrm{U}(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}) - (n-1).\alpha_1, & (V_{1/n} \succ V_1) \\ \\ \mathrm{U}(V_{1/n}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}) - n.\alpha_1 \geq ((n-1)/n).\mathrm{U}(V_{1/n}) + \frac{1}{n}.\mathrm{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) & (V_{1/n} \succ V_{\text{Ins}}) \\ -(n-1).\alpha_1 - \alpha_2, \end{cases}$$

$$\Leftrightarrow \begin{cases} C^a(V_1) \geq (n/(n-1)).A - b.C^b(V_{\text{Ins}}) + (n/(n-1)).\alpha_1, & (V_{1/n} \succ V_1) \\ \\ C^a(V_1) \leq B + (1-b).C^b(V_{\text{Ins}}) + n.(-\alpha_1 + \alpha_2), & (V_{1/n} \succ V_{\text{Ins}}) \end{cases} \quad \text{(A43)}$$

The conditions for a type **2.2)** equilibrium are the following:

a) d = n-1 and 0 < e < n-1

$$\begin{cases} \frac{e+1}{n}.\mathrm{U}(V_{1/n}) + \frac{n-e-1}{n}.\mathrm{U}(V_{\text{Ins}}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}) - (e+1).\alpha_1 - (n-e-1).\alpha_2 \geq \mathrm{U}(V_1) - C^a(V_1) & (V_{1/n} \succ V_1) \\ -b.C^b(V_{\text{Ins}}) - \alpha_1.e - \alpha_2.(n-e-1), \\ \\ \frac{e+1}{n}.\mathrm{U}(V_{1/n}) + \frac{n-e-1}{n}.\mathrm{U}(V_{\text{Ins}}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}) - (e+1).\alpha_1 - (n-e-1).\alpha_2 \geq & (V_{1/n} \succ V_{\text{Ins}}) \\ \frac{e}{n}.\mathrm{U}(V_{1/n}) + \frac{n-e}{n}.\mathrm{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) - \alpha_1.e - (n-e).\alpha_2, \end{cases}$$

$$\Leftrightarrow$$

$$\begin{cases} C^a(V_1) \geq \frac{n}{n-1}.A + \frac{n-e-1}{n-1}.B - b.C^b(V_{\text{Ins}}) + \frac{n}{n-1}.\alpha_1, & (V_{1/n} \succ V_1) \\ \\ C^a(V_1) \leq B + (1-b).C^b(V_{\text{Ins}}) + n.(-\alpha_1 + \alpha_2), & (V_{1/n} \succ V_{\text{Ins}}) \end{cases} \quad \text{(A44)}$$

b) Additionally, also with d = n-1 and 0 < e < n-1:

$$\begin{cases} \frac{e}{n}.U(V_{1/n})+\frac{n-e}{n}.U(V_{Ins})-\frac{1}{n}.C^b(V_{Ins})-\alpha_1.e-\alpha_2.(n-e)\geq U(V_1)-C^a(V_1)-b.C^b(V_{Ins}) \\ -\alpha_1.e-\alpha_2.(n-e-1), & (V_{Ins}\succ V_1) \\ \frac{e}{n}.U(V_{1/n})+\frac{n-e}{n}.U(V_{Ins})-\frac{1}{n}.C^b(V_{Ins})-\alpha_1.e-\alpha_2.(n-e)\geq \frac{e+1}{n}.U(V_{1/n})+\frac{n-e-1}{n}.U(V_{Ins}) \\ -\frac{1}{n}C^a(V_1)-\frac{b}{n}.C^b(V_{Ins})-\alpha_1.(e+1)-\alpha_2.(n-e-1), & (V_{Ins}\succ V_{1/n}) \end{cases}$$

$$\begin{cases} C^a(V_1)\geq A+\frac{n-e}{n}.B-(b-\frac{1}{n}).C^b(V_{Ins})+\alpha_2, \quad (V_{Ins}\succ V_1) \\ C^a(V_1)\geq B+(1-b).C^b(V_{Ins})+n.(-\alpha_1+\alpha_2) \quad (V_{Ins}\succ V_{1/n}) \end{cases} \tag{A45}$$

a) and b) together resume to:

$$\begin{cases} C^a(V_1)=B+(1-b).C^b(V_{Ins})+n.(-\alpha_1+\alpha_2) \\ C^b(V_1)\geq \frac{n}{n-1}.A-\frac{e}{n-1}.B+\frac{n^2}{n-1}.\alpha_1-n.\alpha_2 \end{cases} \tag{A46}$$

The conditions for a type **2.3)** equilibrium are the following:

a) d = n-1 and e = 0:

$$\begin{cases} \frac{1}{n}.U(V_{1/n})+\frac{n-1}{n}.U(V_{Ins})-\frac{1}{n}.C^a(V_1)-\frac{b}{n}.C^b(V_{Ins})-\alpha_1-\alpha_2.(n-1)\geq U(V_1)-C^a(V_1) \\ -b.C^b(V_{Ins})-\alpha_2.(n-1), & (V_{1/n}\succ V_1) \\ \frac{1}{n}.U(V_{1/n})+\frac{n-1}{n}.U(V_{Ins})-\frac{1}{n}.C^a(V_1)-\frac{b}{n}.C^b(V_{Ins})-\alpha_1-\alpha_2.(n-1)\geq U(V_{Ins}) \\ -\frac{1}{n}.C^b(V_{Ins})-n.\alpha_2, & (V_{1/n}\succ V_{Ins}) \end{cases}$$

$$\Leftrightarrow \begin{cases} C^a(V_1)\geq \frac{n}{n-1}.A+B-b.C^b(V_{Ins})+\frac{n}{n-1}.\alpha_1, \quad (V_{1/n}\succ V_1) \\ C^a(V_1)\leq B-(b-1).C^b(V_{Ins})+n.(-\alpha_1+\alpha_2), \quad (V_{1/n}\succ V_{Ins}) \end{cases} \tag{A47}$$

b) Additionally, with d = n-1 and e = 1,

$$\begin{cases} \frac{1}{n}.\text{U}(V_{1/n}) + \frac{n-1}{n}.\text{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) - \alpha_1 - \alpha_2.(n-1) \geq \text{U}(V_1) - C^a(V_1) & (V_{\text{Ins}} \succ V_1) \\ -b.C^b(V_{\text{Ins}}) - \alpha_1 - \alpha_2.(n-2), & \\ \frac{1}{n}.\text{U}(V_{1/n}) + \frac{n-1}{n}.\text{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) - \alpha_1 - \alpha_2.(n-1) \geq \frac{2}{n}.\text{U}(V_{1/n}) + \frac{n-2}{n}.\text{U}(V_{\text{Ins}}) - \frac{1}{n}.C^a(V_1) - & (V_{\text{Ins}} \succ V_{1/n}) \\ b.C^b(V_{\text{Ins}})/n - 2.\alpha_1 - (n-2).\alpha_2, & \end{cases}$$

$$\Leftrightarrow \begin{cases} C^a(V_1) \geq A + \frac{n-1}{n}B + (\frac{1}{n} - b).C^b(V_{\text{Ins}}) + \alpha_2, \quad (V_{\text{Ins}} \succ V_1) \\ C^a(V_1) \geq B - (b-1).C^b(V_{\text{Ins}}) + n.(-\alpha_1 + \alpha_2), \quad (V_{\text{Ins}} \succ V_{1/n}) \end{cases} \tag{A48}$$

The set of conditions resumes to:

$$\begin{cases} C^a(V_1) = B + (1-b).C^b(V_{\text{Ins}}) + n.(-\alpha_1 + \alpha_2) \\ C^b(V_1) \geq \frac{n}{n-1}.A + \frac{1}{n-1}.B + \frac{n^2}{n-1}.\alpha_1 - n.\alpha_2 \end{cases} \tag{A49}$$

The conditions for a type **3.1)** equilibrium are the following:

a) d = n-1 and e = n-1

a) it must not pay to deviate from playing V_{Ins} when the others play $V_{1/n}$. This happens when:

$$\begin{cases} ((n-1))/n).\text{U}(V_{1/n}) + \frac{1}{n}.\text{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) - (n-1).\alpha_1 - \alpha_2 \geq \text{U}(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}) - (n-1).\alpha_1, & (V_{\text{Ins}} \succ V_1) \\ ((n-1)/n).\text{U}(V_{1/n}) + \frac{1}{n}.\text{U}(V_{\text{Ins}}) - \frac{1}{n}.C^b(V_{\text{Ins}}) \geq \text{U}(V_{1/n}) - \frac{1}{n}.C^a(V_1) - \frac{b}{n}.C^b(V_{\text{Ins}}), & (V_{\text{Ins}} \succ V_{1/n}) \end{cases}$$

$$\Leftrightarrow \begin{cases} C^a(V_1) \geq A + \frac{1}{n}B + (\frac{1}{n} - b).C^b(V_{\text{Ins}}) + \alpha_2, \quad (V_{\text{Ins}} \succ V_1) \\ C^a(V_1) \geq B + (1-b).C^b(V_{\text{Ins}}) + n.(-\alpha_1 + \alpha_2), \quad (V_{\text{Ins}} \succ V_{1/n}) \end{cases} \tag{A50}$$

b) Additionally, with d = n-1 and e = n-2:

$$\begin{cases} \frac{n-1}{n}.U(V_{1/n})+\frac{1}{n}.U(V_{\text{Ins}})-\frac{1}{n}.C^a(V_1)-\frac{b}{n}.C^b(V_{\text{Ins}})-(n-1).\alpha_1-\alpha_2 \geq U(V_1)-C^a(V_1) \\ -b.C^b(V_{\text{Ins}})-\alpha_1.(n-2)-\alpha_2, & (V_{1/n} \succ V_1) \\ \frac{n-1}{n}.U(V_{1/n})+\frac{1}{n}.U(V_{\text{Ins}})-\frac{1}{n}.C^a(V_1)-\frac{b}{n}.C^b(V_{\text{Ins}})-(n-1).\alpha_1-\alpha_2 \geq (\frac{n-2}{n}).U(V_{1/n})+\frac{2}{n}.U(V_{\text{Ins}}) & (V_{1/n} \succ V_{\text{Ins}}) \\ -\frac{1}{n}.C^b(V_{\text{Ins}})-(n-2).\alpha_1-2.\alpha_2, \end{cases}$$

$$\Leftrightarrow \begin{cases} C^a(V_1) \geq \frac{n}{n-1}.A+\frac{1}{n-1}B-b.C^b(V_{\text{Ins}})+\frac{n}{n-1}.\alpha_1, & (V_{1/n} \succ V_1) \\ C^a(V_1) \leq B+(1-b).C^b(V_{\text{Ins}})+n.(-\alpha_1+\alpha_2), & (V_{1/n} \succ V_{\text{Ins}}) \end{cases} \quad \text{(A51)}$$

The set of conditions resumes to:

$$\begin{cases} C^a(V_1) = B+(1-b).C^b(V_{\text{Ins}})+n.(-\alpha_1+\alpha_2) \\ C^b(V_1) \geq \frac{n}{n-1}.A-\frac{n-2}{n-1}.B-n.\alpha_2+\frac{n^2}{n-1}.\alpha_1 \end{cases} \quad \text{(A52)}$$

The conditions for a type **3.2)** equilibrium are the following:

a) d = n-1 and 0 < e < n-1

$$\begin{cases} \frac{e}{n}.U(V_{1/n})+((n-e)/n).U(V_{\text{Ins}})-\frac{1}{n}.C^b(V_{\text{Ins}})-\alpha_1.e-\alpha_2.(n-e) \geq U(V_1)-C^a(V_1)-b.C^b(V_{\text{Ins}})-\alpha_1.e & (V_{\text{Ins}} \succ V_1) \\ -\alpha_2.(n-e-1), \\ \frac{e}{n}.U(V_{1/n})+\frac{n-e}{n}U(V_{\text{Ins}})-\frac{1}{n}.C^b(V_{\text{Ins}})-\alpha_1.e-\alpha_2.(n-e) \geq \frac{e+1}{n}.U(V_{1/n})+\frac{n-e-1}{n}.U(V_{\text{Ins}}) & (V_{\text{Ins}} \succ V_{1/n}) \\ -\frac{1}{n}.C^a(V_1)-\frac{b}{n}.C^b(V_{\text{Ins}})-\alpha_1.(e+1)-\alpha_2.(n-e-1), \end{cases}$$

$$\Leftrightarrow \begin{cases} C^a(V_1) \geq A+\frac{n-e}{n}.B+(\frac{1}{n}-b).C^b(V_{\text{Ins}})+\alpha_2, & (V_{Ins} \succ V_1) \\ C^a(V_1) \geq B+(1-b).C^b(V_{\text{Ins}})+n.(-\alpha_1+\alpha_2) & (V_{Ins} \succ V_{1/n}) \end{cases} \quad \text{(A53)}$$

b) Additionally, also with d = n-1 and 0 < e < n-1:

$$\begin{cases} \frac{e+1}{n}.U(V_{1/n})+\frac{n-e-1}{n}.U(V_{\text{Ins}})-\frac{1}{n}.C^a(V_1)-\frac{b}{n}.C^b(V_{\text{Ins}})-(e+1).\alpha_1-\alpha_2.(n-e-1) \geq U(V_1)-C^a(V_1) & (V_{1/n} \succ V_1) \\ -b.C^b(V_{\text{Ins}})-\alpha_1.e-\alpha_2.(n-e-1), \\ \frac{e+1}{n}.U(V_{1/n})+\frac{n-e-1}{n}.U(V_{\text{Ins}})-\frac{1}{n}.C^a(V_1)-\frac{b}{n}.C^b(V_{\text{Ins}})-(e+1).\alpha_1-\alpha_2.(n-e-1) \geq (\frac{e}{n}).U(V_{1/n})+ & (V_{1/n} \succ V_{\text{Ins}}) \\ \frac{n-e}{n}.U(V_{\text{Ins}})-\frac{1}{n}.C^b(V_{\text{Ins}})-e.\alpha_1-(n-e).\alpha_2, \end{cases}$$

The set of conditions resumes to:

$$\begin{cases} C^a(V_1) = B + (1-b).C^b(V_{\text{Ins}}) + n.(-\alpha_1 + \alpha_2) \\ \\ C^b(V_1) \geq \dfrac{n}{n-1}.A - \dfrac{e}{n-1}.B + \dfrac{n^2}{n-1}\alpha_1 - n.\alpha_2 \end{cases} \quad \text{(A54)}$$

The conditions for a type **3.3)** equilibrium are the following:

d = n-1 and e = 0

$$\begin{cases} \text{U}(V_{\text{Ins}}) - \tfrac{1}{n}.C^b(V_{\text{Ins}}) - n.\alpha_2 \geq \text{U}(V_1) - C^a(V_1) - b.C^b(V_{\text{Ins}}) - (n-1).\alpha_2, & (V_{\text{Ins}} \succ V_1) \\ \\ \text{U}(V_{\text{Ins}}) - \tfrac{1}{n}.C^b(V_{\text{Ins}}) - n.\alpha_2 \geq \tfrac{1}{n}.\text{U}(V_{1/n}) + ((n-1)/n).\text{U}(V_{\text{Ins}}) - \tfrac{1}{n}.C^a(V_1) - \tfrac{b}{n}.C^b(V_{\text{Ins}}) - \alpha_1 - \alpha_2.(n-1), & (V_{\text{Ins}} \succ V_{1/n}) \end{cases}$$

$$\Leftrightarrow \begin{cases} C^a(V_1) \geq A + B + (\tfrac{1}{n} - b).C^b(V_{\text{Ins}}) + \alpha_2, & (V_{Ins} \succ V_1) \\ \\ C^a(V_1) \geq B + (1-b).C^b(V_{\text{Ins}}) + n.(-\alpha_1 + \alpha_2), & (V_{Ins} \succ V_{1/\text{n}}) \end{cases} \quad \text{(A55)}$$

It is now possible that there are equilibria including more than one V_1. We have already stated that when V_1 is played, no one plays V_{Ins}. Therefore, we will now have a type **4.1)** equilibrium: (V_1, V_1, …, V_1), and a type **4.2)** equilibrium: (V_1, V_1, …, $V_{1/n}$). The conditions under which they take place are the following:

Type **4.1)** equilibrium. d = 0, e = 0.

$$\begin{cases} \text{U}(V_1) - \tfrac{1}{n}.C^a(V_1) - b.C^b(V_{\text{Ins}}) \geq \text{U}(V_1) - \alpha_1, & (V_1 \succ V_{1/n}) \\ \\ \text{U}(V_1) - \tfrac{1}{n}.C^a(V_1) - b.C^b(V_{\text{Ins}}) \geq \text{U}(V_1) - \alpha_2, & (V_1 \succ V_{\text{Ins}}) \end{cases} \quad \text{(A56)}$$

These conditions resume to:

$C^a(V_1) \leq n.\alpha_1 - n.b.C^b(V_{Ins})$, as long as $C^a(V_1) \geq 0$. (A57)

Type **4.2)** equilibrium.

a) d < n-1, 0 < e < n-1.

$$\begin{cases} \mathrm{U}(V_1)-\frac{1}{n-d}.C^a(V_1)-b.C^b(V_{\mathrm{Ins}})-\alpha_1.e-\alpha_2.(d-e)\geq \mathrm{U}(V_1)-\alpha_1.(e+1)-\alpha_2.(d-e), & (V_1 \succ V_{1/n}) \\ \mathrm{U}(V_1)-\frac{1}{n-d}.C^a(V_1)-b.C^b(V_{\mathrm{Ins}})-\alpha_1.e-\alpha_2.(d-e)\geq \mathrm{U}(V_1)-\alpha_1.e-\alpha_2.(d-e+1), & (V_1 \succ V_{\mathrm{Ins}}) \end{cases} \quad \text{(A58)}$$

The two conditions resume to:

$$C^a(V_1)\leq(n-d).\left[-b.C^b(V_{Ins})+\alpha_1\right], \text{ as long as } C^a(V_1)\geq 0. \quad \text{(A59)}$$

b) d < n-1, 0 < e < n-1.

$$\begin{cases} \mathrm{U}(V_1)-(e+1).\alpha_1-(d-e).\alpha_2 \geq \mathrm{U}(V_1)-\frac{1}{n-d}.C^a(V_1)-b.C^b(V_{\mathrm{Ins}})-\alpha_1.e-\alpha_2.(d-e), & (V_{1/n} \succ V_1) \\ \mathrm{U}(V_1)-(e+1).\alpha_1-(d-e).\alpha_2 \geq \mathrm{U}(V_1)-e.\alpha_1-(d-e+1).\alpha_2, & (V_{1/n} \succ V_{\mathrm{Ins}}) \end{cases} \quad \text{(A60)}$$

$$\begin{cases} C^a(V_1)\leq(n-d).\left[-b.C^b(V_{Ins})+\alpha_1\right] & (V_{1/\mathrm{n}} \succ V_1) \\ \alpha_1 \leq \alpha_2, & (V_{1/\mathrm{n}} \succ V_{Ins}) \end{cases} \quad \text{(A61)}$$

The set of conditions resumes to:

$$C^a(V_1)=(n-d).\left[-b.C^b(V_{Ins})+\alpha_1\right], \text{ as long as } C^a(V_1)\geq 0.$$

Table 1

Payoffs in the Canonical Model with 2 Children

	V_1	$V_{1/2}$	V_{Ins}
V_1	$U(V_1)$- 0,5.$C(V_1)$; $U(V_1)$- 0,5.$C(V_1)$	$U(V_1) – C(V_1)$; $U(V_1)$	$U(V_1)–C(V_1)$; $U(V_1)$
$V_{1/2}$	$U(V_1)$; $U(V_1) – C(V_1)$	$U(V_{1/2})$ – 0,5.$C(V_1)$; $U(V_{1/2})$ – 0,5.$C(V_1)$	0,5.$U(V_{1/2})$ + 0,5. $U(V_{Ins})$ – 0,5.$C(V_1)$; 0,5.$U(V_{1/2})$ + 0,5. $U(V_{Ins})$
V_{Ins}	$U(V_1)$; $U(V_1) – C(V_1)$	0,5.$U(V_{1/2})$+ 0,5. $U(V_{Ins})$; 0,5.$U(V_{1/2})$ + 0,5. $U(V_{Ins})$ – 0,5.$C(V_1)$;	$U(V_{Ins})$; $U(V_{Ins})$

$U(V_1)$: the parent's utility of always living with the same child.

$U(V_{1/2})$: the parent's utility of living part of the time with each child.

$U(V_{Ins})$: the parent's utility of living in an institution.

$C(V_1)$: the child's cost of effort and time of coresiding with the parent all the time.

Table 1'

Payoffs in the Model with Financial Costs and 2 Children

	V_1	$V_{1/2}$	V_{Ins}
V_1	$U(V_1)$- 0,5.$C^a(V_1)$ – 0,5.b. $C^b(V_{Ins})$; $U(V_1)$ – 0,5. $C^a(V1)$ – 0,5.b. $C^b(V_{Ins})$	$U(V_1)$ – C^a (V_1)- b.$C^b(V_{Ins})$; $U(V_1)$	$U(V_1)$– $C^a(V_1)$- b.$C^b(V_{Ins})$; $U(V_1)$
$V_{1/2}$	$U(V_1)$; $U(V_1)$ – C^a (V_1) – b.$C^b(V_{Ins})$	$U(V_{1/2})$ – 0,5. $C^a(V_1)$ – 0,5.b. $C^b(V_{Ins})$; $U(V_{1/2})$ – 0,5. $C^a(V_1)$ – 0,5.b. $C^b(V_{Ins})$	0,5.$U(V_{1/2})$ + 0,5. $U(V_{Ins})$ – 0,5. $C^a(V_1)$ – 0,5.b. $C^b(V_{Ins})$; 0,5.$U(V_{1/2})$ + 0,5. $U(V_{Ins})$ - 0,5.$C^b(V_{Ins})$
V_{Ins}	$U(V_1)$; $U(V_1)$ – $C^a(V_1)$ – b.$C^b(V_{Ins})$	0,5.$U(V_{1/2})$+ 0,5. $U(V_{Ins})$ – 0,5.$C^b(V_{Ins})$; 0,5.$U(V_{1/2})$ + 0,5. $U(V_{Ins})$ – 0,5. C^a (V_1) - 0,5.b. $C^b(V_{Ins})$	$U(V_{Ins})$ – 0,5.$C^b(V_{Ins})$; $U(V_{Ins})$ – 0,5.$C^b(V_{Ins})$

b is the ratio of the financial cost of keeping the parent at home over the financial cost of having the parent in an institution.

b <1.

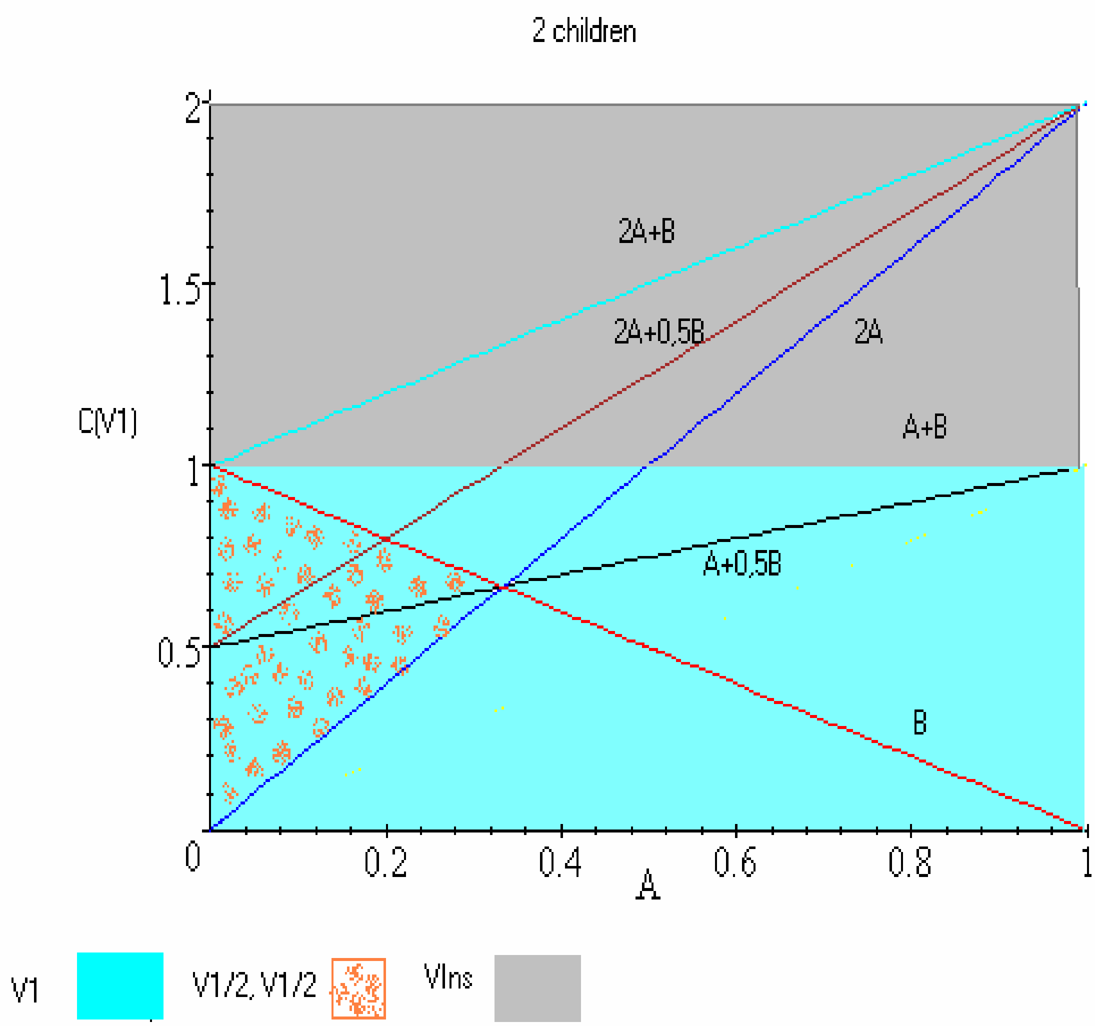
Table 2

Summary of the Impact of Each Considered Effect on the Probability of Each Outcome

Effect	Outcome			
	Full Time Institutionalization	Full Time Coresidence	Part Time Coresidence	Part Time Coresidence and Institutionalization
Cost of effort	+	–	indeterminate	+
Financial cost	– or indeterm.	– or indeter.	+	+
Disappointment effect	–	+ or indeter.	indeterminate	indeterminate

Figure 1

Solutions of the canonical model with 2 children



The area identified with V1 corresponds to the solutions leading to a full time coresidence living arrangement.

The area identified with V1/2,V1/2 corresponds to the solution of equally divided coresidence living arrangement.

The area identified with VIns corresponds to the solution of full time institutionalization.