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4. LESSON STUDY AS A LEARNING CONTEXT IN MATHEMATICS EDUCATION

This chapter presents lesson study, a professional development process that originated in Japan, describing its main features with emphasis on how it may be regarded as a special form of teachers' research on their own practice. We indicate some adaptations of lesson study to fit different purposes and pay attention to the participants' experiences of collaboration, reflection and work in communities of practice. To illustrate these features, we present two case studies, one from Portugal and another from the United Kingdom.

INTRODUCTION

Lesson study is a teacher professional development process that assumes a reflective and collaborative nature and is focused on teaching practice (Fernández, Cannon, & Chokshi, 2003; Perry & Lewis, 2009). In a lesson study, teachers work together, seeking to identify students' difficulties and preparing in detail a lesson that they then observe and analyse thoroughly. The Japanese model of lesson study has received widespread and increasing attention following publication of *The Teaching Gap* (Stigler & Hiebert, 1999) which sought to understand why Japan outperformed other countries, particularly the United States, in international testing of mathematics. Today, lesson study is practised in many countries and receives significant attention from researchers in mathematics education.

The purpose of the chapter is to present the main features of lesson study, indicating also some variations to the Japanese model, with special emphasis in its main characteristics as a learning context. Thus, we strive to show the collaborative and reflective nature of lesson study activities and also that a lesson study group constitutes a community of practice. We illustrate these features in two different cases, one from Portugal and the other from the United Kingdom.

LESSON STUDY PROCESS

Lesson study has been used in all teaching levels, from kindergarten to university. It is carried out in the school environment and the participant teachers play a central role. Usually, a lesson study begins with the identification by the teachers of a significant

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problem in students' learning, very often related to a curriculum topic. Then, the participants plan a lesson, taking into account curriculum documents, research results about the learning of the topic or the issue that they identified, and their former experience. They foresee the difficulties of the students, anticipate possible questions that may emerge on the lesson, design tasks, formulate teaching strategies, anticipate the flow of the lesson, and may prepare instruments for observation. The lesson, often called the "research lesson," is taught by a teacher while the others observe and take notes, paying particular attention to students' learning in relation to the "research question" that is central to the lesson. Next, the teachers and observers meet to analyse and reflect about what they observed. This analysis may lead to a reformulation of the lesson plan for future use, with changes made to the task(s), in the teaching strategies and materials used, in the questions to pose to students, etc. In some models of lesson study, practised in several countries, this re-planned lesson may be taught again to other classes of students by the same or another teacher, in cycles that may repeated several times (for more detailed descriptions, see, e.g., Fujii, 2016; Lewis, Perry, & Hurd, 2009; Murata, 2011).

A fundamental aspect of lesson study is that it centres on students' learning and not on the teachers' work. This makes this professional development process very different from other processes that involve classroom observations but often focus mainly on teachers' actions. The participation in lesson study provides an opportunity for teachers to learn about important issues not just about the selected topic or issue but also about wider curriculum issues, students' reasoning, students' difficulties, and even classroom dynamics. It provides a context for reflection and also to develop professional confidence, essential for teachers' professional development.

Lesson study provides a professional development process closely related to day-to-day practice that enables teachers' learning in several domains – mathematical, didactical, curricular, educational, and organisational. A lesson study may be regarded as a small investigation by teachers into their professional practice. In fact, lesson study as research is driven by a question – the learning problem that is identified at the very beginning of the lesson study cycle. This is why the careful consideration of this driving question is an important aspect of the process. As practised in Japan, such research questions are typically guided by an overarching theme that is determined at a school or wider level that is worked on for a year or more. Also, lesson study is informed by curriculum guidelines and by the results of relevant research. This parallels the usual literature review carried out in scientific research. Relevant knowledge from earlier research and practice must be taken into account, as well as curriculum guidelines concerning the topic or issue identified. The planning of the research lesson and related data collection has a similar role to the planning of an experiment and of the process of assembling evidence. The lesson plan with its underlying strategy is very much like a research hypothesis to be tested in practice. The research lesson is like the critical experiment. Subsequent post-lesson reflection plays the role of data analysis and production of findings. In many cases,

the process of reflection extends for a considerable period and is communicated by the participants to others within their school and in other professional settings. In Japan, this may involve writing up and publishing a school's lesson study activity for the year. Again this is akin to the process of dissemination of scientific research results.

The parallel of lesson study and research is clear, but unlike other forms of research carried out in professional settings (including action research), lesson study follows a pre-specified format and is to be carried out in small scale – usually in six or seven sessions –, unless it involves re-teaching the lesson in successive cycles. This small scale of the process is definitely an advantage regarding other more complex forms of professional research, since it may be aggregated in larger units for the participants that want to do it continuously and at the same time fits the needs of participants that just want to take a single experience.

Lesson study as a practice meets all of the requirements of what research tells us makes for effective professional learning for mathematics teachers (Guskey, 2002; Joubert & Sutherland, 2009; Villegas-Reimers, 2003). That is, lesson study, as a form of professional learning, is:

- *Experiential*: stimulating and drawing on teachers' experiences.
- *Sustained*: cycles of planning, predicting, enactment and reflection.
- *Grounded*: practical, well-resourced; related to context and culture.
- *Safe*: teachers able to speak their minds, permission to take risks.
- *Collaborative*: involving networks of teachers and administrators.
- *Informed*: by outside expertise and research.
- *Provocative*: involving both pressure and support.
- *Focused*: attentive to the development of the students' mathematics knowledge.

The Japanese tradition of lesson study is by far the most well-known. An important feature of this tradition is that it is connected to a curriculum movement that sought to transform "traditional teacher-centered instructional practice to student-centered instruction that focuses on mathematical thinking and problem solving" (Takahashi & McDougal, 2018, p. 144). In Japan, it is carried out at a large scale and is part of the professional environment of most teachers (especially, at elementary schools). There are many forms of lesson study, from relatively informal, within school, groups through to much larger open-house events and demonstration lessons that attract attention widely and sometimes at a national level. Teachers find it natural to participate in many lesson studies during their teaching career. In countries in which there is no tradition of lesson study, it strikes teachers as a rather strange professional development process. They ask questions such as, "Why concentrate on a single topic?," "How come so many observers do not disturb the students?" and so on. Processes that are natural in Japan are not always natural elsewhere and may require adaptation. In fact, it has been argued that lesson study must be adapted to each particular environment, so that it suits the needs and culture of the participants and adjusts to the educational ecosystem (Stigler & Hiebert, 2016).

There are several adaptations of lesson study in different places and purposes. For example, in the United Kingdom, a form of lesson study that has been widely experimented with involves the development of case studies of a few pupils, during and after the research lesson, as a strategy of data collection that allows for a deeper observation of student learning (Cajkler, Wood, Norton, & Pedder, 2014). Another adaptation that has been made is for initial teacher education. In this case, sometimes, in place of regular teaching, the prospective teachers practise microteaching among themselves (Fernandez & Zilliox, 2011). On other occasions the research lesson is carried out in fieldwork schools and is taught by a cooperating teacher, and on still other occasions by the prospective teachers themselves (Ponte, 2017).

LESSON STUDY AS A LEARNING CONTEXT

Collaboration

In collaboration, teachers work jointly in order to achieve a common goal, sharing experiences, constructing new ideas and making decisions together (Robutti et al., 2016). Research about professional development stresses the benefits of joining teachers and researchers in collaborative relationships (Hollingsworth & Clarke, 2017). In collaborative activities, the roles of participants may be different – the essential is that all work together, in horizontal relationships, so that there is mutual support to attain the goals of the group (Boavida & Ponte, 2002). When involved in collaboration and reflection processes about their own practice, teachers make their own knowledge explicit and create new knowledge with the support of more experienced colleagues.

Lesson study provides a natural context for a collaborative activity which requires much responsibility, commitment, and time from teachers but has high potential for solving problems and for developing knowledge in order to improve their professional practice. The joint work of teachers inherent in lesson study represents an important possibility for professional collaboration, as they jointly identify learning problems, discuss ideas, and develop teaching resources. In many cases, this professional collaboration is a new experience for the participating teachers. For example, in a study with primary school teachers, led by Baptista, Ponte, Velez, and Costa (2014), a teacher recognised that, before this experience, she had never worked collaboratively with other teachers. She also indicated that, as the group was thinking together, this led to the creation of a climate of confidence, in which the participants felt at ease to pose questions and discuss issues. For another teacher, working in collaboration led to mutual enrichment of all participants. This teacher acknowledged that there was a commitment of all participants, indicating that the team work led them to experiment in a new situation, "creating something together" (p. 75). Still another teacher indicated that, in the beginning, the group experienced several difficulties as they had not worked together before. However, as the work progressed, the teachers developed confidence in each other and could express, with

no fear, the difficulties that they faced. This creation of a safe environment in which teachers develop confidence to expose themselves in asking questions and making reflections is an important feature of this professional development process.

The intense collaboration that takes place in lesson study leads to changes not only in terms of the teachers' views of teaching and learning but also in the way they work together in professional settings. For example, in the study of Puchner and Taylor (2006), the results suggest that carrying out lesson study, in kindergarten and primary schools in the United States, led to changes related to teachers' professional autonomy and isolation, promoting a collaborative relationship. In this study, the participant teachers considered that the work that they undertook led to a radical change in their teaching and learning. Also, in the research of Lewis, Perry, and Hurd (2009), the changes in professional practice were remarkable and the collaborative work become a regular practice. In this case, six years after the initial work, the teachers were still doing lesson study on their own initiative. It is very common that the teachers who participate in lesson study explicitly assign great value the collaborative work that they experienced (Baptista et al., 2014; Burroughs & Luebeck, 2010; Ponte, Quaresma, Mata-Pereira, & Baptista, 2016).

Recognising the important role of lesson study for teachers' professional development, some authors point to the need to pay attention to the establishment of conditions that allow for collaborative work. For example, Puchner and Taylor (2006) point to the need to consider the agenda of all participants, assure the autonomy of all of them, and take into account that the path for collaboration may be rough. These authors also underline that an essential feature of lesson study is that the social, emotional and cognitive impact of the collaborative process is, at least in an initial stage, more important than the content of the lessons.

Reflection

In lesson study, professional learning depends on teachers' questioning about learning, teaching and classroom practice (Wake, Swan, & Foster, 2016). This idea of questioning is clearly related to what Dewey (1933) regards as reflection and that involves the framing of a problem ("a state of doubt, hesitation, perplexity, mental difficulty," p. 12) and a problem-solving process ("an act of searching, hunting, inquiring," p. 12). These ideas from Dewey combine two essential aspects: first, to recognise issues to investigate, identify problematic aspects in teaching and, second, through a process of inquiry, to seek solutions or new forms of understanding the identified problems. Seen in this way, reflection is essential for teachers' professional development because it generates knowledge based in practice (Clarke, 2000).

Reflection is more than just a process through which teachers look at their experiences from practice, looking back at episodes, emotions and events, it also projects itself in future practice. Schön (1983) considers reflection as a process of developing and testing ideas on action. Clarke (2000) speaks of reflection on, about and for practice. Reflection on practice occurs in a specific context, in or outside

the classroom, simultaneously with the activities that constitute that practice and, therefore, largely depends on the way the teacher interprets events that he/she lives and his/her professional aims. Reflection about practice may occur in the school and in other settings, in conversations with colleagues and other people, and may refer to the teacher's own practice or to the practice of other teachers. Reflection for practice is a goal-directed action that refines knowledge, yields new information, and provides elements for the teacher's action. Therefore, an essential aspect of a teacher's practice is the way in which the teacher anticipates, and informs his or her actions, yielding to a refinement of that practice (Clarke, 2000).

Reflection can, of course, take different forms. A basic form of reflection just involves the recall of events that took place. It tends to associate difficulties that students face with factors such as the abstract nature of mathematics or the students' immaturity. A quite distinct way of reflecting involves not only identifying problems and proposing solutions, but also making these in a reasoned way, taking into account the most salient factors related to the issue, often from a fresh point of view.

The results of several studies suggest that lesson study may influence the development of teachers' reflective capacity as well as many aspects of their professional practice. For example, in the study of Sack and Vasquez (2011) that involved primary school teachers in the United States, the results indicate that the reflection about professional practice yielded by the lesson study was very important for improving teachers' professional practice. In a study carried out in initial teacher education, Burroughs and Luebeck (2010) indicate that prospective teachers become more critical and reflective as they participated in this activity. This experience enabled them to reflect on the work that they carried out during the lesson study process. Lewis, Perry, and Hurd (2009) found similar results, highlighting that reflection led to the revision of the participant teachers' lesson plans in order to promote students' reasoning in a more efficient way. Also, Baptista et al. (2014), in a study with primary school teachers, indicated that the lesson study enabled them to reflect on their own practice. The teachers considered that the lesson study created a situation of research, action and reflection, leading them to a more attentive stance regarding students' possible answers to the tasks and most likely difficulties. As one of the teachers indicated, this deeper way of reflecting led them to "provide opinions in a more constructive way and with more personal commitment than what they use to do" (p. 76).

Making productive reflections requires several conditions that are pointed out by several studies. An interesting aspect referred to by the participant teachers in the lesson study by Sack and Vasquez (2011) is the recognition that some time is necessary so that reflection become productive. Also, in the study of Puchner and Taylor (2006), the teachers valued the reflection about practice and become surprised with the time that they needed to reflect and discuss issues related to the lessons. Ponte, Baptista, Velez, and Costa (2014) indicate that the lesson study gave a positive space for teacher reflection, allowing the participants to analyse issues of their practice from different angles. The basis for this reflection was the interaction

with their colleagues and with the researchers. The fact that the participants observed the students, collecting their data in the classroom, enabled them to make deep reflections based on observations.

Communities of Practice

Central to lesson study, no matter where it is practised, is the initiation and development of communities that have as an important feature a shared goal of addressing professional learning. We may consider these as communities of practice in the sense of Wenger (1998). Wenger defines a community of practice as a group of people who develop a common activity and who interact regularly to learn how to do it better. Indeed, in lesson study, participants work together in ways that involve the three main principles of communities of practice: mutual engagement, joint enterprise, and shared repertoire. We may say that the aim of lesson study is to develop and sustain communities of practice that focus on a shared venture of professional learning.

Wenger suggests that when developing communities of practice attention needs to be paid to elements of domain, community, and practice. In the case of communities of practice that focus on lesson study in terms of the domain this requires that the community defines clearly their area of shared inquiry and key issues. In terms of community, all need to understand and to be sensitive to the relationships among members and develop a sense of belonging. Finally, in terms of shared practice, it is important that the community pays attention to the body of knowledge that is central to the practice of the group and provide supporting methods, stories, cases, tools, documents that facilitate the joint enterprise. This latter aspect of practice is deeply embedded in the well-established communities that carry out lesson study in Japan, whereas in countries that attempt to adopt and adapt lesson study from scratch often focus on developing supporting toolkits that can facilitate the development of what is a new practice for an emergent community (Lewis, 2009). Another aspect of the development of new communities of practice that require some attention is that of key individuals in the community, "brokers" in Wenger's terms. These key networkers need to have a vision of what is to be achieved and have the wherewithal to facilitate the development of a new community in the context and culture in which they work. Their activity requires that they are able to draw on resources, including human resources, to ensure that the proposed model of operation will be appropriately supported. This is clearly an important role that needs to be undertaken by a key person if lesson study is able to be initiated in situations where there are not already well-established working practices.

Lesson study as an activity, therefore, provides a community of practice that has as central collaborative professional learning. Wenger suggests a social theory/model of such learning that recognises that 'learning' is focused on a number of different aspects:

- developing practice,
- identity development (that is a sense of participants becoming someone),
- meaning (making sense of practice),
- community (developing a sense of shared understandings).

It is clear that lesson study as a collaborative activity between teachers and other educators may embody and support such social learning, but as Wenger further points out, such learning has to be designed for, and that in such design care has to be taken as to what is globally fixed and what is designed to be locally varied.

Furthermore, Wenger (2010) emphasises the dynamic nature of communities of practice and how they change path from time to time as membership changes, new policies and imperatives (at a relatively local level) emerge and so on. This is something that is rarely reported in the research of lesson study although the case study from Portugal reported here gives some sense of how some participating teachers in the lesson study community developed important aspects of individual teacher identities and the practice of the group.

In their systematic review of teacher communities as a context for professional development, Vangrieken et al. (2017) highlight that although the term 'community' is often used in the reporting of research that involves lesson study, and that such communities display the fundamental features of communities of practice, only a few explicitly draw on ideas of communities of practice. As Vangrieken and colleagues point out, lesson study communities may be characterised as being one of two types: either communities that have as central to their work the realisation of a goal other than the establishment of the community itself, or communities that have the central aim of initiating, developing and sustaining the professional community as a sustainable entity with different learning goals adopted over time. The United Kingdom case reported here might be considered a hybrid model with the community clearly focused on teaching and learning of problem solving but with another aim that involves the understanding of the potential of establishing lesson study communities in the particular context of schools in the United Kingdom.

In one notable case, Concoran (2011) reports her participant research in establishing lesson study within teacher education school partnerships in Ireland, thus involving both practising teachers and initial teacher education students. Of particular significance in this case was the researcher's sharing of some tools drawn from the research literature, in a way designed to support the development of the communities. As can be imagined, the use of such tools as shared reading to inform the group can help clarify ideas of the group in terms of their mutual engagement, joint enterprise, and shared repertoire.

Hunter and Back (2011) report their lesson study research in the United Kingdom, with a relatively small research base of four lesson study communities, consisting of groups of primary school teachers in each of four schools, through the lens of communities of practice. They highlight four important elements that underpinned their research; the process of lesson study, the notion of professional

learning communities, the nature of effective mathematics pedagogy and reflection on practice. Although they pay considerable attention to issues in relation to mathematics pedagogy, they also engaged in identity work through which the teachers' professional knowledge was enhanced.

The conceptualisation of lesson study communities as communities of practice, as illustrated here, appears to have the potential of informing future work that seeks to better understand how the practice of lesson study becomes operationalised in different settings. Indeed this theoretical framework that focuses on how individuals engage in social work with others in pursuit of shared goals could well provide a unifying approach to research across cultural settings as lesson study increases its hold in a growing number of countries.

A LESSON STUDY WITH PRIMARY TEACHERS

The Setting

This case study concerns a group of primary teachers (teaching grade 3) of a school in Lisbon that participated in a lesson study in 2013–2014. The lesson study emerged from a request from the Director of the school to the research team to support its development project. At the beginning, the group was made of seven teachers who taught several grades. A first decision was to conduct the lesson study in grade 3, and this led several teachers to leave the group. Some of them expressed little interest in participating, as they did not teach this grade, and others indicated other personal and professional reasons. In this way, only three teachers, Irina, Manuela and Antónia (fictitious names), participated in the lesson study. They all had a teaching diploma for primary education and between 10 and 15 years of teaching experience. However, Irina had a specialisation to teach grades 5–6 mathematics and science and was the teacher with the strongest mathematical knowledge. She was familiar with inquiry or exploratory approaches to mathematics teaching because she attended a year-long in-service teacher education program based on this perspective. Manuela also specialised in teaching grades 5–6, but in Portuguese and French, and was the teacher with the weakest preparation in mathematics. Antónia had no specialisation but formerly attended several short duration professional development activities in mathematics. This lesson study was led by two authors of this chapter (Quaresma and Ponte). They assumed the role of 'experts,' conducting all working sessions, including the post-lesson discussion. During the sessions, they proposed the activities to carry out and raised questions for group reflection.

The lesson study had just one cycle with nine initial sessions plus three follow-up sessions. Session 1 included the participants' introductions, the establishment of the general agenda for the work, and the definition of the topic to study. Sessions 2 to 7 were dedicated to deepen the knowledge about the topic as well as to the planning of the research lesson; session 8 was the research lesson and session 9 the post-lesson reflection; in sessions 10, 11, and 12, the teachers planned, taught, and reflected

on two further classes as a means of consolidating and deepening the work already carried out. In this chapter, we pay attention to issues of collaboration and reflection during the lesson study process.

First Part of the Lesson Study

Study of the topic. In session 1, it was decided that, given the usual difficulties of students and the new approach advocated by the recent curriculum, the topic to study would be addition and subtraction of non-negative rational numbers. Sessions 2 to 5 were dedicated to the study of mathematical and didactical issues related to the topic. In session 2, the group analysed curriculum materials, solved tasks and identified students' difficulties. In session 3, there was a first discussion about students' knowledge and the joint elaboration of a diagnostic test for the classes of the participant teachers. In session 4, the answers of the students were analysed, seeking to identify difficulties and surprising strategies. In session 5, possible students' generalisations in addition and subtraction of rational numbers were identified. In these sessions, Manuela was always very reserved in her participation and later, in an interview, she acknowledged that she asked no questions, even when she did not understand the mathematical discussions that often took place, indicating that she did not feel comfortable in the group. Antónia acknowledged that her little involvement in the sessions was because she felt that the statements of the tasks and the possible students' responses were analysed in such fine detail. Finally, Irina was a very active participant, sharing materials, ideas and experiences. However, she recognised that she often felt uneasy about how little her colleagues participated. In this phase, an issue that emerged and perturbed the activity of the group was that none of the teachers was willing to teach the research lesson. The situation was only resolved in session 5, when Irina finally decided that she would take on that role.

Planning the research lesson. This planning took place in sessions 6 and 7. In session 6, Irina, who was taking something of a leadership role, shared that she did not feel happy with the textbook materials and challenged the group to design a task for the research lesson. The task aimed to support the learning of addition and subtraction of fractions by linear juxtaposition of line segments (as indicated in the curriculum) and had the context of an 'animal relay.' It was developed by Irina that thought about the questions together with the researchers.

In session 7, the research lesson was prepared in detail, discussing the questions of the task, the possible students' solutions and difficulties, the way students should work, and the observation procedures. Antónia and Manuela were attentive but made few suggestions for the design of the task and for the preparation of the lesson. Irina worked jointly with us but Antónia and Manuela were only making occasional suggestions. They did not give many suggestions for the lesson, mainly addressing issues of students' organisation and management, perhaps because they did not want to interfere in planning a lesson that they would not teach. Irina, on the contrary, was heavily

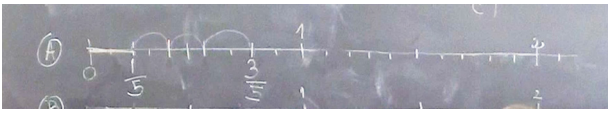

Figure 4.1. One representation in the number line of $\frac{1}{5}$ and $\frac{3}{5}$ made by a student (arcs added during the discussion)

involved, addressing the mathematics content and its teaching and learning, informing this with her practical and professional knowledge of students' learning processes.

Post lesson reflection. The reflection of the research lesson took place in session 9. The whole group considered that the lesson developed according to what was planned. Some video excerpts were analysed to discuss students' strategies and difficulties. It was noticed that several students had difficulties in representing their results on a number line (see Figure 4.1); this surprised Irina and led to much discussion about the possible origin of those difficulties, apparent in the measurement meaning of rational numbers. The conclusion was that it was possible to use the understanding that students had of the part-whole meaning of fractions to develop their understanding in the measurement meaning, leading the students to correctly represent fractions on the number line.

The group's reflection about practice did not consider issues for future practice. Invited to give their opinion, Antónia and Manuela described several events supporting the comments made by the researchers. They were not very expansive, but narrated the observations that they made of the lesson and helped the group to draw a general idea of students' learning. Irina considered that the difficulties identified resulted from the abstract nature of the number line and from students' lack of attention.

Later, we asked teachers about the changes that they would make to the task if they would use it again. Their responses were in sharp contrast. Antónia and Manuela considered that the task required no changes. Irina, on the contrary, made a suggestion for practice, saying that the presentation of the task should be reformulated to solve the students' difficulty with the representations using the number line and their understanding of addition by juxtaposing line segments: "I think that it would have been important [to present that task in another way] because of what we indicated about the distance [from the origin to the point]."

Follow-up Sessions

A new relationship among participants. Between sessions 9 and 10 there were individual interviews with all teachers in order to know their opinion about the lesson study and their learning. Surprisingly, these interviews were moments of deep reflection, with a very spontaneous and open discourse from the teachers who presented their views about the process. Manuela and Antónia referred to their anxiety

of teaching the research lesson and the fear of participating in the sessions given their weak knowledge about the teaching of rational numbers in the measurement meaning. This process of reflection brought all participants together. The teachers were no longer concerned that they were exposing themselves to each other and the researchers, and this changed the climate of the sessions so that everyone felt more comfortable and willing to participate.

Manuela and Antónia used the task of the research lesson in their classes. In the beginning of session 10, they reported this experience, referring to students' achievements and difficulties. Therefore, the two teachers used the materials formerly produced in the group. They also narrated the lesson episodes, speaking with enthusiasm of the work of their students.

Planning. Still in session 10, we asked the teachers to plan a new lesson on a topic that they were going to teach. Manuela, in contrast with her former lack of participation, suggested that the group could plan the lesson together and she and Antónia would teach it. Manuela now showed great willingness to assume an active role because she was more confident with what she was learning. It was decided that both would propose the tasks to be used in their classes and Irina would help them to reflect on the results to present and discuss in the following lesson study session. In this way, the three teachers prepared a lesson about the relationship between decimal fractions and decimal numbers in a climate of shared responsibility and joint work.

However, Antónia and Manuela began their planning with much uncertainty. It was noticeable that they were uncomfortable with the idea of choosing tasks from the textbook, perhaps because they felt that such an approach would be regarded as rather improper. They appeared not to know what to do. Given their difficulty, one of the researchers suggested that, instead of selecting a task, they could adapt a task, for example, asking students to relate the representations of decimal fractions and decimal numbers. Manuela accepted immediately the challenge of adapting a task in order to "make it more challenging" and Antónia and Irina also began giving suggestions and recording ideas to design the task. So, in joint work, the first question was constructed (Figure 4.2).

Two aspects stand out. The first is the change of attitude of Manuela and Antónia that, from a position of restraint, changed to a position of strong participation in the common activity. The second is the teachers' difficulty in assuming a critical authorship role regarding the tasks. However, this difficulty was overcome with a suggestion from us. The former work in the lesson study seemed to have prepared the teachers to assume this manner of working, that is, not with complete autonomy, but with the support of a more experienced partner. All teachers shared ideas, materials, and opinions but, even more, they showed responsibility and commitment in the development of a common activity, as joint work.

Reflection. In session 11, the teachers reflected on their classroom experience in conducting the task planned in the previous session. Antónia reported that her

1. Paint in the figure 0.4 in green, $\frac{40}{100}$ in blue, and "four hundredths" in yellow.

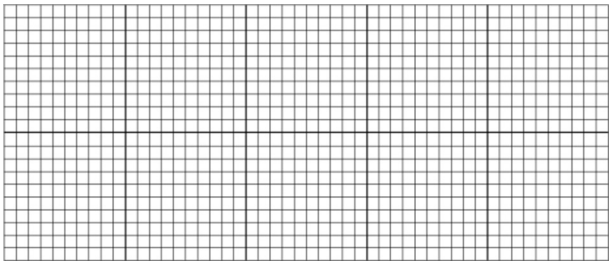

Figure 4.2. Task on representing decimal fractions

students had many difficulties in solving questions; this led to deep reflections about the representations that they proposed in tasks involving tenths, hundredths, and thousandths. In discussions with much input from all participants, the teachers concluded that the students' mistakes were related to the difficulty that they had in visualising the hundredth part of the figure at which they were looking.

Having identified the students' difficulties, Irina reflected about her practice. She considered that the way she used to present the unit submultiples does not facilitate the students' understanding of the unit and of its relationship with the submultiples. All participants agreed and Irina challenged the group to find a solution to the problem: "But, and now? This is a reflection. And now what do we do?" Antónia suggested that they could do what she had done in the discussion of the task, cutting the rectangle in ten, hundred, and thousand parts, and everybody agreed. The teachers concluded that they could use the rectangle representation as unit and its conservation would help students to understand the representation of different decimal submultiples.

In this discussion, the teachers engaged in deep reflection about their own teaching practice, questioned their students' solutions and difficulties, and sought to find explanations for what happened, analysing in detail the origin of such difficulties. Besides seeking to understand the problem, they also sought to find ways of overcoming the students' difficulties in a deep reflection for practice. The change of attitude of Antónia and Manuela transformed the activity in joint work in which everyone participated with commitment.

The Follow-up as an Adaptation

During the lesson study, the authors/researchers had an active role in challenging and questioning the teachers. Irina, the teacher who taught the research lesson and who had most confidence in terms of her knowledge of mathematics was always

much involved in the activities. From very early on, she worked jointly with the researchers to design tasks for the research lesson. In moments of reflection for practice she suggested designing a challenging task in order to promote students' learning and understanding. In this initial stage, Manuela and Antónia were not much involved in the group's activities. When the post-lesson reflection was carried out, Irina and the researchers were working jointly, whereas Antónia and Manuela were still in a mode of describing events. This relationship between the participants was certainly an inhibitor for their engagement in a deeper reflection.

In the follow-up sessions, all of the teachers were invited to plan, teach and reflect about two lessons. Manuela and Antónia became much active and participative. This occurred perhaps because these lessons had no observers, because they were now teaching their own students, or because the spontaneous talk in the interviews gave them new confidence. In this phase, supported by Irina and the researchers, Manuela and Antónia felt confident to adapt tasks from textbooks, making them more challenging for their students. With this involvement of Antónia and Manuela, the lesson study group meetings (including teachers and researchers) became meetings that saw participants working jointly and sharing a common responsibility for the development of the work.

During the follow-up sessions, the teachers were involved in situations of reflection about practice and for practice. In moments of reflection for practice, regarding a new teaching problem – how to promote learning with understanding of the relationship between decimal fractions and decimal numbers – the teachers made suggestions to adapt tasks, but still did not support their proposals with powerful teaching ideas. Later, in moments of reflection about practice, the teachers discussed in detail the work of students, identified mistakes and difficulties common to students of different classes, and this took them to a deeper reflection about their own practice. In this reflection, the teachers questioned the representation that they used when teaching the decimal number system. Accordingly, they proposed to change their practice suggesting that they should always use more consistent iconic representations for decimal numbers.

Therefore, the work carried out in the lesson study favored the creation of an environment of integrating knowledge (Lewis, 2016). In this environment, the teachers actively constructed their own knowledge by collecting data from their students, establishing connections among different sources of data, identifying problems of their own practice and of their students' learning, and also proposing solutions to those problems and justifying them. This lesson study, emphasising a collaborative and reflexive setting including teachers and researchers, favoured the development of relations of joint work among the participants. This evolution was possible because the teachers felt confident to question their conceptions and practices and decided to jointly put into practice new ideas (Cajkler et al., 2015; Fujii, 2016). The follow-up sessions appear to have brought the group together, and to involve all teachers in the development of the work. Therefore, the follow-up

made a good fit to the local culture of the participant teachers, and was helpful in increasing the efficacy of this professional development process.

A LESSON STUDY ON MATHEMATICAL PROBLEM SOLVING

The Setting

The *Lessons for Mathematical Problem Solving* (LeMaPS) project sought to research the potential of developing sustainable and scalable models of lesson study communities to support the professional learning of mathematics teachers. Central to the work of the project was the research question of knowing what supporting tools would help collaborative partnerships to implement lesson study for mathematical problem solving in effective ways that are both sustainable and scalable.

In the period 2014–2016, the university research team led by one of the authors (Wake), worked with clusters of schools and wider networks in England to introduce lesson study processes with an emphasis on developing the teaching of mathematical problem solving. Fundamental to the project was the intention to use a high-quality lesson study approach incorporating the essential principles of the Japanese model. The project was informed by close collaboration with the IMPULS (International Math-teacher Professionalization Using Lesson Study) team of researchers based at Tokyo Gakugei University in Japan. Over the course of the project 28 educators, in three cohorts, raised funding to attend the IMPULS 10-day annual immersion programme.

The issue of adaptation of lesson study to different cultural contexts (Stigler & Hiebert, 2016) is one that needs careful consideration and was of crucial importance in the LeMaPS project that sought to effect the introduction of lesson study at a time of considerable turbulence in school governance at a national level. At a structural level, lesson study communities were initiated, developed and sustained in ways that brought mathematics teachers and other educators together across schools at a time when schools were being encouraged to become autonomous and re-develop their usual patterns of collaboration. Inter-school collaboration was found to be an important aspect of the work as it helped support the development of networks at local, regional and national levels in ways that transcended usual forms of collaboration. In terms of developing lesson study as the focus of the activity of new professional communities it was found important to work with brokers who were able to make new connections across existing communities of practice (Wenger, 1998). These individuals were most important in ensuring that the work of the project was able to infiltrate and be seen to add value to well-established practices as well as dealing with the idiosyncrasies that individuals bring to any community's endeavours.

Important to forming the collaborative lesson study groups was the inclusion of 'outside expertise.' In Japan, an 'external expert,' 'knowledgeable other' or 'koshi' is involved in the lesson study process, mainly contributing the final expert commentary at the end of the post-lesson discussion following the lesson study

research lessons. In our own model, given the early stage of development, we sought to involve those who could offer such expertise but involved them throughout each lesson study cycle as much as possible in a supportive role.

Closer to the classroom, the important elements of Japanese lesson study that were fundamental to the model advocated by the research team were:

- A *research focus* that informed the 'bigger picture,' that is the overall context of the endeavour. In the LeMaPS project, the focus was problem solving, so the research questions were related to the teaching of problem solving processes, rather than mathematical content and concepts. For example, a research question might ask "How can we better enable students to select and use mathematical representations when solving problems?"
- A *detailed lesson plan* that is produced by a small planning team from the wider lesson study group. The intention of this plan was to provide a lesson which will enable the group to answer the research question. At the heart of the plan was: (i) careful anticipation of how students will respond to the task and how the teacher might respond in turn; and (ii) an outline of what pupil progress in problem solving may look like in the particular aspect of problem solving being investigated. The plan also aimed to anticipate what the teacher would do at vital moments in the lesson to progress learning and help students overcome their difficulties.
- The *research lesson* being taught by one of the planning team with the lesson being observed carefully by all members of the lesson study group, including teachers from other schools, an outside expert and student teachers.
- The *post-lesson discussion* involving the teacher and all observers in the analysis of the lesson with an outside expert making a particularly significant contribution to the post-lesson discussion by providing insights informed by research and in-depth knowledge of mathematical problem solving.

The Research Lesson

Here we illustrate the nature of collaborative learning that was central to the work of the project in the context of the lesson *Dance Moves* (Figure 4.3) carried out with students aged 13 to 14.

In a previous lesson, the students had discussed the idea of notating a phenomenon. They were introduced to music notation and produced a set of success criteria by which to assess notation and used this to judge the London underground map as a communicative representation. Their agreed criteria were: Is it easy to read? Is it easy to follow? Is it clear what the notation/symbols mean? The notion of geographical accuracy had emerged during whole-class discussion and it was agreed that although the London underground map is not accurate in this sense, it is still 'fit for purpose' as it provides a schematic view of stations, lines and intersections that inform journey planning.

Research focus: Developing mathematical representations as a means of communication

The task: Students were each given one of two short movie clips. One showed part of a traditional Indian dance, the other part of a robotic dance.

Task instructions:

– You need to produce a way of representing the dance moves you have been working on.
– Your work should allow someone else to perform the moves successfully.

Figure 4.3. The lesson: Dance moves

The teacher had asked students to tackle the task individually in the lesson prior to the research lesson and, having looked at their work, students were placed in groups according to how they had chosen to approach the task. They were seated in groups of four around a laptop with each individual student having their own initial attempt returned and with each group being provided with a printed copy of three questions taken from the five below. Different groups had a different set of questions allocated by the planning group of teachers according to the students' initial attempts:

- How would you make it really clear exactly what each part of the body should be doing?
- How would someone be able to check they have got the moves right?
- How could you make your work easier to follow at a glance?
- Is it clear how you get from one position to another?
- Do the limbs always move together? How can you include this in your work?

The students talked about what they had done and decided how to proceed. For example, one group had produced mostly verbal descriptions of dance moves. "You need diagrams but we are not allowed to draw" was one student's interpretation. The groups then worked together to "produce a set of instructions or some way of representing the dance move that will allow people in the other half of the class to do it."

It was noticeable that the students spent little time discussing how to proceed: they quickly selected a method, then began using and adapting it. The teacher prompted thinking by asking questions involving the timing of moves: "How are you going to show which limbs are moving together?," "Think about the question: How do you get from one position to another?"

All but one group developed methods involving diagrams and words: the exception was a group that developed a parallel line notation (Figure 4.4). In this the top line represented the upper body and the lower line represented the lower body. The L and J symbols represent the right and left hands raised and the E symbols represent the fingers being spread.

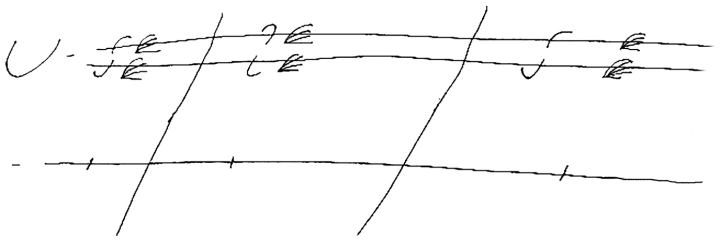

Figure 4.4. Parallel line notation for dance moves

The groups of students then peer-assessed each other's work by answering the following questions:

- Is it easy to read?
- Is it easy follow?
- Is it clear what the notation/symbols mean? If so how have they made it clear?
- Is it clear what to do with different parts of the body? How could their method be clearer?
- How have they included the speed of the movements?
- What improvements might you make to your own work after seeing this group's?

In general, the level of student feedback was poor: it seemed clear that students were not well-rehearsed in providing such feedback. Following this, a student from each group then attempted to use the instructions to act out the other group's dance moves. This did have the effect of highlighting inadequacies in the representations that had been developed. Discussion of this was stimulated at a whole-class level by discussion of the potential of the representation in Figure 4.4.

Learning from the Lesson

Immediately following the research lesson, the lesson study community of seven teachers from across four schools and three members of the research team, focused their discussions on learning from this experience, guided by the research focus of the lesson. The issue of 'task mutation' (Stigler & Hiebert, 2016), that often occurs, was first discussed. It seemed that some students had initially (mis-)understood that they were not allowed to draw diagrams. Perhaps the reason for this was that "stick figures" had been discouraged in the preliminary lesson. Thus, for some students the task had mutated from "devise a representation" into "devise a method that doesn't involve diagrams."

Much discussion focused around issues of representing and representations in mathematics. The discussion considered the different purposes that different representations serve. Some such representations and consequently the notations used are required to be succinct, whereas others are simply intended to be clear.

Issues of efficiency (for example, how to represent repeated moves) and clarity were considered and 'having a sense of audience' was considered to be important. The group worked at ease and was well-focused on their joint endeavour, much as in latter stages of the work of the Portuguese group: there had been the establishment of an environment of integrating knowledge (Lewis, 2016).

In further discussion, the issue of the balance of time used by the students in terms of their planning and implementation was addressed. A teacher commented, and others agreed, that little time was spent on planning by the groups and it was noted that some groups had not planned which notation to use at all. It was remarked that some groups rapidly agreed to use diagrams, for example, but had then spent little time discussing and refining how their diagrams would look. One teacher suggested that perhaps there needed to be a teacher intervention to force the students to consider alternative approaches.

This was one of the later research lessons in the first year of the project and by this stage the modus operandi of the group had been established and importantly individuals felt comfortable in the roles that had been established and their position in, and the contribution they could make to, the emergent community. Important in this regard was the issue of power relationships in the group. For example, relationships between teachers and their colleagues who may have different roles and levels of authority within a particular school, and relationships between teachers and the university researchers. Wenger (1998) identifies this as an important issue in any community of practice and considers this in terms of developing a "social ecology of identity." In his consideration of the development of a community of practice he conceptualises the individual's relationship within the community as one of identity formation that involves both identification and negotiability. We found this particularly helpful and pertinent in our consideration of the development of the lesson study communities. Indeed, this was of particular concern, at the outset of the project, where the different use of terminology regarding problem solving (the focus of the work), led to some tensions. A half-day meeting of the lesson study group was organised at the university and the nature of the focus of the classroom-based research was discussed. In this meeting it emerged that the researchers were indeed researching the learning of problem solving in classrooms and they did not have "the answers" for the teachers. This discussion proved important in clarifying the nature of the group's joint enterprise and the different types of expertise that different participants in the community could offer. Indeed, it was clarified that lesson study can be considered as a genuine process of collaborative action research where all parties are learning.

One aspect of Japanese lesson study that the project wished to preserve, although in a modified form, was that of 'outside expert.' That is, that each lesson study community should be able to call on expertise, outside of the day-to-day running of the group, to bring new/different thinking to stimulate the group to continue to make progress. In the particular research lesson described, this role fell to one of the university researchers, although increasingly, as the lesson study communities

became more well-established, other teachers who were acknowledged as being able to take on such a role, have taken this responsibility. In this case, as well as being asked to be the person who summarises and draws the discussion to close, the outside expert tended to work with the group from the outset, advising on issues in relation to research question, task, lesson design, and so on with such contributions often being made by email. It was implicit in taking on this role that the outside expert would consider how to best contribute to the ongoing learning of the group, not only in terms of the particular lesson, but also in terms of the longer term aims of the group and its involvement in lesson study.

Designing for Learning from Lesson Study

The research team reflected together with the teacher partners what had been learnt more generally and what might by design better facilitate future learning. Regarding how best to facilitate effective practice in each part of the cycle the team considered the role of a number of boundary objects (Star & Greisemer, 1989). An obvious example of such a boundary object is the lesson plan. This acts as a communicative device that is developed by the lesson planning team, used as something akin to a script in the classroom and then used to facilitate discussion of planned intentions and proposed actions in the post-lesson discussion. At first, the team had little understanding of what a lesson plan should contain and how it should be structured. However, experience, and our reflections, suggested ways in which we might develop the lesson plan template so that it allowed for localisation in each setting to meet important needs and also better facilitate our learning in the post lesson discussion. For example, an important aspect of teaching is for the teacher to know how students are likely to respond to mathematical ideas and to be able to draw on specialised content knowledge (Ball, Thames, & Phelps, 2008) and pedagogies in ways that will support them make progress. Within our lesson plans we therefore developed the notion of the "anticipated issues table." The planning group complete it anticipating likely student responses at key moments of the lesson and anticipating how the teachers of the lesson might respond. It is informed by careful planning, for example, drawing on the research literature to anticipate the likely prevalence of common misconceptions, and further to this considering how to respond in ways that meet the agreed intentions of the lesson. In the lesson, the teacher will draw upon this to make well-informed decisions in the moment. Observers use it as an advance organiser of what the planning team expected to happen, and in the post lesson it can help guide the discussion. In this setting it acts to provide insight into the planning team's rationale for the overall structuring of the lesson and the associated pedagogies.

In the latter stages of the project, this conceptualisation of the potential of boundary objects to facilitate the activity of the lesson study groups became important as discussions of the groups focused on how to better support the different phases of the lesson study cycle in ways that brought insight to participants. The

group, particularly the researchers, saw this as a design challenge and it stimulated discussion about how to best support collaborative learning for those who were becoming experienced members whilst also inducting newcomers into the emergent lesson study communities.

CONCLUSION

Lesson study provides a context for teachers' professional development that allows them a significant experience of collaboration and reflection centered on key problems of practice. During the lesson study process the participants act as a community of practice. At the heart of this practice, as illustrated in the cases reported in this chapter, is looking at students' learning, finding ways to represent students' thinking, relating it to learning goals, figuring out what tasks and classroom settings promote or inhibit students' development. As the case from the United Kingdom shows, this may require a deeper look at fundamental notions related to teaching and learning.

Collaboration is indeed a strong aspect of lesson study. Its intensity may depend on many factors, such as the way the teachers get involved in this process (voluntarily, by invitation, by appointment of the school Director …) and the nature of the relationships established among participants, including teachers and experts. An initial carefully focused discussion about the central issue to be considered in the lesson study may lead the group to assume clearly a shared goal, an important condition for a fruitful collaboration to develop. As the case from Portugal shows, collaborative relationships may take some time to emerge, and many reasons related to the participants' concerns may slow down their establishment. However, as this case also shows, the persistent effort of the leaders to ensure the inclusion of everyone in the work of the group may be effective in finally creating a fruitful environment for joint work that involves all participants. The United Kingdom case also brings to the fore the role of experts as part of the lesson study communities and raises issues of how the success or otherwise of collaborative professional learning through lesson study can be enhanced by drawing on the different types of expertise that different participants bring to such communities.

Reflection processes permeate all lesson study, as the participants systematically focus on students' learning processes, bringing to collaborative discussions their experiences (classroom data), and adopt an approach that values empirical evidence (students' written work, students' speech, records of classroom events) over opinion based on informal impressions. The participants in a lesson study consider in detail tasks and situations, but this is framed by the group's shared vision of the educational aims being pursued. It is this combined careful attention to detail within a jointly understood philosophy and model of learning that is important in providing conditions for a professional learning community that can support the development of reflection for practice. As the case from Portugal shows, this may create some discomfort for teachers who initially do not feel ready to explore teaching issues in this way, but, nevertheless, it appears crucial for professional learning. Importantly,

it is up to the leaders of the lesson study process to fine tune the nature of the work taking into account the characteristics of the participants. Many of these issues were also reflected in the United Kingdom case that clearly illustrates the role of the expert as a "knowledge broker."

There has been many successful lesson studies in Japan and in other countries as well as many problematic ones. As in any professional development process the problems may originate in many issues. To adjust to the local culture of each country adaptations may be necessary to make this process fruitful and sustainable. Cultural adaptations to different countries may be necessary but they should not remove the essential nature of lesson study as an intensive collaborative and reflective process in which teachers work as a community of practice in which they pay close attention to students' learning and to the processes by which they can improve it.

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