Lisbon School of Economics & Management
Universidade de Lisboa

MASTER IN MATHEMATICAL FINANCE

MASTER´S FINAL WORK
PROJECT

1-YEAR PERFORMANCE OF OPTIMAL PORTFOLIOS: A COMPARISON AGAINST AN INDEX TRACKER ETF

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SUPERVISION:
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ABSTRACT

The objective of this empirical project is to examine and test some of the investment choices a passive investor may face when deciding to invest in the United Kingdom's Stock Market. For this exercise, the object of study is the one-year performance of mean-variance based portfolios set against an index tracker exchange-traded fund (ETF) between May 2020 and May 2021. This is accomplished by analyzing the UK's main stock index (FTSE 100) between May 2015 and May 2020 and constructing portfolios based on the return and volatility characteristics of its stocks during this period, considering both the case when short selling is forbidden and when it is allowed. In particular, we look at the performance of the tangent or optimal portfolios calculated through the classic Mean Variance Theory (MVT) and also through some of the known return generating models: the Single-Index Model (SIM) and the Constant Correlation Model (CCM). Additionally, the "Naïve" portfolio, where the investor invests equally in all stocks available, is also analyzed in this project. For all portfolios, first we compared the estimated vs actual performance, volatility, and beta. Further to this, we compared the actual performance and volatility against the ones produced by the iShares Core FTSE 100 UCITS ETF. When short sell is forbidden, the "Naïve" portfolio and the CCM tangent portfolio outperformed the tracking ETF while MVT and SIM tangent portfolios have underperformed it. On the case where short selling is allowed ("Naïve" portfolio excluded), all the tangent portfolios have produced extremely negative returns. In terms of estimation, the CCM optimal portfolio was the one which most accurately estimated the performance and volatility for the year of investment.

KEYWORDS

Passive investing, index tracking, mean variance theory, constant correlation model, single index model, "naïve" portfolio, optimal portfolio, tangent portfolio, portfolio composition, performance, volatility, beta.

RESUMO

O objetivo deste projeto empírico é examinar e testar algumas das escolhas que um investidor passivo poderá enfrentar quando decide investir no mercado de ações do Reino Unido. Para este exercício, o objeto de estudo é a comparação entre a performance de portfolios construídos através da MVT e um ETF cujo objetivo é replicar a performance do índice FTSE 100 durante o ano entre 7 Maio 2020 e 7 Maio 2021. Isto é conseguido através da análise do principal índice de ações do Reino Unido (FTSE 100) entre 7 Maio 2015 and 7 Maio 2020 e, posteriormente, da construção de portfolios com base nas características de retorno e risco dos seus componentes, onde são considerados os cenários onde venda a descoberta é proibida e permitida. Em particular, iremos olhar para a performance dos portfolios tangentes, ou ótimos, calculados através da clássica MVT e também através dos modelos bastantes conhecidos: SIM e CCM. Adicionalmente, o portfolio "Naive", onde o investidor investe de forma igual em todas as ações disponíveis, é também analisado neste projeto. Para todos os portfolios, primeiro comparamos os valores esperados e obtidos para retorno, volatilidade e beta. Numa fase posterior, iremos comparar a performance e volatilidades obtidas com aquelas produzidas pelo iShares Core FTSE 100 UCITS ETF. No caso em que venda a descoberto é proibida, os portfolios "Naive" e CCM tiverem melhor performance que o ETF, enquanto que os portfolios MVT e SIM tiverem uma menor performance. No caso em que venda a descoberta é permitida (portfolio "Naive" excluído), todos os portfolios tangentes produziram retornos extremamente negativos. Em termos de estimações, o portfolio CCM foi aquele que produziu retorno e volatilidade mais em linha com os valores estimados para o ano de investimento.

PALAVRAS-CHAVE

Investimento passivo, replicação de índice, teoria média-variância, modelo correlação constante, modelo de fator único, portfolio "Naive", portfolio ótimo, portfolio tangente, composição de portfolio, performance, volatilidade, beta.

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List of Abbreviations

MVT: Mean Variance Theory

CCM: Constant Correlation Model

SIM: Single Index Model

ETF: Exchange-Traded Fund

FTSE 100: Financial Times Stock Exchange 100

1. Introduction

During the last decades, investing in the stock market has become increasingly more accessible to everyone, not only in terms of the uncountable and easy-to-access trading platforms but also in terms of cost, being the transaction of a stock almost free of any type of commission or fee in some cases. This may have led many retail investors to invest their savings without seeking any kind of advice about the risk-return characteristics of the stocks nor about the adequacy of their asset allocation strategy (if any) to achieve their investment goals under a suitable level of risk. Looking at this, it is very crucial to talk about the importance of having an efficient, adequate, and diversified investment strategy. Markowitz (1952) developed the Mean Variance Theory (MVT) that has been the basis of the modern portfolio analysis, with the search for the optimal portfolio in its core. Introducing the concept of diversification into the investment and security analysis world, the theory is still widely accepted by the financial industry. However, due to the limited ability to efficiently estimate the inputs for the MVT, we will also study the famous return generating models that, under some assumptions, simplify the calculations of the correlation structures: Single Index Model (SIM) and Constant Correlation Model (CCM).

The case of study is an investor that looks to allocate a part of his savings in FTSE 100 stocks and is under the dilemma of investing through an index tracking ETF or build an efficient frontier of portfolios and chose the optimal risk-return trade-off. In this project, we look at the risk-return characteristics of the FTSE 100 stocks and, by applying the classic MVT, SIM and CCM, find each optimal portfolio for which we compare the performance and volatility with a FTSE 100 tracking ETF. The “Naïve” portfolio is also shown in this analysis and compared with the tracking investment vehicle. During the year of analysis, the investor follows a passive strategy where rebalance to the initial weights is done monthly and no active management or fundamental analysis is considered. In addition to the one-year comparisons, each optimal portfolio will be assessed based on the accuracy of its estimations.

Despite the limitations in the time series, which comprises Brexit and Covid-19 periods, and the inability to extrapolate our results, these project shows the relevance of the MVT and the return models in creating diversified and robust equity portfolios. It is possible to observe that when short sell is forbidden, the CCM not only outperforms the market as it has the more accurate forecasting among all observed strategies.

In chapter 2 we present a literature review, followed by a focus in the data and methodology in chapter 3 and the results in chapter 4. In chapter 5, we present the conclusions and limitations of this project.

2. Literature Review

The study of performance of optimal portfolios is key for investors to take this option into account when deciding to invest. An investor that wants to allocate part of its savings in equity, may face the dilemma presented in this project. Should the investor build an efficient frontier of portfolios and pick what suits him better in terms of risk-return relation or just opt to invest in the market as a whole through a tracking ETF? By choosing to build the efficient frontier, the investor may face some problems of computation due to the number of correlations to calculate [N(N-1)/2] under the classic MVT framework. In order to overcome this issue, the investor can also reach out to the simplest return generating models, SIM and CCM.

The present project focus in the performance of optimal portfolios built under different assumptions, but all derived from the classic Mean Variance Theory framework, against the performance of a ETF which aims to track the FTSE 100. These may present a balanced option for people wanting to invest in the equity market but don't intend to track the index, they just want to hold the stocks that provide the best risk-return tradeoff when combined in a portfolio.

Section 2.1 presents the concept of passive investing and index tracking vehicles. Section 2.2 provides a view on the market index chosen for this project, the FTSE 100. The subsequent sections review the different approaches to construct the optimal portfolio. Section 2.3 reviews the pure Mean Variance Theory, Section 2.4 approach the analysis of the Constant Correlation Model, Section 2.5 approach the analysis of the Single Index Model and, finally, Section 2.6 presents the "Naïve" approach to build a portfolio.

2.1 Passive Investing and Index Tracking vehicles

Index tracking is the most common way of passive investing. The investor, who does not want to pay high fees to asset managers or take risks associated with stock picking, may decide to invest in an index which reflects the entire equity market of the location he is interested in. Nowadays, if the goal is to pursue an index tracking strategy, the investor has at his disposal many different vehicles, among which we can find mutual funds and exchange-traded funds. The first, being the most common one, may represent higher costs than the later. Mutual funds have low costs compared to other type of more sophisticated funds but still carry some management fees. These funds aim to track an index and usually are composed by the same stocks at the same weightings as the index. The mutual funds are then sold to a pool of investors which can purchase a share of those funds and get exposure to the index while receiving dividends for any of the companies that distribute it. More recently, the investors have at their disposal exchange-traded funds (ETFs), which are usually composed by many different types of financial instruments, including derivatives and sophisticated structured products, that also intend to replicate the performance of an index. Generally, these don't pay dividends but do not carry any management fee. The ETFs market has been rising considerably in the last decade and is now a solid alternative for investors seeking exposure to almost any sector or market through replication of performance.

2.2 FTSE 100

The Financial Times Stock Exchange 100 contains the biggest companies in the UK equity market. This is a cap-weighted index, which means that the weight of each company is based on its market capitalization (number of stocks x price of stock). This is opposed to price weighted indexes where the weight of each company is determined by the price of its stock. As of December 2020, this index contains 101 companies, comprising sectors such as: Communications (4.5%), Consumer Discretionary (8.3%), Consumer Staples (19.7%), Energy (8.8%), Financials (18.4%), Health Care (10.5%), Industrials (10.4%), Information Technology (1.3%), Materials (13.4%), Real Estate (1.2%) and Utilities (3.6%). The largest individual constituent is Unilever plc (6.3%), followed by AstraZeneca plc (5.8%). The five largest constituents represent approximately 25% of the entire index.

2.3 Mean Variance Theory (MVT)

MVT is the basis of modern portfolio analysis. Its introduction by Markowitz in 1952 allowed investor, for the first time, to define the investment opportunity set and the notion of efficient frontier. With this, investors could, then, choose the portfolio from the efficient frontier that better suits his return and risk preferences.

Additionally, MVT served as ground base for several developments that improved the performance of financial investments through portfolio selection. Among them, we recall MVT as the foundation of equilibrium models, such as CAPM, developed by Lintner (1965), Mossin (1966) and Sharpe (1964).

The first step when implementing this framework is the estimation of the inputs, which as previously stated, can become very difficult to calculate given the number of correlation coefficients to calculate. The second step is then constructing the efficient frontier of portfolios based on the return, volatility, and correlation structure of each security. In this frontier, we can find the optimal portfolio which will have the best risk-return trade-off.

2.4 Constant Correlation Model (CCM)

In order to overcome the above-mentioned computational barriers of the MVT related to the number of correlation coefficients, some additional assumptions need to be incorporated in terms of the correlation structures. Elton, Gruber and Urich (1978) tested the forecast of future correlations by smoothing the historical correlation matrix data with averaging. They tested both aggregate and disaggregate type of averaging techniques. The aggregate averaging assumes that future pairwise correlation coefficients are the average of all correlation coefficients in the past correlation structure. The disaggregate type of averaging technique splits the entire population of securities in groups of similar historical pairwise correlation coefficients and then use the average of these coefficients as the forecasted correlation between each pair of securities within these groups. The idea of this disaggregation is to reduce the difference between the average coefficient and the historical correlation structure for each pair of securities. They found that a forecast of all correlation coefficients equal to the aggregated mean performed as well or better than other widely accepted forecasting technique. The Constant Correlation Model uses the aggregate averaging technique, which reduces the necessary computational power in a significant way.

2.5 Single Index Model (SIM)

The SIM of Sharpe, is by far the most popular model when implementing MVT. The primary assumption of Sharpe's model is that there exists one and only one common factor able to explain (systematic) co-movements in returns. This factor, called market, can describe perfectly the co-movement between securities. The previous statement is based on common observation of the behavior of securities relate to that of the market as a whole. The correlation structure across securities is then assumed as the correlation between the securities' return and the index. Implicitly, the assumption is that there are no specific correlations across securities' returns.

2.6 "Naïve" Portfolio

The "Naïve" approach is an interesting baseline for input estimation as it assumes nothing is known about the risk, return, and correlation structure between securities. Without using any statistical data to provide any expectation or risk-return framework, the optimum portfolio is a portfolio that allocates the same amount in every stock available, with weights given by 1/N. As shown by DeMiguel, Garlappi, and Uppal (2005) and other authors the 1/N portfolio of stocks performs surprisingly well out of sample, particularly for small sample sizes. Although these results may not be applicable to allocation across multiple asset classes, as opposed to identifying an optimal stock-only portfolio, they nevertheless challenge the efficacy of standard approaches to input estimation.

3. Data and Methodology

Having the FTSE 100 as benchmark and universe, to see the accuracy of the estimations and the performance of the optimal portfolios against the index tracking ETF, we propose working with in-sample data.

Based on daily log returns of the FTSE 100 and its components during the period between May 7th 2015 and May 7th 2020, we compute the annualized expected returns, volatilities, betas, alphas, and correlation matrix for the year between May 7th 2020 and May 7th 2021. The log returns of the year after are then used to calculate the actual return, volatility, and beta of the portfolios and the ETF and compare with the expected figures. The investment starting date is May 7th 2020, and the investor has the last five years of data available to form a decision as he assumes these are a good estimator for the year to come.

Section 3 presents all the data used for this project. First, we describe the universe and the time frames for the data series, as well as the data itself and some of the criteria used to select it. Second, we present the inputs required and the method to achieve the optimal portfolio for each approach. In the end, we describe the process to compare the estimations and actual results of each model and also to find the performance of each portfolio against the ETF.

3.1 Data

The FTSE 100 is the universe chosen for this analysis as it provides sector diversification, exposure to the core equity market in the UK and an appropriate set of stocks to look at (101 in total). 4 stocks were excluded from the analysis due to lack of data, reducing the total sample of stocks to 97. The daily adjusted closing prices were used and retrieved from Yahoo finance. We opted for adjusted closing prices since it already accounts for any corporate action such as dividend payments or stock splits, which helps avoiding biased returns.

In a first instance, the data used to build the portfolios is the series of adjusted closing prices between May 7th 2015 and May 7th 2020 of the FTSE 100 and its components. Secondly, the adjusted closing prices between May 7th 2020 and May 7th 2021 are used assess the accuracy of the estimations and to calculate the performance of the portfolios against the iShares Core FTSE 100 UCITS ETF. Given the investor rebalances the portfolio to the initial allocation on a monthly basis, transaction costs may play an important role in these results. However, considering the multiple options we have today to access the market with low or zero fees and the degree of deepness that would add to the project, namely with the short selling, we have decided to leave the transaction costs out of this analysis.

The optimal portfolios generated by MVT and the return models (SIM and CCM) are built under the assumption that there is no risk free asset available in the market, for both the cases where short selling is forbidden and allowed. The absence of riskless asset is a necessary assumption for this project given that the main goal is to compare the performance of the portfolios with an index composed solely by equity. Our focus is on risky portfolios and not in combinations of a risky portfolio with a risk-free asset.

3.2 Computation of models' inputs

The inputs to find the portfolios, optimal and "Naïve", are the securities' annualized return (R_i), volatility (σ_i), correlation structure (ρ_{ij}), beta (FTSE 100) (β_i) and alpha (α_i) as shown in Table 1 and Table A of the Appendix. Each of the models uses the necessary inputs under its own specifications, as shown before in the literature review. The formulas to calculate the expected return (1) and volatility (2) of the portfolios are as follows:

$$\overline{R}_p = \sum_{i=1}^{N} x_i \overline{R}_i \tag{1}$$

$$\sigma_p = \left[\sum_{i=1}^{N} x_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} x_i x_j \sigma_i \sigma_j \rho_{ij} \right]^{\frac{1}{2}} \tag{2}$$

Where:

R_i : Return of security i
R_p : Return of portfolio
σ_i : Volatility of security i
σ_p : Volatility of portfolio
ρ_{ij} : Correlation between securities i and j
x_i : Weight of security i
N : Number of securities

3.2.1 Mean Variance Theory Inputs

To apply the MVT we use the expected returns, volatilities, and the pairwise correlation structures of each stock. With this, we create a vector composed of the annual return and a vector composed of the annual volatility of the stocks and a matrix composed with the correlations between the log returns of each pair of stocks. The resulting figures are the inputs of the MVT scenario.

This data would serve to build the entire efficient frontier, although for this project we are just interested in calculating the optimal portfolio.

3.2.2 Constant Correlation Model Inputs

For the Constant Correlation Model (CCM), a part of the inputs are the same as for MVT, the annual return and volatility of each security. However, the correlation matrix (which has a diagonal equal to the MVT matrix because the volatility is the same) was computed using an aggregate averaging technique, as shown in the below formula 3, in order to replace the pairwise correlation between each securities return that is used under the MVT.

$$\rho = \frac{\sum_{i=1}^{N} \sum_{j>i}^{N} \rho_{ij}}{N(N-1)/2} \tag{3}$$

Where:
ρ_{ij} : Correlation between securities i and j
ρ_i : Average correlation between securities
N : Number of securities

3.2.3 Single Index Model Inputs

For the Single Index Model (SIM), the inputs are the return and volatility of each security and the market, with the addition of beta and alpha for each stock that were calculated through a linear regression between the return of each security and the return of the index chosen (FTSE 100). The before mentioned parameters, β and α, represent the part of the securities' return which is dependent and independent from the market, respectively. Hence, volatility is composed of the systematic risk (dependent on the market) and the specific risk (independent from the market). The annual mean return (4) and variance of each security (5) and the correlation (6) structure under this model were computed using the following formulas, respectively:

$$R_i = \alpha_i + \beta_i R_M \tag{4}$$

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{e_i}^2 \tag{5}$$

$$\rho_{ij} = \frac{\beta_i \beta_j \sigma_M^2}{\sigma_i \sigma_j} \tag{6}$$

Where:
R_i : Return of security i
R_M : Return of market
σ_i : Volatility of security i
σ_{ei} : Specific risk of security i
σ_M : Volatility of market
ρ_{ij} : Correlation between securities i and j
β_i : Beta of security i
α_i : Alpha of security i

3.2.4 "Naïve" Approach Inputs

For the "Naïve approach, there is no calculation to be made other than using the number of securities to calculate the amount to invest in each of them. The investor will buy the same amount of every stock available, which in this case means that the amount to be invested in each security is 1/97 (~1%).

3.3 Optimal Portfolio

Once the inputs of each model were computed accordingly, we proceed to finding the optimal portfolio. This section does not apply to the "Naïve" approach.

For the **MVT**, the optimal portfolio was obtained such that the Sharpe Ratio was maximized. The Sharpe Ratio or, as introduced by Sharpe (1966), reward-to-variability ratio, is a measure of the expected return per unit of volatility (R / σ). For this, we have used Excel Solver to find the combination of the stocks that would maximize the Sharpe Ratio of the resulting portfolio (the tangent or optimal portfolio).

For the **SIM,** having all the inputs calculated (R_i, R_M, σ_i, σ_M, β_i, α_i), we follow the approach described in Elton, Gruber, Brown, Goetzmann (2013) where we start by ranking the securities by the Treynor Ratio, which measures the expected return by unit of non-diversifiable risk (R / β), as it can be seen in Table 2 of the Appendix. Under this model, which assumes that there exists one and only one common factor able to explain systematic co-movements in returns, the correlation structure across securities is assumed as the correlation between the securities' return and the index (β). This means that if a security with a certain Treynor Ratio is included in the optimal portfolio, all the securities with equal or higher ratio should also be included. Given this, the selection of stocks is dependent on a single cut off rate (C*) for which higher ratio stocks will be included and lower ratio stocks excluded (or shorted if allowed). We use the below formula to calculate the C for every stock and our C* will be the highest of these (Table 2).

$$C^* = \frac{\sigma_M^2 \sum_{i=1}^{N} \frac{\left(\bar{R}_i - R_F\right)\beta_i}{\sigma_{ei}^2}}{1 + \sigma_M^2 \sum_{i=1}^{N} \frac{\beta_i^2}{\sigma_{ei}^2}} \quad (7)$$

With these data, we then proceed to finding the absolute optimal allocation for each of the selected stocks (8) and, finally, the optimal relative allocation (9).

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2}\left(\frac{\overline{R}_i - R_F}{\beta_i} - C^*\right) \tag{8}$$

$$X_i = \frac{Z_i}{\sum_{j=1}^{N} Z_j} \tag{9}$$

Where (for formulas 7, 8 and 9):
R_i : Return of security i
R_F : Return of risk free asset
σ_i : Volatility of security i
σ_{ei} : Specific risk of security i
σ_M : Volatility of market
β_i : Beta of security i
C^* : Cut off
Z_i : Absolute optimal weight of security i
X_i : Relative optimal weight of security i
N : Number of securities

For the **CCM**, we use the same inputs as in the MVT, however, as described in 3. 2. 2., rather than using the matrix with all pairwise correlation between the securities, we use the average correlation (ρ) of all pairs of securities. Similar to the SIM framework, we start by ranking the securities but now by their individual Sharpe Ratio, as it can be seen in Table 3 of the Appendix. Again and now because the correlation between securities is constant, the selection of stocks is dependent on a single cut off rate (C*) for which higher Sharpe Ratio stocks will be included and lower ratio stocks excluded (or shorted if allowed). We use the below formula to calculate the C for every stock and our C* will be the highest of these (Table 3).

$$C^* = \frac{\rho}{1-\rho+N\rho}\sum_{i=1}^{N}\frac{\overline{R}_i - R_F}{\sigma_i} \tag{10}$$

With these data, we then proceed to finding the absolute optimal allocation for each of the selected stocks (11) and, finally, the optimal relative allocation (12).

$$Z_i = \frac{1}{(1-\rho)\sigma_i}\left(\frac{\overline{R}_i - R_F}{\sigma_i} - C^*\right) \quad (11)$$

$$X_i = \frac{Z_i}{\sum_{j=1}^{N} Z_j} \quad (12)$$

Where (for formulas 10, 11 and 12):
R_i : Return of security i
R_F : Return of risk free asset
σ_i : Volatility of security i
ρ : Average correlation coefficient
C^* : Cut off
Z_i : Absolute optimal weight of security i
X_i : Relative optimal weight of security i
N : Number of securities

3.4 Measuring estimations and performance

For the purpose of this project, we assess the estimations by just comparing it with the actual figures during the year between May 7th 2020 and May 7th 2021. Each portfolio has an expected return, volatility and beta. We will take these figures and compare it with the actual return, volatility and beta during the year of investment. Once this step is concluded, we proceed to comparing all the performances from the constructed portfolios with the performance of the market reflected through the ETF.

4. Results

During the present chapter, for each of the approaches presented in the previous chapters, we show the estimations and the actual returns, volatilities, and betas as well as the comparison of the performance with the ETF during the year of investment.

In section 4.1, we start by showing each model's estimations and the actual results. Moving to section 4.2 we present the data for the ETF and, to finalize, in section 4.3 we provide some discussion and comparison between the portfolios built by the investor and the ETF.

4.1 Estimations and actual results

In the following section, the results obtained by applying MVT, SIM, CCM and "Naïve" approaches are compared with the estimations produced under each model, for both the cases where short sell is forbidden and allowed. Please note that all the portfolios are rebalanced to the initial allocation after each completed month of investment, 11 times in total. On another note, the market showed a positive return of roughly 11% during the month of November 2020 after also a strong initial month of investment (May 2020) where it has returned 9.2%, which was reflected in almost every stock. Due to this fact and the rebalancing policy, when short selling is allowed, all the portfolios have lost all its money in November 2020, and, for that reason, we do not show the actual figures for volatility, beta, and p-value. If we have continued these strategies, the investor would need to invest more after losing all the initial investment.

4.1.1 MVT

The pure MVT approach, where short sell is forbidden, resulted in a portfolio of 10 stocks out of the 97 under the analysis. The composition and weights can be seen in Table 4 of the Appendix. The largest position is in Rentokil Initial plc RTO.L (19.4%) and the lowest in BHP Group ltd BHP.L (0.0%). The expected and actual returns, volatilities and betas for the optimal portfolio, and also the p-value associated with the t-test for the difference between the expected and actual returns, are as shown in the table below:

Table 1 – MVT with no short sell

Short sell forbidden	Return (R)	Volatility (σ)	Beta (β)
Estimation	24.82 %	17.18 %	0.75
Actual (May 2020 to May 2021)	18.56 %	19.01 %	0.51
P-Value (5%)	0.34		

Where short sell is allowed, all the 97 stocks under the analysis are selected. If a stock is not good to buy is then good to short sell and use the leverage to buy more of the "good" ones. The composition and weights can be seen in Table 4 of the Appendix. The largest long position is in SGRO.L (121.9%) and the largest short position in RDSB.L (-103.0%). The expected and actual returns, and only expected volatility and beta for the optimal portfolio are as shown in the table below:

Table 2 – MVT with short sell

Short sell allowed	Return (R)	Volatility (σ)	Beta (β)
Estimation	234.89 %	58.27 %	0.10
Actual (May 2020 to May 2021)	-100.00%	N.A.	N.A.

4.1.2 SIM

The SIM approach, where short sell is forbidden, resulted in a portfolio of 8 stocks out of the 97 under the analysis. The composition and weights can be seen in Table 5 of the Appendix. The largest position is in Rentokil Initial plc RTO.L (22.0%) and the lowest in AstraZeneca plc AZN.L (4.0%). The expected and actual returns, volatilities and betas for the optimal portfolio, and also the p-value associated with the t-test for the difference between the expected and actual returns, are as shown in the table below:

Table 3 – SIM with no short sell

Short sell forbidden	Return (R)	Volatility (σ)	Beta (β)
Estimation	24.64 %	16.07 %	0.75
Actual (May 2020 to May 2021)	18.11 %	19.21 %	0.50
P-Value (5%)	0.34		

Where short sell is allowed, all the 97 stocks under the analysis are selected. If a stock is not good to buy is then good to short sell and use the leverage to buy more of the "good" ones. The composition and weights can be seen in Table 5 of the Appendix. The largest long position is in Halma plc HLMA.L (29.75%) and the largest short position in BT Group plc BT-A.L (-20.36%). The expected and actual returns, and only expected volatility and beta for the optimal portfolio are as shown in the table below:

Table 4 – SIM with short sell

Short sell allowed	Return (R)	Volatility (σ)	Beta (β)
Estimation	112.97 %	22.80 %	0.04
Actual (May 2020 to May 2021)	-100.00 %	N.A.	N.A.

4.1.3 CCM

The CCM approach, where short sell is forbidden, resulted in a portfolio of 8 stocks out of the 97 under the analysis. The composition and weights can be seen in Table 6 of the Appendix. The largest position is in Halma plc HLMA.L (26.3%) and the lowest in Expedia plc EXPN.L (2.9%). The expected and actual returns, volatilities and betas for the optimal portfolio, and also the p-value associated with the t-test for the difference between the expected and actual returns, are as shown in the table below:

Table 5 – CCM with no short sell

Short sell forbidden	Return (R)	Volatility (σ)	Beta (β)
Estimation	23.10 %	16.66 %	0.80
Actual (May 2020 to May 2021)	22.65 %	19.66 %	0.59
P-Value (5%)	0.46		

Where short sell is allowed, all the 97 stocks under the analysis are selected. If a stock is not good to buy is then good to short sell and use the leverage to buy more of the "good" ones. The composition and weights can be seen in Table 6 of the Appendix. The largest long position is in Halma plc HLMA.L (36.0%) and the largest short position in BT Group plc BT-A.L (-29.0%). The expected and actual returns, and only expected volatility and beta for the optimal portfolio are as shown in the table below:

Table 6 – CCM with short sell

Short sell allowed	Return (R)	Volatility (σ)	Beta (β)
Estimation	142.37 %	28.75 %	-0.07
Actual (May 2020 to May 2021)	-100.00 %	N.A.	N.A.

4.1.4 "Naïve"

The "Naïve" approach, where short sell is forbidden, resulted in a portfolio of 97 stocks out of the 97 under the analysis. All the stocks have a long position with the same weight (1.03 %). The expected and actual returns, volatilities and betas for the optimal portfolio, and also the p-value associated with the t-test for the difference between the expected and actual returns, are as shown in the table below:

Table 7 – "Naïve" with no short sell

Short sell forbidden	Return (R)	Volatility (σ)	Beta (β)
Estimation	2.59 %	18.59 %	1.01
Actual (May 2020 to May 2021)	43.90 %	20.51 %	1.04
P-Value (5%)	0.05		

Under this approach, as mentioned before, we have not considered the scenario where short sell is allowed as it compromises the principle of the approach itself.

4.2 FTSE 100 tracking ETF

The iShares Core FTSE 100 UCITS ETF, which intends to replicate the exact performance of the FTSE 100 and be our benchmark in terms of performance for this analysis, have significantly outperformed the estimation for the year of investment. It is important to note that the market estimation for return is negative since our time series not only includes all the downside caused by Covid-19 but also the Brexit effect in the UK stock market since 2015/2016. This is not visible in the models' estimations because only the combination of stocks with better risk adjusted returns are picked, independently of their weight in the FTSE 100 index.

Based on the five years of data to May 7th 2020 and the year of investment, the following table shows the expected and actual returns and volatilities of the ETF:

Table 8 - iShares Core FTSE 100 UCITS ETF

ETF	Return (R)	Volatility (σ)	Beta (β)
Estimation	-3.38 %	17.20 %	0.99
Actual (May 2020 to May 2021)	20.26 %	18.70 %	0.99

During this period of investment, the FTSE 100 index itself has returned 20.11% with an associated volatility of 18.78%. The ETF is a good proxy of the market and provides a solid tracking vehicle to invest in the UK index.

4.3 Discussion

In order to properly compare all the investment choices, we need to look at the table below where all the portfolios and the ETF are presented alongside their Sharpe Ratio:

Table 9 – Sharpe Ratio (expected)

Portfolio	Sharpe Ratio (Expected)
MVT	1.44
MVT (Short Sell)	4.03
SIM	1.53
SIM (Short Sell)	4.95
CCM	1.39
CCM (Short Sell)	4.95
Naïve	0.13
FTSE 100 ETF	-0.19

It is quick to observe that when short sell is allowed the portfolios show a much higher Sharpe ratio. This is a consequence of the leverage that these portfolios present. Additionally, by holding all the stocks available, either long or short positions, the volatility of the resulting portfolios, by definition, becomes lower. Amongst the portfolios where short sell is not allowed, the discussion appears to be around the MVT, SIM and CCM portfolios as the Naïve portfolio not only has a lower expected return but also a lower Sharpe ratio. The tracking ETF reflects the expected return and volatility of the market index, which is not appealing for the investor given the negative expected return with similar levels of volatility as most of the other approaches (where short sell is forbidden). By looking at this table, where short sell is allowed, the investor would choose the SIM or CCM portfolios. Where short sell is forbidden, the investor would opt for the SIM optimal portfolio.

Having decided which investment to do and after the year of May 2020 to May 2021, the investor would see that the CCM portfolio where short sell is forbidden was the one producing actual results more in line with the estimations. Not only this can be seen by the p-values shown in this section for each of the portfolios when short sell is forbidden, where the CCM is the one with the lowest probability of rejecting the equality between the expected return and actual return but also by looking at the differences between the expected Sharpe Ratios and the actual results obtained during the year of investment. In the table below we show the actual Sharpe ratios during the year of investment:

Table 10 – Sharpe Ratio (actual)

Portfolio	Sharpe Ratio (Actual)
MVT	0.98
MVT (Short Sell)	N.A.
SIM	0.94
SIM (Short Sell)	N.A.
CCM	1.15
CCM (Short Sell)	N.A.
Naïve	2.14
FTSE 100 ETF	1.08

In terms of performance, the market index, hence, the ETF, has returned c. 20% during this period while the optimal portfolios have returned between 18 and 23% and the “Naïve” portfolio above c. 44%. It is very interesting to note that the optimal portfolios’ expected return and volatility were more in line with the actual behavior of the market during the year of investment than the market’s own expected return and volatility.

5. Conclusion

Based on the methodology applied and results obtained, we are now able to determine what would be the best option for the investor: an optimal portfolio, "Naïve" approach or a tracking ETF on the index. It is interesting to note that the expected return of the market was not appealing for the investor. A negative expected return of -3.38% could have demotivated our investor from investing in UK equity. However, when building the optimal portfolios under the pure MVT, SIM and CCM, one would be excited to invest given the expected returns of 20 to 25% with relatively low volatilities. Furthermore, if we consider the extremely high expected returns presented by the short sell opportunities across the models, the investor would also consider it when deciding for his strategy.

As shown in the results section, given the relatively good conditions in the market during the year of investment, the rebalancing policy applied and also the high leverage, when short selling is allowed, all the portfolios have lost its money by November 2020, hence none of the optimal portfolios would be the best option for the investor. Where short sell is forbidden, the investor would be satisfied with the CCM optimal portfolio, not only because it has outperformed the market in absolute and "return by unit of risk" terms, but also because it was the most accurate when compared to the estimations. In terms of performance and "return by unit of risk", the "Naïve" portfolio has significantly outperformed the market and every optimal portfolio.

To conclude, given the clear uptrend in the market during the year of investment, the investor would be better off if no short sell is used in his strategy. Any of the portfolios built under the assumption that short sell is forbidden, and the ETF, would provide the investor with a satisfying return relative to the market. For the future, given the consistency of the CCM and despite the higher return of the "Naïve" approach, the investor may opt for the CCM optimal portfolio with no short sell.

Regardless the relevancy of my results, they are true for the specific year under analysis. In fact, to better access the impact of random variables and reach more general conclusions, one could simulate the results obtained in the year of investment through Monte Carlo or other simulation method. By doing so, I could have tested the different techniques in a myriad of alternatives futures, which would allow me to extrapolate de results, overcoming what is, perhaps its strongest limitation: the impossibility of generalize results.

Additionally, it is important to note that the assumption that the five years between May 2015 and May 2020 are a good estimator for the year of investment is a strong and very disputable one namely due to disruptive effects caused by the Covid-19 pandemic in the stock market during the years of 2020 and 2021.

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Appendix

Table 1 (Securities' return, volatility, alpha and beta)

Security/Market	R	σ	α	β
FTSE	-3.38%	17.20%	N.A.	N.A.
AAL.L	8.76%	52.30%	0.15	1.84
ABF.L	-8.24%	27.46%	-0.05	0.82
ADM.L	11.90%	22.30%	0.14	0.67
AHT.L	14.35%	37.30%	0.19	1.43
ANTO.L	3.16%	41.51%	0.08	1.42
AUTO.L	11.43%	31.42%	0.14	0.80
AV.L	-8.88%	28.85%	-0.05	1.24
AVV.L	16.23%	44.12%	0.19	0.95
AZN.L	16.69%	24.60%	0.19	0.70
BA.L	4.12%	22.90%	0.07	0.84
BARC.L	-15.04%	35.32%	-0.10	1.43
BATS.L	0.89%	25.46%	0.04	0.83
BDEV.L	3.15%	38.25%	0.07	1.10
BHP.L	4.35%	38.45%	0.10	1.59
BKG.L	12.68%	32.64%	0.16	0.89
BLND.L	-10.55%	26.79%	-0.08	0.90
BNZL.L	0.22%	21.69%	0.03	0.71
BP.L	-0.65%	30.25%	0.04	1.36
BRBY.L	-2.52%	32.75%	0.01	1.17
BTA.L	-24.65%	29.06%	-0.22	0.89
CCH.L	9.09%	28.40%	0.12	0.98
CCL.L	-20.46%	46.68%	-0.15	1.52
CNA.L	-30.17%	35.26%	-0.27	1.07
CPG.L	3.99%	24.11%	0.07	0.83
CRDA.L	12.86%	20.81%	0.15	0.73
CRH.L	8.02%	28.70%	0.12	1.21
DCC.L	7.55%	25.89%	0.11	0.92
DGE.L	11.43%	19.98%	0.14	0.77
EVR.L	15.03%	56.23%	0.21	1.66
EXPN.L	17.78%	24.01%	0.21	1.02
EZJ.L	-20.70%	43.61%	-0.17	1.15
FERG.L	9.69%	27.69%	0.13	1.12
FLTR.L	11.07%	33.92%	0.14	0.78
FRES.L	1.06%	42.81%	0.03	0.64
GLEN.L	-9.53%	52.48%	-0.03	1.91
GSK.L	7.72%	20.02%	0.10	0.73
HIK.L	5.35%	35.46%	0.08	0.65
HL.L	5.34%	33.87%	0.09	1.20
HLMA.L	23.43%	22.76%	0.26	0.80
HSBA.L	-2.07%	22.16%	0.01	0.95
IAG.L	-16.57%	40.34%	-0.12	1.21
ICP.L	17.76%	39.74%	0.23	1.43
IHG.L	7.00%	32.60%	0.11	1.24
III.L	11.33%	29.57%	0.16	1.29
IMB.L	-5.72%	24.27%	-0.04	0.62

INF.L	-0.42%	26.64%	0.03	0.91
ITRK.L	13.21%	24.91%	0.16	0.87
ITV.L	-19.73%	35.28%	-0.16	1.05
JD.L	30.44%	39.06%	0.34	1.01
JMAT.L	-9.59%	28.68%	-0.06	1.05
LAND.L	-10.40%	25.47%	-0.08	0.81
LGEN.L	0.96%	34.30%	0.06	1.44
LLOY.L	-15.57%	31.22%	-0.12	1.12
MGGT.L	-10.24%	36.70%	-0.06	1.12
MNDI.L	4.62%	28.12%	0.08	1.12
MRO.L	16.88%	41.93%	0.21	1.22
MRW.L	2.66%	23.96%	0.05	0.61
NG.L	5.09%	20.41%	0.07	0.63
NWG.L	-20.09%	35.13%	-0.16	1.14
NXT.L	-4.77%	34.01%	-0.01	1.00
OCDO.L	31.64%	48.32%	0.34	0.72
PHNX.L	5.22%	25.26%	0.09	0.98
PNN.L	8.99%	23.31%	0.11	0.58
POLY.L	26.32%	33.25%	0.28	0.54
PRU.L	-1.48%	33.04%	0.04	1.52
PSN.L	12.09%	36.98%	0.16	1.12
PSON.L	-17.33%	34.78%	-0.15	0.81
RDSA.L	-2.19%	30.12%	0.02	1.34
RDSB.L	-3.50%	31.42%	0.01	1.41
REL.L	13.21%	20.33%	0.16	0.79
RIO.L	9.94%	33.70%	0.14	1.29
RKT.L	5.18%	21.57%	0.07	0.59
RMV.L	11.02%	26.95%	0.14	0.84
RR.L	-22.68%	38.34%	-0.19	1.22
RSA.L	1.40%	27.75%	0.05	0.93
RTO.L	25.52%	24.69%	0.28	0.77
SBRY.L	-3.18%	28.28%	-0.01	0.61
SDR.L	-0.16%	29.13%	0.04	1.25
SGE.L	5.18%	24.59%	0.08	0.76
SGRO.L	18.01%	21.73%	0.21	0.74
SKG.L	2.21%	35.66%	0.06	1.08
SLA.L	-11.38%	34.06%	-0.07	1.43
SMDS.L	2.39%	28.89%	0.06	1.03
SMIN.L	4.74%	28.33%	0.09	1.18
SMT.L	19.99%	24.89%	0.23	1.01
SN.L	9.47%	23.59%	0.12	0.85
SPX.L	22.19%	23.33%	0.25	0.80
SSE.L	1.08%	24.68%	0.04	0.92
STAN.L	-15.61%	33.67%	-0.11	1.31
SVT.L	6.54%	21.50%	0.09	0.58
TSCO.L	1.41%	27.73%	0.04	0.72
TW.L	4.50%	37.48%	0.08	1.04
ULVR.L	10.12%	291.46%	0.12	0.68
UU.L	2.44%	22.91%	0.04	0.60
VOD.L	-8.62%	24.71%	-0.05	0.94
WPP.L	-14.50%	30.59%	-0.11	1.10
WTB.L	-9.85%	30.14%	-0.07	0.97

Table 2 (Securities' Treynor Ratio)

Security	Treynor Ratio
POLY.L	0.49
OCDO.L	0.44
RTO.L	0.33
JD.L	0.30
HLMA.L	0.29
SPX.L	0.28
SGRO.L	0.24
AZN.L	0.24
SMT.L	0.20
ADM.L	0.18
CRDA.L	0.18
EXPN.L	0.17
AVV.L	0.17
REL.L	0.17
PNN.L	0.16
ITRK.L	0.15
DGE.L	0.15
ULVR.L	0.15
AUTO.L	0.14
BKG.L	0.14
FLTR.L	0.14
MRO.L	0.14
RMV.L	0.13
ICP.L	0.12
SVT.L	0.11
SN.L	0.11
PSN.L	0.11
GSK.L	0.11
AHT.L	0.10
CCH.L	0.09
EVR.L	0.09
RKT.L	0.09
III.L	0.09
FERG.L	0.09
HIK.L	0.08
DCC.L	0.08

NG.L	0.08
RIO.L	0.08
SGE.L	0.07
CRH.L	0.07
IHG.L	0.06
PHNX.L	0.05
BA.L	0.05
CPG.L	0.05
AAL.L	0.05
HL.L	0.04
MRW.L	0.04
TW.L	0.04
MNDI.L	0.04
UU.L	0.04
SMIN.L	0.04
BDEV.L	0.03
BHP.L	0.03
SMDS.L	0.02
ANTO.L	0.02
SKG.L	0.02
TSCO.L	0.02
FRES.L	0.02
RSA.L	0.02
SSE.L	0.01
BATS.L	0.01
LGEN.L	0.01
BNZL.L	0.00
SDR.L	0.00
INF.L	0.00
BP.L	0.00
PRU.L	-0.01
RDSA.L	-0.02
BRBY.L	-0.02
HSBA.L	-0.02
RDSB.L	-0.02
NXT.L	-0.05
GLEN.L	-0.05
SBRY.L	-0.05
AV.L	-0.07

SLA.L	-0.08
JMAT.L	-0.09
MGGT.L	-0.09
VOD.L	-0.09
IMB.L	-0.09
ABF.L	-0.10
WTB.L	-0.10
BARC.L	-0.11
BLND.L	-0.12
STAN.L	-0.12
LAND.L	-0.13
WPP.L	-0.13
CCL.L	-0.13
IAG.L	-0.14
LLOY.L	-0.14
NWG.L	-0.18
EZJ.L	-0.18
RR.L	-0.19
ITV.L	-0.19
PSON.L	-0.21
BTA.L	-0.28
CNA.L	-0.28

Table 3 (Securities' Sharpe Ratio)

Security	Sharpe Ratio
RTO.L	1.03
HLMA.L	1.03
SPX.L	0.95
SGRO.L	0.83
SMT.L	0.80
POLY.L	0.79
JD.L	0.78
EXPN.L	0.74
AZN.L	0.68
OCDO.L	0.65
REL.L	0.65
CRDA.L	0.62
DGE.L	0.57
ADM.L	0.53
ITRK.L	0.53
ICP.L	0.45
RMV.L	0.41
MRO.L	0.40
SN.L	0.40
BKG.L	0.39
GSK.L	0.39
PNN.L	0.39
AHT.L	0.38
III.L	0.38
AVV.L	0.37
AUTO.L	0.36
FERG.L	0.35
PSN.L	0.33
FLTR.L	0.33
CCH.L	0.32
SVT.L	0.30
RIO.L	0.30
DCC.L	0.29
CRH.L	0.28
EVR.L	0.27
NG.L	0.25
RKT.L	0.24

IHG.L	0.21
SGE.L	0.21
PHNX.L	0.21
BA.L	0.18
AAL.L	0.17
SMIN.L	0.17
CPG.L	0.17
MNDI.L	0.16
HL.L	0.16
HIK.L	0.15
TW.L	0.12
BHP.L	0.11
MRW.L	0.11
UU.L	0.11
SMDS.L	0.08
BDEV.L	0.08
ANTO.L	0.08
SKG.L	0.06
TSCO.L	0.05
RSA.L	0.05
SSE.L	0.04
BATS.L	0.03
ULVR.L	0.03
LGEN.L	0.03
FRES.L	0.02
BNZL.L	0.01
SDR.L	-0.01
INF.L	-0.02
BP.L	-0.02
PRU.L	-0.04
RDSA.L	-0.07
BRBY.L	-0.08
HSBA.L	-0.09
RDSB.L	-0.11
SBRY.L	-0.11
NXT.L	-0.14
GLEN.L	-0.18
IMB.L	-0.24
MGGT.L	-0.28

ABF.L	-0.30
AV.L	-0.31
WTB.L	-0.33
SLA.L	-0.33
JMAT.L	-0.33
VOD.L	-0.35
BLND.L	-0.39
LAND.L	-0.41
IAG.L	-0.41
BARC.L	-0.43
CCL.L	-0.44
STAN.L	-0.46
WPP.L	-0.47
EZJ.L	-0.47
PSON.L	-0.50
LLOY.L	-0.50
ITV.L	-0.56
NWG.L	-0.57
RR.L	-0.59
BTA.L	-0.85
CNA.L	-0.86

Table 4 (MVT portfolio composition)

MVT	Short Sell Forbidden	Short Sell allowed
Security	**Weight (X_i)**	**Weight (X_i)**
AAL.L	-	31.60%
ABF.L	-	-40.27%
ADM.L	-	47.29%
AHT.L	-	29.64%
ANTO.L	-	-18.90%
AUTO.L	-	11.22%
AV.L	-	-43.16%
AVV.L	-	-1.82%
AZN.L	9.35%	30.97%
BA.L	-	25.75%
BARC.L	-	12.03%
BATS.L	-	-22.10%
BDEV.L	-	-45.93%
BHP.L	0.00%	-4.26%
BKG.L	-	10.84%
BLND.L	-	-52.91%
BNZL.L	-	-45.41%
BP.L	-	-19.36%
BRBY.L	-	-28.64%
BTA.L	-	-52.50%
CCH.L	-	-0.93%
CCL.L	-	-35.28%
CNA.L	-	-30.48%
CPG.L	-	30.13%
CRDA.L	-	24.53%
CRH.L	-	1.78%
DCC.L	-	-18.87%
DGE.L	-	-2.21%
EVR.L	-	11.24%
EXPN.L	-	13.17%
EZJ.L	-	-5.87%
FERG.L	-	12.45%
FLTR.L	-	20.19%
FRES.L	-	-26.76%
GLEN.L	-	-17.75%
GSK.L	-	13.72%
HIK.L	-	-32.98%
HL.L	-	-3.60%
HLMA.L	13.02%	8.98%
HSBA.L	-	16.95%
IAG.L	-	-24.06%
ICP.L	-	34.83%
IHG.L	-	18.21%
III.L	-	32.55%
IMB.L	-	10.35%
INF.L	-	21.09%
ITRK.L	-	-0.53%

ITV.L	-	-26.04%
JD.L	12.20%	52.13%
JMAT.L	-	-31.41%
LAND.L	-	-27.29%
LGEN.L	-	67.06%
LLOY.L	-	-12.82%
MGGT.L	-	-38.39%
MNDI.L	-	-2.58%
MRO.L	-	26.22%
MRW.L	-	2.15%
NG.L	-	2.95%
NWG.L	-	-2.58%
NXT.L	-	-7.88%
OCDO.L	9.02%	13.64%
PHNX.L	-	26.31%
PNN.L	-	38.44%
POLY.L	17.88%	31.38%
PRU.L	-	-12.96%
PSN.L	0.00%	73.84%
PSON.L	-	-14.79%
RDSA.L	-	103.77%
RDSB.L	-	-103.03%
REL.L	-	21.98%
RIO.L	-	15.59%
RKT.L	-	7.05%
RMV.L	-	3.32%
RR.L	-	-29.55%
RSA.L	-	-11.38%
RTO.L	19.39%	42.44%
SBRY.L	-	-4.34%
SDR.L	-	-26.23%
SGE.L	-	-36.23%
SGRO.L	6.13%	121.95%
SKG.L	-	-16.25%
SLA.L	-	-48.02%
SMDS.L	-	14.67%
SMIN.L	-	2.39%
SMT.L	-	30.33%
SN.L	-	-2.69%
SPX.L	13.00%	49.48%
SSE.L	-	-10.79%
STAN.L	-	-12.79%
SVT.L	-	53.03%
TSCO.L	-	11.81%
TW.L	-	1.69%
ULVR.L	-	0.49%
UU.L	-	-59.42%
VOD.L	-	-19.20%
WPP.L	-	-32.58%
WTB.L	-	-19.83%

Table 5 (SIM portfolio composition)

SIM	Short Sell Forbidden	Short Sell allowed
Security	**Weight (X_i)**	**Weight (X_i)**
AAL.L	-	1.00%
ABF.L	-	-8.64%
ADM.L	-	12.74%
AHT.L	-	6.14%
ANTO.L	-	-0.29%
AUTO.L	-	5.34%
AV.L	-	-14.96%
AVV.L	-	3.74%
AZN.L	3.95%	14.82%
BA.L	-	2.69%
BARC.L	-	-13.53%
BATS.L	-	-1.40%
BDEV.L	-	0.07%
BHP.L	-	0.01%
BKG.L	-	5.68%
BLND.L	-	-12.53%
BNZL.L	-	-2.45%
BP.L	-	-5.43%
BRBY.L	-	-3.94%
BTA.L	-	-20.36%
CCH.L	-	5.67%
CCL.L	-	-7.57%
CNA.L	-	-16.79%
CPG.L	-	2.11%
CRDA.L	-	18.19%
CRH.L	-	5.55%
DCC.L	-	5.56%
DGE.L	-	19.16%
EVR.L	-	2.07%
EXPN.L	-	25.73%
EZJ.L	-	-7.28%
FERG.L	-	7.70%
FLTR.L	-	4.25%
FRES.L	-	-0.18%
GLEN.L	-	-4.04%
GSK.L	-	10.84%
HIK.L	-	1.46%
HL.L	-	1.33%
HLMA.L	20.40%	29.75%
HSBA.L	-	-9.61%
IAG.L	-	-7.65%
ICP.L	-	6.57%
IHG.L	-	2.75%
III.L	-	9.37%
IMB.L	-	-7.13%
INF.L	-	-2.85%

ITRK.L	-	12.59%
ITV.L	-	-11.33%
JD.L	7.59%	10.43%
JMAT.L	-	-11.46%
LAND.L	-	-12.73%
LGEN.L	-	-2.44%
LLOY.L	-	-14.11%
MGGT.L	-	-6.24%
MNDI.L	-	1.73%
MRO.L	-	4.74%
MRW.L	-	0.99%
NG.L	-	5.20%
NWG.L	-	-12.56%
NXT.L	-	-4.02%
OCDO.L	7.79%	6.27%
PHNX.L	-	3.32%
PNN.L	-	7.68%
POLY.L	15.30%	11.22%
PRU.L	-	-6.34%
PSN.L	-	4.18%
PSON.L	-	-8.84%
RDSA.L	-	-7.17%
RDSB.L	-	-8.52%
REL.L	-	22.20%
RIO.L	-	4.61%
RKT.L	-	4.55%
RMV.L	-	7.76%
RR.L	-	-11.61%
RSA.L	-	-0.99%
RTO.L	22.03%	24.85%
SBRY.L	-	-3.24%
SDR.L	-	-4.20%
SGE.L	-	3.30%
SGRO.L	7.53%	23.76%
SKG.L	-	-0.37%
SLA.L	-	-12.58%
SMDS.L	-	-0.36%
SMIN.L	-	1.80%
SMT.L	-	24.95%
SN.L	-	9.65%
SPX.L	15.40%	25.84%
SSE.L	-	-1.81%
STAN.L	-	-14.20%
SVT.L	-	6.30%
TSCO.L	-	-0.42%
TW.L	-	0.71%
ULVR.L	-	0.04%
UU.L	-	0.89%
VOD.L	-	-14.66%
WPP.L	-	-13.87%

WTB.L	-	-9.16%

Table 6 (CCM portfolio composition)

CCM	**Short Sell Forbidden**	**Short Sell allowed**
Security	**Weight (X_i)**	**Weight (X_i)**
AAL.L	-	1.07%
ABF.L	-	-13.04%
ADM.L	-	17.04%
AHT.L	-	6.66%
ANTO.L	-	-0.60%
AUTO.L	-	7.31%
AV.L	-	-12.66%
AVV.L	-	5.29%
AZN.L	-	20.67%
BA.L	-	2.92%
BARC.L	-	-13.29%
BATS.L	-	-2.42%
BDEV.L	-	-0.51%
BHP.L	-	0.20%
BKG.L	-	7.71%
BLND.L	-	-16.47%
BNZL.L	-	-3.84%
BP.L	-	-3.69%
BRBY.L	-	-4.90%
BTA.L	-	-29.03%
CCH.L	-	6.72%
CCL.L	-	-10.29%
CNA.L	-	-24.10%
CPG.L	-	2.24%
CRDA.L	-	21.87%
CRH.L	-	5.40%
DCC.L	-	6.41%
DGE.L	-	20.74%
EVR.L	-	2.57%
EXPN.L	2.91%	23.47%
EZJ.L	-	-11.76%
FERG.L	-	7.85%
FLTR.L	-	5.79%
FRES.L	-	-1.65%
GLEN.L	-	-4.83%
GSK.L	-	12.44%
HIK.L	-	1.16%
HL.L	-	1.39%
HLMA.L	26.33%	35.99%
HSBA.L	-	-7.91%
IAG.L	-	-11.31%
ICP.L	-	7.63%
IHG.L	-	3.00%
III.L	-	8.35%
IMB.L	-	-12.40%
INF.L	-	-4.00%
ITRK.L	-	15.14%

ITV.L	-	-16.65%
JD.L	3.61%	15.30%
JMAT.L	-	-13.55%
LAND.L	-	-17.83%
LGEN.L	-	-1.97%
LLOY.L	-	-17.10%
MGGT.L	-	-9.25%
MNDI.L	-	1.88%
MRO.L	-	6.30%
MRW.L	-	0.25%
NG.L	-	6.29%
NWG.L	-	-17.05%
NXT.L	-	-6.37%
OCDO.L	-	10.08%
PHNX.L	-	3.58%
PNN.L	-	10.68%
POLY.L	4.91%	18.30%
PRU.L	-	-4.00%
PSN.L	-	5.33%
PSON.L	-	-15.34%
RDSA.L	-	-5.21%
RDSB.L	-	-6.08%
REL.L	-	23.75%
RIO.L	-	5.01%
RKT.L	-	5.58%
RMV.L	-	10.01%
RR.L	-	-16.08%
RSA.L	-	-1.72%
RTO.L	24.61%	33.34%
SBRY.L	-	-6.79%
SDR.L	-	-3.34%
SGE.L	-	3.83%
SGRO.L	10.64%	29.50%
SKG.L	-	-1.05%
SLA.L	-	-11.40%
SMDS.L	-	-0.67%
SMIN.L	-	1.96%
SMT.L	7.43%	24.87%
SN.L	-	11.15%
SPX.L	19.56%	32.15%
SSE.L	-	-2.18%
STAN.L	-	-14.94%
SVT.L	-	8.22%
TSCO.L	-	-1.71%
TW.L	-	0.37%
ULVR.L	-	-0.21%
UU.L	-	0.08%
VOD.L	-	-16.25%
WPP.L	-	-16.75%
WTB.L	-	-12.67%