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ASSESSING ECONOMIC COMPLEXITY WITH INPUT-OUTPUT BASED MEASURES

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Economic complexity can be defined as the level of interdependence between the component parts of an economy. In input-output systems, intersectoral connectedness is a crucial feature of analysis, and there are many different methods for measuring it. Most of the measures, however, have drawbacks that prevent them from being used as a good indicator of economic complexity, because they were not explicitly made with this purpose in mind. In this paper, we present, discuss and compare empirically different indexes of economic complexity as intersectoral connectedness, using the interindustry tables of several OECD countries.

Keywords: *input-output analysis; intersectoral connectedness; economic complexity*

JEL classifications: C67, D57, R15

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1. INTRODUCTION

Complexity is a multidimensional phenomenon with several approaches and many theoretical definitions that we will not discuss in detail here (see Waldrop 1992; Adami 2002). Originating in the physical and biological sciences, the notion of complexity has been usefully extended to the analysis of social and economic systems (see e.g., Arthur 1999; Rosser 1999; Durlauf 2003; LeBaron and Tesfatsion 2008).

In the economic context, one interesting dimension of complexity is the level of interdependence between the component parts of an economy. The Leontief input-output model is, by its very nature, one of the best theoretical and empirical methodologies for studying this.

In fact, intersectoral connectedness is the central feature of input-output analysis, and there are, as expected, many different ways of measuring it, from the earlier and classical indicators of Chenery and Watanable (1958), Rasmussen (1956) and Hirschman (1958) to more sophisticated methods, such as the interrelatedness measure of Yan and Ames (1963), the cycling measure of Finn (1976) and Ulanovicz (1983), the dominant eigenvalue measure of Dietzenbacher (1992) and many others. Among the more recent examples of interconnectedness measures, proving the resurgence of interest in this kind of research, are the average propagation length (weighted or unweighted) proposed by Dietzenbacher and Romero (2007) and the complexity as interdependence measures of Amaral et al. (2007).

The study of economic complexity in an input-output framework has been an interesting subject for economic analysis and policy-making purposes (see e.g., Robinson and

Markandya 1973; Sonis et al. 1998; Dridi and Hewings 2002; Amaral et al. 2007). For example, in a more complex economy, the effects of (global) policy measures tend to be easily and rapidly propagated and more evenly distributed among sectors, and the same goes for unexpected (desirable or undesirable) shocks of any nature (Sonis et al. 1995, Dietzenbacher and Los 2002, Steinback 2004, Okuyama 2007).

On the other hand, one might expect the complexity of an economy to be negatively correlated with the relative weight of its so-called key sectors and this may eventually make (dominant sectors directed) policy interventions less efficient. (Laumas 1975, Dietzenbacher 1992, Sonis et al. 1995, Muñiz et al. 2008).

For understandable reasons, it is also to be expected that, in general, regional economies will be less complex than national economies, small economies less complex than large economies and open economies less complex than closed economies, but the exhaustive study of these comparisons would need careful theoretical and empirical research into areas that are well beyond the scope of this paper.

It is also predictable that the effects of measurement errors in collecting interindustry data and the robustness of input-output projections from ESA and SNA Tables are in some sense related to the complexity of an economy. This may be an important issue for empirical researchers and statistical units, and so an appropriate measure of sectoral complexity can be supplemented with these input-output tables, in line with the robustness measure proposed by Wolff (2005).

The intersectoral measures of complexity analyzed and quantified in this paper can also be useful in other fields of research, namely for studying the ecological complexity of natural (living) systems (Finn 1976, Zucchetto 1981, Bosserman 1982, Ulanovicz 1983) and the complexity of social networks (Wasserman and Faust 1994, Jackson 2006).

These measures were chosen from among those input-output methodologies directly giving (or making it possible to deduce) holistic indexes of connectedness that can be considered good indicators or proxies of complexity as sectoral interdependence. In order to fully understand and quantify economic complexity in this sense, these measures should be complemented with other forms of uncovering structures, such as qualitative input-output analysis based on the theory of directed graphs (Czamansky 1974, Campbell 1975, Aroche-Reys 2003), minimum flow analysis (Schnable 1994, 1995), fields of influence and feedback loops analysis (Sonis and Hewings 1991, Sonis et al. 1997, van der Linden et al. 2000), the concept of important coefficients (Jensen and West 1980, Aroche-Reys 1966), the fundamental economic structure approach (Simpson and Tsukui 1965, Jensen et al. 1987, Thakur 2008), and the neural network approach to input-output analysis (Wang 2001), among others.

The structure of this paper is as follows: in section 2, the measures of complexity are presented and briefly discussed; in section 3, a detailed quantification is made of economic complexity as connectedness, applying the rich menu of (input-output) measures presented in the previous section and confronting them empirically, using the interindustry tables of several OECD countries; and section 4 concludes the paper.

2. MEASURES OF INPUT-OUTPUT CONNECTEDNESS

There are several measures of connectedness in input-output analysis. Although not explicitly made for this purpose, they can be considered as alternative measures of economic complexity as sectoral interrelatedness. And it is also an interesting exercise *per se* to rank economies according to the level of interrelatedness obtained for each of them.

In this section, we present a (not exhaustive) list of measures, from the traditional ones to some that are more recent and more theoretically elaborate. Most of these measures have been proposed by authors writing in the field of economics, but there are also some that have been proposed by biologists and have an ecological content (useful surveys of some of these measures are to be found in Hamilton and Jensen 1983, Szyrmer 1985, Basu and Johnson 1996, Cai and Leung 2004, Amaral et al. 2007).

One of the first indicators of the connectedness of an input-output system is the Percentage Intermediate Transactions (M1 – PINT) of Chenery and Watanable (1958), defined as "the percentage of the production of industries in the economy which is used to satisfy needs for intermediate inputs", and defined as:

$$\mathrm{PINT} = 100 \frac{i' \mathbf{A} x}{i' x} \tag{1}$$

where **A** is the production (technical) coefficients matrix, x is the vector of sectoral gross outputs, i is a unit vector of appropriate dimension, and ′ means transpose.

Another classical measure of connectedness is the Average Output Multiplier (M2 – AVOM), based on Rasmussen (1956) and Hirschman (1958):

$$\text{AVOM} = \frac{1}{n} i'(\mathbf{I} - \mathbf{A})^{-1} i \qquad (2)$$

with *n* representing the number of sectors and **I** the unit matrix.

A similar measure is used by Blin and Murphy (1974), with n^2 in the denominator.

Useful only in highly disaggregated matrices is the Percentage of Nonzero Coefficients measure (M3 – PNZC) of Peacock and Dosser (1957):

$$\text{PNZC} = \frac{100}{n^2} i'\mathbf{K}i \qquad (3)$$

where **K** is a Boolean matrix, such as: $k = [k_{ij}], k_{ij} = \begin{cases} 1, a_{ij} \neq 0 \\ 0, \textit{otherwise} \end{cases}$

A simple but useful measure is the Mean Intermediate Coefficients Total per Sector (M4 – MICT, Jensen and West 1980):

$$\text{MIPS} = \frac{1}{n} i'\mathbf{A}i \qquad (4)$$

Based on the work of Wang (1954) and Lantner (1974) is the idea that the smaller the value of the determinant of the Leontief matrix, |**I**-**A**|, the larger the elements of the Leontief inverse and the interrelatedness of the IO system, and so we can use the (Inverse) Determinant measure (M5 – IDET):

$$\mathrm{IDET} = \frac{1}{|\mathbf{I} - \mathbf{A}|} \tag{5}$$

A more elaborate measure is the Yan and Ames (1963) interrelatedness measure (M6 – YAAM), defined as:

$$\mathrm{YAAM} = \frac{1}{n^2} \sum_{i,j} \frac{1}{\mathbf{O}_{ij}^{YA}} \tag{6}$$

where $\mathbf{O}_{ij}^{YA}$ is the Order Matrix, with each entry representing the smallest order of interrelatedness between i and j, i.e. given the series $\mathbf{A}$, $\mathbf{A}^2$, $\mathbf{A}^3$, …, $\mathbf{A}^k$, and with k consisting of the exponent necessary to convert the corresponding cell to nonzero.

Dietzenbacher (1992) proposed as an alternative measure of connectedness the Dominant Eigenvalue of Matrix **A** (M7 – DEVA):

$$\mathrm{DEVA} = \lambda \tag{7}$$

with λ being the dominant eigenvalue of matrix **A**. This measure was recently used and refined by Midmore et al. (2006).

Of particular importance for the study of ecological systems are the following measures of connectedness proposed by Finn (1976) and Ulanovics (1983): the Mean Path Length and the Cycling Index.

The Mean Path Length (M8 – MPLE) is:

$$\mathrm{MPLE} = \frac{i'\mathbf{X}i}{i'y} \tag{8}$$

where $i'\mathbf{X}i = t$ is the total system (gross) output and $i'y$ is the system's final demand flow (with y representing the vector of sectoral final demands).

The Cycling Index (M9 – CYCI) is:

$$\text{CYCI} = \frac{b}{t} \tag{9}$$

where: $b = \sum_j (1 - \frac{1}{l_{jj}}) x_j$ is the sum of the cycling flows, l_{ii} are the main diagonal elements of the Leontief inverse matrix and t was defined above.

A recent measure of input-output connectedness that can be used as an indicator of economic complexity is the Average Path Length (unweighted or weighted) proposed in Dietzenbacher et al. (2005) and Dietzenbacher and Romero (2007).

This measure is based on matrices $\mathbf{L} = (\mathbf{I}\text{-}\mathbf{A})^{-1}$ and $\mathbf{H}$, with $\mathbf{H}$ being defined as:

$$\mathbf{H} = 1 \times \mathbf{A} + 2 \times \mathbf{A}^2 + 3 \times \mathbf{A}^3 + \ldots$$

Dietzenbacher and Romero (2007) show that:

$$\mathbf{H} = \mathbf{L}(\mathbf{L} - \mathbf{I})$$

and define the Sectoral Average Propagation Lengths (APLs, which we can represent on a *n*x*n* matrix; let us call it the **APL** matrix):

$$APL_{ij} = \frac{h_{ij}}{l_{ij}}, for : i \neq j$$

$$APL_{ii} = APL_{jj} = \frac{h_{ii}}{(l_{ii}-1)} = \frac{h_{jj}}{(l_{jj}-1)}, for : i = j$$

These values are the base of the M10 - APLU: Average Propagation Lengths (Unweighted) measure:

$$\frac{1}{n}\sum_i \left[\frac{1}{n}\sum_j APL_{ij}\right] = \frac{1}{n}\sum_j \left[\frac{1}{n}\sum_i APL_{ij}\right] \tag{10}$$

Another recent measure, explicitly made for quantifying economic complexity as input-output interdependence, is proposed by Amaral et al. (2007), based on Amaral (1999).

This measure considers i) a "network" effect, which gives the extent of the direct and indirect connections of each part of the system with the other parts, and where more connections corresponds to more complexity; and ii) a "dependency" effect, i.e. how much of the behavior of each part of the system is determined by internal connections between the elements of that part – which means more autonomy and less dependency – and how much of that behavior is determined by external relations, i.e. relations with other parts of the system – which means less autonomy and greater dependency.

A brief description of this measure is presented here, closely following the work of Amaral et al. (2007).

Consider a system represented by a square matrix **A**, of order *N* and with all values non negative. A part of the system of order *m* (*m* = 1, …, *N*-1), is a square block **A*** of order *m*, which has its main diagonal formed by *m* elements of the main diagonal of **A**.

Let **A*** be a part of the system. For example:

$$\mathbf{A}^* = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

A* can be considered a sub-system of system **A**. This sub-system is the more autonomous (or, equivalently, the less dependent) the greater the values of its elements (a_{11}, a_{12}, a_{21}, a_{22}) relative to the elements (a_{1j}, a_{2j}, a_{j1} a_{j2}), for all $j>2$.

In order to measure the greater or lesser autonomy of the sub-system **A***, the degree of autonomy of sub-system **A*** can be defined as:

$$G_a(\mathbf{A}^*) = \frac{\|\mathbf{A}^*\|}{\|\mathbf{A}^*\| + \|\mathbf{A}^{**}\| + \|\mathbf{A}^{***}\|}$$

where $\|\mathbf{M}\|$ represents the "sum of the elements of matrix **M**", **A**** is the block of all the elements of the columns belonging to **A*** with the exception of the elements of **A*,** and **A***** represents the same thing for the rows. For example, if **A*** is the block defined above:

$$\|\mathbf{A}^{**}\| = \sum(a_{j1} + a_{j2}) \text{ and } \|\mathbf{A}^{***}\| = \sum(a_{1j} + a_{2j}) \text{ for } j = 3, 4, \ldots, N$$

Based on the degree of autonomy, a degree of block dependency can be defined as:

$$G_d(\mathbf{A}^*) = 1 - G_a(\mathbf{A}^*)$$

It is easy to see that in a matrix **A** of order N there are $2^N - 2$ blocks **A*** (because there are $\sum\binom{N}{k}$ blocks **A*** with $k = 1, \ldots, N\text{-}1$).

So, the degree of (raw) dependency of system **A** is defined as:

$$G^*(\mathbf{A}) = \frac{\sum_k G_d(\mathbf{A}_k{}^*)}{2^N - 2}$$

for which k varies from 1 to $2^N - 2$ and $\mathbf{A_k}$***** represents a square block that includes the main diagonal.

After correcting by the scaling factor given by the maximum value of $G^*(\mathbf{A})$ (which is a function of N):

$$\frac{2^N - 2^{N-2} - 1}{2^N - 2}$$

the **dependency degree** $G(\mathbf{A})$ of **A** is:

$$G(\mathbf{A}) = \frac{(2^N - 2)G^*(\mathbf{A})}{2^N - 2^{N-2} - 1}$$

The **network effect indicator**, $H(\mathbf{A})$ is:

$$H(\mathbf{A}) = 1 - h(\mathbf{A})$$

with $h(\mathbf{A}) = \frac{Z(\mathbf{A})}{N^2 - N}$,

in which $Z(\mathbf{A})$ is the number of zeros of matrix $\mathbf{L} = (\mathbf{I}\text{-}\mathbf{A})^{-1}$.

Finally, the index of complexity as interdependence, combining the dependency and the network effects is:

$$I(\mathbf{A}) = G(\mathbf{A}) \text{ x } H(\mathbf{A}) \tag{11}$$

This measure can be based on the technical coefficients matrix, **A** (M11 – CAIA) or on the Leontief inverse, substituting, in $G(\mathbf{A})$, **L** for **A** (M12 – CAIL).

3. MEASURING CONNECTEDNESS AND COMPLEXITY WITH OECD IO DATA

From the previous section, we end up with 12 measures of complexity as input-output connectedness, listed in the table presented in Appendix 1.

In this section, we present the results of an empirical application of all these measures using the Input-Output Tables of nine OECD economies in the early 1970s and early 1990s.

For the convenience of analysis, the original data is aggregated into the 17 sectors presented in Appendix 2.

Tables 1 and 2 show the main results, i.e. the values of all the measures for all the countries in the early 1970s and early 1990s.

INSERT TABLES 1 AND 2

A broad inspection of these values confirms the expected conclusions that the large economies (Japan and USA) were in fact more complex, and that smaller economies tended to be less complex (the Netherlands and Denmark), both in the 1970s and 1990s.

This can be clearly seen in Tables 3 and 4, where we present the rankings of countries for each measure (9 points for the largest value, 1 point for the smallest) and the final ranking based on all the measures (total number of points and relative position of each country).

INSERT TABLES 3 AND 4

Looking at the absolute values of the connectedness measures and their percentage changes (Table 5) we also note a slight (and perhaps unexpected) reduction in the average economic complexity, with a decreasing dispersion of countries along the "interrelatedness scale function" but no significant relative changes, except in the UK, which rose from 8^{th} in the 1970s to 4^{th} in the 1990s.

INSERT TABLE 5

A closer inspection of the absolute values and rankings calls for a careful association of the measures corresponding to different methodologies or conceptualizations of economic complexity. This task can be better accomplished by analyzing the correlation coefficients presented in Tables 6 and 7 and using the following definitions and results.

INSERT TABLES 6 AND 7

Let M be the set of the measures m_i, r(i,j) the absolute value of the correlation coefficient between m_i and m_j, and c the number $0 \leq c \leq 1$.

Definition 1: A **bundle B** of measures of M is a set of elements of M such that, for every pair (m_i , m_j) of B, we have r(i,j) ≥ c and, for every m_k of M-B, we have at least one m_i of B such that r(i,k) < c.

Two bundles B_1 and B_2 are **perfectly separated** when, for every m_k of B_1, we have r(k,i) < c for every m_i of B_2.

Definition 2: An **isolated measure** m_l is one in which the bundle where it belongs is the **degenerate** bundle $\{m_l\}$.

It is easy to see that the family of bundles of the measures of M is a partition of M as the union of disjoint sets. However the set M may be partitioned in several ways.

Assumption (emergent concepts*):* For a set M that is partitioned into perfectly separated bundles, each bundle B is interpreted as the emergence at the surface of a hidden concept of interrelatedness.

When the bundles are not perfectly separated, the hidden concepts of interrelatedness are called fuzzy concepts.

It is easy to see that if there is a perfectly separated partition it is the only perfectly separated partition that exists.

Applying these concepts to the results of Tables 6 and 7, and taking as the value of *c* for each of the years respectively the average of all the correlation coefficients, we have two perfectly separated bundles for the 1990s:

B_1 = {PINT, AVOM, MICT, IDET, DEVA, MPLE, CYCI, APLU, CAIA, CAIL}

B_2 ={ PNZC, YAAM}.

This result indicates a clear distinction (at this level of aggregation – 17 sectors) between measures based on Boolean (B_2) and non-Boolean (B_1) methods, which is probably only interesting, or useful, for highly disaggregated matrices.

Another, more useful distinction is obtained by considering, within the domain of strongly correlated non-Boolean measures, the correlation coefficients with positive and negative signs, pointing to a further separation of bundles of this kind:

B_{11} = {PINT, AVOM, MICT, IDET, DEVA, MPLE, APLU, CAIL}

B_{12} = {CYCI, CAIA}

The closely related behavior of measures CYCI and CAIA is supposedly explained by the fact that they explicitly exclude (direct) intra-dependence flows (the values of self-supplying inputs or the coefficients in the main diagonal of matrix A), and that, in the sense of complexity as (sectoral) interdependence, these are probably the most appropriate measures.

To explore this distinction further, it is useful to calculate the rankings of economic complexity based on bundle B_{11} (Tables 8 and 9) and bundle B_{12} (Tables 10 and 11).

INSERT TABLES 8, 9, 10 AND 11

The interesting result is, of course, that, according to this particular notion of complexity as interdependence, large economies appear to be less complex than small ones and complexity does not necessarily increase as economies grow and develop. But the full understanding of all the forces behind this surprising result would require further research.

Looking at the correlation coefficients of the early 1970s, a degenerate bundle exists with the isolated measure M11 - CAIA, pointing to an autonomous emergent concept of economic complexity, which did not persist into the 1990s.

4. CONCLUDING REMARKS

Connectedness is a crucial feature of input-output analysis, which can be used for studying economic complexity as sectoral interdependence.

There are many ways to quantify connectedness, and it is a useful exercise to compare different measures, both theoretically and empirically.

In this paper, a menu of twelve measures is presented and briefly discussed. All these measures are quantified using an input-output database of nine OECD countries in the early 1970s and 1990s, which gives us an interesting inter-country comparison and shows us two decades in the evolution of economic complexity as sectoral interdependence.

Looking at the absolute values of the measures, it appears that large economies (Japan and USA) are more "intensely connected" (and so, more complex) than small ones (the Netherlands, Denmark). It also appears that there is a slight reduction in complexity and a decreasing dispersion of countries along the "interrelatedness scale", with one peculiar exception of complex upgrading (the UK).

A closer inspection of the values, applying a method of identifying emergent concepts using the correlation coefficients, points to the emergence of three bundles of measures: a Boolean-based group of two measures with weak correlation with all the others; a group of eight measures based on all technical coefficients (and production multipliers), with strong positive correlations between them and weak positive correlations with the Boolean group; a bundle of two measures that explicitly exclude intra-sectoral flows, negatively correlated with all the others, but probably the most appropriate for measuring complexity as (sectoral) interdependence.

According to the majority bundle of (more conventional) measures of connectedness, large economies seem to be more complex than small ones. The bundle of two measures excluding (direct) intra-sectoral flows, on the other hand, points to the opposite conclusion, but this surprising result needs to be confirmed with further theoretical and empirical research.

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APPENDIX 1. Input-Output Connectedness Measures

Number:	Designation:	Formula:	Proponents:
M1	PINT	$100\frac{i'Ax}{i'x}$	Chenery and Watanable (1958)
M2	AVOM	$\frac{1}{n}i'(I-A)^{-1}i$	Rasmussen-Hirschman (1958)
M3	PNZC	$\frac{100}{n^2}i'Ki$	Peacock and Dosser (1957)
M4	MICT	$\frac{1}{n}i'Ai$	Jensen and West (1980)
M5	IDET	$\frac{1}{\lvert I-A \rvert}$	Wang(1954) Lantner(1974)
M6	YAAM	$\frac{1}{n^2}\sum_{i,j}\frac{1}{O_{ij}^{YA}}$	Yan and Ames (1963)
M7	DEVA	λ : dominant eigenvalue of A	Dietzenbacher (1992)
M8	MPLE	$\frac{i'Xi}{i'y}$	Finn (1976) Ulanovicz(1983)
M9	CYCI	$\frac{b}{t}$	Finn (1976) Ulanovicz(1983)
M10	APLU	$\frac{1}{n^2}\sum_i\left[\sum_j APL_{ij}\right]$	Dietzenbacher (2007)
M11	CAIA	$G(A)\times H(A)$ A_{based}	Amaral, Dias and Lopes (2007)
M12	CAIL	$G(L)\times H(L)$ L_{based}	Amaral, Dias and Lopes (2007)

APPENDIX 2. Sectors used in analysis

1	Agriculture, mining & quarrying
2	Food, beverages & tobacco
3	Textiles, apparel & leather
4	Wood and paper
5	Chemicals, drugs, oil and plastics
6	Minerals and metals
7	Electrical and non-electrical equipment
8	Transport equipment
9	Other manufacturing
10	Electricity, gas & water
11	Construction
12	Wholesale & retail trade
13	Restaurants & hotels
14	Transport & storage
15	Communication
16	Finance & insurance
17	Other sectors

TABLE 1. Connectedness Measures – values from the early 1970s

Country	Year	PINT	AVOM	PNZC	MICT	IDET	YAAM	DEVA	MPLE	CYCI	APLU	CAIA	CAIL
Australia	1968	44.667	1.742	91.004	0.420	8.539	0.955	0.428	1.807	0.864	2.188	0.747	0.406
Canada	1971	42.287	1.685	100.000	0.395	5.231	1.000	0.416	1.733	0.899	2.041	0.779	0.404
Denmark	1972	31.755	1.473	99.654	0.318	3.381	0.998	0.329	1.465	0.926	1.705	0.786	0.328
France	1972	41.030	1.678	96.886	0.407	8.639	0.984	0.395	1.691	0.869	1.959	0.737	0.381
Germany	1978	40.939	1.757	99.308	0.424	10.537	0.997	0.458	1.693	0.857	2.046	0.732	0.402
Japan	1970	50.524	1.956	97.232	0.484	15.087	0.986	0.501	2.021	0.825	2.272	0.740	0.450
Netherlands	1972	29.758	1.449	91.350	0.304	4.298	0.957	0.368	1.424	0.909	1.735	0.754	0.301
UK	1968	37.560	1.683	93.426	0.393	9.180	0.967	0.427	1.602	0.868	2.014	0.729	0.379
USA	1972	41.916	1.898	100.000	0.478	18.285	1.000	0.465	1.722	0.858	2.187	0.712	0.430

TABLE 2. Connectedness Measures – values from the early 1990s

Country	Year	PINT	AVOM	PNZC	MICT	IDET	YAAM	DEVA	MPLE	CYCI	APLU	CAIA	CAIL
Australia	1989	38.391	1.722	100.000	0.429	6.629	1.000	0.399	1.623	0.897	1.919	0.767	0.410
Canada	1990	40.764	1.686	100.000	0.399	6.068	1.000	0.414	1.688	0.891	2.037	0.765	0.400
Denmark	1990	31.656	1.532	99.654	0.356	3.638	0.998	0.315	1.463	0.922	1.725	0.791	0.355
France	1990	37.051	1.683	95.848	0.416	8.087	0.979	0.411	1.586	0.883	1.914	0.748	0.386
Germany	1990	41.064	1.769	99.654	0.446	8.737	0.998	0.416	1.697	0.864	1.963	0.753	0.416
Japan	1990	45.999	1.912	95.502	0.483	18.497	0.978	0.473	1.852	0.844	2.212	0.719	0.432
Netherlands	1986	29.986	1.473	91.696	0.324	3.601	0.959	0.330	1.428	0.921	1.728	0.782	0.324
UK	1990	40.389	1.743	100.000	0.428	9.562	1.000	0.421	1.678	0.859	1.944	0.737	0.403
USA	1990	40.153	1.849	100.000	0.468	12.871	1.000	0.442	1.671	0.864	2.077	0.732	0.429

TABLE 3. Connectedness Measures – rankings from the early 1970s

Country	Year	PINT	AVOM	PNZC	MICT	IDET	YAAM	DEVA	MPLE	*CYCI*	APLU	*CAIA*	CAIL	Total	**FR**
Australia	1968	8	6	1	6	4	1	6	8	*4*	8	*6*	7	65.0	**5**
Canada	1971	7	5	8.5	4	3	8.5	4	7	*7*	5	*8*	6	73.0	**3**
Denmark	1972	2	2	7	2	1	7	1	2	*9*	1	*9*	2	45.0	**7**
France	1972	5	3	4	5	5	4	3	4	*6*	3	*4*	4	50.0	**6**
Germany	1978	4	7	6	7	7	6	7	5	*3*	6	*3*	5	66.0	**4**
Japan	1970	9	9	5	9	8	5	9	9	*1*	9	*5*	9	87.0	**1**
Netherlands	1972	1	1	2	1	2	2	2	1	*8*	2	*7*	1	30.0	**9**
UK	1968	3	4	3	3	6	3	5	3	*5*	4	*2*	3	44.0	**8**
USA	1972	6	8	8.5	8	9	8.5	8	6	*2*	7	*1*	8	80.0	**2**
Total		45	45	45	45	45	45	45	45	***45***	45	***45***	45		

TABLE 4. Connectedness Measures – rankings from the early 1990s

Country	Year	PINT	AVOM	PNZC	MICT	IDET	YAAM	DEVA	MPLE	*CYCI*	APLU	*CAIA*	CAIL	Total	**FR**
Australia	1989	4	5	7.5	6	4	7.5	3	4	*7*	4	*7*	6	65.0	**6**
Canada	1990	7	4	7.5	3	3	7.5	5	7	*6*	7	*6*	4	66.5	**5**
Denmark	1990	2	2	5	2	2	4.5	1	2	*9*	1	*9*	2	41.0	**8**
France	1990	3	3	3	4	5	3	5	3	*5*	3	*4*	3	43.5	**7**
Germany	1990	8	7	5	7	6	4.5	6	8	*2*	6	*5*	7	71.0	**3**
Japan	1990	9	9	2	9	9	2	9	9	*1*	9	*1*	9	78.0	**2**
Netherlands	1986	1	1	1	1	1	1	2	1	*8*	2	*8*	1	28.0	**9**
UK	1990	6	6	7.5	5	7	7.5	7	6	*3*	5	*3*	5	68.0	**4**
USA	1990	5	8	7.5	8	8	7.5	8	5	*4*	8	*2*	8	79.0	**1**
Total		45	45	45	45	45	45	45	45	45	45	45	45		

TABLE 5. Percent changes of absolute values, between the 1970's and 1990's

Country	PINT	AVOM	PNZC	MICT	IDET	YAAM	DEVA	MPLE	CYCI	APLU	CAIA	CAIL
Australia	-14.051	-1.179	9.886	1.987	-22.371	4.710	-6.872	-10.187	3.828	-12.264	2.669	1.103
Canada	-3.601	0.086	0.000	1.099	16.001	0.000	-0.556	-2.570	-0.205	-0.205	-1.809	-1.043
Denmark	-0.312	3.975	0.000	11.956	7.596	0.000	-4.353	-0.145	1.179	1.179	0.703	8.362
France	-9.699	0.350	-1.071	2.007	-6.387	-0.527	4.140	-6.182	-2.304	-2.304	1.562	1.320
Germany	0.304	0.681	0.348	5.249	-17.082	0.174	-9.262	0.211	-4.053	-4.053	2.887	3.639
Japan	-8.956	-2.266	-1.779	-0.178	22.601	-0.877	-5.684	-8.379	-2.632	-2.632	-2.799	-4.187
Netherlands	0.764	1.627	0.379	6.547	-16.214	0.181	-10.211	0.325	-0.405	-0.405	3.712	7.702
UK	7.533	3.554	7.037	8.853	4.170	3.399	-1.262	4.747	-3.465	-3.465	1.117	6.333
USA	-4.205	-2.560	0.000	-1.991	-29.611	0.000	-4.960	-2.945	-5.037	-5.037	2.817	-0.282

TABLE 6. Correlation coefficients – 1970's

Country	PINT	AVOM	PNZC	MICT	IDET	YAAM	DEVA	MPLE	CYCI	APLU	CAIA	CAIL
PINT	1.000	0.910	0.181	0.905	0.665	0.181	0.848	0.992	-0.847	0.938	-0.370	0.951
AVOM	0.910	1.000	0.277	0.993	0.906	0.277	0.948	0.897	-0.912	0.956	-0.635	0.972
PNZC	0.181	0.277	1.000	0.294	0.250	1.000	0.142	0.152	-0.001	0.080	0.112	0.341
MICT	0.905	0.993	0.294	1.000	0.905	0.294	0.921	0.881	-0.904	0.949	-0.659	0.972
IDET	0.665	0.906	0.250	0.905	1.000	0.250	0.850	0.656	-0.838	0.796	-0.817	0.797
YAAM	0.181	0.277	1.000	0.294	0.250	1.000	0.142	0.152	-0.001	0.080	0.112	0.341
DEVA	0.848	0.948	0.142	0.921	0.850	0.142	1.000	0.845	-0.928	0.923	-0.668	0.900
MPLE	0.992	0.897	0.152	0.881	0.656	0.152	0.845	1.000	-0.844	0.919	-0.334	0.925
CYCI	-0.847	-0.912	-0.001	-0.904	-0.838	-0.001	-0.928	-0.844	1.000	-0.881	0.749	-0.843
APLU	0.938	0.956	0.080	0.949	0.796	0.080	0.923	0.919	-0.881	1.000	-0.568	0.961
CAIA	-0.370	-0.635	0.112	-1.659	-0.817	0.112	-0.668	-0.334	0.749	-0.568	1.000	-0.489
CAIL	0.951	0.972	0.341	0.972	0.797	0.341	0.900	0.925	-0.843	0.961	-0.489	1.000

Note: mean absolute values below main diagonal = 0.637

TABLE 7. Correlation coefficients – 1990's

Country	PINT	AVOM	PNZC	MICT	IDET	YAAM	DEVA	MPLE	CYCI	APLU	CAIA	CAIL
PINT	1.000	0.943	0.357	0.914	0.834	0.357	0.957	0.937	-0.931	0.938	-0.913	0.966
AVOM	0.943	1.000	0.351	0.991	0.909	0.351	0.957	0.937	-0.931	0.938	-0.913	0.966
PNZC	0.357	0.351	1.000	0.398	0.012	1.000	0.224	0.301	-0.222	0.221	-0.087	0.553
MICT	0.914	0.991	0.398	1.000	0.873	0.398	0.931	0.900	-0.913	0.890	-0.888	0.973
IDET	0.834	0.909	0.012	0.873	1.000	0.012	0.875	0.860	-0.895	0.889	-0.933	0.777
YAAM	0.357	0.351	1.000	0.398	0.012	1.000	0.224	0.301	-0.222	0.221	-0.087	0.553
DEVA	0.957	0.957	0.224	0.931	0.875	0.224	1.000	0.943	-0.937	0.959	-0.940	0.907
MPLE	0.937	0.937	0.301	0.900	0.860	0.301	0.943	1.000	-0.910	0.958	-0.851	0.907
CYCI	-0.931	-0.931	-0.222	-0.913	-0.895	-0.222	-0.937	-0.910	1.000	-0.868	0.962	-0.857
APLU	0.938	0.938	0.221	0.890	0.889	0.221	0.959	0.958	-0.868	1.000	-0.872	0.888
CAIA	-0.913	-0.913	-0.087	-0.888	-0.933	-0.087	-0.940	-0.851	0.962	-0.872	1.000	-0.803
CAIL	0.966	0.966	0.553	0.973	0.777	0.553	0.907	0.907	-0.857	0.888	-0.803	1.000

Note: mean absolute values below main diagonal = 0.719

TABLE 8. Connectedness Measures - rankings from the early 1970s – B_{11} (all interindustry flows)

Country	Year	PINT	AVOM	MICT	IDET	DEVA	MPLE	APLU	CAIL	Total	**FR**
Australia	1989	4	5	6	4	3	4	4	6	36.0	**6**
Canada	1990	7	4	3	3	5	7	7	4	39.5	**5**
Denmark	1990	2	2	2	2	1	2	1	2	14.0	**8**
France	1990	3	3	4	5	5	3	3	3	28.5	**7**
Germany	1990	8	7	7	6	6	8	6	7	55.0	**3**
Japan	1990	9	9	9	9	9	9	9	9	72.0	**1**
Netherlands	1986	1	1	1	1	2	1	2	1	10.0	**9**
UK	1990	6	6	5	7	7	6	5	5	47.0	**4**
USA	1990	5	8	8	8	8	5	8	8	58.0	**2**
Total		45	45	45	45	45	45	45	45		

TABLE 9. Connectedness Measures – rankings from the early 1990s: B_{11} (all interindustry flows)

Country	Year	PINT	AVOM	MICT	IDET	DEVA	MPLE	APLU	CAIL	Total	**FR**
Australia	1989	4	5	6	4	3	4	4	6	36.0	**6**
Canada	1990	7	4	3	3	5	7	7	4	39.5	**5**
Denmark	1990	2	2	2	2	1	2	1	2	14.0	**8**
France	1990	3	3	4	5	5	3	3	3	28.5	**7**
Germany	1990	8	7	7	6	6	8	6	7	55.0	**3**
Japan	1990	9	9	9	9	9	9	9	9	72.0	**1**
Netherlands	1986	1	1	1	1	2	1	2	1	10.0	**9**
UK	1990	6	6	5	7	7	6	5	5	47.0	**4**
USA	1990	5	8	8	8	8	5	8	8	58.0	**2**
Total		45	45	45	45	45	45	45	45		

TABLE 10. C.M. – rankings from the early 1970s: B_{12}: (off-main diagonal flows)

Country	Year	CYCI	CAIA	Total	**FR**
Australia	1968	*4*	*6*	10.0	**4**
Canada	1971	*7*	*8*	15.0	**2**
Denmark	1972	*9*	*9*	18.0	**1**
France	1972	*6*	*4*	10.0	**4**
Germany	1978	*3*	*3*	6.0	**7**
Japan	1970	*1*	*5*	6.0	**7**
Netherlands	1972	*8*	*7*	15.0	**2**
UK	1968	*5*	*2*	7.0	**6**
USA	1972	*2*	*1*	3.0	**9**
Total		*45*	*45*		

TABLE 11. C.M. – rankings from the early 1990s: B_{12}: (off-main diagonal flows)

Country	Year	CYCI	CAIA	Total	**FR**
Australia	1989	*7*	*7*	14.0	**3**
Canada	1990	*6*	*6*	12.0	**4**
Denmark	1990	*9*	*9*	18.0	**1**
France	1990	*5*	*4*	9.0	**5**
Germany	1990	*2*	*5*	7.0	**6**
Japan	1990	*1*	*1*	2.0	**9**
Netherlands	1986	*8*	*8*	16.0	**2**
UK	1990	*3*	*3*	6.0	**7**
USA	1990	*4*	*2*	6.0	**7**
Total		45	45		