Lesson study and curriculum development

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Lesson study is a teacher development process focused on professional practice that is receiving increasing recognition in many countries around the world. Although very popular in mathematics, there are experiences in many other school subjects, both for in-service and pre-service teachers. Originated from Japan, it receives local adaptations regarding the national cultures and conditions and also the agendas of those that promote it. Lesson studies constitute reflexive and collaborative activities as teachers work together, identifying students' difficulties on a given topic or issue, documenting curriculum objectives and guidelines, analyzing tasks, and planning what those teachers regard as an "exemplary" lesson. This lesson (called "research lesson") is taught by one member of the group whereas the other participants observe it with a focus on student learning. So, the teachers seek to verify to what extent this lesson achieved the sought objectives and what difficulties arose. We may regard this process as a small investigation of the teachers on their own professional practice. Lesson studies take into account the curricular guidelines and the research results on the topic or issue under study. In this paper our aim is to discuss how teachers consider curriculum guidelines in three lesson study experiences carried out in mathematics, with special attention to teachers' views on the nature of the tasks, students' reasoning processes, and classroom communication.

The methodology is qualitative, using participant observation, as practitioner research. The participants are three groups of teachers from a cluster of schools in Lisbon, at 1st, 2nd and 3rd cycles of basic education(corresponding to grades 1 to 9), and the four authors of this paper. Three of the authors conducted the lesson studies sessions and one author was responsible for data collection which included research journal, video recording with transcription of the sessions, teachers' written reflections, and individual interviews. The lesson study at grade 3 focused on representing rational numbers as fractions and on the number line, at grade 5 focused on ordering and comparing rational numbers, and at grade 7 focused on solving first degree equations.

The results show that the current Portuguese curriculum documents (Programa de Matemática / Metas de Aprendizagem) were very important to map the development of the topics under study. However, the teachers did not agree with the formalistic orientation of these curriculum documents, and decided to emphasize intuitive elements such as connections with students' reality and pictorial representations. With their participation in the lesson study, the teachers developed a sense for the importance of the wording of tasks and for possible students' difficulties in solving them. The teachers recognized that tasks with some degree of challenge were important to stimulate students' reasoning and that in such cases unforeseen responses could be given by students. The teachers also got a clearer notion of students' difficulties in reasoning processes (especially generalizing and justifying) and to aspects of classroom communication that promote or inhibit such processes. As teachers did not identify themselves with the curriculum documents, these did not play a significant role, besides the initial planning of the research lesson.

Introduction

Lesson study is a teacher professional development process that originated in Japan, in the beginning of the 20th century, and is receiving great attention in many countries around the world. It is carried out in the context of professional practice and has a strong collaborative and reflexive nature (Fernandez, Cannon & Chokshi, 2003; Lewis, Perry & Hurd, 2009; Perry & Lewis, 2009). There are many lesson studies carried in mathematics, and also in other school subjects, such as science, social science, and language. There are experiences at all school levels, from infant school to university and both with in-service and prospective teachers (Burroughs & Luebeck, 2010). There is an international organization – The World Association of Lesson Studies (WALS) (http://www.walsnet.org) – which aim is to promote the exchange of experiences and the collaboration of researchers, teacher educators, and teachers aiming to develop lesson studies (Clivaz, 2015).

In a lesson study, teachers work together, identifying students' difficulties on a given concept or curricular issue, study curriculum documents and research results, analyse tasks, and plan a "research lesson" that they expect will help students to learn the sought objectives (Murata, 2011). The leader may be a practicing teacher or an external person, as a university teacher educator or researcher. We regard this process as a small investigation of the participants on their own professional practice given the fact that this work develops from an issue identified as a professional problem, stands on studying the related literature, draws on collecting relevant data (the observation from the lesson), and seeks to arrive at conclusions by careful analysis of this data (Ponte, 2002; Zeichner, 2001).

Lesson studies yield a strong combination of practice and theory as they promote a close look at students' thinking at the same time as the need to take into account the curricular guidelines and other curricular documents and the research results on the concept or issue under study. In this paper, our aim is to discuss how participants in three lesson studies carried out in Portugal in the subject of mathematics deal with curricular guidelines. We pay special attention to their views on (i) the nature of the tasks to propose in the classroom, (ii) students' reasoning, and (iii) classroom communication.

Lesson study and mathematical curriculum approaches

In a lesson study, the observation and discussion of the research lesson are key phases of the process but we must not underplay the role of the planning phase. A suitable definition of the topic for the lesson to guide all work from the beginning to the final reflection is a critical issue for the success of the whole activity. Furthermore, the identification and study of relevant literature is also very important, as well as the practical work of analyzing mathematical tasks and samples of students' work that may suggest reasoning strategies and difficulties. Very often, a preliminary diagnostic study is conducted (Murata, 2011) to figure out students' previous knowledge, especially regarding important pre-requisites for the concept or issue under study.

The participants plan together the research lesson aiming to promote students' learning regarding the proposed aims, which may concern a concept, a representation, a procedure, an important capacity (such as reasoning, problem solving or communication) or other curriculum aim. They select or construct one or more tasks to propose to the students, anticipate possible responses and foresee difficulties that may arise when the students are working on these tasks. They pay attention not only to students' errors and difficulties but also to their different strategies

that may lead or not to the correct solution. This anticipation of students' responses and possible difficulties and strategies is a central feature of lesson studies (Meyer & Wilkerson, 2011).

The analysis of the research lesson focuses on students' responses, taking into account the features of the tasks and of the classroom communication processes. This analysis may lead to the reformulation of the lesson plan with changes in the task, in the supporting materials, in the strategy to follow, in the questions to pose to students, and so on, and this cycle of planning/observing/analyzing may be repeated several times (Lewis, Perry & Hurd, 2009).

Carrying out a lesson study requires a careful preparation and conduction that may be undertaken by experienced teachers, teacher educators, or researchers. In Japan, there are teachers who specialize as leaders of lesson studies. The other participants also have a high responsibility since in between lesson study sessions they need to carry out specific work, such as selecting tasks and reading relevant literature. The dynamics of the sessions depends both on the leadership and on the high involvement and capacity of the other participants to interact in a professional way, raising questions, critiquing, listening to critiques, and supporting each other as necessary. In order that a lesson study reaches the sough aims, it is necessary that all participants assume a strong commitment regarding this professional development process, both in the sessions and in carrying out the individual work collectively decided by the group.

Lesson studies have as a focal activity the teaching and the observation of a research lesson, with its subsequent analysis and discussion. The activity of teacher that teaches this lesson is in the spotlight. However, a central aspect of lesson studies is that they focus on the students' learning and not on the teachers' work. In fact, lesson studies aim to examine students' learning and to observe the way they participate in the lesson. As the goal is to produce a successful research lesson, this is usually taught by an experienced teacher; however, lesson studies may be also targeted to support the development of prospective teachers and unexperienced teachers. By participating in lesson studies, teachers may learn about mathematics, curriculum guidelines, students' thinking processes and difficulties, classroom dynamics, and many other professional issues.

Regarding the subject of mathematics, lesson studies provide opportunities for teachers to reflect on the possibilities of an exploratory approach to mathematics teaching, an approach that has an increasing support in curriculum orientations for this subject (e.g., NCTM, 2000, 2014). In this approach students are called to deal with tasks such as problems and explorations for which they do not have an immediate solution, constructing their own methods by using their previous knowledge (Ponte, 2005; Ponte, Branco & Quaresma, 2014). This contrasts with the teaching in which the teacher introduces a new idea, demonstrates a few examples and then presents many exercises for the students to practice. Exploratory lessons also include exercises but there are combined with problems and explorations. Communication processes are also important in an exploratory approach by promoting students' opportunities to participate in the discourse. That is, communication becomes much more dialogical than univocal, as it tends to happen in many classrooms.

Usually, an exploratory lesson develops in three phases: (i) introduction of the task, which is a collective moment where the relevant information and terminology is discussed; (ii) students' autonomous work on the task, which may be carried out individually, in pairs, or in small groups, and (iii) whole class discussion, in which the students present their solutions and have an opportunity to argue with which other. This discussion may end with a final synthesis of the most relevant aspects of the concepts, representations, and procedures involved in the work done as well as signalling connections to other topics and issues. Exploratory work creates many opportunities for

students to build or deepen their understanding of concepts, procedures, representations, and mathematical ideas. Students are thus called to play an active role in interpreting the questions, representing of the information given, and designing and carrying out solving strategies which they must justify to their colleagues and to the teacher. Exploratory teaching seeks to develop students' reasoning processes, especially developing strategies to solve problems, making conjectures, generalizations and justifications (Lannin, Ellis, & Elliot, 2011; Mata-Pereira & Ponte, 2013). However, conducting exploratory mathematics teaching is a serious challenge for teachers, demanding specific knowledge, competency and disposition.

To analyse the work and the professional development of teachers, one must take into account the different levels of curriculum development and implementations (see, e.g., Gimeno-Sacristán, 1989): (i) the official curriculum documents issued by the educational authorities, (ii) the textbooks and other materials that offer written interpretations of these official documents, (iii) the teachers' interpretations of the curriculum, textbooks and other documents and lesson plans, (iv) what happens in the classroom, as result of the teacher/students' interactions; (v) what students learn, and (vi) what is accessed in large scale exams and other evaluation programs. In this paper we are mostly concerned with level (iii) but we also explore links with levels (i), (ii), (iv) and (v).

The work of the mathematics teacher in the classroom is always made in the frame of curriculum orientations. In the last few years, the mathematics curriculum in Portugal experienced a strong change, from a perspective largely aligned with a problem solving and exploratory approach that was developing since the 1990s (Ministério da Educação, 1990, 1991a, 1991b, 2007), to a back to basics approach (Ministério da Educação e Ciência, 2013), that emphasizes abstract concepts and representations and often presents the mathematical content at very demanding level for most students. This situation poses great problems for the teachers, most of whom were supportive of the previous curriculum orientations: Therefore, the teachers face the issue of attending to the new curriculum demands and still be able to lead their students in experiencing success in mathematics learning.

Research methodology

This study is a practitioner's research carried out by the four authors who designed, conducted and analysed the lessons studies. The methodology is qualitative, using participant observation. The participants are teachers at 1st, 2nd and 3rd cycles of basic education, and the four authors of this paper. The 1st and 2nd cycle teachers were from the same cluster of schools (*agrupamento*) in Lisbon and the participants of the 3rd cycle were seven prospective teachers (for teaching grades 7-12), studying at the University of Lisbon, and the mentor teacher from the school where they were doing fieldwork. The authors took part in conducting the lesson study sessions and were responsible for data collection which included video recording, teachers' written reflections, and individual and group interviews. The video records of the sessions and the audio records of the interviews were transcribed. A research journal was also made by a member of the research team, recording the most salient reflections from each lesson study session. The curriculum document used by the participants was the *Programa e metas curriculares Matemática: Ensino básico* (Ministério da Educação e Ciência, 2013) which includes, in a first part, a syllabus with a condensed presentation of the content and, in second part, detailed standards for each topic and grade level. In this paper our analysis concerns: (i) the participants' views on the nature of the tasks to propose to students, (ii) their perspective on students' reasoning processes, (iii) their views on classroom communication,

especially concerning whole class discussions, and (iv) their perspectives regarding the curricular guidelines and how they were interpreted in the lesson plan and put into practice in the classroom.

Learning addition of rational numbers (grade 3)

The lesson study. This lesson study was carried out in a cluster of schools in Lisbon, involving three teachers from a 1st cycle school.[53] In the first session, all participants (teachers and teacher educators/researchers) decided that the topic would be the learning of addition and subtraction of rational numbers. The new curriculum required this topic to be studied using fractions and number line representations, two rather abstract representations, and moved it from grade 5 to grade 3. The fact that the students in this school had a weak level of attainment made this topic particularly challenging to teach.[54]

One of the teachers (Irina) had a strong interest in mathematics teaching and assumed in practice a leadership role regarding her colleagues. She did most of the preparation work for the research lesson and was selected to teach this lesson. Given the fact that the two other teachers (Elsa and Manuela) had a weak mathematical preparation, a considerable time of the sessions was devoted to solving mathematical tasks and reflecting about possible students' difficulties. Some of this work was based on a diagnostic test the teachers made in their classes. The three teachers reported great satisfaction with this professional development process, indicating that it provided a deeper understanding of didactical issues. In addition, Manuela referred that this study was important for her mathematics preparation:

> It enriched me a lot. No doubt about that. I think it was a new world that just opened for me . . . I have no doubt that I feel [now] much more comfortable in the area of fractions and I am able to help them [the students] much better. (Manuela)

In her final reflection, Irina writes the following about this experience:

> With the new curriculum standards and related introduction of new contents there is still a scarcity of sources to search for ideas of tasks to use in the classroom, especially regarding rational numbers, so I find very important [the attention given to tasks] in this professional development activity… When I decided to participate in this professional development I did it… because I have the notion that the current curriculum standards would bring increased difficulties for teachers and students… [Rational numbers are] no doubt the critical point of the standards, the point that implies more fear in the teachers and difficulties in the students. (Irina)

Curriculum interpretation. The research lesson was supported by a very detailed and ambitious lesson plan with two *tasks* involving a context of competition among wild animals. There was a strong attempt to follow the curriculum guidelines regarding doing addition and subtraction in the two representations (by finding equivalent fractions and by juxtaposing segments in the number line), immersing those tasks on an appealing imaginary situation. Some of the questions posed to the students were rather simple, just requiring the use of the conventions of the representations (exercises), whereas others were more complex, requiring interpretation and combination of

[53] There were more teachers in the first session, who later decided not to participate in the lesson study.

[54] More information about this lesson study may be seen in Ponte et al. (2015).

different kinds of information (explorations). Both the lesson plan and the actual lesson show that the teachers put a high value on *students' reasoning* processes, especially concerning generalizations (how to add fractions when denominators are equal and different) and justifications (using the meanings of fractions as part-whole and as measures on the number line). A particular difficulty that was perceived during the lesson was students' tendency to see fractions as represented by dots on the number line and not by line segments. Ways to support students' *communication* in the classroom were also object of attention, organizing students' autonomous work in pairs and leading a whole class discussion at the end of each task. These discussions were widely participated by students of different ability levels. A few students in the class appeared to have understood the aims of the lesson, but many of them showed trouble with one of both rational number representations. In the final reflection, the participants considered that the major change needed in the task was the introduction of more questions to make sure that students would interpret correctly the meaning of the different representations of fractions.

Summary. The lesson study provided an opportunity for the teachers to take a close look at the current curriculum for grade 3 that they had trouble using as a guide for teaching practice. Their reflections indicate that they become more confident in dealing with the curriculum documents and related teaching resources. The overall strategy of the lesson study strived to combine the rather demanding and formalist approach dictated by the official curriculum with an appealing context and an exploratory approach regarding the students' learning of the topic. For both the teachers and the researchers the way the students reacted to this lesson was a confirmation that rational numbers are a very difficult topic for most of the students that is hardly suitable to be taught using the proposed approach and representations at grade 3.

Comparing and ordering rational numbers (grade 5)

The lesson study. This study was carried out in a 2nd cycle school in Lisbon, involving all its 5 mathematics teachers. Three of these teachers knew each other since a long time but rarely worked together and the other two were on short term contracts and felt as outsiders in the school. The topic of rational numbers was selected for the research lesson in the first session. The teachers considered that the students had strongest difficulties in this topic and that it was very appropriate given the school calendar. In a later session, the specific issue of comparing and ordering rational numbers was chosen as the aim of the research lesson.[55]

In an initial phase of exploration of the topic the teachers analyzed the curriculum standards. Afterwards, they solved several mathematical tasks and analyzed students' solutions. They also did a diagnostic test in their classes to know the previous knowledge and the difficulties of their students. It was not simple to decide what class should be used for the research lesson, since all the teachers felt uncomfortable with the idea of being observed by a large group of people. Finally, it was decided that the research lesson should be made in the class of Luísa, one of the new teachers in the school. This teacher was concerned with the weak attainment level and behavioral problems of her class, but the other possible classes still presented more complex problems. The lesson was base d in several tasks seeking that students discovered some of the properties of ordering rational numbers given as fractions. The aims of this lesson were only partially attained, as the teachers concluded that it was necessary to make a deep revision of the task for future applications. Even so, there were interesting moments in which students elaborated

[55] More information on this lesson study may be seen in Ponte et al. (2014).

generalizations and justifications. The teachers considered that the lesson study was helpful to learn about the teaching of rational numbers but stressed that its most important feature was the collaboration that took place among the participants.

Curriculum interpretation. The teachers were surprised with some of the *tasks* that we suggested could be proposed to the students. They felt that their students only could handle quite straightforward tasks posed in mathematical language (that they called exercises) or in real-life contexts (that they called problems). The work on the lesson study led them to accept that another productive distinction concerned the low or high level of challenge of a task. They saw several examples of students designing interesting solutions to quite challenging tasks, and the level of change become the main feature to classify a task as an exercise or a problem. In their research lesson the teachers included some challenging tasks.

> I think that it is [useful]… We have to open the spread and, it is what we say, we have students that [need to do] the exercises, exercises and more exercises, but we have to think that there are students that have capacity for much more than that, and we must not think just in the students that have difficulties, and very often we have classes in which there are possibilities for the students to make generalization and justifications. (Tânia)

The *reasoning processes* of the students were object of attention in this lesson study. The teachers identified situations of generalization and justification in the students' work and included tasks aiming to promote these processes in their research lesson. One of the highlights of the lesson was a justification made by a student regarding the validity of a statement about the equivalence of fractions based on a pictorial representation which was clearly perceived by the teachers:

> That girl… Berta represented the aimed $\frac{4}{6}$. From $\frac{2}{3}$ she went to $\frac{4}{6}$, I think that she explained it in a really simple way, putting a line at the middle and the others really saw… Very well… And she led the others to understand. (Inês)

Classroom communication was also object of attention and the teachers recognized the value of conducting whole class-discussions in which students have the possibility of arguing with each other. The interesting moment that we just mentioned occurred during a whole class discussion.

Summary. As with the 1st cycle teachers, those that participated in this lesson study faced the need to implement the new mathematical curriculum. They preferred the previous curriculum but accepted as a matter of fact that this new curriculum was officially approved and were interested in learning how to deal with it or, at least, to adapt it to the particular conditions of their classes. The lesson plan had some exploratory flavour but the actual lesson was not as successful as sought by the teachers. They considered that the tasks proposed in the lesson plan should be reformulated in order to achieve the aim of having students making generalizations by themselves.

Similarity of triangles (grade 7)

The lesson study. This experience was undertaken with seven prospective teachers for the 3rd cycle of basic education and secondary education and the school mentor teacher. All the students in this teacher's class had a very weak mathematics attainment and were taking grade 7 for a second year. The topic of similarity of triangles

was chosen attending to the school calendar and also to the perception that the students usually experience significant difficulties in learning it.

In a first phase, a thorough analysis was made of the current curriculum guidelines concerning similar figures and, in particular, similar triangles. The prospective teachers solved several mathematical tasks involving similar triangles, many of which they selected and brought to the class. They discussed the features of these tasks and analyzed possible students' difficulties. Two lessons were planned, one aiming to teach the criteria for similar triangles and the other to promote the ability of solving problems using this criteria. As the prospective teachers were in the first part of their teacher education program, we decided since the very beginning that the research lesson would be taught by the mentor teacher. The prospective teachers were divided in two groups and all of them observed the two lessons. Each group was responsible to analyze one lesson. Both the mentor teacher and the prospective teachers took an active participation in the planning and in the reflection phase. In their final reflection, the prospective teachers recognized that this work helped them to develop a much better understanding of the processes of identifying students' difficulties and of planning a lesson which strived for a strong involvement of the students.

The mentor teacher made the most important decisions regarding the adaptation of the mathematical content to the class. She decided not to present the formal definition of similar figures stated in the curriculum that she considered too formal and out of reach of her students' understanding. So, she presented a less mathematically rigorous but more intuitive definition. She also decided that the criteria for similar triangles (that are the main tools that students have to solve problems on this topic) should be presented in an intuitive way and not derived from other mathematical theorems regarding relations of segments in parallel lines, as the curriculum established. She made these decisions on the grounds of her knowledge of the students and her willingness to take this into account in making the adaptations that she regarded as necessary in the curriculum documents. The prospective teachers were intrigued at first with such decisions but with time they accepted them as the most suitable for that particular students' class.

Curriculum interpretation. The participants strived to use several kinds of *tasks* in the two lessons, including problems, explorations, and exercises. The tasks required very significant practical work, including measuring line segments and angles and computing ratios between line segments that proved to be productive entry points to support students' participation in the classroom activity. These tasks were taken from a variety of sources, including textbooks and the internet, and most were extensively adapted during the lesson study with the prospective teachers. One of them recognized that, in this experience, the nature of tasks and the way of working with the tasks constituted an approach to mathematics teaching that is very different from what she experienced as a student:

> It was more the way of introducing the task, as an exploration… Because usually this is not the way one works… As we worked when we were at school… In planning [a lesson], we have to take into account students' difficulties and the choice of the task we want to propose. (Ana)

Reasoning played a significant role in the lesson plan. One of the criteria for similarity of triangles was presented in a worksheet and it was intended that the students would formulate two other criteria, thus making generalizations. Reasoning was also important in some problems which solutions required a rather elaborated strategy. *Classroom communication* was object of attention, as the mentor teacher decided that the students could work individually, in

pairs or in small groups, depending on their will. The whole class discussion provided ample opportunity for the students to presents their strategies and results. Another prospective teachers acknowledged the value of discussions:

> I think that the systematization is very important in the final discussion. They [the students] may have some questions, that accumulate, and this is the moment in which, OK, they learn this… Therefore, I think that this structure is very important to improve learning. (Carla)

This participant also indicates that this curriculum approach was different from her previous experience:

> It is more exploratory tasks, and they are much more interesting… What we presented was more towards problems, not towards exercises, and I think it develops reasoning better. (Carla)

Summary. The overall strategy for the interpretation of the curriculum was developed by the mentor teacher and the researcher that conducted the lesson study. The prospective teachers, who had already courses in which curriculum issues were discussed, did not present any problem in accepting their interpretations. All the participants had already a critical view regarding the current curriculum orientations for teaching this topic, and this lesson study just reinforced their perspectives. Therefore, they took the most visible elements of the curriculum (knowing criteria for similar triangles and solving problems involving the use of such criteria) as key points for those two lessons, approaching them in an intuitive way and dispensing the abstract and complex approach indicated in the official curriculum documents.

Discussion and conclusion

The development of these lesson studies shows that the current Portuguese curriculum documents (Ministério da Educação e Ciência, 2013) were essentially used to map the content to teach. The participant teachers and prospective teachers were nor comfortable with the formalistic orientation of these documents, and decided to emphasize intuitive elements such as connections with the students' reality and use of pictorial representations. As the new curriculum documents do not have much more to offer besides the specification of content, they did not play a significant role besides the initial planning of the research lesson.

By participating in this lesson study, focusing on mathematical tasks and on students' reasoning, the participant teachers had many opportunities to get involved in doing mathematics through an exploratory approach. They discussed in depth the features of tasks that make them simple exercises or more engaging problems or explorations as well as features of reasoning processes such as justification and generalization (Lannin, Ellis, & Elliot, 2011; Mata-Pereira & Ponte, 2013). The teachers developed a sense for the importance of the wording of tasks and for possible students' difficulties in solving them. They recognized that tasks with some degree of challenge were important to stimulate students' reasoning and that in such cases unforeseen responses could be given by students. The teachers also got a clearer notion of students' difficulties in reasoning processes (especially generalizing and justifying) and to aspects of classroom communication that promote or inhibit such processes. The key feature of lesson study of anticipating possible difficulties of students and looking at what they actually do in the classroom was effective in leading the participants to reflect and consider how to carry out their practice within an exploratory approach, taking into account the nature of the tasks, students' reasoning processes, and classroom communication.

Lesson studies aim the development of teaching practices and of the reflexive capacity of teachers. The three experiences that we report in this paper had distinct dynamics but they all show that this formative process involves a strong relationship with teaching practice, yields a deep look about the mathematical and didactical issues of the selected topic, enables a deep look about students' difficulties as well as of their reasoning processes. It is a professional development process framed by collaboration and reflection, in an investigative framework. In addition, it constitutes a process of production of professional knowledge deeply rooted in schools and from which it is possible to draw important elements for classroom practice (Murata, 2011). However, this professional development process does not guarantee the participants' learning. It only creates conditions for this to occur. As in any learning process, to reach the aimed goals a deep personal engagement is required. In order that the generalization of lesson studies may lead to significant transformations in teaching practices, it will be necessary that they assume a prominent role in educational policies as it happens since a long time in Japan (Takahashi & McDougal, 2014). The experiences that we presented show that lesson studies have a potential to lead to significant professional learning, supporting teachers in their interpretation of curriculum frameworks, but they are not immune to problems and difficulties. This creates an interesting agenda for those that are interested in the professional development of teachers.

Acknowledgement

This work is supported by national funds through FCT – Fundação para a Ciência e Tecnologia by grants to Marisa Quaresma (SFRH/BD/97702/2013) and Joana Mata-Pereira (SFRH/BD/94928/2013).

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