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Population density and economic development

By

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Abstract: We describe formally the relationship between population density and per capita income along the two growth regimes put forward by KUZNETS (1960), BOSERUP (1965) and TAMURA (2002). We consider a spatial economy where an undifferentiated consumer good is produced by a continuum of competitive agents. Each agent requires one unit of a capital good to produce the final good and the two goods are assumed to be perfect substitutes in production.

Under the first growth regime (called "classical" or "Malthusian"), each agent self-produces the capital good by shifting resources that would otherwise produce one unit of the final good. This economy shows marginal decreasing returns of labour. Population growth brings about congestion and the elasticity of output per worker (and income per capita) is negative.

A structural change takes place following a sufficient increase in population density and decrease in transport costs. Then, the supply of capital goods is outsourced to a specialized industry, operating in spatial monopolistic competition in line with SALOP (1979).

Under this "modern" growth regime, the specialization of the capital goods production is a source of increasing returns in the aggregate economy. The elasticity of per capita income with respect to population density becomes positive just after the structural transition, but this effect may not persist in the long run.

If the outsourcing of capital goods is matched by a rising importance in final production of activities which are *not* land-based, then aggregate increasing returns can be sustained in the long run. Otherwise, the positive sign of elasticity will be transitory and a further increase in population density will revert the economy to a situation where the use of increasing amounts of labour with non-reproducible resources determines mainly a congestion effect.

JEL Classification: O12, O33, R11.

Keywords: Economic Development, Population Density, Industrialization, Spatial Competition, Technological Change, Economies of Agglomeration.

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1. Introduction

It is widely accepted that there is clear cut relationship between population growth and economic development. Authors such as TOURNEMAINE (2007) and BUCCI (2008) explain this indeterminacy by two contradicting forces.

On the one hand, the aggregate income of a region or country is limited by the availability of primary factors of production, which are either non-reproducible (such as fertile land or other natural resources) or exist in fixed quantity in the short run (i.e., different types of capital either physical or human). A fast population growth leads to the division of the current income among a higher number of individuals.

On the other hand, demographic growth makes the production of new technical knowledge more profitable since outlays associated with R&D are usually fixed and most output of research activity is a non-rival good.

Under a different approach, the econometric estimation of the elasticity of labour productivity (or median wage) with respect to population or employment density across small territorial units or cities lead usually to positive outcomes although modest around $0.05 \text{ or } 0.06$ (see SVEIKAUSKAS, 1975; CICCONE and HALL, 1996; and CICCONE, 2002). According to PROOST and THISSE (2019), most productivity/density estimates fall between $0.03 \text{ and } 0.09$.

Nevertheless, negative estimates of productivity/density elasticity are possible. A meta-analysis by MELO and Al. (2009), which is defined over 729 elasticity estimates drawn from 34 studies covering the 1965-2002 period, finds out elasticities contained in the interval between $-0.800 \text{ and } 0.658$. Although the non-weighted mean of the estimates is 0.058, with a standard deviation 0.115, negative elasticities are not infrequent.

Furthermore, if structural variables that are correlated with population density, namely the average skills level of the labour force or the sectoral composition of output, are controlled for in the regressions, the elasticity value, while being still positive, may fall slightly below the lower bound 0.03 (see PROOST and THISSE, 2019).

The sectoral composition of the productive activity matters on two grounds. Controlling for the share of agriculture in GDP allows to consider the relative importance of land (and other natural resources as well) in aggregate production. In addition, service activities show a much higher elasticity than manufacturing since their inputs and output must be often delivered in person, thus entailing much higher transport costs.

The elasticity value of labour productivity with respect to employment density is often rationalized trough the interplay of two contradicting forces, namely the “congestion effects” and the “agglomeration economies” (see CICCONE and HALL, 1996; DURANTON and PUGA, 2004).

“Congestion effects” emerge when a rising number of workers is employed in land intensive activities, such as agriculture and the production of housing services, within a fixed geographical area. In this case, the output per worker decreases steadily with size of the labour force, so that the estimated elasticity is negative.

“Agglomeration effects” account for the increase in output per worker derived from a rising number of individuals working in a fixed area of land. DURANTON and PUGA (2004) recognize three different sources of agglomeration forces, namely *learning, sharing and matching*.

“Learning” concerns the creation and diffusion of technical knowledge that arises if the density of educated workers is high thus enabling them to contact face-to-face often over long time periods. Hence, these workers can exchange and assemble heterogeneous bits of professional experience. “Sharing” a fixed input, such as a school, a hospital, an infrastructure for transport or telecommunication, by many neighbouring individuals allows each one of them to acquire a service at a low unit cost. The “matching” externality concerns the enhancement of the bilateral relationship between a supplier and a buyer of a differentiated service following from a rise in the number of involved agents. For instance, if the youngster population expands in a region, the density of the school network will increase and the average distance between a student’s residence and the nearest school will diminish thereby raising the quality of the bilateral assignment.

In this paper, we adopt the viewpoint of authors such as KUZNETS (1960), BOSERUP (1965) and TAMURA (2002) on the relationship between population growth and output per worker. Hence, we discriminate two successive growth regimes.

The first regime is usually called the *classic* (or *Malthusian*) regime and is typical of low density, low productivity areas, where most individuals operate in land intensive activities such as agriculture or the production of housing services and have basic skills. If population grows due to exogenous factors, land cultivation becomes more labour intensive and the output per worker falls steadily.

When the density of educated workers passes a threshold, “industrial” techniques, which are more intensive in human capital, substitute for land based, “agricultural” techniques. Under such new techniques, the productivity of labour becomes an increasing function of demographic (capital human) density. Population and aggregate income tend to grow faster spurred by the positive correlation between labour force density and output per worker. This structural transition takes place earlier the higher the initial level of labour skills is.

In this paper, we purport to describe the transition the two growth regimes. In addition, we try to rationalize the importance of the overall transport cost level as a determinant of the elasticity of productivity with respect to density. On the one hand, it is intuitive that highways and high-speed trains entail significant fixed costs that can be made profitable more easily in densely populated areas. (see among others SIMON and GLOVER, 1975). On the other hand, if the manufacturing of a final good is based on the assembly of differentiated components whose production require fixed inputs (as in ETHIER, 1982), density and productivity will be positively correlated only if the parts are produced in the same region as the final product. This requirement can be met more likely if the intermediates have “infinitely high” transport costs. Otherwise, the economies of scale associated with the sharing of differentiated inputs would be “international” in nature, as in ETHIER (1982), so that they would not generate *per se* any agglomerative effects.

2. Some stylized facts about the relationship between labour productivity and population density in Europe

A tentative approach was done for 27 European countries (24 EU countries, plus the UK, Norway and Switzerland). In Table 1 located in the appendix we recorded data for GDP per employee, population density and highway density for these countries. We estimated by OLS the following cross-section model for the year 2019.

$$\ln y = \alpha + \beta \ln D_{pop} \quad (1)$$

where

$y \equiv$ GDP per person employed in 2019 (in PPS)

$D_{pop} \equiv$ Population density (people per Km^2)

The source of these data is the AMECO database

We estimated the elasticity β for the whole country set and separately for the two subsets composed by countries whose population density is below and above the median (which is about 102 people per Km^2).

Given the limitations of the data, the results are unsurprisingly not statistically significant, but they fit the theoretical expectations about their sign and size. The elasticity estimated for the whole sample with a R^2 close to zero (about 0.012). Population density does not explain at all the variation in labour productivity across countries, but it is nevertheless interesting that the estimate is in line with the values surveyed in the literature for much smaller territorial units.

The estimated elasticity for the group of countries with low population density is about -0.111, hence it has the theoretically expected sign although the R^2 is only 0.052. Density explains a little more than 5 percent of the variation in labour productivity for underpopulated countries.

For the high-density group, we find out an estimate of the elasticity of 0.264, a value which lies within the broad bounds defined by MELO et Al. (2009). The R^2 is 0.395, so that density explains almost forty percent in the output per worker for the densely populated countries, thus becoming significant under the 0.01 confidence level.

Hence, this empirical result confirms the viewpoint that an economy passes through two successive growth regimes, where the elasticity of labour productivity with respect to density switches from a negative to a positive sign as population and aggregate human capital increase sufficiently.

We also regressed the GDP per worker with respect to the country density of motorways, with the latter data being provided by the *European Road Federation* for 2017. While the density of highways is expected to be correlated with population density as their high fixed costs are economically shared by many users (see SIMON and GLOVER, 1975), such a density also approximates inversely the overall level of transport costs in the economy.

The results of the regressions with the highway density are close to those made with the population density as the explaining variable. Again, the density of transport infrastructure is a significant causal factor of output per worker for dense countries, but it seems not to matter much for low density ones. This suggests that dense countries have also a more concentrated geographical structure and thus they require a more efficient transportation system.

If we consider the whole set of 27 countries, although highway density is still not a statistically significant explanation of GDP per worker, the fit is clearly better than the one obtained with the population density as an independent variable. The estimated elasticity and the R^2 are much higher (≈ 0.07 instead of 0.04 , and 0.072 instead of 0.012 , respectively). This drives us to believe that the overall level of transport costs and fixed costs is an important factor of economic development.

3. A spatial economy with two growth regimes

3.1. The "classical" growth regime

We assume an economy where L individuals are uniformly distributed along a circle with length S . The individuals are competitive producers of an undifferentiated consumer good, which works as the *numéraire* in this economy. For this purpose, they use labour and land, under a constant returns to scale technology described by the aggregate Cobb-Douglas production function.

$$Q = AL^{\alpha} S^{(1-\alpha)} \tag{2}$$

where

$Q \equiv$ output of composite consumer good
$L \equiv$ labour
$S \equiv$ land
$\alpha \equiv$ parameter such that $0 < \alpha < 1$
$A \equiv$ "Hicks neutral" aggregate productivity term

By choosing adequate measure units, the aggregate productivity term can be set equal to 1, so that expression (2) becomes

$$Q = L^{\alpha} S^{(1-\alpha)} \tag{3}$$

We assume that the transport cost of the composite good is positive. Together with the assumption of a constant returns to scale technology, positivity of transport costs of the final good determines that each person self-produces and consumes the undifferentiated good without needing to engage in trade with other individuals.

A careful distinction should be made between the output of consumer good of this economy, Q, and the income Y, perceived by the workers. The reason is that each worker must use one unit of a "capital" good (either physical or human capital) to produce the consumer good. The production cost of the capital good should be subtracted from the output of the composite good to obtain the income earned by individual consumers.

We assume from the start that each competitive firm self-produces a unit of the capital good by shifting primary resources (land and labour) that would otherwise produce one unit of the consumer good. Since the latter good is the *numéraire* of the economy, the capital good has a unit production cost and price equal to 1.

We will introduce later the conditions for the capital good to be produced under increasing returns to scale by a specialized industry composed by firms under monopolistic competition.

In this initial "classical" setting, the aggregate income earned by the producers is,

$$\begin{aligned} Y &= Q - L \\ &= L^{\alpha} S^{(1-\alpha)} - L \end{aligned} \tag{4}$$

Per capita income in this traditional economy is,

$$y = \frac{Y}{L} = \left(\frac{S}{L}\right)^{(1-\alpha)} - 1 \tag{5}$$

As TAMURA (2002) contended, this traditional economy is also implicitly dominated by sectors such as "agriculture" or the "production of housing services", where the income earned by an individual producer is almost completely determined by the amount of land or other natural resources that is available to him, i.e., by the ratio $\frac{S}{L}$.

Per capita income in (5) can also be written in terms of the population or labour force density $D \equiv \frac{L}{S}$, thus becoming,

$$y = D^{(\alpha-1)} - 1 \tag{6}$$

If we assume that the capital cost is small in relation to the value of the consumer good output, the elasticity of per capita income with respect to population density can computed approximately through the following steps.

Firstly, we factor (6) to get,

$$y = D^{(\alpha-1)}\left[1 - D^{(1-\alpha)}\right]$$

Taking logs, we obtain,

$$\ln y = (\alpha - 1)\ln D + \ln\left[1 - D^{(1-\alpha)}\right] \tag{7}$$

We can make a linear approximation of the second log.

$$\ln\left[1-D^{(1-\alpha)}\right] \approx -D^{(1-\alpha)} = -e^{(1-\alpha)\ln D}$$

Hence, (7) becomes,

$$\ln y \approx (\alpha-1)\ln D - e^{(1-\alpha)\ln D}$$

By differentiation, we obtain an approximation σ to the elasticity of per capita income with respect to labour force density.

$$\sigma \equiv \frac{d\ln y}{d\ln D} \approx (\alpha-1)\left[1+D^{(1-\alpha)}\right] \quad (8)$$

It is clear that σ is negative and decreasing in the labour force density. Basically, the addition of workers to a labour force operating within a fixed area of land determines congestion and this impact becomes more severe as the population density increases.

3.2. The "modern" growth regime

Let us now assume that exogenous forces bring about a strong rise in population. According to BUCCI (2008), this assumption is realistic, since nowadays a major part of world population growth is concentrated in the poorest countries and this trend is likely to persist even in the long run.

An increase in population density can make the production of capital goods by a specialized, increasing returns industry more profitable than their former in-house production. To model this structural transition, we feature a capital good industry working in spatial monopolistic competition inspired in SALOP (1979).

The capital good is thus produced by n symmetric firms that only incur a fixed cost F, variable production costs being zero. It is assumed that each firm charges a fob mill price p and each consumer bears the transport cost between his address and the nearest firm location, where t labels the unit transport cost.

With such a symmetric setting, the price and location equilibrium of this industry will also be fully symmetric. In equilibrium, n^* firms will enter the market and they will distribute themselves symmetrically along the circle and each one will quote the same fob mill price p^*.

We deal now with the structural change from the self-production of the capital good at the constant unit cost or price 1 to its manufacturing by firms operating under economies of scale. Basically, we view this transition as the outcome of a choice made by each individual while playing two different roles.

Firstly, as a *producer* of the composite good, an individual opts to outsource the capital good to increasing returns producers instead of producing it in-house under constant returns to scale methods of production. For instance, an individual who must travel to some destination chooses between going "on foot" or by purchasing a public transportation service.

Secondly, since the capital and the consumer good are perfect substitutes in production when they are made by the same person, this option can be viewed as a choice between two consumer goods, namely the undifferentiated consumer good and the outsourced capital item, which is properly regarded here as a "consumption good". For instance, an individual consumer might face a choice between attending "college education" or buying a "decent car".

For the sake of simplicity, we presuppose that the commodities over which the consumer makes a choice are perfect substitutes, so that the individual always takes the cheaper option. Furthermore, we assume that the individuals must be *unanimous* in their choices for a transition from in-house produced to outsourced capital goods to take place. In what follows, we detail this option.

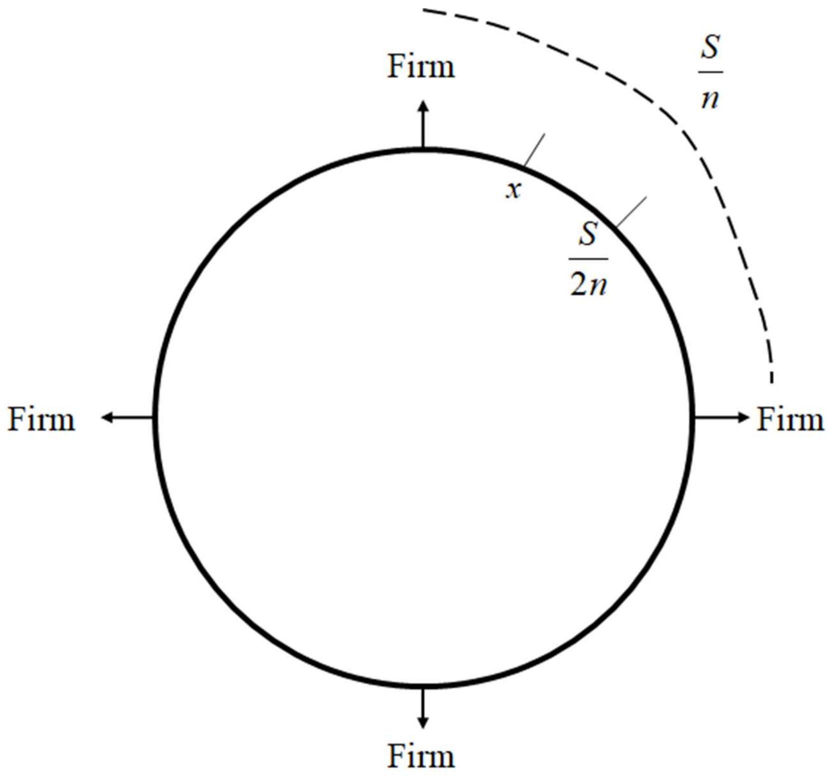

Figure 1: Spatial economy with $n = 4$ firms

In Figure 1, we plot the circle with perimeter S and n^* firms symmetrically located, which charge the same equilibrium fob mill price p^*.

We presuppose that the delivered price paid by every consumer should not exceed 1, the unit cost and price of the outside good. A necessary and sufficient condition for this assumption to be fulfilled is,

$$p^* + t\left(\frac{S}{2n^*}\right) \leq 1 \tag{9}$$

In (9), $\frac{S}{2n^*}$ is the maximal distance between a consumer and its nearest supplying firm. To compute the equilibrium values p^* and n^*, we assume from the start that a representative firm quotes the fob mill price p while its two competing neighbours charge $\overline{p}$.

Then, the boundary x of the representative firm's market area (see Figure 1) is given by the equation,

$$p + tx = \overline{p} + t\left(\frac{S}{n} - x\right) \qquad (10)$$

whose solution is,

$$x = \frac{1}{2t}\left(\overline{p} - p + \frac{tS}{n}\right) \qquad (11)$$

Since an individual acquires one unit of the capital, provided that its delivered price does not exceed the price 1 of the outside good, we can write the representative firm's profit function depending on p and n as,

$$\pi(p,n) = (2x)Dp - F$$

or

$$\pi(p,n) = \frac{D}{t}\left[p\left(\overline{p} - p + \frac{tS}{n}\right)\right] - F \qquad (12)$$

As the profit function in (12) is strictly concave with respect to p ,we can easily compute the price p that is a best reply to the neighbouring firm's price $\overline{p}$, for a given number n of symmetrically located firms as,

$$p = \frac{1}{2}\left(\overline{p} + \frac{tS}{n}\right)$$

Since the price equilibrium is also symmetric, we substitute $\overline{p} = p$ and obtain each firm's profit maximizing fob mill price.

$$p^* = \frac{tS}{n} \qquad (13)$$

If we insert $p^* = p = \overline{p}$ from (13) into (12), the profit function can be written as depending only on the number n of symmetrically located firms.

$$\pi(n) = \frac{D}{t}\left(\frac{tS}{n}\right)^2 - F$$

The equilibrium number of firms under free entry, n^*, can be found by solving equation $\pi\left(n^*\right)=0$, which yields,

$$n^* = S\left(\frac{Dt}{F}\right)^{\frac{1}{2}} \tag{14}$$

By substituting n^* from (14) into (13), the equilibrium fob mill price appears to be

$$p^* = \left(\frac{tF}{D}\right)^{\frac{1}{2}} \tag{15}$$

Therefore, the condition (9) of unanimous outsourcing of the capital good to a specialized industry composed by increasing returns firms can be written by substituting p^* from (15) and n^* from (14) to yield the inequality,

$$\frac{D}{t} \geq \left(\frac{9}{4}\right)F \tag{16}$$

We can also write this condition as it it defines a lower bound of population density $\tilde{D}$ for the economy to switch to the production of capital goods by a specialized, increasing returns industry.

$$D \geq \tilde{D} \equiv \left(\frac{9}{4}\right)\left(tF\right) \tag{17}$$

Inequality (16) means that, starting from a traditional, resource-based economy, a structural transition to a modern economy with a specialized capital goods production can be achieved *both* by increasing population density and decreasing transport costs. Condition (16) rationalizes the empirical findings in section 2 that transport infrastructure density has the same qualitative effect on labour productivity as population density with an additional explanatory power.

It remains to compute the elasticity of per capita income with respect to labour force or population density. We should be aware that aggregate output of the composite consumer good is still given by Cobb-Douglas function in (3), so that the output per worker equals $D^{(\alpha-1)}$.

We have still to calculate the per capita cost that the specialized input industry must incur to supply the final producers. Total fixed costs are,

$$TF = n^* F$$

Substituting n^* from (14), total fixed costs become,

$$TF = S\left(tDF\right)^{\frac{1}{2}} \tag{18}$$

Total transport costs are (see Figure 1),

$$TT = 2n^* \int_0^{\frac{S}{2n^*}} (tr)\,dr$$

By substituting n^* from (14) and solving the definite integral, total transport costs become,

$$TT = \frac{S}{4}\left(\frac{tF}{D}\right)^{\frac{1}{2}} \tag{19}$$

Hence, the aggregate cost borne by the specialized capital good industry to supply all the final producers is,

$$TC = TF + TT = S\left(tDF\right)^{\frac{1}{2}} + \frac{S}{4}\left(\frac{tF}{D}\right)^{\frac{1}{2}}$$

The per capita cost C incurred by the capital goods industry is obtained by dividing the aggregate cost by the total labour force in the final sector $L = DS$.

$$C = \left(\frac{tF}{D}\right)^{\frac{1}{2}} \cdot \left(1 + \frac{1}{4D}\right) \tag{20}$$

Since the per capita cost in (20) is measured in composite good units, it can be subtracted from the output per worker $q = D^{(\alpha-1)}$ to obtain the per capita income in this two sector economy.

$$\begin{aligned} y &= q - C \\ &= D^{(\alpha-1)} - \left(\frac{tF}{D}\right)^{\frac{1}{2}} \cdot \left(1 + \frac{1}{4D}\right) \end{aligned} \tag{21}$$

By factoring out $D^{(\alpha-1)}$, the per capita income becomes,

$$y = D^{(\alpha-1)}\left[1 - \left(\frac{tF}{D}\right)^{\frac{1}{2}} D^{(1-\alpha)}\left(1 + \frac{1}{4D}\right)\right]$$

which expands to,

$$y = D^{(\alpha-1)}\left\{1 - (tF)^{\frac{1}{2}}\left[D^{\left(\frac{1}{2}-\alpha\right)} + \frac{D^{-\left(\frac{1}{2}+\alpha\right)}}{4}\right]\right\}$$

If we take logs and use the linear approximation $\ln(1+x)\approx x$ for x close to zero, we obtain,

$$\ln y=(\alpha-1)\ln D-(tF)^{\frac{1}{2}}\left[D^{\left(\frac{1}{2}-\alpha\right)}+\frac{D^{-\left(\frac{1}{2}+\alpha\right)}}{4}\right]$$

If we differentiate $\ln y$ with respect to $\ln D$, we compute the elasticity of per capita income in relation to population density as a function of the density.

$$\begin{aligned}\varepsilon(D,\alpha)&\equiv\frac{d\ln y}{d\ln D}\\&=(\alpha-1)-(tF)^{\frac{1}{2}}\Gamma(D,\alpha)\end{aligned}\qquad(22)$$

where we define the function,

$$\Gamma(D,\alpha)\equiv\left(\frac{1}{2}-\alpha\right)D^{\left(\frac{1}{2}-\alpha\right)}-\frac{1}{4}\left(\frac{1}{2}+\alpha\right)D^{-\left(\frac{1}{2}+\alpha\right)}\qquad(23)$$

By computing $\frac{\partial\Gamma(D,\alpha)}{\partial D}>0$, we can be sure that Γ is a strictly increasing function of D and hence that ε from (22) decreases strictly with the population density.

It harder to assess the sign of $\varepsilon(D,\alpha)$. If $\alpha<\frac{1}{2}$, the sign of $\Gamma(D,\alpha)$ is indeterminate. If $\alpha>\frac{1}{2}$, then $\Gamma(D,\alpha)$ is negative, but the sign of $\varepsilon(D,\alpha)$ in expression (22) is indeterminate.

We recall from (17) that final producers chose to outsource capital goods to a specialized increasing returns if and only if population density falls in the interval $\left[\frac{9}{4}tF,\infty\right)$. We can be sure that both Γ and ε are continuous functions for values of D within this interval.

Due to the indeterminacy of the elasticity ε sign, we will limit ourselves to two polar cases which relate to the growth regimes, “agricultural” and “industrial” described by TAMURA (2002).

The “agricultural” economy corresponds to the situation where $\alpha\approx 0$, so that the output per worker is,

$$q\approx D^{(-1)}=\frac{S}{L}\qquad(24)$$

Labour productivity depends exclusively on the average area of land (or quantity of any other "natural resource") that is available to each final producer.

From (23), it is clear that for $\alpha \approx 0$

$$\Gamma\left(D,0\right)=\frac{D^{\frac{1}{2}}}{2}\left(1-\frac{1}{4D}\right) \tag{25}$$

We can compute the value of ε in the lower bound $\tilde{D}\equiv\frac{9}{4}tF$ of the feasible interval from (22) and (25).

$$\varepsilon\left(\tilde{D},0\right)=\frac{1}{3}-\frac{3}{4}\left(tF\right) \tag{26}$$

Moreover, from (25) and (22), we have $\lim_{D\to+\infty}\varepsilon\left(D,0\right)=-\infty$. Since $\varepsilon\left(D,0\right)$ is a continuous and strictly decreasing function of D, if $\varepsilon\left(\tilde{D},0\right)$ in (26) is positive, then there is a level $\hat{D}$ of the population density that exhibits the following property.

$$\begin{cases}\varepsilon\left(D\right)>0 & if \quad \tilde{D}<D<\hat{D}\\ \varepsilon\left(D\right)=0 & if \quad D=\hat{D}\\ \varepsilon\left(D\right)<0 & if \quad \hat{D}<D\end{cases} \tag{27}$$

It remains to assess the boundary condition $\varepsilon\left(\tilde{D},0\right)>0$, which from (26) appears to be equivalent to,

$$\tilde{D}\equiv\frac{9}{4}\left(tF\right)<1\leftrightarrow tF<\frac{4}{9} \tag{28}$$

This means that transport and fixed costs should be low enough to allow a transition to a specialized production of capital inputs. Furthermore, the scope for overall increasing returns is rather limited and transient. Property (27) shows that when population density passes the threshold $\hat{D}$, the economy regresses to a decreasing returns state, where additional population just causes more congestion in agriculture with a diminishing effect on per capita income.

We deal now with the other polar case where $\alpha\approx 1$, so that the output of composite good per worker is about,

$$q\approx D^{0}=1 \tag{29}$$

Hence, the output per worker does not depend upon the area of land (or the amount of any kind of natural resource). According to TAMURA (2002), although the final good is still produced under competitive conditions, it is essentially an industrial good.

From (22), the income per capita elasticity with respect to population density is,

$$\varepsilon(D,1) = -(tF)^{\frac{1}{2}}\,\Gamma(D,1) \tag{30}$$

From (23), the function $\Gamma(D,1)$ is easily calculated.

$$\Gamma(D,1) = -\frac{1}{2}D^{\left(-\frac{1}{2}\right)}\left(1+\frac{3}{4D}\right) \tag{31}$$

By substituting (31) into (30) and simplifying, we obtain the elasticity for the case where $\alpha \approx 1$.

$$\varepsilon(D,1) = \left(\frac{1}{2}\right)\left(\frac{tF}{D}\right)^{\frac{1}{2}}\left(1+\frac{3}{4D}\right) \tag{32}$$

As before, $\varepsilon(D,1)$ is a continuous function of D over the interval $\left[\frac{9}{4}tF,+\infty\right)$. It is always positive and strictly decreasing in relation to the density. It converges to zero when $D \to +\infty$.

Therefore, if the final production is of the industrial type, once capital goods become provided by specialized firms the scope for an economy with aggregate increasing returns is unbounded. However large the demographic growth, it will always remain consistent with a positive elasticity of income per capita with respect to labour force density.

4. Concluding remarks

In this paper, we described the relationship between population density and per capita income along the two growth regimes put forward by KUZNETS (1960), BOSERUP (1965) and TAMURA (2002). We considered a spatial economy where an undifferentiated consumer good is produced by a continuum of competitive agents. Each agent requires one unit of a capital good to produce the final composite good and the two goods are assumed to be perfect substitutes in production.

Under the first growth regime (usually called “classical” or “Malthusian”), each agent self-produces the capital good by shifting resources that would produce one unit of the final good. The economy shows marginal decreasing returns of labour. Population growth brings about congestion and the elasticity of output per worker (and income per capita) is negative.

Empirical evidence shows that population tends to grow faster in low-income regions and countries. If the rise in population is matched by a decrease in transport costs, then a structural change with the outsourcing of capital goods to a specialized industry takes place. Then, input production will become concentrated in a small number of plants due to the existence of fixed costs and each final producer will have to bear the transport cost to the closest supplier of the capital good.

Under the “modern” growth regime, the specialization of capital good production generates an overall increasing returns factor. A fast rising population brings about effects of both “sharing” of industrial fixed costs and better “matching” concerning the quality of the relation between supplier and user of a capital good as they become closer.

While the elasticity of per capita income with respect to density always becomes positive just after the structural transition, the persistence of increasing returns in the aggregate economy in the long run is crucially dependent on the sectoral composition of the final production. If the specialization of capital goods production is matched by a change in the composition of final production with a rising share of activities which are *not* resource-based, aggregate increasing returns can be sustained in the long run. Otherwise, if activities such as agriculture or production of housing services remain dominant in final production, it is likely that elasticity of per capita income with respect to density will switch again to be negative if population density continues to grow in the long run.

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Appendix: Data on labour productivity and density for 27 European countries

1. Low density countries (population density lower than or equal to 102 people per square Km).

Country	y	x	z
Austria	109	101	2.1
Bulgaria	46	66	0.7
Croatia	68	76	2.3
Estonia	74	29	0.3
Finland	100	16	0.3
Greece	68	82	1.6
Ireland	186	66	1.3
Latvia	65	31	0.1
Lithuania	73	45	0.5
Norway	120	13	0.2
Romania	68	84	0.3
Slovenia	77	102	3.9
Spain	93	93	3.1
Sweden	106	22	0.5

2. High density countries (with population density higher than or equal to 102 people per square Km).

Country	y	x	z
Belgium	122	367	5.8
Czechia	80	133	1.6
Denmark	110	132	3.0
France	110	104	1.8
Germany	98	226	3.6
Hungary	68	106	2.1
Italy	99	202	2.3
The Netherlands	102	406	6.6
Poland	76	123	0.5
Portugal	72	113	3.3
Slovakia	72	110	1.0
Slovenia	77	102	3.9
Switzerland	118	198	3.5
United Kingdom	94	263	1.5

Meaning of variables:

$y \equiv$ GDP per employed person in 2019 (unit: PPS). Source: AMECO database.
$x \equiv$ Population density in 2020 (unit: people per Km^2). Source: AMECO database.
z $\equiv$ Motorway density in 2017 (unit: Km of motorway per Km^2 in percentage).
Source: "European Road Federation"