



In memoriam: Tomas Björk (1947–2021)

On his career and beyond

Raquel M. Gaspar¹ · Mariana Khapko²

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Abstract

This article celebrates the legacy of Tomas Björk. As we reflect on Tomas' life and career, we explore his personal and professional journey, highlighting his most significant contributions to mathematical finance.

Keywords In memoriam · Legacy · Financial mathematics · Interest rate theory · Time-inconsistent control · Pedagogical impact · Mentorship

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1 Introduction

We are honoured to commemorate the remarkable life and contributions of Tomas Björk, a brilliant scholar, teacher, mentor, caring colleague and friend whose impact on the field of mathematical finance and on those who knew him will never be forgotten. Tomas was a beloved member of the academic community, admired for both his research achievements and his kind and generous spirit. As former PhD students of Tomas, we have had the privilege of witnessing his exceptional intellect, guidance and unwavering support firsthand.

In this article, we seek to celebrate the life and accomplishments of Tomas, honouring his memory and the valuable contributions he made to the world. While it is

✉ M. Khapko
mariana.khapko@rotman.utoronto.ca

R.M. Gaspar
Rmgaspar@iseg.ulisboa.pt

¹ ISEG, Universidade de Lisboa and CEMAPRE/REM Research Center, Rua do Quelhas 6, 1200-078 Lisboa, Portugal

² Department of Management UTSC and Rotman School of Management, University of Toronto, 105 St. George Street, Toronto, ON, M5S 3E6, Canada

impossible to capture the full breadth of Tomas' life and career in a single article, we hope to provide a glimpse into his exceptional journey, from his childhood in Fagersta to his many achievements in academia and beyond.

2 Early life

Tomas was born in 1947 in Fagersta, in the northern part of the Västmanland province in central Sweden. He was the second child of Georg and Margareta, and had an older sister named Birgitta. Tomas lived in the countryside until the end of high school. Fagersta, located less than 200 km from Stockholm, is a small town known for its steel industry. It was here that Tomas spent the formative years of his life.

Despite growing up in a small town, Tomas was a curious and ambitious child. He showed a particular interest in mathematics, a passion he would pursue throughout his life. His love for exploration and adventure led him to spend much of his childhood outdoors, in the beautiful natural surroundings of Fagersta. Even during harsh winters, he would ski to school, eager to learn and discover more about the world. Tomas was also a scout, along with his mother and sister. He was very proud of his scouting and orientation skills, which he developed through his love of nature and exploration.

Tomas' curiosity extended beyond academics and nature. From a young age, he had a fascination with magic. When he was just eight years old, he began assisting his father with magic shows at local events and gatherings. By his early teens, Tomas had developed his own magic show and became quite popular. He was always proud to show the local newspaper articles mentioning his shows, and it was clear that his passion for magic had a special place in his heart. In addition to his interest in magic, Tomas developed a passion for music at a young age and continued to pursue it throughout his life. During middle and high school, he played the clarinet in the local street band Bruksmusiken. In his early teens, Tomas also began playing chess and quickly became skilled at the game. He participated in national competitions for school teams, achieving great success.

Tomas' exceptional academic ability became evident in high school, where he excelled in a variety of subjects, including mathematics, physics and chemistry. His teachers and peers recognised his potential early on, and he quickly became known as one of the brightest students in the school. By the time he graduated from high school, it was clear to everyone who knew him that Tomas was destined for great things and would go on to achieve great success in whatever he chose to do.

3 Education and career journey

3.1 Tomas the mathematician

Tomas was a man driven by his passion for the exact sciences. His academic journey began at Stockholm University, where he earned his bachelor's degree in physics. However, Tomas wanted to delve deeper into mathematics and its applications. So

he decided to pursue a PhD in optimisation theory at the prestigious KTH, the Royal Institute of Technology in Stockholm.

During his PhD studies, Tomas focused on the intricate problems of filtering, prediction and smoothing. His thesis, titled *On Finite Dimensional Filtering, Prediction and Smoothing* [8], was written under the supervision of Bengt Rosén and Lars Erik Zachrisson. Upon completing his studies in 1981, he remained at KTH as a faculty member, starting as a research associate and becoming a senior lecturer in optimisation and systems theory in 1987. He was then awarded the title of “Docent”, equivalent to an associate professor in the United States, in 1994. However, Tomas felt drawn to the field of mathematical finance which was emerging as a discipline at the time.

In 1995, Tomas made a bold move by joining the finance department at the Stockholm School of Economics (SSE) as an associate professor. The options market was becoming increasingly important in Sweden, and SSE needed experts who could conduct research in this area. Although his former department at KTH made efforts to retain him, including an offer of a salary that matched the one from SSE, Tomas decided to accept the position at SSE. He was promoted to professor in 1998 and later went on to lead the department for several years as its head. In recognition of his significant contributions to the field, he was named Professor Emeritus of Mathematical Finance in 2014. Tomas remained a faculty member at SSE until his passing in 2021, but his ties to KTH remained strong throughout his career. He sustained his affiliation with the institute and, for many years, continued to teach and inspire students there.

3.1.1 1995–1998: “the” book

Tomas’ transition from KTH to SSE marked a significant turning point in his academic journey, particularly in the field of financial mathematics. His arrival at SSE was met with great enthusiasm and anticipation as he embarked on a new chapter of his career that would leave a lasting impact.

One of the defining moments of this period was the publication of his magnum opus, *Arbitrage Theory in Continuous Time*. This highly successful book, first published in 1998, has been widely acclaimed for its comprehensive coverage of arbitrage theory and its applications in pricing financial derivatives. Designed as a textbook for graduate and advanced undergraduate students in finance, economics, mathematics and statistics, it also offered practical insights for practitioners.

In reflecting on the book’s genesis, Tomas fondly recalled the opportunity to deliver a series of lectures at the Monte Verità summer school in Ascona in 1995. This experience proved to be a turning point, marking the beginning of a new phase in his career and providing the foundation for what would become *Arbitrage Theory in Continuous Time*. The lecture notes from that transformative summer school served as the basis for the book, showcasing Tomas’ remarkable insights and contributions to the field.

3.1.2 1995–2010: interest rate theory

Throughout his time at the Stockholm School of Economics, Tomas established a distinguished career marked by exceptional achievements and impactful contributions.

His pioneering work in interest rate theory in particular has earned him widespread recognition within the academic community.

Tomas is widely known for his seminal paper co-authored with Yuri Kabanov and Wolfgang Runggaldier [49] on the bond market structure in the presence of marked point processes. This paper remains one of his most highly cited works to this day. Additionally, Tomas' collaboration with the same co-authors and Giovanni Di Masi [39] has made significant contributions to the theory of bond markets. His Springer lecture notes on interest rate theory [15] (part of the Lecture Notes in Mathematics book series) have also gained tremendous popularity.

However, it was the geometric view of interest rate theory that was particularly close to his heart. Tomas formed a close collaboration with Lars Svensson, a mathematician at KTH and his friend and colleague. Together, they explored the existence of finite-dimensional realisations for nonlinear forward rate models [55, 63]. In addition to his work with Lars, Tomas engaged in collaborations on related topics with Bent Jesper Christensen [35, 36] and Andrea Gombani [36, 42], as well as his PhD students Camilla Landén [34, 54, 55], Raquel M. Gaspar [40] and Magnus Blix [34].

Tomas and his co-authors delved into the geometric properties underlying the evolution of the forward rate curve within an arbitrage-free bond market. He was one of the first to view the so-called forward rate equation not as an infinite system of SDEs (as it was standard in interest rate theory), but as a single equation for an infinite-dimensional object—the forward rate curve, i.e., the curve $x \mapsto r_t(x)$. This shift in perspective opened the door to investigating the geometric characteristics of the forward rate curve process r as a process evolving in some infinite-dimensional function space \mathcal{H} of forward rate curves.

To emphasise the geometric point of view, Tomas rewrote the forward rate equation. The forward rate curve at time t was denoted as r_t , and the entire forward rate curve process was represented by r . The dynamics were expressed as

$$dr_t = \left(\mathbf{F}r_t + \sigma(r_t)\mathbf{H}\sigma^*(r_t) - \frac{1}{2}\sigma'_r(r_t)\sigma(r_t) \right) dt + \sigma(r_t) \circ dW_t. \quad (3.1)$$

Here, the symbol $*$ denotes transpose, \circ represents the Stratonovich type of integration, and σ'_r denotes the Fréchet derivative of σ with respect to r . The operators \mathbf{F} and \mathbf{H} , both defined on \mathcal{H} , are given by

$$\mathbf{F} = \frac{\partial}{\partial x},$$

$$(\mathbf{H}f)(x) = \int_0^x f(s)ds \quad \text{for } f \in \mathcal{H}.$$

This geometric perspective was an absolute novelty within the mathematical finance community, and its discovery and application required the adeptness of a true mathematician.

Tomas' geometric analysis focused on two main problems. Suppose that we are given a concrete forward rate model \mathcal{M} (i.e., suppose that we are given a concrete specification of the volatility process σ from (3.1)).

1) Take in addition to \mathcal{M} also a given parametrised family \mathcal{G} of forward rate curves. Under which conditions is the family \mathcal{G} consistent with the dynamics of the model \mathcal{M} ? Here, consistency is interpreted in the sense that given an initial forward rate curve, the interest rate model \mathcal{M} will only produce forward rate curves belonging to the given family \mathcal{G} .

2) When does the given, infinite-dimensional interest rate model \mathcal{M} admit a finite-dimensional realisation? More precisely, what are the conditions under which the forward rate process $(r_t(x))$ induced by the model \mathcal{M} can be realised by a system of the form

$$dZ_t = a(Z_t)dt + b(Z_t)dW_t,$$

$$r_t(x) = G(Z_t, x),$$

where Z (interpreted as the state vector process) is a finite-dimensional diffusion, $a(z), b(z)$ and $G(z, x)$ are deterministic functions, and W is the same Wiener process as in (3.1).

No matter how abstract these problems may seem, their significance in the daily lives of traders operating in the fixed-income market cannot be overstated. The solution to Problem 1) highlights the pitfalls that traders face when using a combination of stochastic interest rate models and deterministic parametrisations to fit the term structure. In other words, relying solely on a parametrisation, such as the Nelson–Siegel model, without considering its consistency with the stochastic model’s dynamics can lead to issues like the frequent need for recalibration. Tomas’ research has demonstrated that popular parametrisations, including Nelson–Siegel, are not compatible with widely used stochastic affine term structure models, and similar problems can be expected with other ad-hoc parametrisations.

The solutions to Problem 2) shed light on addressing this issue, emphasising the importance of starting from the stochastic model and then exploring if it allows finite-dimensional realisations (i.e., parametrisations). The Björk and Christensen 5-factor parametrisation (see [35]) extends the Nelson–Siegel model and answers the question of which parametrisation to use when considering short or forward interest rate models.

The general results for both problems are as follows:

1) Given some technical smoothness conditions on the drift and diffusion terms of (3.1), as well as the requirement of smooth embedding of the mapping $z \mapsto G(z)$, the forward rate curve manifold \mathcal{G} is locally invariant for the forward rate process $(r_t(x))$ in \mathcal{M} if and only if

$$G'_x(z) + \sigma_r(r)\mathbf{H}\sigma^*(r) - \frac{1}{2}\sigma'_r(r)\sigma(r) \in \text{Im}(G'_z(z)), \tag{3.2}$$

$$\sigma(r) \in \text{Im}(G'_z(z)) \tag{3.3}$$

hold for all $z \in \mathcal{Z}$, an open connected subset of \mathbb{R}^d , with $r = G(z)$, where G'_z and G'_x denote the Fréchet derivatives of G with respect to z and x , respectively. The conditions (3.2) and (3.3) are the drift and volatility consistency conditions, and the latter is to be interpreted componentwise for σ .

2) The forward rate in (3.1) admits a finite-dimensional realisation if and only if each component of σ in (3.1) is quasi-exponential. Thus, one needs each component of σ to be of the form

$$f(x) = \sum_i e^{\lambda_i x} + \sum_j e^{\alpha_j x} (p_j(x) \cos(w_j x) + q_j(x) \sin(w_j x)),$$

where λ_i, α_i, w_j are real numbers, whereas p_j and q_j are real polynomials.

Tomas' contributions to the geometric view of interest rate theory extended to various handbooks and books. In recognition of his expertise, he was awarded the prestigious "Cattedra Galileiana" at the Scuola Normale Superiore in Pisa, Italy, in 2000. This award led to the creation of his Cattedra Galileiana lectures on the subject [17]. In addition to these lectures, Tomas contributed overview chapters on the geometry of interest rate models in notable publications such as [18, 22, 24]. In particular, his involvement in the Paris–Princeton Lectures in Financial Mathematics [20] is regarded as a pedagogic masterpiece on a challenging subject.

3.1.3 2008–2021: time-inconsistent control

In the latter part of his career, Tomas turned his attention to the topic of time inconsistency. It is important to note that in this context, the issue of inconsistency does not pertain to the consistency between the dynamics of a particular interest rate model and the employed forward curve family. Rather, in the realm of stochastic optimal control, a problem is deemed dynamically consistent when the Bellman optimality principle holds: If a control law is optimal over the entire time interval $[0, T]$, then it is also optimal when restricted to any subinterval $[t, T]$. Tomas was intrigued by the challenge of finding solutions for problems that do *not* admit a Bellman optimality principle and are not susceptible to standard optimisation methods. Tomas' work extended the existing body of knowledge on time-inconsistent control problems considerably and further strengthened his reputation as a leading figure in the field.

In a typical continuous-time stochastic optimal control problem, the goal is to maximise (or minimise) a functional of the form

$$E_{t,x} \left[\int_t^T H(s, X_s, u_s) ds + F(X_T) \right].$$

Here, X is a controlled Markov process, u_s is the control applied at time s , and H, F are given functions. A common example is when X is given by a controlled SDE of the form

$$dX_t = \mu(t, X_t, u_t) dt + \sigma(t, X_t, u_t) dW_t.$$

To solve a problem like this, dynamic programming is often used. However, one must make several restrictive assumptions to have access to Bellman's optimality principle. Specifically, the instantaneous utility H and the terminal utility F cannot depend on the initial point (t, x) , and the terminal evaluation term can be of the form

$E_{t,x}[F(X_T)]$, but not of the form $G(E_{t,x}[F(X_T)])$, meaning that a nonlinear function of the expected value cannot be considered.

However, in financial economics, there are quite a few important problems where these assumptions do not hold and time consistency fails. Some examples include:

1) Non-exponential discounting,

$$\max_u E_{t,x} \left[\int_t^T \varphi(s-t)h(X_s, u_s)ds + \varphi(T-t)F(X_T) \right],$$

where h is the local utility, F is the utility of terminal wealth and φ is the discounting function. Importantly, every choice of the discounting function φ , apart from the exponential case, leads to a time-inconsistent problem.

2) Mean–variance utility,

$$\max_u E_{t,x}[X_T] - \frac{\gamma}{2} \text{Var}_{t,x}[X_T],$$

where the presence of the variance term introduces time inconsistency in this dynamic version of a standard Markowitz problem.

3) Endogenous habit model,

$$\max_u E_{t,x}[\ln(X_T - x + \beta)], \quad \beta > 0.$$

The lowest acceptable level of terminal wealth is given by $x - \beta$ and depends on the agent’s wealth $X_t = x$ at time t , thus making the control problem dynamically inconsistent.

4) Non-expected utility,

$$\int_0^\infty w(P_{t,x}[U(X_T) > z])dz,$$

where $w : [0, 1] \rightarrow [0, 1]$ is a probability distortion function.

So how can one handle time-inconsistent problems? In his work, Tomas studied this question within a game-theoretic framework, which takes an agent’s strategies as the outcome of an intrapersonal game played by successive incarnations of the same agent. In this framework, each point in time corresponds to a participant in a non-cooperative game. Referred to as “player t ,” each participant controls the process X at their respective time t employing the chosen control function $\mathbf{u}(t, \cdot)$. The collective control functions of all players form a feedback control law $\mathbf{u} : [0, T] \times \mathbb{R}^n \rightarrow \mathbb{R}^k$, guiding decision-making throughout the time interval. This approach replaces the usual concept of optimality with that of an intrapersonal equilibrium and looks for a Nash subgame-perfect equilibrium control law $\hat{\mathbf{u}}$.

Utilising the game-theoretic approach, Tomas developed a comprehensive theory encompassing Markovian time-inconsistent control problems in both discrete- and continuous-time settings. The key outcome of his research is an extension of the conventional dynamic programming equation, presented as a system of nonlinear equations. In contrast to the standard time-consistent case, one does not have a direct

recursive equation for the equilibrium value function. Instead, due to the equilibrium nature of the problem, the extended dynamic programming result constitutes a system of equations.

Consider as an example a simple time-inconsistent reward functional of the form $J(t, x, \mathbf{u}) = E_{t,x}[F(X_T^{\mathbf{u}})] + G(E_{t,x}[X_T^{\mathbf{u}}])$ which features no state dependence and no instantaneous utility term (see [52, Sect. 16.2]). This model accommodates the mean–variance objective, with $F(x) = x - \frac{\gamma}{2}x^2$ and $G(x) = \frac{\gamma}{2}x^2$. The time inconsistency is introduced by the presence of the G -term, which is not the expectation of a nonlinear function, but a nonlinear function of the expected value. In this case, the extended Hamilton–Jacobi–Bellman (HJB) system takes the following form: The equilibrium value function $V = J(t, x, \hat{\mathbf{u}})$ is determined by

$$\sup_u \left(\mathcal{A}^u V(t, x) - \frac{1}{2}\sigma^2(t, x, u)G''(g(t, x))g_x^2(t, x) \right) = 0, \quad 0 \leq t \leq T, \quad (3.4)$$

$$V(T, x) = F(x) + G(x), \quad (3.5)$$

where \mathcal{A}^u is the usual controlled infinitesimal operator and the function $g(t, x)$ is defined by

$$\mathcal{A}^{\hat{\mathbf{u}}}g(t, x) = 0, \quad 0 \leq t \leq T, \quad (3.6)$$

$$g(T, x) = x. \quad (3.7)$$

Notably, the equations (3.6), (3.7) for g correspond to the Kolmogorov backward equation for the expectation, i.e., $g(t, x) = E_{t,x}[X_T^{\hat{\mathbf{u}}}]$. We observe that the boundary condition (3.7) is dictated by the structure of the nonlinear G -term in the reward functional. Specifically, (3.7) needs to be modified to $g(T, x) = h(x)$ if the term $G(E_{t,x}[X_T^{\mathbf{u}}])$ is replaced by $G(E_{t,x}[h(X_T^{\mathbf{u}})])$ for some function h . If the nonlinear G -term does not appear, the problem simplifies to a time-consistent one, and (3.4) and (3.5) reduce to the standard HJB equation.

It is important to highlight that previous literature has primarily focused on studying specific cases and developing tailored methodologies for particular applications of time-inconsistent control. In Tomas’ work, however, a pioneering effort was made to establish a general theory applicable across various contexts, including problems with non-exponential discounting, mean–variance objectives, dynamically inconsistent linear–quadratic regulators, probability distortion, and market equilibrium with time-inconsistent preferences. Tomas was also the first to show that for every time-inconsistent problem, there exists an associated time-consistent problem such that the optimal control and the value function for the consistent problem coincide with the equilibrium control and value function, respectively, for the time-inconsistent problem.

In his research on time inconsistency, Tomas collaborated with his PhD students Agatha Murgoci and Mariana Khapko, as well as with Xunyu Zhou. The discrete-time results for his theory of Markovian time-inconsistent stochastic control can be found in [57], while the continuous-time setting is covered in [51]. An important special case of the general theory, the mean–variance portfolio choice problem with wealth-dependent preferences, is considered in [58], one of his most cited papers.

3.1.4 Other work

In addition to his pioneering research in interest rate theory and time-inconsistent control, Tomas made significant contributions to the mathematics of stochastic processes. His initial breakthroughs came in filtering theory, as evidenced by his early research works [7, 9] and his teaching interests [5, 6]. Tomas also explored topics in systems theory, investigating optimal maintenance in [61] and modelling signals with low multiplicity for interest rate applications in [41]. Collaborating with Jan Grandell, Tomas delved into actuarial science, where they studied important topics such as insensitivity in actuarial models [43] and developed exponential approximations in insurance mathematics [44]. He collaborated with Björn Johansson on several impactful studies. Together, they investigated prediction models with unknown probability measures [46], characterised statistical models compatible with specific data reduction through sufficient statistics [47], and established connections between parameter estimation and reverse martingales [48].

Tomas' extensive research in mathematical finance encompassed a wide array of topics. He worked extensively on arbitrage theory and derivative pricing, collaborating with Fredrik Armerin and Bjarne Astrup Jensen [1], Simon Benninga [2], Francesca Biagini [3], Eric Clapham [37], Henrik Hult [45], Camilla Landén [53] and Bertil Näslund [60]. In collaboration with Anders Szepessy, Raul Tempone and Georgios Zouraris [64], Tomas even ventured into the numerical methods associated with the Heath–Jarrow–Morton (HJM) term structure model. He made notable advancements in the research on good deal bounds in collaboration with Irina Slinko [62]. Furthermore, his exploration of optimal consumption and investment problems included collaborations with Mark Davis and Camilla Landén [38], as well as Johan Myhrman and Mats Persson [59]. Tomas also contributed several encyclopedia and survey articles, including [12, 26, 27].

Through his collaborations and diverse research interests, Tomas made significant strides in various areas of mathematics and mathematical finance, leaving a lasting impact on the field.

3.2 Tomas the book author

When discussing Tomas' work, one cannot overlook the significance of his books, which showcase his expertise and his ability to communicate complex concepts in an accessible way.

His most well-known book, *Arbitrage Theory in Continuous Time*, has been a staple in graduate financial mathematics courses since its first publication in 1998 [16], with subsequent editions released in 2004 [19], 2009 [23] and the latest fourth edition in 2020 [32]. Translations of the book are also available in other languages [21, 25].

Tomas wrote the book based on his experience teaching the subject, drawing on his lectures at KTH in Stockholm [11, 13] and an ETH Summer School in Ascona (other precursors to the book possibly include [10, 14]), and it is known for its clear and approachable style. The book covers measure-based probability theory, stochastic calculus, arbitrage-free option pricing (Black–Scholes), the abstract martingale

approach (fundamental theorem of asset pricing and change of numeraire) and numerous extensions, such as interest rate modelling and stochastic control problems.

In 2021, Tomas' body of work expanded with the publication of two additional books [33, 52], both drawing on years of research. Despite his declining health, Tomas remained dedicated to his passion for mathematics until the end, working on finalising both books in the fall of 2020.

Point Processes and Jump Diffusions: An Introduction with Finance Applications [33] grew from lecture notes that Tomas wrote for teaching a course on point processes at KTH [28]. The book provides an intuitive understanding of the central concepts of the theory of marked point processes, presents the mathematical theory rigorously and discusses applications in filtering theory and financial economics.

Meanwhile, *Time-Inconsistent Control Theory with Finance Applications* [52] offers the first comprehensive treatment of time-inconsistent dynamic choice problems in both continuous and discrete time and in the context of financial economics. This book goes beyond the articles Tomas and his co-authors published on the subject [51, 57, 58] and includes materials and results that did not make it into the published versions [50, 56], as well as many new results that Tomas continued to work on over the years. Notably, the book extends the game-theoretic approach to time-inconsistent stopping problems.

3.3 Tomas the teacher

Tomas will be remembered as a beloved and inspiring teacher renowned for his unique approach to challenging his students. His exceptional talent for simplifying complex concepts and making them accessible and intuitive was a hallmark of his teaching style.

It was evident early in his career that Tomas was destined to become a great pedagogue. When he was a teaching assistant, students from other courses flocked to his classes, leading to him being reassigned to a larger classroom due to the high demand. Tomas' passion for teaching was apparent, and his students reciprocated it. He was elected Teacher of the Year in KTH, manifesting his popularity and effectiveness among the student body. At the Stockholm School of Economics, Tomas quickly became a popular teacher among undergraduate and graduate students and professionals in the executive education program. He was widely regarded as one of the best teachers by those who had the privilege of taking a course with him.

Tomas' pedagogical approach was to provide as much intuition as possible while maintaining a reasonable level of precision. His mastery of the subject matter allowed him to know when to simplify the technical details and when to provide in-depth explanations. Tomas had a remarkable ability to leave a lasting impression on his audience. His talks, like his papers, were always clear and structured, reflecting his attention to detail. This trait extended to his notation choices as well. For example, he reserved the letter 'B' for a bank account and 'W' for Wiener processes, and he would get amusingly annoyed with academic papers that messed up this system.

Tomas' pedagogical impact extended far beyond Stockholm, as he inspired and taught students all over the world. He gave lecture series at numerous leading academic institutions, including Lisbon, Vienna, Aarhus, Oxford and Barcelona, where

he taught multiple courses over the years. He also shared his knowledge and expertise in many other cities, including Amsterdam, Ascona, Austin, Bologna, Budapest, Bressanone, Coimbra, Dubrovnik, Eindhoven, Kaiserslautern, Kyoto, Lahti, Linköping, Marrakesh, Montreal, New York, Princeton and Toronto.

Tomas' most regular lectures were in Lisbon in the fall and in Vienna in the spring, a tradition he maintained for over 15 years. He also had an affiliated position in Aarhus and gave a memorable lecture series at the University of Technology Sydney in 2008, which he would reminisce about for years.

3.4 Tomas the mentor

Tomas was a dedicated supervisor, and he had a profound impact on the lives of his 11 PhD students: Camilla Landén (graduated in 2001), Magnus Hyll (2001), Magnus Blix (2004), Raquel Gaspar (2006), Irina Slinko (2006), Mia Hinnerich (2007), Mikael Elhouar (2008), Agatha Murgoci (2009), Linus Kaisajuntti (2011), Kristoffer Lindensjö (2013) and Mariana Khapko (2015). He affectionately referred to them as his “Math Finance Gang,” emphasising the closeness he shared with each one. Tomas had a truly unique relationship with his students. He took a genuine interest in their lives, not just their academic progress, and was always there to offer advice.

Whenever his students doubted themselves, Tomas was incredibly supportive, often repeating “Of course, you can!” He took time to help and teach them, going together to conferences, pushing them when they needed to be pushed, and consoling them when they got stuck. Tomas was also always willing to help his students understand complex topics. He often gave one-on-one lectures to fill knowledge gaps and provided valuable advice for studying and understanding the subject matter. His famous question “What exactly do you mean?” made his students sweat, but it forced them to present arguments in a formal and structured way.

Besides being an incredible mentor, Tomas actually “adopted” his PhD students as his own. Most of his students placed him between a best friend and a father, taking the best of both worlds. During the times when his students were in Sweden, he would often be invited to their homes for dinner, or he would host them at his own home. He challenged his students to run or play golf with him and went with them for lunches, “fika” (a Swedish social coffee break with pastries), and even opera events. He was a great listener and would talk with his students about life in general or anything in particular.

Tomas celebrated his students' victories with them and remained invested in their lives beyond their academic pursuits. He was present at his students' weddings, met and befriended their parents and even met their kids, who, in a way, became his academic grandkids. Tomas also kept organising Math Finance Christmas Dinners, to which he invited all his former PhD students. This level of personal investment made his students feel valued and supported, and they remained in close contact with him even after graduating.

3.5 Tomas the eternal student

Tomas was a true eternal student, always eager to expand his knowledge and explore new subjects. As a student at KTH, he took courses in philosophy and Swedish literature at Stockholm University. Later, as an associate professor at the Stockholm School

of Economics, he took elementary courses in economics and finance to broaden his understanding even further.

Tomas' curiosity and drive to learn never waned. He often initiated a reading group with his colleagues and students to delve deeper into a particular paper. However, these discussions often lead to a broader exploration of the subject matter. He would then compile comprehensive lecture notes on the entire topic using his notes from these discussions. Some of his most notable lecture notes include *Equilibrium Theory in Continuous Time* [4], a nearly 200-page set of lecture notes that was partly incorporated into the latest edition of his arbitrage theory book. He also wrote *The Pedestrian's Guide to Local Time* [31] and *Comments on the Ross Recovery Theorem* [30]. When Piketty's best-selling book, *Capital in the Twenty-First Century*, became the talk of the town, Tomas wrote *Piketty for the Pedestrian* [29] to, as he put it in the abstract, "explain the theory to himself and his old friend Lasse [Lars Svensson]".

Tomas was an avid reader and participated in various reading clubs, including a club he dubbed the 'Philosophical Beer Club' with his friend Lars Svensson. They would meet once a week in a restaurant and study Kirkegaard, Kant, Schopenhauer, Spinoza and other philosophers over a pint of beer. One of his favourite philosophers, Baruch Spinoza, said, "The endeavour to understand is the first and only basis of virtue." This quote truly embodied Tomas' approach to life. He remained curious and eager to learn new things until his last day. In fact, during his final months, he was finalising his books and studying the continuous-time form of recursive utility with a small study group. It was just one more example of Tomas' insatiable thirst for knowledge and his unwavering dedication to understanding the world around him.

4 Personal life

4.1 Tomas the family man

It is often said that behind every great man, there is an even greater woman. In the case of Tomas, that woman was Agneta. They first met at KTH, where they were both students, and their relationship survived all of life's challenges. Together they raised two wonderful children, Kajsa and Stefan, born 3.5 years apart, just like Tomas and his sister.

Tomas was a dedicated, fun-loving father who enjoyed spending time with his children. He believed a good father must be willing to embarrass his children to give them perspective on what truly matters, especially during their early teen years. To this end, Tomas would watch lousy TV series with them and tell stories that sometimes stretched the truth. For example, he once convinced Kajsa and Stefan that he had worked doing tricks on a horse in a circus when he was young, going so far as to adopt the stage name "El Bravo" and learn special riding terms from Kajsa's horse riding lessons to make the story more convincing. The children were so confused by his antics that they even asked his mother for confirmation.

Tomas enjoyed spending time with his children and creating lasting traditions with his family. They decorated their Sundbyberg home like a Christmas postcard and travelled the world together, sharing many unforgettable experiences. And when Tomas

became a grandfather in 2009, it was no surprise that he embraced his new role with enthusiasm and joy. He described it as “like being a father of small children again, but only with the good things, without the responsibility and stress.” One of his proudest projects with his grandchildren was building a tree house with his grandson Alex in 2014, which created a special bond between them. He also loved to have fun with his granddaughter Julia, blowing soap bubbles and sharing magic tricks that made her laugh.

4.2 Tomas the sportsman

Tomas’ passion for sports was an integral part of who he was. He had a natural affinity for sports and physical activity, which he cultivated throughout his life. Tomas’ dedication to sports was contagious, inspiring those around him to pursue their own passions and reach their full potential.

Undoubtedly, golf held a special place in his heart. Although he began playing at the age of 18, it was not until later in life that he truly immersed himself in the sport, becoming a skilled and passionate single handicapper. He played as often as he could, relishing the opportunity to spend time with friends and family on the greens. One of his fondest memories was when his wife surprised him with her golfing abilities after secretly taking lessons for a few months.

In addition to golf, Tomas was an avid runner. He began running at a young age and maintained a consistent running schedule throughout his life, running 5 km several times per week and occasionally challenging himself to run 10 km or more. He was always striving to improve, gradually increasing his distance and speed over time. Even as he aged, Tomas continued to run, maintaining his stamina and determination. For him, running was a source of joy and a way to clear his mind, and he never lost his enthusiasm for it.

Tomas was also no stranger to cross-country skiing. In 1972, he participated in the Vasaloppet race, a gruelling 90-kilometre cross-country skiing event that follows the route taken by King Gustav Vasa from Mora to Sälen, although the modern race is run in the opposite direction, finishing in Mora. This was a remarkable achievement which demonstrated Tomas’ strength and determination. It was a clear reflection of his unwavering commitment to his goals and his ability to push himself to the limits.

4.3 Tomas the music lover

Alongside his love for sports, Tomas also had a strong passion for music that began in his youth and continued to thrive throughout his life. After leaving Fagersta, he continued to play the clarinet in a quintet in Stockholm and the KTH student orchestra. Later, when his daughter Kajsa expressed an interest in playing the piano, he decided to take lessons himself. He fell in love with the instrument and never stopped playing, eventually acquiring two pianos – one for his Sundbyberg apartment and another for his country house.

In addition to being a talented musician, Tomas was also a skilled dance teacher. While studying at KTH, he worked at several dance studios in Stockholm and was known for his impressive skills on the dance floor. Agneta’s friends even envied her for having such a great dancer as a husband.

As he entered his later years, Tomas discovered a new interest in singing. At the urging of his friend Kjell, he joined a local choir and began taking lessons to improve his skills. Tomas worked hard to improve his singing and eventually, he could showcase his versatility as a performer by singing an Italian aria.

Tomas' passion for music did not stop at playing and singing. He was also an avid opera enthusiast, regularly attending performances at the Royal Swedish Opera. His love for music and the arts was evident in all aspects of his life, and he took great pleasure in sharing his enthusiasm with those around him.

4.4 Tomas the multifaceted

Tomas was multifaceted, and one of his most endearing qualities was his great sense of humour. He was always up for a good challenge or bet, often coming up with quirky and unexpected competitions. From balancing on a tightrope with his first PhD student Camilla to betting on mundane things like which elevator would arrive first or which suitcase would come out of the baggage claim first, Tomas kept things light-hearted and fun. He was competitive, but loved the process of a competition or a bet even more than the outcome. There would be strict rules and stakes decided upon beforehand, usually small stakes like a dinner or a bottle of wine. And even when he did not win, he never took things too seriously and always found a way to make everyone around him laugh.

Tomas' humour extended beyond competitions, though. He was interested in languages and enjoyed learning a minimum set of words from different languages. He would often use this to his advantage by pretending to know a language that he did not, making people unsure whether he was joking. One year when he was teaching a class, he brought a Japanese translation of his book [21] and tried to convince his students that it was the textbook they should be using. When one student admitted they did not know Japanese, Tomas jokingly replied, "Well, what you should do is obvious: first learn Japanese, and then study the book!" His humour and wit always kept people on their toes.

In addition to his love of competitions and languages, Tomas had a unique passion for collecting original objects that showcased his personality. He boasted a collection of about 15 cow creamers, inspired by a P. G. Wodehouse novel, and had amassed over 40 shot glasses from various universities. Tomas also enjoyed doing rankings and lists, like the "MPP" (Most Pitiful Person) list he kept with his students. Anyone with a problem could claim to be at the top and either get immediate sympathy from the others or enter a debate about who was, in fact, on top of that list at any time. It was just for fun, but it also helped to put problems and "dramas" into perspective.

Tomas' wit and humour were essential to his personality and made him a delight to be around. But beyond just providing laughter, his bets, competitions and lists were a way of connecting with others. For those who knew him, Tomas' sense of humour will always be remembered as one of his most endearing qualities.

While Tomas had a warm heart, he had a low tolerance for incompetence combined with arrogance. He was passionate about research and was always ready to engage in a heated argument about it. This passion and intensity could sometimes make him appear intimidating to those who did not know him well. However, as Tomas himself

would say, he often got angry at things, but never at people. His frustration could sometimes manifest itself in amusingly angry outbursts towards non-living things, such as the printer that stopped printing or the pen that stopped writing. Despite his fiery spirit, he always saw himself as a teddy bear at heart. And those who knew him well knew that he truly lived up to that description.

5 Legacy

Tomas left a profound mark on mathematical finance through his extensive work, including numerous papers, books and book chapters. He was a highly productive author, co-authoring over 40 papers, and a sought-after speaker, giving over 100 conference and seminar presentations to audiences worldwide.

Tomas was also a highly valued member of the Bachelier Finance Society (BFS), serving on its Council from 2000 to 2003, as a member of the Executive Committee from 2008 to 2013, and as its President from 2009 to 2011. During his presidency, Tomas helped to introduce the Bruti Liberati Prize and played a key role in the success of several BFS Congresses. In addition to his contributions to the BFS, Tomas served as Co-Editor of *Mathematical Finance* and as a member of the editorial boards of *Finance and Stochastics* and *Annals of Applied Probability*.

Tomas' influence on the field of mathematical finance will be felt for years to come. As we remember Tomas and honour his memory, we should like to share some of the many tributes and testimonials that have poured in from those who knew him. These words offer a glimpse into the lasting impact that Tomas had on the academic community.¹

“He was a very determined man. He had his opinions, and he stood for them.”
– Knut Kristian Aase

“He was a very clear researcher, asking very sharp, well-posed questions. He brought a lot of rigour and precision to the discipline [...]. He was very generous in sharing his knowledge and very open about what he thought was important [...]. That was very inspiring and very fruitful for young researchers.”
– Bent Jesper Christensen

“He really brought us extreme sincerity for the field, mixed with an aspect of joy and pleasure. I find that unique of Tomas.” – Paul Embrechts

“He was very helpful. He gave me good input and encouraged me to write my first paper.” – Damir Filipović

“He made difficult theories accessible to a large audience.” – Mia Hinnerich

“I think his impact is very important, particularly on two points [...]. I think his book on arbitrage theory is the best one ... and also the work he did on marked point processes.” – Monique Jeanblanc

¹Most of the quotes included below were collected during the *Conference in memory of Tomas Björk* held in Stockholm on October 10–11, 2022, at the Swedish House of Finance.

“He seemed to know a bit of everything. He could discuss books, movies, math, finance, politics . . .” – Linus Kaisajuntti

“I will always admire his interest in people.” – Camilla Landén

“My favourite memory of Tomas is when he was explaining the term structure of interest rates, the geometry of it, with his hand, his fingers and a sheet of paper [. . .]. It was very charming and very inspiring [. . .]. I will remember his sense of humour and the warmth of his personality.” – Ali Lazrak

“Tomas was sometimes quite stern and demanding, but also very fair and nice [. . .]. Tomas had a crucial impact on my development and career.” – Kristoffer Lindensjö

“He said that being a PhD advisor is like being a parent: You do your best, and then you have to send the kid into the world [. . .]. With us, he was very kind, always caring for his PhD students and helping us a lot.” – Agatha Murgoci

“He was a remarkable combination of being an excellent mathematician, an excellent finance expert, and an excellent pedagogue.” – Bernt Øksendal

“He was an excellent mathematician, but also open to the challenges we have in economics and finance.” – Eckhard Platen

“The book [*Arbitrage Theory in Continuous Time*] is a pedagogical masterpiece; it is written in a lucid style that makes continuous-time mathematical finance easy – and easy in the best possible way.” – Rolf Poulsen

“He was such a brilliant lecturer. He also had quite strong opinions and was very quick at understanding the essence of things.” – Wolfgang Runggaldier

“The [scientific] impact was huge [. . .]. And I will always remember that he was such an excellent golf player, way beyond my abilities, but he was still very patient with me, and we had some lovely golf rounds.” – Walter Schachermayer

“My favourite memory is listening to him giving his presentations, explaining things very clearly and at the same time speaking extremely fast. I have never heard a speaker who was so fast and yet so clear at the same time.” – Martin Schweizer

“Tomas was very generous sharing his ideas with others, lifting people and empowering [. . .]. This feature inspired many young researchers and students in the field [. . .]. I will always remember Tomas’ *Of course you can do it.*” – Irina Slinko

“He had the ability to make complex continuous-time mathematics easy, accessible and intuitive.” – Per Strömberg

“Since we met, my fire has been lit for mathematical finance and for Tomas [. . .]. His impact on my career cannot be overestimated. I appreciate his research for simplicity, clarity, his clear notation, but always his view for the big things, to not get lost in details, but have the big questions in mind.” – Josef Teichmann

“He built a bridge between anyone who could handle calculus to the [sophisticated] mathematics that is required to handle arbitrage theory.” – Carlos Veiga

“He combined rigour and style. His work had a foundational impact on our community because he developed truly interdisciplinary research.” – Thaleia Zariphopoulou

“He was a very serious person, very rigorous mathematician, but on the other hand, he was also very intuitive in various finance problems.” – Xunyu Zhou

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Declarations

Competing Interests The authors declare no competing interests.

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