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Pedro Mazeda Gil[*], Oscar Afonso[†], Paulo Brito[‡]

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This paper builds an endogenous growth model of directed technical change with vertical and horizontal R&D and scale effects at the industry level to study an analytical mechanism that is consistent with the observed cross-country pattern in the skill structure, the technology structure and economic growth. We calibrate the model in order to uncover the effect of the skill structure on economic growth by studying how the former affects the technology structure. We find that the small positive elasticity of the economic growth rate regarding the ratio of high- to low-skilled workers that is empirically observed is explained by the combination of moderate levels of the market complexity costs related to vertical R&D and high entry costs in the high- vis-à-vis the low-tech sectors, which dampen the positive direct effect of the absolute productivity advantage of the high-skilled workers on growth.

Keywords: high-tech, low-tech, scale effects, skills, directed technical change

JEL Classification: O41, O31

1. Introduction

Casual empiricism suggests that the evolution of the skill structure, measured by the ratio of high- to low-skilled workers, and economic growth are twin processes. However,

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causality can run both ways: improvements in the skill structure would be associated to higher rates of growth via biased technological change, but growth in income would increase education enrollment rates and therefore the proportion of skilled people in the population. In this paper, we address the first type of causation, which can be isolated because it takes place within a shorter time scale.

We gathered data for a number of European countries between 1995 and 2007 (see Section 2 for details), which shows a large dispersion of values for the ratio of high- to low-skilled workers.[1] By running cross-section regressions, we find that the elasticity with respect to the proportion of high-skilled workers is: (i) positive, but small, in the case of the economic growth rate; (ii) positive in the case of the ratio of production and of the number of firms between the high- and the low-tech sectors; (iii) positive, but small, in the case of the ratio of the firm size between those two types of sectors; and (iv) strongly negative in the case of the skill premium, measured as the wage ratio between high- and low-skilled workers.

We infer from the previous evidence that the skill structure featuring a higher proportion of high-skilled workers is associated with technological change directed towards the high-tech sectors, given the observed positive elasticity of the technology structure (measured either by the number of firms, by production or by the firm size in high- versus low-tech sectors) regarding the relative supply of skills. However, it is also associated with a lower wage premium for the high-skilled workers. The second effect tends to offset the technological-knowledge bias, by creating incentives for a redirection of R&D towards the low-tech sectors, but evidence shows that the first effect apparently dominates.

There is a strand of the literature that analyses the link between the distribution of economic activity across sectors and economic growth, either empirically (e.g., Fagerberg, 2000) or theoretically (e.g., Ngai and Pissarides, 2007; Bonatti and Felice, 2008; Acemoglu and Guerrieri, 2008) and which, by deriving the implications of different sectoral Total Factor Productivity (TFP) growth rates, suggests that countries specialised in "technologically progressive" sectors (high TFP growth) enjoy higher growth rates.

We follow instead an alternative strand of the literature, in particular Acemoglu and Zilibotti (2001), which endogenises R&D activities by introducing a mechanism of endogenous directed technical change, with symmetric TFP growth across sectors, in which high-skilled workers have an absolute productivity advantage. This approach incorporates an analytical mechanism relating the skill structure to the technology structure and to economic growth. We extend the Acemoglu and Zilibotti's (2001) model, featuring endogenous growth with directed technical change and horizontal R&D, by adding vertical R&D through a quality-ladders mechanism and scale effects in both R&D activities. Then, by solving and calibrating the model using the data on the skill structure and the technology structure presented in Section 2, we try to uncover the mechanism through which the skill structure affects economic growth.

The setup of the model presented in this paper features final-goods production using

[1]For example, considering the distribution of the ratio of high- to low-skilled workers, measured as the ratio of college to non-college graduates among persons employed in manufacturing, across a sample of 29 European countries, the ratio of the fourth to the first quartile is as high as 4.5 (see section 2 for more details on the data).

either low- or high-skilled labour with labour-specific intermediate goods, while R&D can be directed to either the low- or the high-skilled labour complementary technology; hence, "sector" herein represents a group of firms producing the same type of labour-complementary intermediate goods. Since the data shows that the high-tech sectors are more intensive in high-skilled labour than the low-tech sectors,[2] we consider the high- and low-skilled labour-complementary intermediate-good sectors in the model as the theoretical counterpart of the high- and low-tech sectors (e.g., Cozzi and Impullitti, 2010).

While the directed technical change setup allows us to have an endogenous high- versus low-tech technological bias, the joint consideration of vertical and horizontal R&D is in line with the view that industrial growth proceeds both along an intensive (increase of product quality) and an extensive margin (introduction of new products). We consider a lab-equipment R&D specification,[3] which implies that the choice between vertical and horizontal innovation is related to the allocation of R&D expenditures, which are fully endogenous. Therefore, we endogenise the rate of extensive growth, and thereby the number of firms in each sector. Given the distinct nature of vertical and horizontal innovation (immaterial versus physical) and the consequent asymmetry in terms of R&D complexity costs, this framework then makes economic growth and firm dynamics closely related: vertical R&D is the ultimate growth engine, while horizontal R&D builds an explicit link between aggregate and industry-level variables (the number of firms and firm size).

A key ingredient of the model is the existence of scale effects. Existing empirical studies clearly indicate the existence of positive scale effects at the industry (manufacturing) level (e.g., Backus, Kehoe, and Kehoe, 1992; Peretto and Smulders, 2002). We allow for flexible scale effects at the industry (sector) level, which implies that the relationship between the number of firms, production and average firm size across sectors is endogenous. We also get a flexible relationship between economic growth and the relative supply of skills. As usual in the R&D-driven growth literature, there are gross positive scale effects connected with the size of profits that accrue to the R&D successful firm; a larger market expands profits and, thus, the incentives to allocate resources to R&D, thereby increasing the economic growth rate. However, an increase in market scale also dilutes the impact of R&D outlays on innovation outcomes, due to a number of costs and rental protection actions by incumbents (e.g., Dinopoulos and Thompson, 1999; Sener, 2008) related to market size. These market complexity costs (which add to the R&D complexity costs) then induce gross negative scale effects, which may partially or totally offset, or even revert, the benefits of scale to innovative activity.

By solving the model for the balanced-growth path (BGP), we show analytically that it provides measurable relationships between the skill structure and the technology struc-

[2] According to the data for the average of the European Union (27 countries), 30.9% of the employment in the high-tech manufacturing sectors is high-skilled ("college graduates"), against 12.1% of the employment in the low-tech manufacturing sectors (see Section 2 for more details on the data).

[3] Following Rivera-Batiz and Romer (1991), the assumption that homogeneous final good is the R&D input means that one adopts the "lab-equipment" version of R&D, instead of the "knowledge-driven" specification, in which labour is ultimately the only input.

ture, which we are able to calibrate with cross-country data. We assume that there are (gross negative) scale effects induced by market complexity costs, affecting both vertical and horizontal innovative activities, and that those complexity costs have an asymmetric impact on the elasticity of the relative number of firms (i.e., the ratio of the number of firms in the high- to the low-tech sectors) and of relative production (idem) with respect to the skill structure. Under this framework, the market complexity costs related to horizontal R&D have a direct negative impact on this type of R&D and an indirect positive impact on vertical R&D (substitution effect); however, the market complexity costs related to vertical R&D have a direct negative impact on this type of R&D, but also a negative impact, although smaller in modulus, on horizontal R&D. This reflects the fact that the extensive margin is endogenous and commanded by the intensive margin, hence generating what we call a roundabout cost effect. We show analytically that the elasticities of relative production, of the relative number of firms and of the relative firm size as regards the skill structure are positive, as in the data, if the market complexity costs are below a certain threshold (i.e., for sufficiently high levels of net scale effects).

We also show that the direct effect of the skill structure on the technology structure and, through the absolute productivity advantage of the high-skilled workers, on the economic growth rate is countervailed by the fact that it also generates a reduction in the skill premium. The elasticity of the skill premium with respect to the skill structure is negative and depends on the market complexity costs regarding vertical R&D, meaning that an increase in the proportion of high-skilled workers generates incentives that benefit the low-skilled workers. We find analytically that if the complexity costs are sufficiently low, then the direct effect dominates and there is a positive relationship between the skill structure and both the technology structure and the economic growth rate.

By calibrating the relationship between the skill structure and the technology structure, i.e., by considering the set of parameter values that determine a high quality of adjustment of the model to the data for the technology-structure variables as a function of the skill structure, we identify the more plausible values for the technology parameters. Then, we use them to compute the predicted values for the economic growth rates and compare them to the values in the data. We find that the small positive elasticity of the economic growth rate regarding the ratio of high- to low-skilled workers that is empirically observed is explained by the combination of moderate levels of the market complexity costs related to vertical R&D and high entry costs in the high- vis-à-vis the low-tech sectors (in particular when entry occurs through vertical innovation),[4] which dampen the positive direct effect on growth arising from the absolute productivity advantage of the high-skilled workers.[5]

[4] The literature on the economics of innovation sheds some light on why entry costs may be, in practice, generally larger in the high- than in the low-tech sectors. Firms in the high-tech sectors tend to face relatively thin markets, less mature and changing more rapidly than in the low-tech sectors, with the appropriation of technology through Intellectual Property Rights being more aggressively pursued; they also rely more heavily on formal planning activities, on customer support and on superior product warranties, and face environments where regulation more frequently plays a structuring role (e.g., the biotech industry) (e.g., Covin, Slevin, and Covin, 1990; Qian and Li, 2003; Tunzelmann and Acha, 2005).

[5] A related result frequently appearing in the empirical literature on the determinants of growth is the

The remainder of the paper has the following structure. The next section presents the evidence on the technology structure, economic growth, the skill premium and the supply of skills for a number of European countries. In Section 3, we present the model of directed technological change with vertical and horizontal R&D and scale effects, derive the general equilibrium and analyse the BGP properties. In Section 4, we detail the comparative statics results, deriving predictions with respect to the relationship between the relative supply of skills, the BGP technology structure and economic growth. In Section 5, we calibrate the model using the data on the skill structure and the technology structure, and study the mechanism through which the skill structure affects economic growth. Section 6 gives some concluding remarks.

2. Empirical evidence: skill structure, technology structure and growth

In this section, we present the cross-country data with respect to the number of firms, production, and average firm size (production by firm) in manufacturing, by considering the OECD classification of high- and low-tech sectors (see Hatzichronoglou, 1997).[6] We also collected data on the ratio of high- to low-skilled workers (which we will henceforth call the relative supply of skills), measured as the ratio of college to non-college graduates among persons employed in manufacturing, and the skill premium, measured as the mean annual earnings of the college graduates employed in manufacturing vis-à-vis the mean annual earnings of the non-college graduates. "College graduates" refers to those who have completed tertiary education (corresponding to the International Standard Classification of Education [ISCED] levels 5 and 6), while "non-college graduates" refers to those who have completed higher-secondary education or less (ISCED levels from 0 to 4).

The data concerns the 1995-2007 average and covers 25, 16 and 29 European countries regarding, respectively, the number of firms, production,[7] and the supply of skills (educational attainment), whereas the data for the mean annual earnings respects to 2002 and covers 28 European countries. The source is the Eurostat on-line database on

weak relationship between educational attainment *in absolute terms* and per capita economic growth. The hypothetical explanations for this result are the existence of, e.g., a pervasive mismatch between skills and jobs that translates into a low impact of human capital on growth at the aggregate level, strongly decreasing marginal returns to education as the supply of educated labour expands, and low education quality such that increasing years of schooling do not correspond to a larger human capital stock (see, e.g., Pritchett, 2001; Easterly, 2005), while an alternative line of work has emphasised the existence of errors in the measurement of human capital, both conceptually and empirically (e.g.,Krueger and Lindahl, 2001; Cohen and Soto, 2007). In contrast to this strand of the literature, which tends to focus on human capital *per se*, our approach focuses on explanations featuring the technical characteristics of the sectors that demand high-skilled labour.

[6] High-tech industries are, e.g., aerospace, computers and office machinery, electronics and communications, and pharmaceuticals, while the low-tech sector comprises, e.g., petroleum refining, ferrous metals, paper and printing, textiles and clothing, wood and furniture, and food and beverages.

[7] According to our theoretical model, we should restrict our analysis to the production of intermediate and capital goods. However, we were not able to find data according to the OECD classification of high- and low-tech sectors detailed by type of good and thus focused on total production in each sector.

Science, Technology and Innovation – tables "Economic statistics on high-tech industries and knowledge-intensive services at the national level" and "Annual data on employment in technology and knowledge-intensive sectors at the national level, by level of education" (available at `http://epp.eurostat.ec.europa.eu`).[8] At the aggregate level, we gathered data on the per capita real GDP growth rates for the same period, also from the same Eurostat on-line database.

While empirical data, as illustrated by Figures 1 and 2, suggests a significant variability of the technological structure across countries by considering the number of firms, total production and average firm size in high- vis-à-vis low-tech sectors, interesting regularities stand out:[9] (i) the number of firms and total production are smaller in high- than in low-tech sectors (i.e., the *relative* number of firms and relative production are below unity) in all countries, while the average firm size is larger in the high-tech sectors (i.e., *relative* average firm size is above unity); (ii) the elasticity of the relative number of firms, relative production and the relative average firm size with respect to the relative supply of skills is positive, although the elasticity of the later is substantially smaller; (iii) the economic growth rate has a mildly positive elasticity and the skill premium has a strong negative elasticity with respect to the relative supply of skills.

[Figure 1 goes about here]

[Figure 2 goes about here]

Table 1 reports the details on the OLS regressions run on the data depicted by Figure 2. Notice that, even though the goodness of fit of the regressions in Table 1 might most likely increase if we added explanatory variables, the bivariate approach followed therein anticipates the fact that the log-log linear relationships between the technology-structure variables, the skill premium and the relative supply of skills have an exact analytical counterpart in terms of the BGP equilibrium of the model developed in Sections 3 and 4. We take advantage of this fact to pursue an identification strategy for the technology parameters of the model later in Section 5 and thereby to calibrate it using the data on the technology-structure variables presented in Table 1. This will allow us to uncover the effect of the skill structure on economic growth by studying how the former affects the technology structure of the economy.

[8] Data with respect to output (Gross Value Added) is also available from the on-line OECD STAN Indicators Database (link at `http://stats.oecd.org`). According to the latter, the patterns depicted by Figure 1 are also verified for the non-European countries belonging to the OECD, namely the US, Canada, Mexico, Australia, Korea and Japan.

[9] In order to get a clearer picture of the behaviour of the number of firms, total production and firm size across sectors and countries, we only focus on the high- and low-tech sectors, which are the extreme OECD categories for manufacturing. That is, we leave out of our empirical illustration the medium-high and the medium-low categories also considered in the OECD classification. This way, we avoid the arbitrariness that involves the computation of the ratios when all categories are considered, since the theory offers no guidance on whether a specific intermediate category should incorporate the denominator or the numerator.

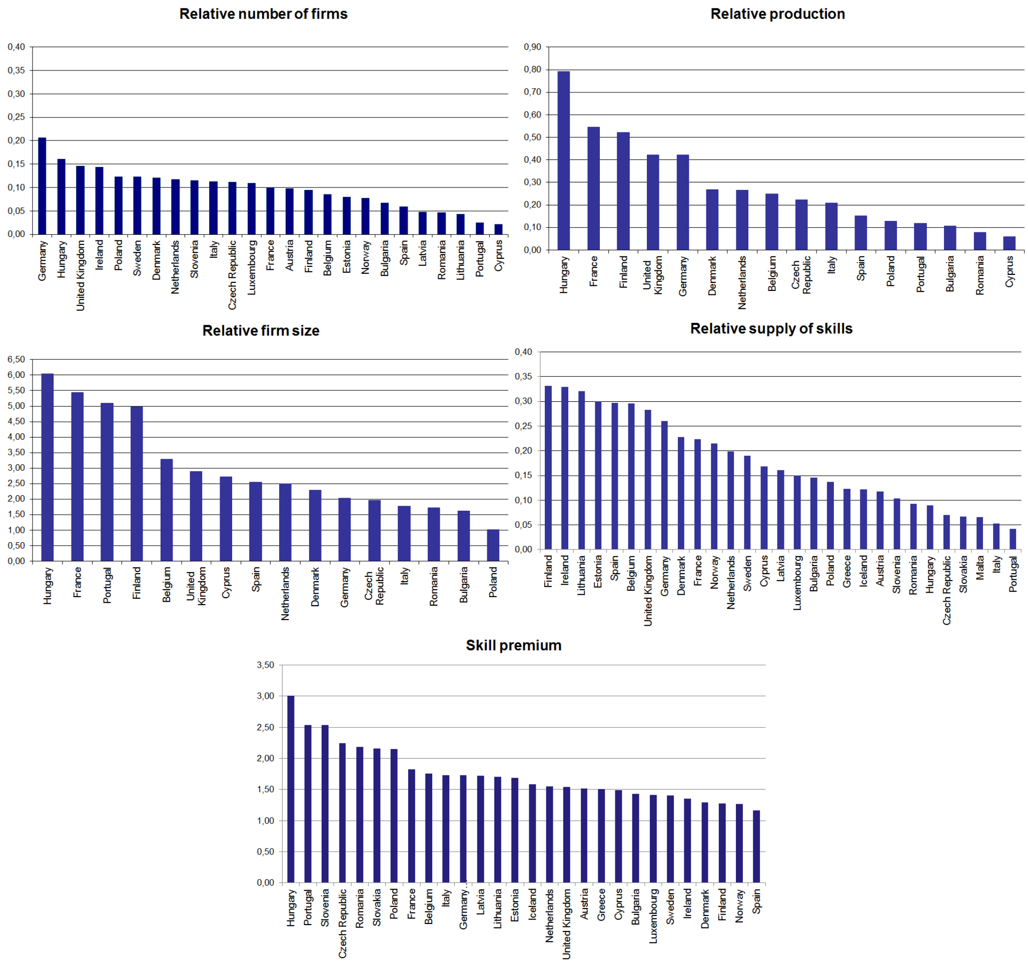

Figure 1: The technology-structure variables (the relative number of firms, relative production and the relative average firm size), the relative supply of skills and the skill premium, in a sample of European countries, 1995-2007 average (except for the skill premium, whose data is from 2002).

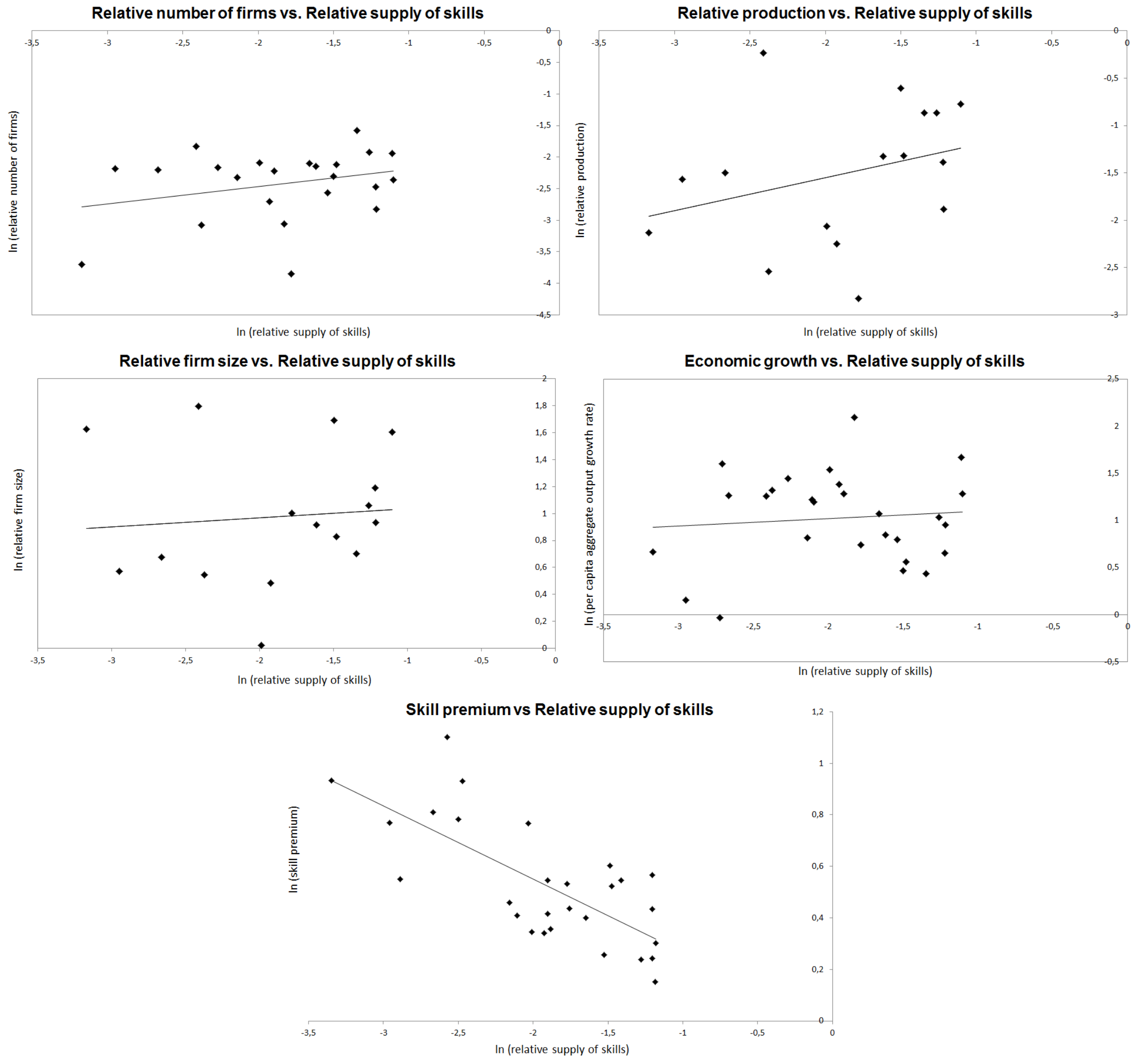

Figure 2: The technology-structure variables, the economic growth rate and the skill premium vis-à-vis the relative supply of skills for a cross-section of European countries, 1995-2007 average (except for the skill premium panel, whose data is from 2002). The straight line that appears in each panel is an OLS regression line (the details on the regressions are given in Table 1).

Dependent variable	ln Relative number of firms	ln Relative production	ln Relative firm size	ln Skill premium	ln Growth
Constant	−1.917	−0.855	1.105	−0.018	1.177
(White s.d.)	(0.399)	(0.387)	(0.367)	(0.097)	(0.336)
ln Relative supply of skills	0.274	0.349	0.067	−0.284	0.079
(White s.d.)	(0.228)	(0.205)	(0.211)	(0.052)	(0.180)
Observations	23	16	16	28	27
Correlation coefficient	0.284	0.312	0.089	0.720	0.095

Table 1: OLS regressions of the technology-structure variables, the economic growth rate, and the skill premium on the relative supply of skills, in logs.

[Table 1 goes about here]

3. The model

Biased technical change is introduced in a closed-economy dynamic general-equilibrium setup. The economy consists in a competitive sector that produces a final good that can be used in consumption, production of intermediate goods and R&D. There are also two intermediate-good sectors, having a large number of firms which operate in a monopolistic competitive framework. There are both vertical and horizontal R&D activities that are subject to flexible scale effects. If an innovation is successful, an incumbent is replaced by a new entrant in a given existing industry, or a monopolist emerges in a new industry, within a particular sector. Then successful R&D introduces, through creative destruction and variety expansion, both internal and external industry-wide limits to market power and generates endogenous economic growth. Thus, the model is an extension of Acemoglu and Zilibotti (2001), augmented with vertical R&D, as introduced in Afonso (2006), under flexible scale effects.

The economy is populated by a fixed number of infinitely-lived households who inelastically supply one of two types of labour to final-good firms: low-skilled, L, and high-skilled labour, H. The final good is produced by a continuum of firms, indexed by $n \in [0,1]$, to which two substitute technologies are available: the "Low" (respectively, "High") technology uses a combination of L (H) and a continuum of L-(H-)specific intermediate goods indexed by $\omega_L \in [0, N_L]$ ($\omega_H \in [0, N_H]$). Potential entrants can devote resources to either horizontal or vertical R&D, and directed to either the high- or the low-skilled labour-complementary technology. Horizontal R&D increases the number of industries, N_m, $m \in \{L, H\}$, in the m-complementary intermediate-good sector,[10] while vertical R&D increases the quality level of the good of an existing industry, indexed by $j_m(\omega_m)$. Then, the quality level $j_m(\omega_m)$ translates into productivity of the final producer

[10]Henceforth, we will refer to the "m-complementary intermediate-good sector" as "m-technology sector".

from using the good produced by industry ω_m, $\lambda^{j_m(\omega_m)}$, where $\lambda > 1$ measures the size of each quality upgrade. By improving on the current best quality j_m, a successful R&D firm will introduce the leading-edge quality $j_m(\omega_m)+1$ and thus render inefficient the existing input. Hence, the monopoly in ω_m is temporary.

3.1. Production and price decisions

As in Acemoglu and Zilibotti (2001), the aggregate output at time t is defined as $Y_{tot}(t) = \int_0^1 P(n,t)Y(n,t)dn$, where $P(n,t)$ and $Y(n,t)$ are the relative price and the quantity of the final good produced by firm n. Every firm n has a constant-returns-to-scale technology and uses, ex-ante, low- and high-skilled labour and a continuum of labour-specific intermediate goods with measure $N_m(t)$, $m \in \{L, H\}$, so that $N_{tot}(t) = N_L(t) + N_H(t)$ and

$$\begin{array}{rcl} Y(n,t) & = & A\left[\int_0^{N_L(t)} \left(\lambda^{j_L(\omega_L,t)} \cdot X_L(n,\omega_L,t)\right)^{1-\alpha} d\omega_L\right] [(1-n)\cdot l \cdot L(n)]^{\alpha} + \\ & & +A\left[\int_0^{N_H(t)} \left(\lambda^{j_H(\omega_H,t)} \cdot X_H(n,\omega_H,t)\right)^{1-\alpha} d\omega_H\right] [n \cdot h \cdot H(n)]^{\alpha} \end{array}, \quad 0<\alpha<1, \tag{1}$$

where $l \cdot L(n)$ and $h \cdot H(n)$ are the efficiency-adjusted labour inputs, with $h > l \geq 1$ capturing the absolute-productivity advantage of H over L, and $\lambda^{j_m(\omega_m,t)} \cdot X_m(n,\omega_m,t)$ is the efficiency-adjusted input of m-complementary intermediate good ω_m, used by firm n at time t.[11] The parameters $A > 0$ and α denote the total factor productivity and the labour share in production. The indexing of firms assigns larger (smaller) n to firms holding a relative productivity advantage of using the H (L)-technology as opposed to L (H)-technology. For every t, there is a competitive equilibrium threshold $\bar{n}(t)$ that is endogenously determined, at which a switch from one technology to the other becomes advantageous, so that every firm n produces exclusively with either the low- or the high-tech (or L- or H-technology).

Final producers take the price of their final good, $P(n,t)$, wages, $w_m(t)$, and input prices $p_m(\omega_m,t)$ as given. From the profit maximisation conditions, the demand of intermediate good ω_m by firm n is

$$\begin{array}{rcl} X_L(n,\omega_L,t) & = & (1-n)\cdot l \cdot L(n) \cdot \left[\frac{A \cdot P(n,t)\cdot(1-\alpha)}{p_L(\omega_L,t)}\right]^{\frac{1}{\alpha}} \lambda^{j_L(\omega_L,t)\left(\frac{1-\alpha}{\alpha}\right)} \\ X_H(n,\omega_H,t) & = & n \cdot h \cdot H(n) \cdot \left[\frac{A \cdot P(n,t)\cdot(1-\alpha)}{p_H(\omega_H,t)}\right]^{\frac{1}{\alpha}} \lambda^{j_H(\omega_H,t)\left(\frac{1-\alpha}{\alpha}\right)} \end{array} \tag{2}$$

when it belongs to the L- or the H-technology sector, respectively.

Intermediate-good m-technology sector consists of a continuum $N_m(t)$ of industries. There is monopolistic competition if we consider the whole sector: the monopolist in industry $\omega_m \in [0, N_m(t)]$ fixes the price $p_m(\omega_m,t)$ but faces an isoelastic demand curve, $X_L(\omega_L,t) = \int_0^{\bar{n}(t)} X_L(n,\omega_L,t)dn$ or $X_H(\omega_H,t) = \int_{\bar{n}(t)}^1 X_H(n,\omega_H,t)dn$ (see (2)). Intermediate goods are non-durable and entail a unit marginal cost of production, in terms

[11] In equilibrium, only the top quality of each ω_m is produced and used.

of the final good, whose price is taken as given. Profit in ω_m is thus $\pi_m(\omega_m, t) = (p_m(\omega_m, t) - 1) \cdot X_m(\omega_m, t)$, and the profit maximising price is a constant markup over marginal cost,

$$p_m(\omega_m, t) \equiv p = \frac{1}{1-\alpha} > 1,\ m \in \{L, H\}\,. \tag{3}$$

Given $\bar{n}$ and (3), the final-good output for firm n can be rewritten as

$$Y(n,t) = \begin{cases} A^{\frac{1}{\alpha}} P(n,t)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} \cdot (1-n) \cdot l \cdot L(n) \cdot Q_L(t) & , 0 \leq n \leq \bar{n} \\ A^{\frac{1}{\alpha}} P(n,t)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} \cdot n \cdot h \cdot H(n) \cdot Q_H(t) & , \bar{n} \leq n \leq 1 \end{cases}, \tag{4}$$

Defining the quality index associated to industry ω_m by $q_m(\omega_m, t) \equiv \lambda^{j_m(\omega_m,t)\left(\frac{1-\alpha}{\alpha}\right)}$, we denote the aggregate quality index by

$$Q_m(t) = \int_0^{N_m(t)} q_m(\omega_m, t) d\omega_m,\ m \in \{L, H\}\,, \tag{5}$$

which measures the technological-knowledge level associated to using the L- or the H-technology. Thus, $Q_H(t)/Q_L(t)$ measures the technological-knowledge bias. The allocation of the low- and high-skilled labour inputs to the L- or the H-technology sector verify $L = \int_0^{\bar{n}} L(n)dn$ and $H = \int_{\bar{n}}^1 H(n)dn$.

With competitive final-good producers, economic viability of either L- or H-technology relies on the relative productivity and price of labour, as well as on the relative productivity and prices of intermediate goods, due to complementarity in production. The endogenous threshold $\bar{n}(t)$ then follows from equilibrium in the inputs markets, such that $\bar{n}(t) = \left[1 + (h/l \cdot H/L \cdot Q_H(t)/Q_L(t))^{1/2}\right]^{-1}$. Again, L- (H-)technology is exclusively adopted by final-good firms indexed by $n \in [0, \bar{n}(t)]$ ($n \in [\bar{n}(t), 1]$), which use the quantity $L(H)$ of low(high)-skilled labour and X_L- (X_H-) of complementary-intermediate goods. The relative price of final goods produced with L- and H-technologies is also a function of $\bar{n}(t)$,

$$\frac{P_H(t)}{P_L(t)} = \left(\frac{\bar{n}(t)}{1-\bar{n}(t)}\right)^{\alpha}, \text{ where } \begin{cases} P_L(t) = P(n,t) \cdot (1-n)^{\alpha} = e^{-\alpha}\bar{n}(t)^{-\alpha} \\ P_H(t) = P(n,t) \cdot n^{\alpha} = e^{-\alpha}(1-\bar{n}(t))^{-\alpha} \end{cases}. \tag{6}$$

If we define the price indices, $P_L(t)$ and $P_H(t)$, by recognising that, in equilibrium, the marginal value product, $\frac{\partial}{\partial m(n)}\left(P(n,t)Y(n,t)\right)$, must be constant over n, then $P(n,t)^{\frac{1}{\alpha}} \cdot (1-n)$ and $P(n,t)^{\frac{1}{\alpha}} \cdot n$ are also constant over $n \in [0, \bar{n}(t)]$ and $n \in [\bar{n}(t), 1]$, respectively. Thus, by considering that at the switching point $\bar{n}(t)$ both L- and the H- technology firms must break even, we get $P_L(t)$ and $P_H(t)$ as in equation (6).

From equations (2), (3) and (6), the profit accrued by the monopolist in ω_m becomes

$$\pi_L(\omega_L,t) = \pi_0 \cdot l \cdot L \cdot P_L(t)^{\frac{1}{\alpha}} \cdot q_L(\omega_L,t), \quad \pi_H(\omega_H,t) = \pi_0 \cdot h \cdot H \cdot P_H(t)^{\frac{1}{\alpha}} \cdot q_H(\omega_H,t) \tag{7}$$

where $\pi_0 \equiv A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{2}{\alpha}} \alpha/(1-\alpha)$ is a positive constant. Total intermediate-good optimal production, $X_{tot}(t) \equiv X_L(t)+X_H(t) \equiv \int_0^{N_L(t)} X_L(\omega_L,t)d\omega_L + \int_0^{N_H(t)} X_H(\omega_H,t)d\omega_H$, and total final-good optimal production, $Y_{tot}(t) \equiv Y_L(t)+Y_H(t) \equiv \int_0^{\bar{n}(t)} P(n,t)Y(n,t)dn + \int_{\bar{n}(t)}^1 P(n,t)Y(n,t)dn$, become, respectively,

$$X_{tot}(t) = A^{\frac{1}{\alpha}} \cdot (1-\alpha)^{\frac{2}{\alpha}} \cdot \left(P_L(t)^{\frac{1}{\alpha}} \cdot L \cdot Q_L(t) + P_H(t)^{\frac{1}{\alpha}} \cdot H \cdot Q_H(t)\right) \tag{8}$$

and

$$Y_{tot}(t) = A^{\frac{1}{\alpha}} \cdot (1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} \cdot \left(P_L(t)^{\frac{1}{\alpha}} \cdot L \cdot Q_L(t) + P_H(t)^{\frac{1}{\alpha}} \cdot H \cdot Q_H(t)\right). \tag{9}$$

Finally, by considering the condition that the real wage, W_m, must equal the marginal productivity of labour in equilibrium in the m-technology sector $m \in \{L, H\}$, we get, from equation (9), the skill premium

$$\frac{W_H(t)}{W_L(t)} = \left(\frac{h}{l}\right)^{1/2} \left(\frac{H}{L}\right)^{-1/2} \left(\frac{Q_H}{Q_L}\right)^{1/2} \tag{10}$$

3.2. R&D

We assume there are two types of R&D, one targeting vertical innovation and the other targeting horizontal innovation, because the pools of innovators performing each type of R&D are different. Each new design (a new variety or a higher quality good) is granted a patent and thus a successful innovator retains exclusive rights over the use of his/her good. We also postulate, to simplify the analysis, that both vertical and horizontal R&D are performed by (potential) entrants, and that successful R&D leads to the set-up of a new firm in either an existing or in a new industry (as in Howitt, 1999; Strulik, 2007; Gil, Brito, and Afonso, 2012). There is perfect competition among entrants and free entry in the R&D businesses.

3.2.1. Vertical R&D

By improving on the current top quality level $j_m(\omega_m,t)$, a successful vertical R&D firm earns monopoly profits from selling the leading-edge input of $j_m(\omega_m,t)+1$ quality to final-good firms. A successful innovation will instantaneously increase the quality index in ω_m from $q_m(\omega_m,t) = q_m(j_m)$ to $q_m^+(\omega_m,t) = q_m(j_m+1) = \lambda^{(1-\alpha)/\alpha} q_m(\omega_m,t)$. In equilibrium, lower qualities of intermediate good ω_m are priced out of business.

Let $I_m^i(j_m)$ denote the Poisson arrival rate of vertical innovations (vertical-innovation rate) by potential entrant i in industry ω_m, at a cost of $\Phi_m(j_m)$ units of the final good, when the highest quality is j_m. The rate $I_m^i(j_m)$ is independently distributed across firms, across industries and over time, and depends on the flow of resources $R^i_{v,m}(j_m)$ committed by entrants at time t. As in, e.g., Barro and Sala-i-Martin (2004, ch. 7), $I^i_m(j_m)$ features constant returns in R&D expenditures, $I^i_m(j_m) = R^i_{v,m}(j_m)/\Phi_m(j_m)$. The cost $\Phi_m(j_m)$ is assumed to be symmetric within sector m, such that

$$\Phi_m(j_m) = \zeta_m \cdot m^{\epsilon} \cdot q_m(j_m + 1), \; m \in \{L, H\}, \tag{11}$$

where $\zeta_m > 0$ is a constant fixed (flow) cost, and $\epsilon \geq 0$. Equation (11) incorporates three types of effects. First, there is an R&D complexity effect according to which the larger the level of quality in an industry of sector m, q_m, the costlier it is to introduce a further jump in quality.[12] This effect has been considered in the literature (e.g., Barro and Sala-i-Martin, 2004, ch. 7; Etro, 2008) and implies vertical-R&D is subject to dynamic decreasing returns to scale (i.e., decreasing returns to cumulated R&D). Second, in equation (11) there is also a market effect: an increase in the market scale of m-technology sector, measured by L and H respectively, dilutes the effect of R&D outlays on the innovation probability. In the literature, the market size effect is measured by employed labour (e.g., Barro and Sala-i-Martin, 2004) and is attributed to coordination, organisational and transportation costs (e.g., Dinopoulos and Thompson, 1999), as well as rental protection actions by incumbents (e.g., Sener, 2008).[13] The dilution effect, generated by those costs and actions, can partially ($0 < \epsilon < 1$) or totally ($\epsilon = 1$) counterbalance, or revert ($\epsilon > 1$) the scale benefits on profits (see (7)), which accrue to the R&D successful firm. Thus, as shown later, there may be positive, null or negative net scale effects on industrial growth, as measured by $1 - \epsilon$. At last, for any given supply of labour and quality index, the cost of vertical R&D also depends on a fixed flow cost, which can be specific to the type of production technology that is targeted by vertical R&D, H-complementary, measured by ζ_H, or L-complementary, measured by ζ_L.

Aggregating across firms i in ω_m, we get $R_{v,m}(j_m) = \sum_i R^i_{v,m}(j_m)$ and $I_m(j_m) = \sum_i I^i_m(j_m)$, and thus

$$I_m(j_m) = R_{v,m}(j_m) \cdot \frac{1}{\zeta_m \cdot m^{\epsilon} \cdot q_m(j_m + 1)}, \; m \in \{L, H\}. \tag{12}$$

As the terminal date of each monopoly arrives as a Poisson process with frequency $I_m(j_m)$ per (infinitesimal) increment of time, the present value of a monopolist's profits

[12] As usual in the literature, the fact that Φ_m depends linearly on q_m implies that the increasing difficulty of creating new product generations over t exactly offsets the increased rewards from marketing higher quality products; see (11) and (7). This allows for constant vertical-innovation rate over t and across ω_m in BGP (on *asymmetric* equilibrium in quality-ladders models and its growth consequences, see Cozzi, 2007).

[13] Sener (2008) contrasts the effects of rental protection actions with the expanding variety and the dynamic decreasing returns to R&D as scale-removal mechanisms within a quality-ladders model with knowledge-driven R&D specification. Observe, however, that the dynamic decreasing returns to R&D, as first introduced by Segerstrom (1998), and represented by the term $1/q_m(j_m + 1)$ in (11), are neither necessary nor sufficient for the purpose of scale removal in a model with lab-equipment specification (though it plays a crucial role in guaranteeing a Poisson rate constant over ω_m and hence the existence of a symmetric equilibrium; see fn. 12). The same applies to the expanding variety mechanism, as it is clear if we let $\epsilon = 0$ in our results below.

is a random variable. Let $V_m(j_m)$ denote the expected value of an incumbent with current quality level $j_m(\omega_m, t)$,[14]

$$\begin{array}{c} V_L(j_L) = \pi_0 \cdot l \cdot L \cdot q_L(j_L) \int_t^{\infty} P_L(s)^{\frac{1}{\alpha}} \cdot e^{-\int_t^s (r(v)+I_L(j_L(v)))dv} ds \\ V_H(j_H) = \pi_0 \cdot h \cdot H \cdot q_H(j_H) \int_t^{\infty} P_H(s)^{\frac{1}{\alpha}} \cdot e^{-\int_t^s (r(v)+I_H(j_H(v)))dv} ds \end{array}, \tag{13}$$

where r is the equilibrium market real interest rate, and $\pi_0 \cdot l \cdot L \cdot q_L(j_L) = \pi_L(j_L) \cdot P_L^{-\frac{1}{\alpha}}$ and $\pi_0 \cdot h \cdot H \cdot q_H(j_H) = \pi_H(j_H) \cdot P_H^{-\frac{1}{\alpha}}$, given by (7) and (6), are constant in-between innovations. Free-entry prevails in vertical R&D such that the condition $I_m(j_m) \cdot V_m(j_m+1) = R_{v,m}(j_m)$ holds, which implies that

$$V_m(j_m+1) = \Phi_m(j_m) = \zeta_m \cdot m^{\epsilon} \cdot q_m(j_m+1), \; m \in \{L, H\}. \tag{14}$$

Next, we determine $V_m(j_m+1)$ analogously to (13), then consider (14) and time-differentiate the resulting expression. Therefore, if we also consider (7), we get the arbitrage condition facing a vertical innovator,

$$r(t) + I_L(t) = \frac{\pi_0 \cdot l \cdot L^{1-\epsilon} \cdot P_L(t)^{\frac{1}{\alpha}}}{\zeta_L}, \; r(t) + I_H(t) = \frac{\pi_0 \cdot h \cdot H^{1-\epsilon} \cdot P_H(t)^{\frac{1}{\alpha}}}{\zeta_H}, \tag{15}$$

which then implies that the rates of entry are symmetric across industries, $I_m(\omega_m, t) = I_m(t)$.[15]

Equating the effective rate of return for both sectors, e.g., by considering (15), the no-arbitrage condition obtains

$$I_H(t) - I_L(t) = \pi_0 \left(\frac{h}{\zeta_H} \cdot H^{1-\epsilon} \cdot P_H(t)^{\frac{1}{\alpha}} - \frac{l}{\zeta_L} L^{1-\epsilon} \cdot P_L(t)^{\frac{1}{\alpha}} \right). \tag{16}$$

After solving equation (12) for $R_{v,m}(\omega_m, t) = R_{v,m}(j_m)$ and aggregating across industries ω_m, we determine total resources devoted to vertical R&D, $R_{v,m}(t) = \int_0^{N_m(t)} R_{v,m}(\omega_m, t) \, d\omega_m = \int_0^{N_m(t)} \zeta_m \cdot m^{\epsilon} \cdot q_m^{+}(\omega_m, t) \cdot I_m(\omega_m, t) \, d\omega_m$. As the innovation rate is industry independent, then

$$R_{v,m}(t) = \zeta_m \cdot m^{\epsilon} \cdot \lambda^{\frac{1-\alpha}{\alpha}} \cdot I_m(t) \cdot Q_m(t), \; m \in \{L, H\}. \tag{17}$$

[14] We assume that entrants are risk-neutral and, thus, only care about the expected value of the firm.

[15] Observe that, from (7) and (12), we have $\frac{\dot{\pi}_m(\omega_m,t)}{\pi_m(\omega_m,t)} - \frac{1}{\alpha}\frac{\dot{P}_m(t)}{P_m(t)} = I_m(\omega_m, t) \cdot \left[\dot{j}_m(\omega_m, t) \cdot \left(\frac{\alpha}{1-\alpha} \right) \cdot \ln \lambda \right]$ and $\frac{\dot{R}_{vm}(\omega_m,t)}{R_{vm}(\omega_m,t)} - \frac{\dot{I}_m(\omega_m,t)}{I_m(\omega_m,t)} = I_m(\omega_m, t) \cdot \left[\dot{j}_m(\omega_m, t) \cdot \left(\frac{\alpha}{1-\alpha} \right) \cdot \ln \lambda \right]$. Thus, if we time-differentiate (14) by considering (13) and the equations above, we get $r(t) = \frac{\pi_m(j_m+1) \cdot I_m(j_m)}{R_{vm}(j_m)} - I_m(j_m+1)$, which can then be re-written as (15).

3.2.2. Horizontal R&D

Variety expansion arises from R&D aimed at creating a new intermediate good. Under perfect competition and constant returns to scale at the firm level, the instantaneous entry rate is obtained as $\dot{N}_m^e(t)/N_m^e(t) = R_{h,m}^e(t)/\eta_m(t)$, where $\dot{N}_m^e$ is the contribution to the instantaneous flow of new m-complementary intermediate goods by R&D firm e at a cost of η_m units of the final good and $R_{h,m}^e$ is the flow of resources devoted to horizontal R&D by innovator e at time t. The cost η_m is assumed to be symmetric within sector m. Then, $R_{h,m}(t) = \sum_e R_{h,m}^e(t)$ and $\dot{N}_m(t) = \sum_e \dot{N}_m^e(t)$, implying

$$R_{h,m}(t) = \eta_m(t) \cdot \dot{N}_m(t)/N_m(t),\ m \in \{L, H\}\,. \tag{18}$$

We assume that the cost of setting up a new variety (cost of horizontal entry) is increasing in the number of existing varieties, N_m,

$$\eta_m(t) = \phi_m \cdot m^{\delta} \cdot N_m(t)^{1+\sigma},\ m \in \{L, H\}\,, \tag{19}$$

where $\phi_m > 0$ is a constant fixed (flow) cost, and $\sigma > 0$ and $\delta \geq 0$. Similarly to vertical R&D, equation (19) also incorporates three types of effects. First, an R&D complexity effect arises through the dependence of η_m on N_m (e.g., Evans, Honkapohja, and Romer, 1998; Barro and Sala-i-Martin, 2004, ch. 6), implying horizontal-R&D dynamic decreasing returns to scale. That is, the larger the number of existing varieties, the costlier it is to introduce new varieties. Second, (19) also implies that an increase in market scale, measured by L and H respectively, dilutes the effect of R&D outlays on the innovation rate. Again, this may happen due to coordination, organisational and transportation costs related to market size (e.g., Dinopoulos and Thompson, 1999), which may partially ($0 < \delta < 1$), totally ($\delta = 1$) or over ($\delta > 1$) counterbalance the scale benefits on profits. Finally, for any given supply of labour and number of varieties, the cost of horizontal R&D also depends on a fixed flow cost, which can be specific to the type of production technology that is targeted by horizontal R&D, ϕ_H and ϕ_L.

Each horizontal innovation results in a new intermediate good whose quality level is drawn randomly from the distribution of existing varieties (e.g., Howitt, 1999). Thus, the expected quality level of the horizontal innovator is

$$\bar{q}_m(t) = \int_0^{N_m(t)} \frac{q_m(\omega_m, t)}{N_m(t)} d\omega_m = \frac{Q_m(t)}{N_m(t)},\ m \in \{L, H\}\,, \tag{20}$$

As his/her monopoly power will be also terminated by the arrival of a successful vertical innovator in the future, the benefits from entry are

$$\begin{array}{c} V_L(\bar{q}_L) = \pi_0 \cdot l \cdot L \cdot \bar{q}_L(t) \int_t^{\infty} P_L(s)^{\frac{1}{\alpha}} \cdot e^{-\int_t^s [r(\nu) + I_L(\bar{q}_L(v))] d\nu} ds \\ V_H(\bar{q}_H) = \pi_0 \cdot h \cdot H \cdot \bar{q}_H(t) \int_t^{\infty} P_H(s)^{\frac{1}{\alpha}} \cdot e^{-\int_t^s [r(\nu) + I_H(\bar{q}_H(v))] d\nu} ds \end{array}\,, \tag{21}$$

where $\pi_0 l L \bar{q}_L = \bar{\pi}_L P_L^{-\frac{1}{\alpha}}$ and $\pi_0 h H \bar{q}_H = \bar{\pi}_H P_H^{-\frac{1}{\alpha}}$. The free-entry condition, $\dot{N}_m \cdot V(\bar{q}_m) = R_{hm}$, by (18), simplifies to

$$V_m(\bar{q}_m) = \eta_m(t)/N_m(t),\ m \in \{L, H\}\,. \tag{22}$$

Substituting (21) into (22) and time-differentiating the resulting expression, yields the arbitrage equation facing a horizontal innovator

$$r\,(t) + I_m(t) = \frac{\bar{\pi}_m(t)}{\eta_m\,(t)\,/N_m(t)},\ m \in \{L, H\}\,. \tag{23}$$

3.2.3. Intra-sector no-arbitrage condition

No-arbitrage in the capital market requires that the two types of investment – vertical and horizontal R&D – yield equal rates of return; otherwise, one type of investment dominates the other and a corner solution obtains. Thus, if we equate the effective rate of return $r + I_m$ for both types of entry, from (15) and (23), we get the *intra-sector* no-arbitrage conditions

$$\bar{q}_m(t) = \frac{Q_m(t)}{N_m(t)} = \frac{\eta_m(t)}{\zeta_m \cdot m^{\epsilon} \cdot N_m(t)} = \frac{\phi_m}{\zeta_m} \cdot m^{\delta-\epsilon} \cdot N_m(t)^{\sigma}\ ,\ m \in \{L, H\} \tag{24}$$

which is a key ingredient of the model. No arbitrage conditions, within the H- and L-technology R&D sectors, equate the average cost of horizontal R&D, η_m/N_m, to the average cost of vertical R&D, $\bar{q}_m\zeta_m m^{\epsilon}$.

3.3. General equilibrium

The economy is populated by a fixed number of infinitely-lived households who consume and collect income from investments in financial assets (equity) and from labour. Households inelastically supply low-skilled, L, or high-skilled labour, H. Thus, total labour supply, $L + H$, is exogenous and constant. We assume consumers have perfect foresight concerning the technological change over time and choose the path of final-good aggregate consumption $(C(t))_{t\geq 0}$ to maximise discounted lifetime utility

$$U = \int_0^{\infty} \left(\frac{C(t)^{1-\theta} - 1}{1-\theta}\right) e^{-\rho t} dt, \tag{25}$$

where $\rho > 0$ is the subjective discount rate and $\theta > 0$ is the inverse of the intertemporal elasticity of substitution, subject to the flow budget constraint

$$\dot{a}(t) = r(t) \cdot a(t) + w_L(t) \cdot L + w_H(t) \cdot H - C(t), \tag{26}$$

where a denotes households' real financial assets holdings. The initial level of wealth $a(0)$ is given and the non-Ponzi games condition $lim_{t\to\infty} e^{-\int_0^t r(s)ds} a(t) \geq 0$ is imposed. The optimal consumption path Euler equation and the transversality condition are standard,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta} \cdot (r(t) - \rho) \tag{27}$$

$$\lim_{t \to \infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot a(t) = 0. \tag{28}$$

The aggregate financial wealth held by households is composed by equity of intermediate good producers $a(t) = a_L(t) + a_H(t)$, where $a_m(t) = \int_0^{N_m(t)} V_m(\omega_m, t) d\omega_m$, $m \in \{L, H\}$. From the arbitrage condition between vertical and horizontal entry, we have equivalently $a(t) = \eta_L(t) \cdot N_L(t) + \eta_H(t) \cdot N_H(t)$. Taking time derivatives and comparing with (26), the aggregate flow budget constraint is equivalent to the final product market equilibrium condition

$$Y_{tot}(t) = X_{tot}(t) + C(t) + R_h(t) + R_v(t) \tag{29}$$

where $R_h(t) = R_{h,L}(t) + R_{h,H}(t)$ and $R_v(t) = R_{v,L}(t) + R_{v,H}(t)$ are the aggregate horizontal and vertical R&D expenditures, respectively.

The dynamic general equilibrium is defined by the paths of allocations and price distributions $(\{X_m(\omega_m, t), p_m(\omega_m, t)\}, \omega_m \in [0, N_m(t)])_{t \geq 0}$ and aggregate number of firms, quality indices and vertical-innovation rates $(\{\ N_m(t), Q_m(t), I_m(t)\}\ )_{t \geq 0}$ for sectors $m \in \{L, H\}$, and by the aggregate paths $(C(t), r(t))_{t \geq 0}$, such that: (i) consumers, final-good firms and intermediate-good firms solve their problems; (ii) free-entry and no-arbitrage conditions are met; and (iii) markets clear. Total supplies of skilled- and unskilled-labour are exogenous.

3.4. The balanced-growth path

A general-equilibrium balanced growth path (BGP) exists only if the following conditions hold among the asymptotic growth rates, which are all constant: (i) the growth rates for consumption and for the quality indices are equal to the endogenous growth rate for the economy g, $g_C = g_{Q_L} = g_{Q_H} = g$; (ii) the growth rates for the number of varieties are equal, $g_{N_L} = g_{N_H}$; (iii) the vertical-innovation rates and the final-good price indices are asymptotically trendless, $g_{I_L} = g_{I_H} = g_{P_L} = g_{P_H} = 0$; and (iv) the growth rates for the quality indices and for the number of varieties are monotonously related as $g_{Q_L}/g_{N_L} = g_{Q_H}/g_{N_H} = 1 + \sigma$. Then $g_{N_L} = g_{N_H} = g/(1 + \sigma)$.

Necessary conditions (i) and (ii) imply that the trendless levels for the vertical-innovation rates verify $I_L = I_H = I$, along the BGP. Introducing this in equation (16) we derive P_H/P_L. Substituting, in turn, in equation (6) we can get the long-run technological-knowledge bias, $Q \equiv Q_H/Q_L$, as (henceforth $\sim$ denotes BGP magnitudes)

$$\tilde{Q} = \left(\frac{H}{L}\right)^{1-2\epsilon} \left(\frac{h}{l}\right) \left(\frac{\zeta_H}{\zeta_L}\right)^{-2}. \tag{30}$$

Moreover, by considering equations (10) and (30), we get the long-run skill premium

$$\left(\tilde{\frac{W_H}{W_L}}\right) = \frac{h}{l}\left(\frac{\zeta_H}{\zeta_L}\right)^{-1}\left(\frac{H}{L}\right)^{-\epsilon} \tag{31}$$

The larger the market complexity costs pertaining to vertical R&D, ϵ, the stronger the (negative) impact of the skill structure on the skill premium.

If we assume that the number of industries, N, is large enough to treat Q as time-differentiable and non-stochastic, then we can time-differentiate (5) to get $\dot{Q}_m(t) = \int_0^{N_m(t)} \dot{q}(\omega,t)d\omega + q(N,t)\dot{N}(t)$, which is well-defined if $\sigma > 0$. After some algebraic manipulation of the latter, we can write, for the case in which $I_m > 0$, another asymptotic relationship between the long-run growth rate of the quality indices and of the number of varieties, $g_{Q_m} = \Xi I_m + g_{N_m}$ $m \in \{L, H\}$, where $\Xi \equiv \left(\lambda^{\frac{1-\alpha}{\alpha}} - 1\right)$ denotes the quality shift. Then $g = \Xi I + (1+\sigma)g$ should hold, from the above conditions (i) and (iv). From the Euler equation (27) and the necessary condition (i), we get the familiar relationship between the long-run real interest rate and the endogenous growth rate, $r = \rho + \theta g$. The transversality condition holds if $g > 0$. The non-arbitrage condition for vertical R&D allows us to get the endogenous long-run economic growth rate

$$\tilde{g} = \frac{\tilde{r} - \rho}{\theta}\left(1 - \frac{1}{1+\theta\mu}\right), \tag{32}$$

where the long-run real interest is constant,

$$\tilde{r} = \frac{\pi_0}{e}\left(\frac{l}{\zeta_L}L^{1-\epsilon} + \frac{h}{\zeta_H}H^{1-\epsilon}\right)$$

and $\mu \equiv \Xi(1+\sigma)/\sigma > 0$. The long-run vertical-innovation rate is homogeneous across H and L-technology sectors and is proportional to the economic growth rate

$$\tilde{I}_L = \tilde{I}_H = \tilde{I} = \frac{1}{\mu}\tilde{g} \geq 0, \tag{33}$$

and the endogenous long-run growth rates for the quality indices and the varieties are equalized across sectors and become

$$\tilde{g}_{Q_L} = \tilde{g}_{Q_H} = \tilde{g} > 0, \tag{34}$$

and

$$\tilde{g}_{N_L} = \tilde{g}_{N_H} = \frac{1}{1+\sigma}\,\tilde{g} > 0. \tag{35}$$

It is clear from (32) that the long-run economic growth rate is constant and positive and displays positive, null or negative net scale effects, depending upon the magnitude of the market complexity costs pertaining to vertical R&D, ϵ. These costs have a negative effect on growth *per se*. In addition, our model predicts, under a sufficiently productive technology, that g_{Q_m} exceed g_{N_m}, where the difference is equal to the expected value of the shift in the intermediate-good quality (recall that $g_{Q_m} = \Xi I_m + g_{N_m}$), if the probability of introducing successful vertical innovations is positive. Thus, the economic growth rate equals the growth rate of the number of varieties plus the growth rate of intermediate-good quality, in line with the well-known view that industrial growth proceeds both along an intensive and an extensive margin.

However, given the distinct nature of vertical and horizontal innovation (immaterial versus physical) and the consequent asymmetry in terms of R&D complexity costs (see (11) and (19)), vertical R&D is the ultimate growth engine, whereas variety expansion is sustained by the endogenous quality upgrade: the expected growth of intermediate-good quality due to vertical R&D makes it attractive, in terms of intertemporal profits, for potential entrants to always put up an horizontal R&D complexity cost, in spite of its more than proportional increase with N_m. Thus, there is a negative relationship between the economic growth rate and both the horizontal R&D complexity cost parameter, σ, and the flow fixed costs to vertical R&D, ζ_H and ζ_L, while there is no impact from the flow fixed cost to horizontal R&D, ϕ_H and ϕ_L, and the market complexity cost pertaining to horizontal R&D, δ. There is also a positive relationship between the economic growth rate and the productivity parameters, h and l.

4. Analysis

4.1. Growth, technological structure and skill structure

The long-run economic growth rate, in equation (32), is a function of the absolute magnitude of the economy's endowments of both high- and low-skilled labour. If we assume that the endowment of low-skilled labour does not decay we can also sign its relationship with changes in the skill structure.

Proposition 1.

(i) Assume there is an increase in either high- or low-skilled labour endowments (with the other endowment constant). Then, the long-run economic growth rate, $\tilde{g}$, increases if $0 \leq \epsilon < 1$ or decreases if $\epsilon > 1$.

(ii) Assume there is an increase in the relative supply of skills, H/L. Then,

$$\frac{d\tilde{g}}{\tilde{g}} > (1-\epsilon)\frac{dL}{L}$$

and the long-run economic growth rate increases, if $0 \leq \epsilon < 1$ and the low-skilled labour supply is not decreasing.

The cross-country evidence in Section 2 suggests that the economic growth rate has a positive elasticity with respect to the skill structure. By ranking countries from the lowest to the highest proportion of high-skilled workers, if the low-skilled labour is not decreasing across countries, our model is compatible with the previous empirical fact if $0 \leq \epsilon < 1$. On the other hand, by considering the negative relationship between the skill structure and the skill premium in the data (see, again, Section 2), we infer that the effect of the skill structure on the long-run economic growth rate is reduced by the impact of the former on the skill premium. In fact, according to equation (31), the long-run elasticity of the skill premium with respect to the skill structure reflects the complexity costs regarding vertical R&D, ϵ, meaning that an increase in the supply of high-skilled workers benefits the low-skilled workers. If the complexity costs are sufficiently low (smaller than unity), then the first effect dominates and there is a positive relationship between the skill structure and economic growth, as stated in Proposition 1.

The long-run technology structure of our model is characterised by the technological-knowledge bias, $\tilde{Q}$, the relative intermediate-good production, $\tilde{X}$, and the relative number of firms $\tilde{N}$ (i.e., H- vis-à-vis L-technology sector). In equation (30), we show that the technological-knowledge bias is a function of the skill structure, H/L. The same can be proved as regards relative production and the relative number of firms. From the expressions for X_L and X_H (see (8)), we get

$$\tilde{X} \equiv \widetilde{\left(\frac{X_H}{X_L}\right)} = \left(\frac{H}{L}\right)^{1-\epsilon} \cdot \left(\frac{h}{l}\right) \cdot \left(\frac{\zeta_H}{\zeta_L}\right)^{-1} = \frac{H}{L} \cdot \frac{W_H}{W_L}, \tag{36}$$

and from the expressions for N_L and N_H (see (24)), combined with (30), we derive the relative number of firms

$$\tilde{N} \equiv \widetilde{\left(\frac{N_H}{N_L}\right)} = Z_0 \cdot \left(\frac{H}{L}\right)^{D_0}, \tag{37}$$

where

$$D_0 \equiv \frac{1-\epsilon-\delta}{1+\sigma} \tag{38}$$

$$Z_0 \equiv \left(\frac{h}{l}\right)^{\frac{1}{\sigma+1}} \cdot \left(\frac{\phi_H}{\phi_L}\right)^{\frac{-1}{\sigma+1}} \cdot \left(\frac{\zeta_H}{\zeta_L}\right)^{\frac{1}{\sigma+1}}. \tag{39}$$

Therefore, the technology structure depends on relative productivity, h/l, the relative flow fixed costs in both vertical and horizontal R&D, ζ_H/ζ_L and ϕ_H/ϕ_L, and the skill structure, H/L. The direction and intensity of these effects depend crucially on the complexity costs parameters, ϵ, δ and σ. It is also clear the interplay between the direct positive impact of the skill structure on the technology structure and its indirect negative impact through the skill premium (see (31) and (36)). If the complexity costs ϵ are sufficiently low (smaller than unity), then the direct effect dominates and there is a positive relationship between the skill structure and the technology structure.

In order to take our model to the data and, in particular, to quantitatively associate the empirical facts on growth and the skill structure to the complexity costs and the scale effects on growth, a task to carry out in Section 5, we next relate our theoretical predictions regarding the technology structure to the cross-country data on the skill structure by considering a convenient measure of the technology-structure variables.

4.2. Measuring technology structure and implications

Since we wish to confront our theoretical results with the data on production for a number of countries and the data is presented in a quality-adjusted base by the national statistics offices (see, e.g., Eurostat, 2001), we find it convenient to compute production also in quality-adjusted terms. Reiterating the steps as in Subsection 3.1, we find total intermediate-good quality-adjusted production to be (e.g., with $m = L$) $\mathfrak{X}_L = \int_0^{N_L}\int_0^{\bar{n}} \lambda^{j_L(\omega_L)}\cdot X_L(n,\omega_L)dnd\omega_L = A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{2}{\alpha}} P_L^{\frac{1}{\alpha}} lL\mathcal{Q}_L$, where $\mathcal{Q}_L = \int_0^{N_L}\lambda^{j_L(\omega_L)\frac{1}{\alpha}}d\omega_L$, and $\mathfrak{X}_{tot} = \mathfrak{X}_L + \mathfrak{X}_H$. We cannot find an explicit algebraic expression for the BGP value of $\mathcal{Q}_m$. However, as shown in Appendix A, we can build an adequate proxy for $\mathcal{Q}_m$, $\widehat{\mathcal{Q}}_m = Q_m^{\frac{1}{1-\alpha}} \cdot N_m^{-\left(\frac{\alpha}{1-\alpha}\right)}$ $m \in \{L, H\}$. Accordingly, we define the proxy $\widehat{\mathfrak{X}}_m = X_m \cdot (Q_m/N_m)^{\frac{\alpha}{1-\alpha}}$ for $\mathfrak{X}_m$. Thus, bearing in mind (30), (36) and (37), and by measuring firm size as production per firm, we consider the following quality-adjusted measures of relative production,

$$\tilde{\widehat{\mathfrak{X}}} = \tilde{X} \cdot \left(\frac{\tilde{Q}}{N}\right)^{\frac{\alpha}{1-\alpha}} = Z_1 \cdot \left(\frac{H}{L}\right)^{D_1}, \tag{40}$$

where

$$D_1 \equiv \frac{\alpha\delta + 1 - \alpha + \sigma - \epsilon\left[1 + (1+\alpha)\sigma\right]}{(1+\sigma)(1-\alpha)} \tag{41}$$

$$Z_1 \equiv \left(\frac{h}{l}\right)^{\left[1+\left(\frac{\sigma}{\sigma+1}\right)\left(\frac{\alpha}{1-\alpha}\right)\right]} \cdot \left(\frac{\phi_H}{\phi_L}\right)^{\frac{\alpha}{(\sigma+1)(1-\alpha)}} \cdot \left(\frac{\zeta_H}{\zeta_L}\right)^{-\left[1+\left(\frac{2\sigma+1}{\sigma+1}\right)\left(\frac{\alpha}{1-\alpha}\right)\right]} \tag{42}$$

and of the relative firm size,

$$\frac{\tilde{\widehat{\mathfrak{X}}}}{\tilde{N}} = \left(\frac{\tilde{X}}{N}\right) \cdot \left(\frac{\tilde{Q}}{N}\right)^{\frac{\alpha}{1-\alpha}} = Z_2 \cdot \left(\frac{H}{L}\right)^{D_2}, \tag{43}$$

where

$$D_2 \equiv \frac{\delta + \sigma - \epsilon\left[\alpha + (1+\alpha)\sigma\right]}{(1+\sigma)(1-\alpha)} \tag{44}$$

$$Z_2 \equiv \left(\frac{h}{l}\right)^{\left(\frac{\sigma}{\sigma+1}\right)\left(\frac{1}{1-\alpha}\right)} \cdot \left(\frac{\phi_H}{\phi_L}\right)^{\frac{1}{(\sigma+1)(1-\alpha)}} \cdot \left(\frac{\zeta_H}{\zeta_L}\right)^{-\left[\left(\frac{\sigma}{\sigma+1}\right)+\left(\frac{2\sigma+1}{\sigma+1}\right)\left(\frac{\alpha}{1-\alpha}\right)\right]} \tag{45}$$

Moreover, from $\widehat{\mathfrak{X}}_m = X_m \cdot (Q_m/N_m)^{\frac{\alpha}{1-\alpha}}$, the quality-adjusted long-run economic growth rate is

$$\tilde{\mathcal{G}} = \left(1 + \frac{\alpha}{1-\alpha}\frac{\sigma}{1+\sigma}\right) \cdot \tilde{g}. \tag{46}$$

As one can see, in addition to its impact on the BGP economic growth rate (see (32)), the elasticity of m in the vertical R&D cost function, ϵ, plays an important role in the determination of the sign of the relationship between the skill structure and the technology-structure variables. Thus, we define a vector of critical values for ϵ, $\bar{\epsilon} = [\bar{\epsilon}_0, \bar{\epsilon}_1, \bar{\epsilon}_2]$, $\bar{\epsilon} \subset \mathbb{R}^3$, such that $D_0(\bar{\epsilon}_0) = 0$, $D_1(\bar{\epsilon}_1) = 0$ and $D_2(\bar{\epsilon}_2) = 0$, where

$$\bar{\epsilon}_0 = 1 - \delta, \tag{47}$$

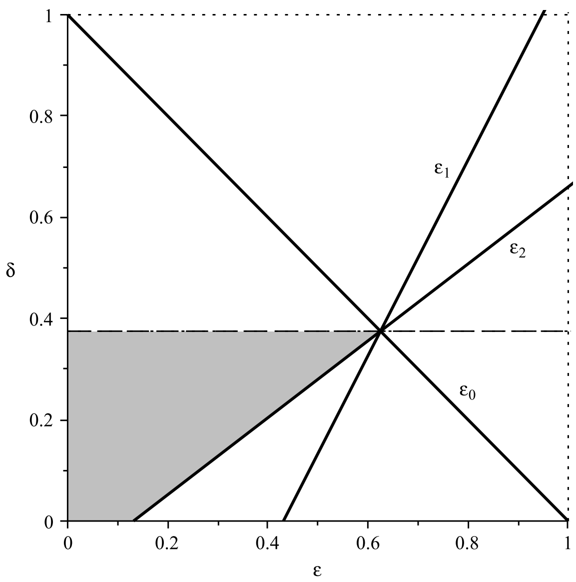

Figure 3: Set of values for the market complexity-cost parameters (ϵ, δ) that are qualitatively consistent with the technology-structure elasticities found in the cross-country data of Figure 2, i.e., that imply D_0, D_1, $D_2 > 0$ and D_2 smaller than D_0 and D_1 in (38), (41) and (44).

$$\bar{\epsilon}_1 = \frac{1 - \alpha + \sigma + \alpha\delta}{1 + (1+\alpha)\sigma}, \tag{48}$$

$$\bar{\epsilon}_2 = \frac{\sigma + \delta}{\alpha + (1+\alpha)\sigma}. \tag{49}$$

According with the cross-country evidence in Section 2, the elasticity of relative production, of the relative number of firms and of the relative firm size are all positive, which corresponds in our model to D_0, D_1, $D_2 > 0$. Thus, the model produces results that are consistent with the sign of the cross-country elasticities if $\epsilon << \bar{\epsilon}$. However, the empirical evidence also suggests that the elasticity of the relative firm size is noticeable smaller than the other two elasticities, which corresponds to D_2 smaller than D_0 and D_1. A sufficient condition for this is to have $\bar{\epsilon}_2$ smaller than $\bar{\epsilon}_0$ and $\bar{\epsilon}_1$. Given $0 < \alpha < 1$, and for $0 \leq \delta < \bar{\delta}$, where $\bar{\delta} = \alpha/(1+\alpha)$, we have $0 \leq \delta/\alpha \leq \bar{\epsilon}_2 \leq 1/(1+\alpha) < 1$, $0 < 1 - \alpha(1-\delta) \leq \bar{\epsilon}_1 \leq 1/(1+\alpha) < 1$, for $\sigma \in [0, +\infty)$, and thus $0 \leq \epsilon < \bar{\epsilon}_2$ and $\bar{\epsilon}_2 < \bar{\epsilon}_1 < \bar{\epsilon}_0$ for σ finite. This is depicted by Figure 3, which shows that there is a non-empty set of values for the market complexity-cost parameters (ϵ, δ) which are qualitatively consistent with the cross-country evidence.

[Figure 3 goes about here]

To better understand the mechanism behind the results above, it is useful to see how shifts in the market complexity cost ϵ change the relationship between the skill structure and the technology structure. The next proposition summarises the results.[16]

Proposition 2. Assume that $0 \leq \delta < \alpha/(1+\alpha)$. If a country has a *larger* proportion of high-skilled labour, H/L, then it will have:

(i) A *larger* relative number of firms, production and firm size, if $0 \leq \epsilon < \bar{\epsilon}_2$;

(ii) A *larger* relative number of firms and production but a *smaller* relative firm size, if $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$;

(iii) A *larger* relative number of firms but a *smaller* relative production and firm size, if $\bar{\epsilon}_1 < \epsilon < \bar{\epsilon}_0$;

(iv) A *smaller* relative number of firms, production and firm size, if $\epsilon > \bar{\epsilon}_0$.

The results above stem from the different response of the relative number of firms, N, relative production, $\widehat{\mathfrak{X}}$, and the relative firm size, $\widehat{\mathfrak{X}}/N$, through the technological-knowledge bias channel, to shifts in the skill structure, H/L. This is explained by the asymmetric impact of both market and R&D complexity costs on the elasticity of those technological-structure variables with respect to H/L. The market complexity costs related to horizontal R&D, summarised by δ, have a direct negative impact on this type of R&D and an indirect positive impact on vertical R&D (substitution effect). Consequently, there is a negative effect on horizontal entry and hence on the elasticity of N ($\partial D_0/\partial\delta < 0$, in (37)), whereas, through the positive impact on the quality index, $q(j)$, and thereby on the technological-knowledge bias, Q, there is also a positive effect on the elasticity of $\widehat{\mathfrak{X}}$ and $\widehat{\mathfrak{X}}/N$ ($\partial D_1/\partial\delta > 0$ and $\partial D_2/\partial\delta > 0$, in (40) and (43)).

However, the market complexity costs related to vertical R&D, summarised by ϵ, have a direct negative impact on this type of R&D (and hence $\partial D_1/\partial\epsilon < 0$ and $\partial D_2/\partial\epsilon < 0$), but also have a negative impact, although smaller in modulus, on horizontal R&D ($\partial D_0/\partial\epsilon < 0$, with $|\partial D_0/\partial\epsilon| < |\partial D_1/\partial\epsilon|$). This reflects the fact that the vertical-innovation mechanism ultimately commands the horizontal entry dynamics, meaning that a BGP with increasingly costly horizontal R&D occurs only because entrants expect the incumbency value to grow propelled by quality-enhancing R&D, hence generating a roundabout cost effect connected to ϵ.

Furthermore, the effect of H/L on N is dampened by the horizontal R&D complexity cost, summarised by σ (see $\partial D_0/\partial\sigma < 0$), whereas this cost has an indirect positive impact (substitution effect) on $\widehat{\mathfrak{X}}$ and $\widehat{\mathfrak{X}}/N$ (see $\partial D_1/\partial\sigma > 0$ and $\partial D_2/\partial\sigma > 0$).

In particular, the asymmetric impact of the market complexity costs on the behaviour of the technological-structure variables can be seen by noticing that $\widehat{\mathfrak{X}}/N$ is constant with respect to H/L when $\epsilon = \bar{\epsilon}_2$, $\widehat{\mathfrak{X}}$ is constant when $\epsilon = \bar{\epsilon}_1$, and N is constant when $\epsilon = \bar{\epsilon}_0$, where $\bar{\epsilon}_2 < \bar{\epsilon}_1 < \bar{\epsilon}_0$. The described mechanism also explains the effect of the elasticity of the horizontal entry cost function, σ, on the thresholds $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$: the lower σ, the lower $\bar{\epsilon}_2$ and $\bar{\epsilon}_1$, i.e., the higher the degree of net scale effects below which there is,

[16] Henceforth, the $\tilde{}$ is omitted for the sake of simplicity.

respectively, a decrease in $\widehat{\mathfrak{X}}/N$ and $\widehat{\mathfrak{X}}$ with H/L. Moreover, the dependence of $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$ on σ makes clear the role of the horizontal-entry mechanism in generating a rich pattern of technological structure: without entry, i.e., with $\sigma \to \infty$, $\bar{\epsilon}_2$ and $\bar{\epsilon}_1$ converge to the single point $\epsilon = 1/(1+\alpha)$.

5. Identification and calibration

As shown above, there is a non-empty set of values for parameters ϵ, δ, σ and α that are consistent with the cross-country evidence. In particular, the conditions $0 \leq \delta < \bar{\delta}$ and $0 < \epsilon < \bar{\epsilon}_2$ are required. This implies that the calibration exercise is feasible. We now turn to a quantitative assessment of the model and investigate how important our analytical mechanism may be in accounting for the cross-country pattern in the distribution of firms and production between high- and low-tech sectors. Although it should be clear that our mechanism does not account for all the variation in the technology structure across countries, we abstract from all other potential sources of variation. Thus, this exercise provides an upper bound on how much of the referred to cross-country differences can be explained by the variation in the skill structure. Then, given the estimated effect of the skill structure on the technology structure, we try to uncover the effect of the former on economic growth.

We adopt the following three-step method for calibrating the model.

First step: we run the regressions

$$\ln N = \ln Z_0 + D_0 \ln (H/L) + e_0 \tag{50}$$

$$\ln \mathfrak{X} = \ln Z_1 + D_1 \ln (H/L) + e_1 \tag{51}$$

$$\ln (\mathfrak{X}/N) = \ln Z_2 + D_2 \ln (H/L) + e_2 \tag{52}$$

where e_i, $i = 1, 2, 3$, are the error terms, to get the estimates $\hat{D}_0$, $\hat{D}_1$, $\hat{D}_2$, $\widehat{\ln Z_0}$, $\widehat{\ln Z_1}$, and $\widehat{\ln Z_2}$.

Second step: as D_0, D_1 and D_2 are functions of $(\alpha, \sigma, \epsilon, \delta)$, we use the estimates $\hat{D}_0$, $\hat{D}_1$ and $\hat{D}_2$ to identify those parameters. As there are only two independent relationships, we assume reasonable values for (α, σ) and then choose the calibrated values for the market complexity-costs parameters (ϵ, δ) by minimizing the Squared Quadratic Error (SQE).[17] We choose these parameters because they determine crucially the long-run behaviour of our model.

Third step: as Z_0, Z_1 Z_2 are functions of $(\alpha, \sigma, h/l, \zeta_H/\zeta_L, \phi_H/\phi_L)$, we take the previous values for (α, σ) and determine $(h/l, \zeta_H/\zeta_L, \phi_H/\phi_L)$. At last, we assess the general quality of the calibrated parameters.

Table 1, in Section 2, reports the OLS estimates of the coefficients in regressions (50)-(52). From the point estimates of the slopes, we calibrate the parameters α, σ, δ and ϵ, and the calibration strategy is as follows. The baseline value for α is standard (we set $\alpha = 0.6$, which implies $\bar{\delta} = 0.375$), but the values of the remaining parameters

[17] We compute the SQE as the square root of the sum of the square of the percentage difference between $\hat{D}_i$ and the predicted D_i, which is obtained upon calibration of the theoretical model.

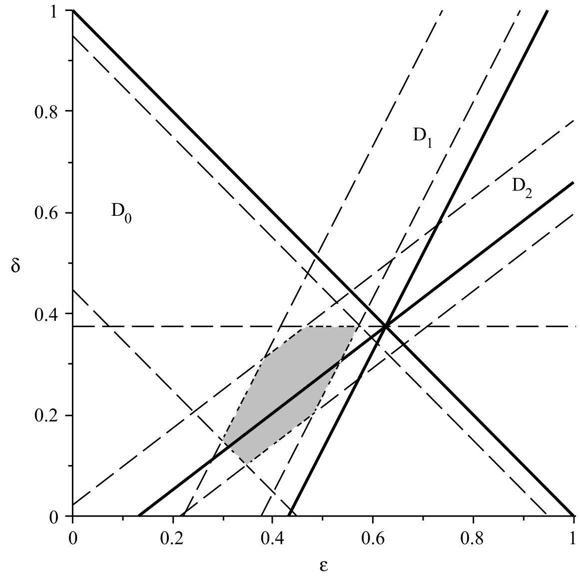

Figure 4: Confidence intervals for ϵ and δ implicit in the estimates for the slopes in Table 1 (dashed lines). Bold lines are the same as in Figure 3. $\alpha = 0.6$ and $\sigma = 0.1$.

have never been discussed in the literature. However, as shown in Section 4, the model establishes specific constraints for the market complexity cost parameters, δ and ϵ, in order to produce results that are qualitatively consistent with the data, whereas the horizontal R&D complexity cost parameter, σ, is free to take any positive value. Figure 4 juxtaposes the confidence intervals for ϵ and δ implicit in the estimates for the slopes in Table 1, together with the subset of values depicted in Figure 3, for σ arbitrarily set to 0.1. We observe that, with a (slight) exception for the relationship regarding the relative firm size, the confidence intervals lie within the subset in which there is a positive relationship between the skill structure and both relative production and the relative number of firms, for values of ϵ between roughly 0.25 and 0.55 and δ between 0.15 and 0.375. Inside that area, we choose the point values for δ and ϵ that are required for the minimisation of the SQE. We repeat this procedure for a wide range of arbitrary plausible values for σ.

[Figure 4 goes about here]

This exercise is summarised in Table 2 which shows an interesting interaction between the complexity-cost parameters, as follows:

(i) Down to a certain threshold (σ slightly larger than unity, for the baseline value of α), the lower the σ, the lower the required (positive) ϵ that minimises the SQE, while the required δ is zero; this movement unequivocally improves the quality of the adjustment (i.e., the minimum SQE for each set of parameters decreases);

σ	δ	ϵ	D_0	D_1	D_2	SQE	$\bar{\epsilon}_2$	$\bar{\epsilon}_1$
4.0	0.00	0.550	0.090	0.165	0.075	0.861	0.571	0.595
2.0	0.00	0.505	0.165	0.233	0.068	0.519	0.526	0.571
1.0	0.01	0.435	0.278	0.344	0.066	0.026	0.455	0.539
0.5	0.17	0.450	0.253	0.320	0.067	0.112	0.479	0.557
0.1	0.25	0.420	0.300	0.370	0.070	0.119	0.461	0.560
point estimates			0.274	0.349	0.067			

Table 2: Calibration exercise for the technology-structure elasticities D_0, D_1 and D_2 (equations (38), (41) and (44)). $\alpha = 0.6$, and SQE $= \sqrt{\sum_i (D_i/\hat{D}_i - 1)^2}$, $i = 0, 1, 2$, where $\hat{D}_i$ denotes the point estimates for the slopes in Table 1.

(ii) Below that threshold, the lower the σ, the higher the δ that minimises the SQE, while the required ϵ and the quality of the adjustment are roughly stabilised.

This result reflects the fact that shifts in σ and ϵ have mainly opposing effects, whereas shifts in σ and δ produce qualitatively similar (substitution) effects between vertical and horizontal R&D activities. To complete the analysis, we note that the threshold value for σ increases with α.

[Table 2 goes about here]

[Figure 5 goes about here]

We now calibrate the model to simultaneously target the three estimates of the intercepts in Table 1. In addition to the ratios h/l, ζ_H/ζ_L and ϕ_H/ϕ_L, the relevant parameters are now α and σ. Again, since the values for σ are arbitrary, we perform the calibration exercise with its lowest value considered in Table 2 ($\sigma = 0.1$), while, in line with the empirical studies, we let $h/l = 1.3$ (see, e.g., Afonso and Thompson, 2011), as the baseline; however, given the uncertainty surrounding the empirical estimates, we also consider values in the range of ± 0.2 around the baseline. As there is no empirical guidance for ζ_H/ζ_L and ϕ_H/ϕ_L, we construct confidence intervals for these variables, given the previous values of the parameters, by using the estimation results for $\ln Z_0$, $\ln Z_1$ and $\ln Z_2$ in Table 1. A graphical depiction is given in Figure 5, which suggests values for ϕ_H/ϕ_L between roughly 2.5 and 4.75 and values for ζ_H/ζ_L between 2.2 and 3.75. Again, we pick values for these parameters that minimise the SQE and display some cases in Table 3.[18]

[18]The exercise in Table 3 was carried out by setting 3.0 as the upper boundary for ζ_H/ζ_L and ϕ_H/ϕ_L; by removing this constraint, the calibration exercise yields values for the SQE of between 0.023 and 0.038. On the other hand, it can be shown that the higher σ, the higher ϕ_H/ϕ_L and/or ζ_H/ζ_L necessary to keep the quality of adjustment unchanged.

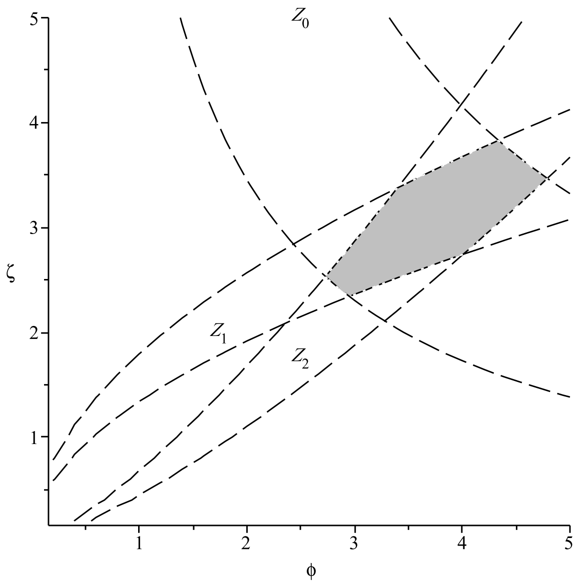

Figure 5: Confidence intervals for $\phi \equiv \phi_H/\phi_L$ and $\zeta \equiv \zeta_H/\zeta_L$ implicit in the estimates for the intercepts in Table 1. $\alpha = 0.6$, $\sigma = 0.1$ and $h/l = 1.3$.

As ϕ_H/ϕ_L and ζ_H/ζ_L have opposing effects on the intercepts, the combination of values for those two ratios allows for an effective reduction of the SQE. However, the ability of the model to replicate the data on the intercept depends crucially on $\phi_H/\phi_L > 1$ and $\zeta_H/\zeta_L > 1$. Moreover, the approximation is acceptable only for relatively high values of both ratios (larger than 2), even when combined with low values of σ.

[Table 3 goes about here]

By combining the results in Tables 2 and 3, we assess the global predictive power of our model as regards the technology structure. Using the data on the skill structure (mea-

h/l	ζ_H/ζ_L	ϕ_H/ϕ_L	$\ln Z_0$	$\ln Z_1$	$\ln Z_2$	SQE
1.1	2.5	3.0	-1.775	-0.765	1.010	0.155
1.3	2.5	2.9	-1.562	-0.666	0.897	0.345
1.5	2.7	3.0	-1.563	-0.615	0.948	0.365
point estimates			-1.917	-0.855	1.105	

Table 3: Calibration exercise for the technology-structure coefficients Z_0, Z_1 and Z_2 (equations (39), (42) and (45)). $\alpha = 0.6$, $\sigma = 0.1$, and SQE $= \sqrt{\sum_i (\ln Z_i / \widehat{\ln Z_i} - 1)^2}$, $i = 0, 1, 2$, where $\widehat{\ln Z_i}$ denotes the point estimates for the intercepts (constants) in Table 1.

h/l		Relative number of firms	Relative production	Relative firm size
1.1	predicted avg.	0.096	0.233	2.402
	$\mathcal{R}^2$	0.852	0.679	0.771
1.3	predicted avg.	0.119	0.257	2.144
	$\mathcal{R}^2$	0.807	0.691	0.738
1.5	predicted avg.	0.119	0.271	2.256
	$\mathcal{R}^2$	0.807	0.693	0.754
observed average		0.100	0.281	2.993

Table 4: Quantitative assessment of the model: the cross-country average of the predicted and of the observed values of the technology-structure variables, and the associated $\mathcal{R}^2$. $\alpha = 0.6$, $\sigma = 0.1$, $\delta = 0.25$ and $\epsilon = 0.42$. The values for ζ_H/ζ_L and ϕ_H/ϕ_L (not shown) are chosen in accordance to the calibration exercise in Table 3.

sured by the relative supply of skills) into equations (37)-(45), Table 4 presents the cross-country average of the predicted values for the technology-structure variables compared to the average observed in the data, and also the constrained R^2, which is a more general measure of goodness of fit. We compute the latter as $\mathcal{R}^2 = 1 - \sum_c (x_c^{obs} - x_c)^2 / \left(x_c^{obs}\right)^2$, where x_c (x_c^{obs}) denotes the predicted (observed) value of each of the technology-structure variables for country c in our sample (see, e.g., Acemoglu and Zilibotti, 2001).[19]

As Table 4 shows, for the selected calibration, the average of the predicted values for each technology-structure variable is quite similar to the average of the values observed in the data (between 96.2 and 119.0 percent of the observed average for the relative number of firms, 82.9 and 96.3 percent for relative production, and 71.6 and 80.2 percent for the relative firm size), whereas the $\mathcal{R}^2$ indicates a rather high goodness of fit in all cases (between 0.679 and 0.852). Nevertheless, Figure 6 shows that the predicted values for the technology-structure variables underestimate the cross-country dispersion observed in the data.

[Table 4 goes about here]

[Figure 6 goes about here]

Finally, we complete the assessment of the model by considering the data on the supply of high- and low-skilled workers in order to compute the predicted BGP growth rates using (46) and then by relating them to the skill structure. Based on the calibration used in Table 4, we compare the results of OLS regressions run on the predicted and on

[19] This is the R^2 from a regression of x^{obs} on x when the slope is constrained to be equal to unity and the constant to be zero; in this case, $\mathcal{R}^2 \in (-\infty, 1]$.

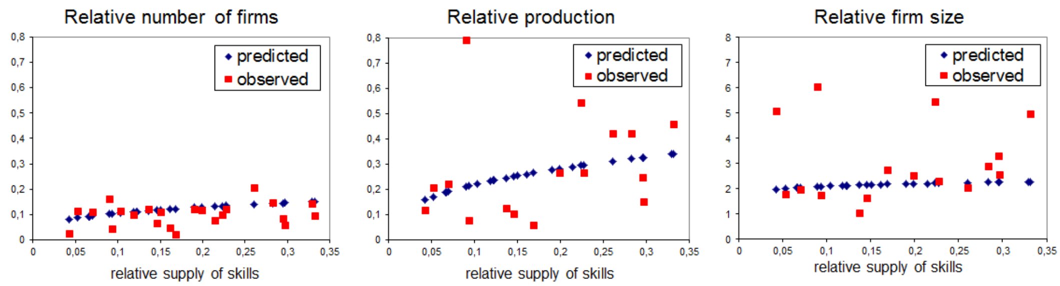

Figure 6: Predicted versus observed values of the technology-structure variables. $\alpha = 0.6$, $\sigma = 0.1$, $\delta = 0.25$, $\epsilon = 0.42$, $h/l = 1.3$, $\zeta_H/\zeta_L = 2.5$, and $\phi_H/\phi_L = 2.9$.

the observed values for the economic growth rate with respect to the relative supply of skills (Table 5). Importantly, we find that the point estimates of the elasticity of growth regarding the relative supply of skills that are based on the predicted values for growth is small positive, falling within the confidence interval for the estimate of the elasticity based on the growth rate observed in the data (second column of Table 5).[20]

[Table 5 goes about here]

As an extension to the calibration exercise in Table 5, we let $\zeta_H/\zeta_L = 3.75$, which is roughly the maximum admissible value shown in Figure 5. For the benchmark case of $h/l = 1.3$, the estimated elasticity of growth with respect to the relative supply of skills is lowered to 0.132 and the intercept to 1.029.

Alternatively, we choose the value for A in order to maximise the constrained R^2, $\mathcal{R}^2$, which is the more general measure of goodness of fit used in Table 4. For the benchmark case of $h/l = 1.3$, we find that $A = 1.31$ implies an $\mathcal{R}^2$ of 0.31, which although still lower than the values obtained for the technology-structure variables in Table 4, is quite higher than the values that obtain from the calibration exercise in Table 5 (between 0.113 and 0.123). However, this result comes at the expense of the ability of the model to target the observed average economic growth rate (we get 1.8 percent instead of 3.1 percent), while the estimated elasticity of the economic growth rate with respect to the relative supply of skills is 0.159, close to the one obtained in Table 5 (the point estimate of the intercept, however, falls considerably, to 0.512).

Figure 7 depicts the predicted versus the observed values of the economic growth rate. It shows that the predicted values somewhat overestimate the cross-country dispersion observed in the data, in contrast to the underestimation that occurs in the case of the

[20] As a robustness check, one may consider the set of values of ϵ that are consistent with the estimate of the elasticity of the skill premium with respect to the relative supply of skills in Table 1 (see (31)) and compare them to the set of values of ϵ that are consistent with the elasticities of the technology-structure variables regarding the relative supply of skills found in the cross-country data, as depicted by Figure 3. We find that there is a considerable juxtaposition of the two set of values (0.23-0.34 versus 0.25-0.55).

Dependent variable	ln Growth (observed)	ln Growth (predicted) $h/l = 1.1$	ln Growth (predicted) $h/l = 1.3$	ln Growth (predicted) $h/l = 1.5$
Constant	1.177	1.057	1.086	1.085
(White s.d.)	(0.336)			
ln Relative supply of skills	0.079	0.146	0.157	0.161
(White s.d.)	(0.180)			
Observations	27	27	27	27
Correlation coefficient	0.095	0.088	0.095	0.098

Table 5: Quantitative assessment of the model: OLS regressions of the economic growth rate (predicted and observed values) on the relative supply of skills, in logs. Predicted values are obtained by setting $\alpha = 0.6$, $\sigma = 0.1$, $\epsilon = 0.42$, and also $A = 1.84$, $\rho = 0.02$, $\theta = 1.5$, and $\lambda = 2.5$. A is chosen such that the average of the (cross-country) predicted economic growth rate matches the average of the observed economic growth rate (3.1 percent); the values for ζ_H/ζ_L (not shown) are chosen in accordance to the calibration exercise in Table 3; the values for λ, θ, ρ are set in line with the standard literature on growth (e.g., Barro and Sala-i-Martin, 2004). Note: the second column repeats the data presented in the last column of Table 1.

technology-structure variables. This disparity of results reflects the fact that the calibration strategy followed throughout this section was designed to match the point estimates of the elasticities of the technological-structure variables with respect to the skill structure, as explained earlier.

To sum up:

- The predicted values of the elasticity of the technology-structure variables with respect to the skill structure are quite similar to the ones observed in the data, for a significant range of the relevant parameters of the model.

- For intermediate levels of market complexity costs, sufficiently low levels of horizontal R&D complexity costs, relatively large fixed flow costs in the vertical and horizontal R&D sectors targeting the H-complementary production technology, and high-skilled workers with an absolute productivity advantage, the global quality of the adjustment of the predicted values to the values observed in the data for the technology-structure variables is quite high.

- The small positive elasticity of the economic growth rate with respect to the proportion of high-skilled labour observed in the data is explained by the combination of moderate levels of the market complexity costs related to vertical R&D and relatively large fixed flow costs in the vertical R&D sector targeting the H-complementary production technology (implying higher entry costs in the high-

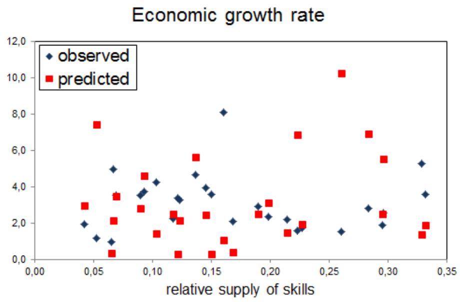

Figure 7: Predicted versus observed values of the economic growth rate. $\alpha = 0.6$, $\sigma = 0.1$, $\delta = 0.25$, $\epsilon = 0.42$, $h/l = 1.3$, $\zeta_H/\zeta_L = 2.5$, $A = 1.84$, $\rho = 0.02$, $\theta = 1.5$, and $\lambda = 2.5$.

than in the low-tech sectors when entry occurs through vertical innovation),[21] which dampen the direct effect arising from the absolute productivity advantage of the high-skilled workers observed in the data.

6. Policy implications and concluding remarks

Naive intuition suggests that there should be a positive relationship between the skill structure and economic growth, since a larger number of high-skilled workers would induce a larger concentration of activity in high- vis-à-vis low-tech sectors, which would then lead to accelerated growth. Anecdotal evidence based on cross-section data for a number of European countries indeed shows that the elasticity of different measures of concentration and of the economic growth rate with respect to the skill structure tends to be positive.

This paper builds an endogenous growth model of directed technical change with simultaneous vertical and horizontal R&D and scale effects at the industry level to study an analytical mechanism that is consistent with the observed cross-country pattern in the skill structure, the technology structure and economic growth. Our calculations indicate that the cross-country differences in the skill structure, combined with the existence of intermediate levels of market complexity costs, sufficiently low levels of horizontal R&D complexity costs, high relative fixed entry costs in the high-tech sectors and an absolute productivity advantage of the high-skilled workers, may be an important factor in explaining the observed pattern in the number of firms, production and firm size in high- versus low-tech sectors and hence the relationship between economic growth and the skill structure.

Furthermore, by linking the determinants of the technology structure to economic growth, our model allows us to derive interesting policy implications: (i) the effects of education policy (e.g., incentives for households to accumulate skills via improvement of

[21] As referred to earlier, the result that entry costs may be, in practice, larger in the high- than in the low-tech sectors finds support in some empirical literature (see fn. 4).

the educational attainment level) on economic growth may be leveraged by industrial policy; (ii) in particular, the latter should aim to reduce both the complexity and the fixed-entry costs pertaining to R&D activities, which may be perceived in practice as barriers to entry. These forms of industrial policy should complement the direct subsidisation of R&D activities usually emphasised in the economic growth literature.

It is also noteworthy the importance of distinguishing between the effects of industrial policies targeted at vertical R&D – which can be seen as pertaining to process innovation and incremental product innovation – and those targeted at horizontal R&D – pertaining to radical product innovation.[22] For instance, a reduction of the market complexity costs related to vertical R&D and of the R&D complexity costs related to horizontal R&D will have a similar, positive, impact on economic growth, but an asymmetric impact on the technology structure: for a given relative supply of skills below unity, a decrease of the first type of costs implies a smaller concentration of activity in high- vis-à-vis low-tech sectors in terms of the number of firms, production and firm size; a decrease of the second type implies an decrease of the proportion of high- versus the low-tech sectors in terms of the number of firms only.

The study of the transitional dynamics should be an objective for future work. The qualitative characterisation of the local dynamics properties might allow one to find to what extent variations in a country's initial conditions lead to non-monotonic time paths of the technology structure and the economic growth rate towards the BGP in face of a shock in the skills structure, as observed in the 80s and 90s in a number of developed countries. Moreover, this will allow us to accommodate the fact that the time-span of the available data is relatively short and hence may encapsulate transitional dynamics effects.

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[22] The importance of analysing the impact of R&D policies separated this way has been emphasised by, e.g., Peretto (1998).

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Appendix

A. Proxy for quality-adjusted production

Assume that j follows a Poisson distribution with parameter $I \cdot t$, $j \sim Po(I \cdot t)$ over $[0, t]$. Then $\mathbb{E}\left(\lambda^{\beta j}\right) = e^{-(1-\lambda^{\beta})It}$. Proof:

$$\begin{aligned} \mathbb{E}\left(\lambda^{\beta j}\right) &= \mathbb{E}\left(\left(\lambda^{\beta}\right)^{j}\right) = \sum_{j=0}^{\infty}\left(\lambda^{\beta}\right)^{j}\frac{e^{-It}\left(It\right)^{j}}{j!} = \\ &= e^{It\lambda^{\beta}}e^{-It}\sum_{j=0}^{\infty}\frac{e^{-It\lambda^{\beta}}\left(It\lambda^{\beta}\right)^{j}}{j!} = e^{It\lambda^{\beta}}e^{-It} = e^{-It(1-\lambda^{\beta})} \end{aligned}$$

Next, consider the random variables $\mathcal{Z} \equiv \lambda^{j\frac{1-\alpha}{\alpha}}$ and $\mathcal{K} \equiv \lambda^{j\frac{1}{\alpha}}$, as well as the sum of the random variables $\mathcal{Z}_i$, i.i.d. of $\mathcal{Z}$, in $Q_m = \sum_i^{N_m} \mathcal{Z}_{mi}$, and $\mathcal{K}_i$, i.i.d. of $\mathcal{K}$, in $\mathcal{Q}_m = \sum_i^{N_m} \mathcal{K}_{mi}$, $m \in \{L, H\}$. Then, for a given N_m, we get

$$\mathbb{E}(Q_m) = N_m e^{-I_m t(1-\lambda^{\frac{1-\alpha}{\alpha}})} \tag{53}$$

$$\mathbb{E}(\mathcal{Q}_m) = N_m e^{-I_m t(1-\lambda^{\frac{1}{\alpha}})} \tag{54}$$

Using $\ln(v+1) \approx v$ for v small enough, (53) and (54) can be rewritten as follows

$$\mathbb{E}(Q_m) = N_m e^{I_m t\left(\frac{1-\alpha}{\alpha}\right)\ln\lambda} = N_m \lambda^{I_m t\left(\frac{1-\alpha}{\alpha}\right)} \tag{55}$$

$$\mathbb{E}(\mathcal{Q}_m) = N_m e^{I_m t\left(\frac{1}{\alpha}\right)\ln\lambda} = N_m \lambda^{I_m t\left(\frac{1}{\alpha}\right)}. \tag{56}$$

Thus, $\mathbb{E}(\mathcal{Q}_m)/\mathbb{E}(Q_m) = \lambda^{I_m t\left(\frac{1}{\alpha} - \frac{1-\alpha}{\alpha}\right)} = \lambda^{I_m t}$, which goes to ∞ as $t \to \infty$. However, given (55) and (56), we also have

$$(\mathbb{E}(Q_m))^{\left(\frac{1}{1-\alpha}\right)} N_m^{-\left(\frac{\alpha}{1-\alpha}\right)} = N_m \lambda^{I_m t\left(\frac{1}{\alpha}\right)} = \mathbb{E}(\mathcal{Q}_m) \tag{57}$$

Since, in our model, Q_m is treated as a continuous deterministic variable, we consider the following proxy, $\widehat{\mathcal{Q}}_m$, as a deterministic version of (57)

$$\widehat{\mathcal{Q}}_m = Q_m^{\frac{1}{1-\alpha}} \cdot N_m^{-\left(\frac{\alpha}{1-\alpha}\right)}$$

It can then be shown that $\mathcal{Q}_m/\widehat{\mathcal{Q}}_m = \text{constant}$.

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