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Introduction

Purchasing Power Parity (PPP), as a long run equilibrium condition, is a frequent assumption in open economy macroeconomic models. Hence, given the importance of PPP as a long run determinant of exchange rate behaviour, testing this theory has become a popular exercise. However, traditional tests of PPP are probably not the best method. Cointegration theory, recently developed, provides a more adequate method to test the long run relationships between exchange rates and prices.

The purpose of this paper is to test the PPP theory using the Portuguese escudo *vis à vis* seven currencies, representing Portugal's principal trade partners, namelythe Deutsche Mark (DM), the Spanish Peseta (PTA), the French Franc (FF), the Italian Lira (LIT), the United Kingdom Pound (GBP), the Swiss Franc (SFR), the Swedish Krona (SKR) and U.S. Dollar (USD).

The remainder of this paper is organised as follows: section one introduces the main theoretical aspects; section two presents the econometric methodology used to test the theory. The empirical results are reported in section three and conclusions are presented at the end of the paper.

1. Purchasing Power Parity Theory

One of the most popular approaches of exchange rate behaviour is PPP theory. It provides an important link between two economies in any open-economy macroeconomic analysis. PPP theory states that there is a close relationship between domestic and foreign prices and the nominal exchange rate.

There are two versions of PPP theory: the absolute and the relative versions.

The absolute version relates the exchange rate to the ratio of price levels in each country which can be defined by:

$$S_t = \frac{PL_t}{PL_t^*} \qquad (1a)$$

S_t - nominal exchange rate in period t (domestic price of foreign currency).

PL_t - domestic price level in period t.

PL^*_t - foreign price level in period t.

This version assumes that the nominal exchange rate totally reflects the ratio of domestic to foreign price levels. Thus currencies' real purchasing power is constant over time. This hypothesis means that the *law of one price* is always verified presumably through the perfect arbitrage of individual goods' prices. Since, in general the price levels do not contain the same bundle of goods as well as the same corresponding weights, a simple constant wedge K is frequently imposed in absolute parity.

$$S_t = K\frac{PL_t}{PL_t^*} \qquad (1b)$$

The absolute PPP is commonly employed in open-economy theoretical specifications. However, empirical tests of the theory usually test for the relative version. Following Pippenger (1993), an expression for relative PPP using price indices may be derived from equation (1b) dividing it by the same equation in a base period (zero). This yields equation (2a).

$$\frac{S_t}{S_0} = \frac{PL_t/PL_0}{PL^*_t/PL^*_0} = \frac{P_t}{P_t^*} \qquad (2a)$$

P_t - Domestic price indice in period t.

P^*_t - Foreign price indice in period t.

This form of PPP is weaker than the previous one, because it only requires that the percentage change in exchange rate between period zero and any arbitrary period t to be equivalent to the percentage change in price levels for the same period. Note that any fixed wedge in the law of one price is eliminated in relative PPP. Essentially, the difference between the two versions is that while absolute PPP concerns price levels and ultimately price indices, relative PPP concerns only price indices.

The logarithmic version of equation (2a) is:

$$s_t = s_0 + p_t - p^*_t \tag{2b}$$

where are s_t, s_0, p_t and p^*_t are the natural logarithms of S_0, S_t, P_t and P^*_t. Temporary deviations in relative PPP can be added by the inclusion of an error term

$$s_t = \alpha + p_t - p^*_t + \varepsilon_t \qquad a(L) . \varepsilon_t = \eta_t \tag{3}$$

where η_t is a white noise process and a (L) is a polynomial in the lag operator L with roots which all lie outside the unit circle, i. e., ε_t is a stationary process. Note that α is not simply the logarithm of the exchange rate in the base period because, in general, there are no guarantees that relative PPP holds without error during that period. If deviations from relative PPP are serially uncorrelated a(L) has degree zero. If deviations persist but relative PPP still holds as a long run equilibrium, a(L) has a degree greater than unity. Finally, if relative PPP does not hold in the long run all the roots lie inside the unit circle, therefore ε_t is a nonstationary process.

The purpose of this paper is to test empirically the relative version of PPP. Recent studies of PPP can be classified in three groups with different degrees of restrictiveness[1]. Some authors only accept PPP provided that the real exchange rate is stationary (univariate model). Others test the PPP hypothesis in a less restrictive way by testing the relationship between nominal exchange rate and relative price ratio (bivariate model). Finally, the less restrictive version of PPP waives all restrictions and concentrates on the existence of a stationary linear combination of the three variables (trivariate model).

This last two group's argument is based on the existence of *measurement errors* in the price series. In fact, the existence of non-traded goods and services, transportation costs, trade restrictions and imperfect competition are some factors

[1] See Cheung and Lai (1993) and Liu (1992).

which might weaken the relation between exchange rates and prices. Supposing it were possible to observe the theoretical price series PPP could be written[2] as

$$s_t = c + g_t - g^*_t + \zeta_t \tag{4}$$

where small letters represent the logarithms, g, g* are the theoretical prices and ζ_t is a stationary process. Depending on how the price indices are constructed a 1 percent change in the observed prices may correspond to more or less than 1 percent change in the theoretical counterparts. This can be captured by the following equations:

$$p_t = \alpha_1 + \beta_1 g_t + \mu_{1t} \tag{5a}$$

$$p^*_t = \alpha_2 + \beta_2 g^*_t + \mu_{2t} \tag{5b}$$

α_1, α_2, β_1, β_2, are constant and systematic measurement errors; μ_{1t}, μ_{2t} are stationary stochastic measurement errors. Since μ_{1t} and μ_{2t} are stationary the observed prices will not drift too far apart from the theoretical ones.

For the purpose of empirical testing, PPP can be written as

$$S_t = A\,(P_t^{\,b1} / P^{*\,b2}_t) \cdot \exp(v_t) \tag{6}$$

where A, b_1 and b_2 are constants, and v_t is an error term. Or, if we consider the logarithmic version of (6),

$$s_t = a + b_1 \cdot p_t - b_2 \cdot p^*_t + v_t \tag{7}$$

where the small letters represent the variables' logarithm. Relative PPP (2b) imposes both symmetry ($b_1 = b_2$) and proportionality ($b_1 = b_2 = 1$) restrictions. However, if there are measurement errors in prices, b_1 and b_2 can both differ from unity but PPP still holds in the long run. In fact, combining (4), (5a) and (5b) gives $a = c - \alpha_1/\beta_1 + \alpha_2/\beta_2$, $b_1 = 1/\beta_1$, $b_2 = 1/\beta_2$, and $v_t = \zeta_t - \mu_{1t}/\beta_1 + \mu_{2t}/\beta_2$ in equation (7). Thus, it seems more adequate to test PPP without imposing the restrictions *à priori*.

2. Cointegration

When testing empirically an economic theory one should always be concerned with the correspondence between the tecnhique employed and the economic relationships under consideration. Since PPP can be understood as a long run

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[2] See Cheung and Lai (1993)

equilibrium the concept of cointegration seems to be an adequate statistical framework to test the theory. In order to illustrate this statement a discussion of the concept of cointegration follows.

A variable x_t is integrated of order d (I(d)) if it has a stationary, invertible, deterministic ARMA representation, i.e., if it tends to return to its mean and flutuate around it within a more or less constant range, after beind differenced d times. The integration order should be understood as the number of necessary differentiations to stationarize a time series.

If there is a long run relationship between *n* nonstationary variables, caracterized by having a different mean at different points in time, but the deviations from that long run path are stationary, then the variables are said to be cointegrated.

Let X_t be an n x 1 vector of time series variables, such as $X_t' = [\, x^1_t , x^2_t , \ldots, x^n_t]$, and $x^i_t \sim I(d)$, $i = 1, 2, \ldots, n$. If there exists $\alpha = [\, \alpha_1, \alpha_2, \ldots, \alpha_n \,]$, such that

$$u_t = \alpha \,.\, X_t \sim I\,(d\text{-}b) \quad \text{where} \quad b > 0 \tag{8}$$

then $x^1_t , x^2_t , \ldots, x^n_t$ are cointegrated of order (d-b), where α' is the cointegrating vector. However, there might be more than one cointegrating vector, in which case α will represent a matrix.

If d-b = 0 then u_t is stationary. Intuitively, this means that the long run components of the elements in X_t 'cancel out' or trend together. The stationary behaviour of u_t is precisely the behaviour of ε_t in equation (3) when it represents stationary deviations from PPP[3]. If two or more variables are cointegrated then there exists an error correction model which gives the adjustment to the long run equilibrium embodied in the cointegrating regression (8).[4]

Testing for cointegration implies two different stages: first, we must determine the order of integration of the varibles and second, if the variables are integrated of the same order, we test for cointegration.

2.1. Non-stationarity Tests

In order to determine the order of integration for each element of X_t we use the following tests: the Integrated Durbin-Watson test (I.D.W.); and the tests for a unit root based on the work of Fuller (1976) and Dickey and Fuller (1979, 1981). From the analysis of the original data, tests are made on consecutive differentiations until the null hypothesis of non-stationarity is rejected.

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[3] See Charemza and Deadman (1992), Cuthbertson (1992) and Pippenger (1993).

[4] Engle and Granger (1987)

The I.D.W. test for x^i_t is based on the analysis of the residuals from equation

$$x^i_t = c + e_t \tag{9}$$

where c is a constant and is e_t an random term. The null hypothesis of non-stationarity is rejected for values higher than 0.3 (Sargan e Bhargava, 1983) using a 5% significance level, for samples with more than one hundred observations. In this case, the rejection of the null hypothesis means the rejection of non-stationarity for x^i_t, i.e., that x^i_t is I (0). If the null hypothesis cannot be rejected an equivalent test is made for $\Delta x^i_t = x^i_t - x^i_{t-1}$.

An alternative test is based on the presence of a unit root in the series x^i_t or Δx^i_t. When x^i_t is I(1) it follows a process which may be described as:

$$x^i_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j x^i_{t-j} + \varepsilon_t \tag{10}$$

To test for a unit root in series x^i_t the following regression is estimated via OLS:

$$(x^i_t - x^i_{t-1}) = \Delta x^i_t = c_0 + b_0(\text{Time}) + b_1 x^i_{t-1} + \sum_{j=1}^{p} c_j \Delta x^i_{t-j} + \mu_t \tag{11}$$

And to test a unit root in the series Δx^i_t the equation estimated by OLS is:

$$(\Delta x^i_t - \Delta x^i_{t-1}) = \Delta^2 x^i_t = c^*_0 + b^* \Delta x^i_{t-1} + \sum_{j=1}^{p} c^*_j \Delta^2 x^i_{t-j} + \mu^*_t \tag{12}$$

where p is chosen to induce the residuals μ_t and μ^*_t into white noise series. Under p=0 this tests is known as the Dickey-Fuller (DF) test and if p is greater than zero the test is known as the Augmented Dickey-Fuller (ADF) test. Equation (11) should contain a linear trend term for two reasons. First, if Time is not included and x^i_t follows a process with drift ($\alpha_0 \neq 0$), the distribution of the test statistic for b_1 contains a nuisance parameter α_0. Second, cointegration tests can only be conducted for series whith compatible stochastic features.

The test statistic used to test the nonstationarity of x^i_t is the likelihood ratio statistic Φ_3 and the null hypothesis to be tested is $b_0 = b_1 = 0$[5]. This statistic is given by the value of the F-statistic although its distribution is not the standard one. The relevant critical values are given in table VI in Dickey and Fuller (1981, p. 1063).

If b* in equation (12) is estimated to be significantly less than zero, the null hypothesis of a unit root for the series is rejected, i.e., Δx^i_t is stationary. The t_μ statistic, proposed in Dickey and Fuller (1979) for testing the hypothesis $b^* = 0$, is

[5] This test is the same used by Pippenger (1993).

given by the usual t-ratio of the coefficient of b*. The relevant critical values are given in Fuller (1976, p. 373).

If b* is estimated as significantly different from zero and $b_0 = b_1 = 0$ is not rejected the hypothesis that x^i_t is an integrated process of order one is accepted.

2.2. Testing for Cointegration

The methodologies used to test cointegration are: Engle and Granger's and Johansen's.

Engle and Granger methodology is based on the static cointegrating regression

$$x_t^1 = \beta_0 + \sum_{j=2}^{n} \beta_j x_t^j + u_t \tag{13}$$

When the variables are not cointegrated the residuals from this regression are nonstationary. Therefore, to test for cointegration is to test for the stationarity of the residuals. The test proposed is the application of the Dickey Fuller unit root test procedure based upon the following OLS regression:

$$(u_t - u_{t-1}) = \Delta u_t^i = \gamma_0 u_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta u_{t-j} + \delta_t \tag{14}$$

where δ_t is a white noise.

The relevant critical values for the null hypothesis $\gamma_0 = 0$ can be found in Engle and Granger (1987) for the bivariate case and in Engle and Yoo (1987) for the 3, 4, and 5 variables case.

Johansen' s methodology is a different approach to determine if a set of variables are cointegrated. This method begins by assuming that the data is generated by a auto regressive vector which can be expressed by:

$$X_t = \mu + \Pi_1 X_{t-1} + \ldots + \Pi_p X_{t-p} + e_t \tag{15}$$

where each of the Π_i is an $n \times n$ matrix of parameters. The system of equations (15) can be reparameterised in an Error Correction form (ECM) with Gaussian errors:

$$\Delta X_t = \mu + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{p-1} \Delta X_{t-p+1} + \Gamma_p X_{t-p} + B Z_t + v_t \tag{16}$$

where $\Gamma_i = -I + \Pi_1 + \ldots + \Pi_i$, $i = 1, \ldots, p$, B is an $n \times s$ matrix, $v_t \sim N(0,\Sigma)$ and Z_t represents stationary variables that are included in the model to ensure that the disturbances v_t are as close to being gaussian as possible. In practice, the long run equilibrium is tested using (16) which represents the short run dynamics of X_t. Thus, this method might be very sensitive to functional form mis-specification errors. Hence, the inclusion of stationary variables (Z_t) that might be important in explaining the short run behaviour of X_t is sometimes necessary to avoid these errors. Γ_p defines the long run levels solution to (15) which can act as the adjustment mechanism proposed by relative PPP[6].

The idea is that if X_t is a vector of I (1) variables then ΔX_t must be I (0). Thus, the right-hand side of equation (16) must also be stationary implying that $\Gamma_p X_{t-p} \sim$ I (0). Either X_{t-p} contains a number of cointegrating vectors or Γ_p must be a matrix of zeros. If Γ_p is not a a matrix of zero then the variables in X_t can be cointegrated and there might be a cointegrating vector as the one proposed by PPP.

To illustrate this fact, consider a n x r matrix of parameters β such that $\beta X_{t-p} \sim$ I (0). If $X_{t-p} \sim$ I (1), β must form the cointegrating parameters vectors for X_t. Since there can only be up to (n-1) cointegration vectors, β must have r less than n. Now consider a suitable matrix α of the same dimension such that

$$-\Gamma_p = \alpha \beta' \tag{17}$$

where the rank $(\Gamma_p) = r < n$, which is known as the reduced rank condition.

Johansen' s approach is based on the maximum likelihood method of estimating all of the distinct cointegration vectors which may exist between a set of variables. The Johansen ML procedure estimates (16) subject to the reduced rank condition. The non-cointegration hypothesis is tested using a statistic based on the trace and maximal eigenvalues proposed by Johansen. Critical values for the test are listed in Johansen and Juselius (1990). In this dynamic framework it is also possible to test the existence of more than one cointegration vector (at most r) as well as certain linear restrictions on their parameters, i. e, whether a known vector, like the one suggested by relative PPP, is contained in the cointegrating space.

While Johansen's methodology might suffer from mis-specification errors, the Engle and Granger's approach might also have some problems. According to Kugler and Lenz (1993), "... the direction of the regression, which is arbitrarily selected, often leads to different results. Furthermore, the approach breaks down when p_t and p^*_t are pairwise cointegrated. Moreover, the approach is static and does not account for the dynamic interrelationship of the variables." (p. 181)

[6] Two possible candidates could be, for instances, the interest rate differentials in a free float, or, when a currency is pegged, foreign reserves. However, in our exercise we shall not include any stationary variables which might bias our results, in particular the values of the coefficients in the cointegrating vector.

Simulation experiments conducted by Cheung and Lai (1993), show that unit root tests can only reject the false null hypothesis of no cointegration 5% of the time, for a 5% level of significance. At the same significance level Johansen test is able to reject the false null hypothesis about 24% of the time. Hence, the latter seems to be a much more powerful test of a long run relationship.

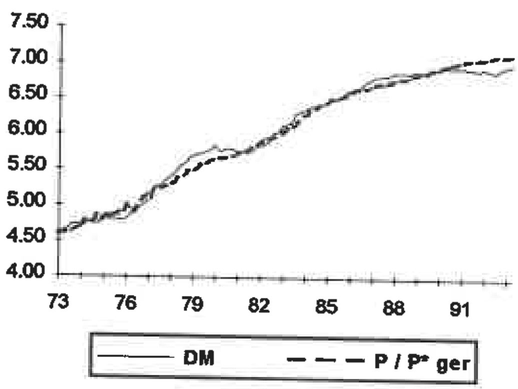
3. Empirical Evidence

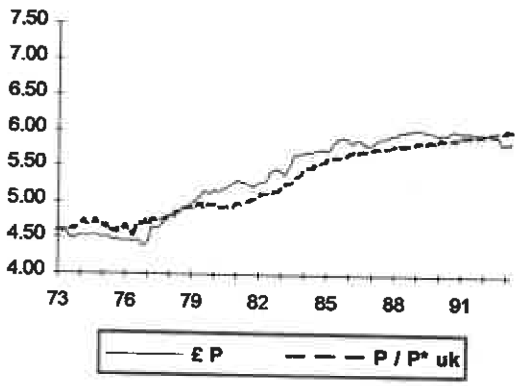
In this section we test PPP theory with reference to Portuguese data. Eight bilateral relations are considered having Portugal as the home country: Germany, Spain, France, Italy, United Kingdom, Sweden, Switzerland and the United States. These countries are representative of Portugal's main trade partners.

The exchange rate series, expressed as domestic price of foreign currency, are monthly average of spot rates taken from the monthly Bundesbank statistics and the consumer price indices from monthly Eurostat and IMF's. The sample covers the period from January 1973 to March 1993. All data series were transformed into monthly indices beginning at 1973.

The logarithm of nominal exchange rates and relative prices are shown in Figure 1 to 8. The graphs give a rough idea whether PPP theory is empirically supported by the data, when symmetry is imposed.

Nominal exchange rate and relative prices





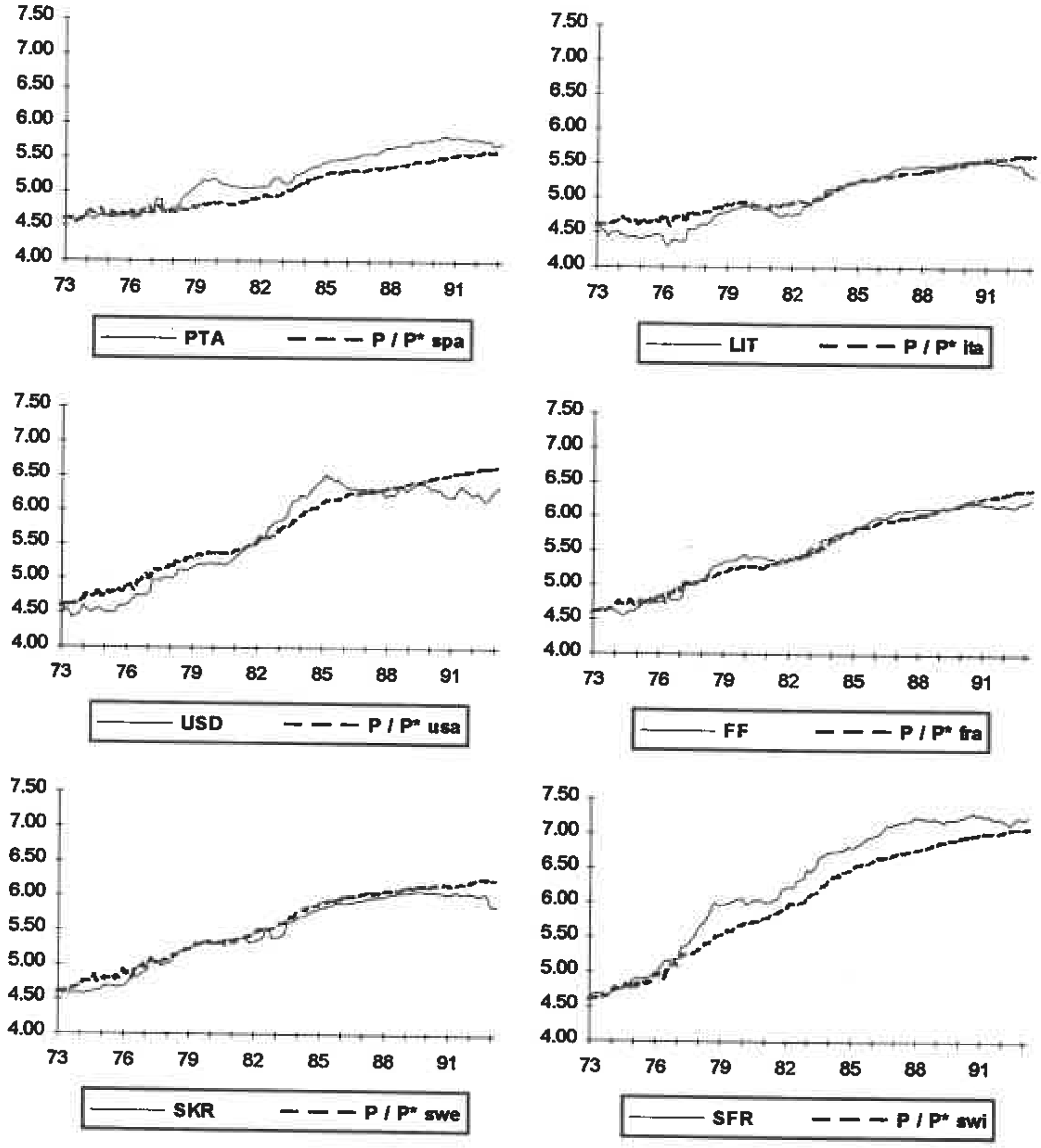


Figure 1 to 8

The exchange rate seems to follow very closely relative prices in two cases: Germany and France. However, in the last three years there seems to be a deviation from that behaviour. The Italian Lira seems to behave accordingly to PPP after 1979 with a similar evolution to the previous cases in the last years of the sample. In the other cases the relationship is less clear. The analysis of the real exchange rate evolution (when symmetry and proportionality are imposed) can provide an additional graphic interpretation (Figure 9 to 16).

Real exchange rate

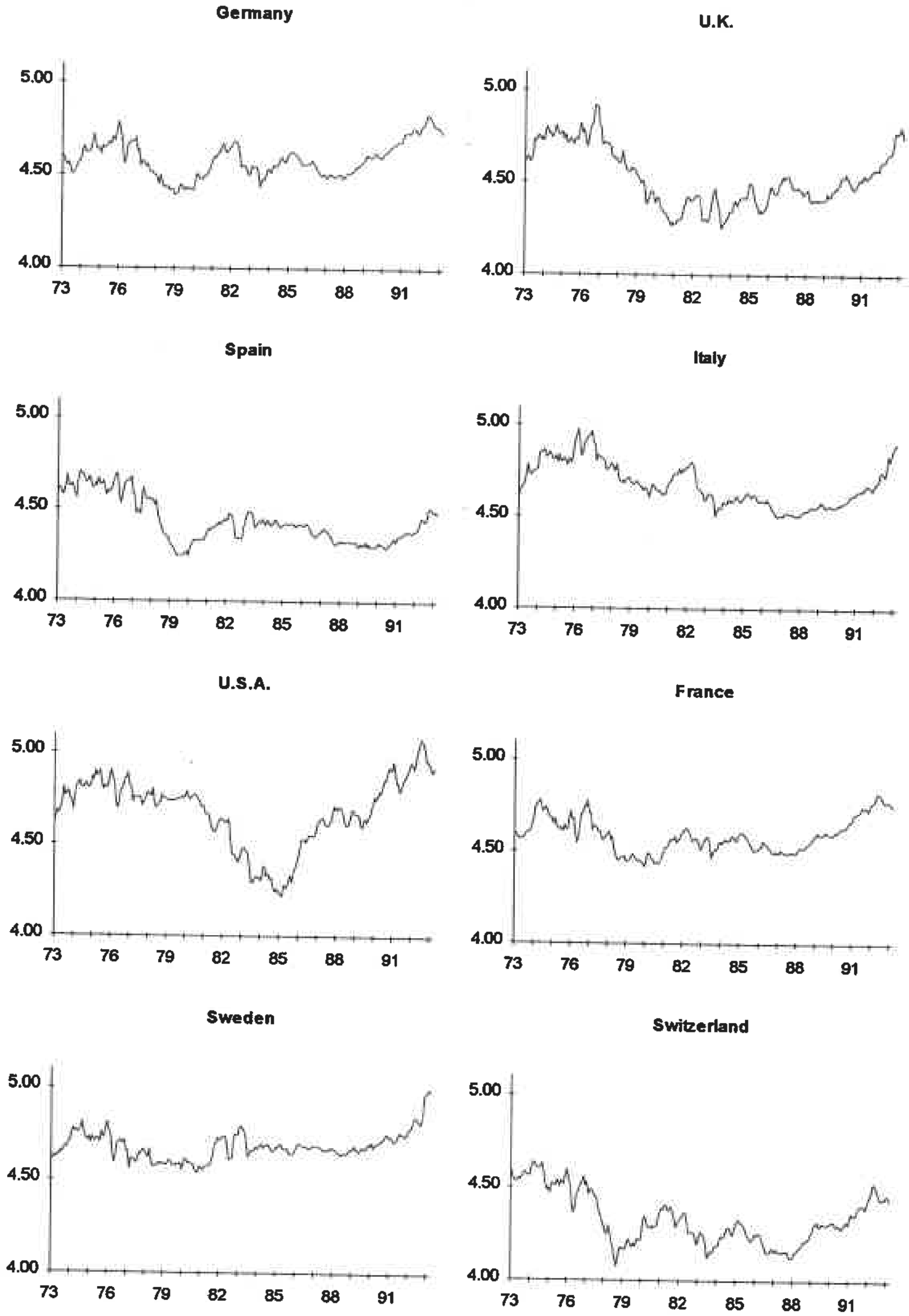


Figure 9 to 16

The German, Swedish and French real exchange rates look like possible candidates to stationary series, thus supporting relative PPP. The other cases exhibit larger deviations from the mean which seem to persist during a finite amount of time as, for instances, in the American case.

3.1 Non-stationarity Tests

In order to determine the order of integration of all time series we use as described above, the IDW, Φ_3 and the DF or the ADF tests. The autoregressive lag length for the ADF test were chosen accordingly to the Box Ljung Q-statistic: we chose the shortest lag for which the hypothesis of white noise of the residuals could not be rejected considering a 10% significance level for autocorrelations up to 36 lags[7].

Table 1 summarizes the tests for non-stationarity of the log levels and the first differences of nominal exchange rates (Δs).

Table 1 - Nominal Exchange Rate

Country	Log levels Φ_3 (a)	First Differences t_μ (b)	First Differences I.D.W. (c)
France	0.91	-7.22 (3)*	1.45*
Germany	2.54	-5.17 (4)*	1.45*
Italy	0.61	-11.46 (0)*	1.41*
Spain	1.15	-9.03 (3)*	1.49*
Sweden	3.51	-11.98 (0)*	1.50*
Switzerland	2.35	-6.37 (3)*	1.25*
U. K.	0.57	-10.99 (0)*	1.34*
U.S.A.	0.92	-11.26 (0)*	1.37*

Note: The symbol " * " means the rejection of the null hypothesis at a 5% significance level.

(a) Critical value for Φ_3 from Dickey and Fuller (1981) is approximately c=6.34 (5 % significance level).

(b) The values in brackets are the lag p of the Dickey-Fuller regression based on the Box-Ljung Q-statistic up to 36 lags. The critical value for t_μ is -2.87

(c) The critical value for Integrated Durbin-Watson test is approximately 0.3 (Sargan e Barghava, 1983).

As shown in Table 1 the non-stationarity null can be rejected for all the first-differenced series. Conversely, we cannot reject the null hypothesis of a unit root and no time trend for the variables' log levels. Therefore we conclude that all exchange rates are I(1) variables.

[7] The choice criterion used is the one suggested by Pippenger (1993).

The stationarity results for the first difference in the price series, reported in Table 2, show that the French, Italian, Portuguese, Swiss and Swede price indices can be considered as I(1) variables. On the other hand, the null hypothesis of a unit root in the first difference series cannot be rejected for the remaining countries, which indicates that for these cases there might be two unit roots instead of one[8]. Therefore, the German, Spanish, the U.K and the American price indices are not suitable for cointegration analysis in a trivariate model.

Table 2 - Consumer Price Indices

Country	Log levels Φ_3 (a)	First Differences t_μ (b)	First Differences I.D.W. (c)
France	8.28*	-2.87 (3)*	1.22
Germany	2.26	-2.17 (11)	1.10
Italy	5.29	-3.04 (4)*	0.74
Portugal	8.01*	-4.12 (7)*	1.80
Spain	5.88	-1.00 (11)	1.32
Sweden	3.52	-3.71 (9)*	1.84
Switzerland	5.68	-4.06 (10)*	1.53
U. K.	6.54*	-1.71 (12)	1.10
U.S.A.	2.26	-2.80 (6)	0.72

Note: See Table 1 for critical values.

Although the Portuguese price index can be considered an I(1) variable, the tests conducted for its log levels show that it has different stochastic features from the rest of the countries (as France and U.K.). In fact, we reject the hypothesis of absence of a time trend in these series as shown by the Φ_3 statistic[9]. Given that the same hypothesis could not be rejected for the other price indices and exchange rates, it is not adequate to test cointegration among them. However, we have decided to test cointegration in a trivariate framework, in order to investigate what results we might find.

The stationarity results for the first difference of relative prices are reported in table 3.

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[8] Pippenger (1993) achieved the same results for U.K and U.S.A, using wholesale price indices.

[9] Additional unit root tests conducted under the null hypothesis $b_1 = 0$ in equation (11) lead to its rejection, i.e., the variables in log levels are nonstationary.

Table 3 -Relative Prices

Country	Log levels Φ_3 (a)	First Differences t_μ (b)	First Differences I.D.W. (c)
France	1.15	-6.88 (6)*	1.91*
Germany	5.40	-4.13 (7)*	1.81*
Italy	2.29	-6.55 (5)*	1.91*
Spain	1.49	-5.71 (7)*	1.83*
Sweden	2.15	-4.46 (7)*	1.95*
Switzerland	5.32	-4.01 (7)*	1.77*
U. K.	1.16	-5.17 (6)*	1.81*
U.S.A.	2.02	-4.58 (7)*	1.82*

Note: See Table 1 for critical values.

Unlike the results of consumer price indices, the ones for relative prices suggest that all series are I (1) and we cannot reject the null hypothesis under Φ_3. Since the relative price series have the same order of integration and stochastic properties as the nominal exchange rates, cointegration tests are applicable and provide an adequate methodology to test PPP theory.

3.2 Cointegration Tests

This section reports the empirical results of the cointegration tests, namely unit root tests and Johansen cointegration analysis. According to the different approaches of relative PPP theory, first we test its broadest interpretation (trivariate model), and then the more restrictive ones (bivariate and univariate models).

Table 4.1 contains the results of unit root tests for cointegration between nominal exchange rate, domestic and foreign prices.

Table 4.1 - Trivariate model

$$s_t = a + b_1 \cdot p_t - b_2 \cdot p^*_t + \eta_t$$

Country	b_1	b_2	D.F. (a)	A.D.F.(1) (b)
France	0.6055	0.1289	-1.80	-2.32
Italy	0.9038	0.7497	-1.51	-1.75
Sweden	1.6780	1.4062	-1.95	-2.27
Switzerland	1.5219	3.3280	-3.76	-4.07*

Note: The symbol " * " means the rejection of the null hypothesis at a 5% significance level.

(a) The critical value for DF is -3.7775.

(b) The critical value for ADF (1) is -3.7776.

The hypothesis of no cointegration could not be rejected at a 5% significance level for any value of the A.D.F. statistic, regardless of its lag, except for Switzerland on the first lag (additional tests indicated that this was not the relevant lag[10]). Hence, we decided to present both D.F. and A.D.F.(1) as examples that illustrate this result, as shown in Table 4.1.

Table 4.2 reports the values for Johansen eigenvalues and trace-based test statistics for the hypothesis of at most *r* linear independent cointegrating vectors. We estimated (15), without any I(0) variables, subject to the reduced rank condition assuming that there are no deterministic trends in the variables in X_t and that the underlying data generation process does not contain a trend term either ($\mu = 0$). This implies the existence of an intercept term in the cointegrating vectors. It should be noticed that as there are no restrictions on the intercept term value for testing the relative version of PPP, its values are not reported. To estimate the VAR model we use a 8th-order auto regressive structure.

Table 4.2 - Trivariate model

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_7 \Delta X_{t-7} + \Gamma_8 X_{t-8} + v_t$$
$$X_t' = [\, s_t \quad p_t \quad p^*_t \quad 1 \,] \quad \text{and} \quad -\Gamma_8 = \alpha \beta'$$

Country		b_1	b_2	**Johansen** (eigenvalues) (a)	**Johansen** (trace) (b)
France	r = 0	-5.506	-11.027	28.41**	49.77**
	r ≤ 1	0.438	-0.438	16.13**	21.36**
Italy		-1012.3	-1237.8	30.50**	44.94**
Sweden		2.03	2.927	43.42**	51.06**
Switzerland	r = 0	1.526	4.540	34.34**	56.34**
	r ≤ 1	1.603	3.694	17.99**	22.00**

The symbols " * " and " ** " means the non-rejection of the null hypothesis at a 5% and 10% significance level.

(a) For r = 0, critical values are 19.7660 (10% significance level) and 22.0020 (5% significance level). For r ≤ 1, critical values are 13.752 (10% significance level) and 15.6720 (5% significance level).

(b) For r = 0, critical values are 32.0030 (10% significance level) and 34.9100 (5% significance level). For r ≤ 1, critical values are 17.8520 (10% significance level) and 19.9640 (5% significance level).

These results differ substantially from those of the residual-based test. In all cases the no cointegration hypothesis (r = 0) is rejected at 5% significance level, which means that there exists at least one cointegrating vector. In the French and Swiss cases one accepts the existence of two cointegrating vectors. However, the sign and magnitude of the estimates of the prices indices coefficients in the cointegrating vector, are far from being the ones predicted by relative PPP. As suggested by Table

[10] The criterion used to test if the first lag was the relevant one was the Akaike informational criterion (AIC) suggested by Pippenger (1993). See Judge et al. (1988, p. 727-29).

2, such results might be due to the fact that the variables have different stochastic properties and illustrate the inadequacy of testing cointegration between these variables.

When symmetry is imposed we obtain a two variable model, with nominal exchange rate and relative prices. Unit root tests conducted for this bivariate model are reported in table 5.1.

Table 5.1 - Unit Root Tests of the Bivariate Model

$$s_t = a + b\,(p_t - p^*_t) + \eta_t$$

Country	$b_1 = b_2 = b$	D.F. (a)	A.D.F. (1) (b)
France	0.980	-1.54	-1.86
Germany	0.972	-1.71	-2.04
Italy	1.194	-1.14	-1.33
U.K.	1.113	-0.93	-1.40
Spain	1.201	-1.91	-2.16
Sweden	0.956	-1.34	-1.82
Switzerland	1.090	-1.64	-2.14
U.S.A	1.038	-0.96	-1.22

(a) The critical value for DF is -3.3624.
(b) The critical value for ADF (1) is -3.3625.

As Table 5.1 shows the null hypothesis of no cointegration cannot be rejected in all cases, i.e., we cannot accept the existence of a long run equilibrium between nominal exchange rate and relative prices according to unit root based-tests, although the coefficients are more reasonable than those on Table 4.2.

The results, using Johansen's methodology as described earlier, are presented in table 5.2.

Table 5.2 - Johansen's Tests for the Bivariate Model

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + ... + \Gamma_7 \Delta X_{t-7} + \Gamma_8 X_{t-8} + v_t$$
$$X_t' = [\, s_t \quad (p_t - p^*_t) \quad 1 \,] \text{ and } -\Gamma_8 = \alpha \beta'$$

Country	$b = b_1 = b_2$	Johansen (eigenvalues) (a)	Johansen (trace) (b)	Hpro χ^2 (c)
France	1.423	26.49**	30.28**	0.84
Germany	1.148	30.51**	35.49**	4.47*
Italy	1.102	17.24**	19.31*	0.07
Spain	1.092	17.58**	21.40**	0.04
Sweden	1.378	24.70**	27.23**	3.10
Switzerland	1.281	26.71**	32.11**	8.84*
U.K.	1.184	14.69*	18.45*	1.26
U.S.A	1.182	39.92**	43.25**	3.88*

Note: The symbols " * " and " ** " means the rejection of the null hypothesis at a 5% and 10% significance level.
(a) For r = 0, critical values are 13.752 (10% significance level) and 15.6720 (5% significance level). For r ≤ 1, critical values are 7.5250 (10% significance level) and 9.2430 (5% significance level).
(b) For r = 0, critical values are 17.8520 (10% significance level) and 19.9640 (5% significance level). For r ≤ 1, critical values are 7.5250 (10% significance level) and 9.2430 (5% significance level).
(c) The statistic for testing the hypothesis of proportionality has a chi-square distribution of one degree of freedom. The critical value is 3.84146 (5% significance level).

In all the cases considered, the hypothesis of no cointegrating vector ($r = 0$) is rejected. Therefore, using the Johansen's methodology (eingenvalues or trace based statistic) s and (p - p*) are cointegrated, i.e., these variables are constrained by a long run equilibrium. The mechanism of adjustment to the long run equilibrium can be represented by an Error Correction Model where the ECM term is given by βX_{t-i}. Moreover, in this model the estimates of the relative price coefficient are much more reasonable than in the trivariate model presented in Table 4.2, being closer to unity as predicted by relative PPP. Nevertheless, all estimates are greater than unity which seems to indicate a tendency for a real depreciation of the Portuguese Escudo.

The hypothesis of proportionality (b = 1) represents a testable linear restriction on the cointegrating vector, using a Chi-Squared test developed by Johansen (1991). For Germany, Switzerland and U.S.A the proportionality hypothesis is rejected. For these cases, although the coefficients are not statistically equal to unity, there still exists a long run relationship between the variables which deviations are stationary. In the other countries, relative PPP seems to hold. Therefore, short run deviations from PPP are expected to return to the equilibrium exchange rate defined by the long run relationship between exchange rate and relative prices.

Finally, when both symmetry and proportionality are imposed upon the data we obtain a univariate model, i.e., the real exchange rate. The univariate test results of non-stationarity of the real exchange rate are reported in Table 6.

Table 6 - Real Exchange Rate

$$s_t - (p_t - p^*_t) = \eta_t$$

Country	D.F. (a)	D.F. (b)	I.D.W. (c)
France	-1.59	-1.89	0.05
Germany	-1.87	-2.18	0.05
Italy	-0.89	-1.12	0.04
Spain	-1.67	-1.99	0.04
Sweden	-1.59	-2.03	0.10
Switzerland	-1.52	-1.95	0.04
U.K.	-0.74	-1.23	0.03
U.S.A	-0.96	-1.23	0.03

(a) The critical value for DF is -3.4295.
(b) The critical value for ADF (1) is -3.4296.
(c) The critical value for Integrated Durbin-Watson test is approximately 0.3 (Sargan e Barghava, 1983).

The nonstationarity null could not be rejected for any of the considered cases according to both unit root tests and I.D.W. Again, the results did not vary over different lag lengths so we decided to present the D.F. and A.D.F.(1) as representative of these results.

The non-stationarity of the real exchange rate, namely for Germany, Switzerland and U.S.A., contradicts the result of the Johansen's cointegration test for these countries, which might be due to the different methodologies used in both tests.

Conclusion

This paper presents empirical tests of relative PPP as long run equilibrium by applying the concept of cointegration. We intended to test PPP theory in three different frameworks: trivariate, bivariate and univariate models. However, due to the stochastic properties of Portuguese data it was only adequate to apply cointegration tests in the last two.

In order to test for cointegration we used two different approaches, namely Engle and Granger's and Johansen's, which lead to different results. First, we could never reject the no cointegration null when using Engle and Granger's approach, contrary to the Johansen's. This could be explained by a power advantage of the latter as suggested by Cheung and Lai (1992).

Second, Johansen's relative price coefficient yields estimates all above unity presumably indicating a real long run depreciation. The comparison of this results with the actual evolution of the real exchange rate (Figure 9 to 16) casts some doubts about the accuracy of such estimates. For example, the Swedish and French cases, where the value of estimated coefficients are 1.38 and 1.42, respectively, do not seem to fit properly. Although we could not find cointegration using Engle and Granger's methodology, their estimates for the same coefficients seem to be more adequate in these cases (0.96 and 0.98, respectively), probably because they are not affected by the short run dynamics, as in the previous case.

The findings of cointegration in the bivariate model might be a supportive result of the theoretical assumption that relative PPP holds as a long run equilibrium. However, this result might not be considered sufficient if we are also concerned with the coefficients estimates, namely if the proporcionality restriction is respected. Then, if one do not reject the proporcionality hypothesis, as in France, Italy, Spain, Sweden and U.K, the validity of PPP seems to hold, in spite of eventual problems related to the power of such tests.

Unit root tests conducted for real exchange rates do not confirm the cointegration results, even in the cases that might support PPP.

There are two aspects which we would like to develop in future work. First, to find stationary variables that could improve the short run dynamics specification of Johansen's autoregressive approach, since mis-specification errors might be causing a bias in the coefficients estimates. Second, to test if the findings of no cointegration in the Engle and Granger's approach is due to a structural change in the data using the test suggested by Husted (1992). Events like joinning the

European Comunity, the EMS or the eliminating of capital controls might be some of the causes of this structural change.

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