Mathematics teachers' professional knowledge*

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This paper addresses the study of teachers' knowledge, beliefs, conceptions and practices, presenting some illustrations from the area of problem solving. In mathematics education, the teacher has attracted much less work than the student. This may be due, in part, to the different knowledge base of interest in each case. Regarding students, we are concerned with their learning of mathematics. The nature of mathematical knowledge is itself problematic, yet that does not seem to raise too many difficulties for our work. Regarding teachers, it is much less clear what is the specific knowledge (necessary for teaching mathematics) that we should be looking at. Is it knowledge of mathematics content? Of mathematics pedagogy? Of students' cognitive processes? Some mixture of several of these?

In recent years, the teacher emerged as a key figure from who depends much of the success of current reform efforts in mathematics education. At our research group, the study of teaching became a major area of interest. Problem solving appeared as an interesting focus for inquiry since (a) it is strongly valued by the new curriculum orientations, (b) there are many different views about it, among both teachers and mathematics educators (Schoenfeld, 1992), and (c) it deals with processes that we all agree to be at the heart of the mathematical activity.

In the first part of the paper I will briefly review work done on teachers' professional knowledge and related concepts within and outside PME. Then, I will present cases taken from empirical research and discuss a few concepts used in our investigations.

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And in the final part I will contrast some general frameworks to study mathematics teachers' professional knowledge and draw some perspectives for future work.

Teachers and research on teaching

The place of the teacher in mathematics education research. Until quite recently, teachers received just a very devaluated view both in curriculum development projects (Elbaz, 1983; Howson, Keitel and Kilpatrick, 1981) and in psychologically oriented studies in mathematics education such as those reported in PME (Hoyles, 1992).

> Research on teaching, like classroom research, has generally viewed teachers in a fragmented way, in terms of isolated characteristics, and from a negative stance... Such approaches reinforce the view of the teacher as an instrument; she is a cog in the educational machine, and one which often seems to fall below the quality-controlled standards of the whole, at that... Part of the problem of such research undoubtedly lies on viewing the teacher and her work in isolation from the substance of what she teaches, that which gives much of its meaning and direction to her work. But the main failing of these approaches is that they view teachers as passive, dependent and often unsuccessful participants in the educational enterprise. (Elbaz, 1983, p. 9-10)

This devaluated view is quite natural within the dominant *individualistic* tradition of educational research, "focusing on learners, their intelligence, their abilities, and their thinking" (Bauersfeld, 1994, p. 133). However, it is not acceptable to an *interactionist* view in which the individual and the society are seen as inseparable units, having a mutually interdependent relationship, as "teacher and students interactively constitute the culture of the classroom" (p. 139).

The interactionist perspective, requiring the consideration of both psychological and sociological theories, is receiving increasing acceptance. Many mathematics educators now concur that a comprehensive understanding of the main issues bearing in the processes of teaching and learning mathematics cannot be studied just by looking at the learner. It is necessary to take into account also other factors, such as the social context of learning (Balacheff, 1990) and the nature of the knowledge being learned — also a social construct (Berger and Luckmann, 1976). The teacher is certainly one of the most important elements of the learning context and a key person in the definition of what is knowledge.

Not surprisingly, the individualistic tradition of educational research, focusing on the learner, basically sees the teacher as a complicating variable. However, a growing awareness of the limitations of this individualistic stream of educational research — for example, making it impossible to guarantee reproducibility conditions in teaching experiments (Artigue, 1992) — and the current pressures for reform in mathematics education all led to a greater awareness of the complexities of the mathematics classroom and of the important role of the teacher.

Recent work in PME. In PME, many studies have addressed (in one way or another) *mathematics teachers' beliefs*. This research mostly derives from the pioneer work of Thompson (1982) and Cooney (1985), standing on the assumption that what teachers do in the classroom mostly depends on their beliefs about mathematics and mathematics teaching. Recently, the view that the classroom environment and social, educational, and personal constraints also shapes teachers' beliefs is favouring a more dialectical perspective of the relationship between beliefs and practices (Hoyles, 1992; Thompson, 1992).

Another line of research in PME has to do with teachers' knowledge of subject matter and teaching strategies, reflecting the influence of Shulman' s (1986) ideas about the key role of *subject matter pedagogy*. These studies investigate teachers' knowledge of mathematical concepts and how to teach them (Even and Markovits, 1991; Llinares and Sánchez,1991; Sánchez and Llinares, 1992). Their assumption is that teachers who do not know well their subject cannot do a good job in teaching it, which is certainly a relevant point. However, this research has a strong focus on declarative aspects of knowledge and may be leaving out of the scene the most important issues regarding teachers' instructional activity, clearly stressed in Shulman's (1992) more recent writings.

Some research about teachers mostly stand on general frameworks of *constructivism* and *activity theory*, usually either presenting these ideas as content for teachers to learn or using them to inform the approach followed in teacher education initiatives (Adler, 1992; Crawford, 1992; Hart and Najee-ullah, 1992; Rice, 1992). Such general frameworks may provide useful starting points, but still need to be developed into more specific concepts and models related to teachers' actual professional roles.

The NCTM *Standards* (1989, 1991) key concepts of mathematical power, discourse, problem solving, etc. are the basic frame for *didactic oriented studies* (Davenport, 1992; Dougherty, 1992; Santos and Kroll, 1992). Such studies are clearly grounded in teachers' subject matter and pedagogical knowledge but seem to require a more elaborated view about the specific ways how such knowledge develops and works in practice.

Finally, recent work has been done in the perspective of teachers as *reflective practitioners*, or, going one step further, of teachers as *researchers* (Chapman, 1993; Jaworski, 1992; Lerman and Scott-Hodgetts, 1991; Mousley, 1992). Teachers, in this perspective, are seen as playing an important role in the definition of the purposes and goals of their work as well as on the means to attain them and, therefore, participate in the production of knowledge about teaching (Zeichner, 1993). Most of these ideas spring from the influential work of Schön (1983), discussing different kinds of reflection, notably reflection-on-action and reflection-in-action.

Work not much represented in PME. However, there are other important lines of work on teaching which, perhaps surprisingly, have not been strongly represented in PME:

a) The study of teachers' thinking within a *cognitive psychology approach*, dealing with teachers' interactive thoughts and decisions, both in planning an in conducting classroom activities. Most of this research has been done contrasting "expert" and "novice" teachers, looking at their squemata or knowledge structures (Berliner et al., 1988; Leinhardt, 1988).

b) Whereas that program of study presupposed that actions are guided by knowledge structures existing in the individual mind, more recently *situated cognition* began defining expertise from the perspective of knowledge use in practice, assuming that the acquisition and use of expert knowledge is essentially bound to particular contexts (Lampert and Clark, 1990).

c) With a similar concern but with a different origin, the study of teachers' practical knowledge has been carried from an *interpretative-phenomenological perspective*. For example, Elbaz (1983) views teachers' knowledge as a complex, practically-oriented set of understandings used to shape and direct the work of teaching. The content of this knowledge is revealed in the responses that teachers give to the situations that they live in their professional activity. Also in this perspective, Clandinin (1986) describes personal practical knowledge as being experiential, value-laden, positive and oriented towards practice. In her view, such knowledge is acquired through trial and error, is subject to change, and implies a dialectical relationship between theory and practice.

In mathematics education research, the teacher has been mostly viewed as a *deficient professional* — a person with deep misconceptions, lack of mathematical knowledge, and inappropriate and inconsistent beliefs, contradictory with current reform efforts. This brief review suggests that research on mathematics teaching may consider alternative perspectives. Outside mathematics education, other lines of study have considered important aspects of teachers' activity and knowledge. Without denying the difficulties that teachers face, both in conceptual and practical grounds, these perspectives may yield new and more interesting ways of looking at them.

Teachers' conceptions and practices regarding mathematical problem solving

In Portugal, recommendations to radical reform in mathematics teaching have been strongly supported by teacher education institutions and the association of mathematics teachers and were partially adopted by the Ministry of Education. In just a few years, positions that were minority became part of the official discourse. Problem solving is at the heart of the new curriculum orientations that also emphasize aspects as enhancing students' attitudes and values, applications of mathematics, use of calculators, active methods, group work, history of mathematics, and new assessment methods.

I will consider several examples of teachers' professional knowledge and practice as it relates to problem solving. The ideas and data presented in this paper were developed using an interpretative qualitative methodology, based in case studies of teachers. Of special concern were their personal and professional experiences, including their

conceptions, motivations, and areas of difficulty[1]. And I will take these examples as starting points to discuss some related theoretical concepts.

Carolina[2]. Carolina has 5 years of experience teaching 7th-11th grade students. She graduated two years ago from the Faculty of Sciences of Lisbon, where she had some difficulty in completing all her mathematics requirements. She felt that in the mathematics methods course there was a stress in just a few ideas (as problem solving) that she did not view as particularly relevant for mathematics teaching. During the internship, under the influence of the supervising teacher, she began viewing more favourably the new orientations for mathematics education. Stimulated by her colleagues, she has participated in meetings of the association of teachers of mathematics, co-leading some sessions on the use of graphic calculators.

Problem solving, as a personal activity, does not attract much of her enthusiasm. As she indicated: "I am not very good in solving problems." Her attempts to introduce specific problem solving activities in her classroom have not been very encouraging. She feels a particular difficulty in the discussion of the solutions:

> Last year... With one of my classes I did some problems... I think that the big interest is the discussion that is generated... And my discussions are terrible! [Each] problem has a discussion, [contrasting] the several ways they did them, and it is a disaster... Those classes go very poorly... And it is something from which I run away, not because I do not find it interesting, but because [I do not feel good].

Carolina also indicates some trouble in finding good materials to support problem solving classes. She considers that her classes follow basically a "traditional" model, with moments for exposition and periods for solving exercises. She does not feel comfortable in unforeseen situations and tries to prevent those from arising. She has trouble in changing her approach if the situation requires so. She likes to present mathematical material in the form of games and is very concerned (and apparently successful) in establishing a good relationship with her students. Notwithstanding all her difficulties and frustrations she has a prevailing feeling of satisfaction — not because of the intensity of the mathematical activity she is able to promote but because her good relationship with students.

A distinction among knowledge, beliefs and conceptions may be helpful in interpreting this case. I take *knowledge* to refer to a wide network of concepts, images, and intelligent abilities possessed by human beings. *Beliefs* are the incontrovertible personal "truths" held by everyone, deriving from experience or from fantasy, having a strong affective and evaluative component (Pajares, 1992). *Conceptions* are the underlying organizing frames of concepts, having essentially a cognitive nature. Both beliefs and conceptions are part of knowledge.

Beliefs are just a part relatively less elaborated of knowledge, kept from confronts with empirical reality. Belief systems do not require social consensus regarding their

validity or appropriateness. Personal beliefs do not even require internal consistency. This implies that beliefs are quite disputable, more inflexible, and less dynamic than other aspects of knowledge (Pajares, 1992). Beliefs play a major role in domains of knowledge where verification is difficult or impossible. Although we cannot live and act without beliefs, one of the most important goals of education is to discuss and promote our awareness of them.

Conceptions, as underlying organizing frames, conditionate the way we tackle tasks, very often in forms that others find far from appropriate. The interest in the study of conceptions stands on the assumption that, as a conceptual substratum, they play an essential role in thinking and acting. Instead of referring to specific concepts, they constitute a way of seeing the world and organizing thought. However, they cannot be reduced to the most immediate observable aspects of behaviour and they do not reveal themselves easily — both to others and to us.

Carolina moved from rather definite beliefs about the non-appropriateness of current curriculum orientations to a somehow middle ground position: these ideas seem all right but they are very difficult to put into practice. She does not know how to conduct several aspects of a problem solving activity (how to get materials? how to lead a discussion?) and so keeps following what she recognizes as a traditional approach. She is perfectly aware that her teaching practice does not conform to the current curriculum orientations which she does not know how to carry out. Her beliefs about the importance of the personal relationships between teacher and student constitute a very strong core around which she constructed her professional role. Her conceptions regarding the educational usefulness of games (that include a game metaphor to describe mathematics learning) provide her an important framework to organize her teaching.

Carolina is a rather insecure teacher — she expects that experience will help her to become more confident. She has many problems left to solve in her relationship with mathematics — an issue that she wants to avoid. But the essential point is the remarkable distance between what she considers didactically desirable and what she does in practice, and this seems to stand mostly on difficulties in the practical knowledge regarding instructional activities and classroom management.

Isaura[3]. Isaura is a mathematics teacher in a middle school (5th and 6th grade) with 18 years of experience. She is a very responsible person, enthusiastic about teaching, who likes to speak about her work. At the age of 15 she already intended to become a mathematics teacher. However, a new mathematics class she had at 10th and 11th grade was a rather negative experience. She disliked the teacher who, she said, “made me feel anxious about everything”. And Isaura chose to study agricultural engineering. But even before completing this degree she was already teaching in a secondary school.

Isaura is now fully certified to teach. She has been a mathematics head teacher in several schools, is a regular active participant in the national meetings of the association of teachers of mathematics and has been involved in the MINERVA project dealing with the introduction of computers in schools.

To this teacher, problem solving is the essence of mathematics and should underlie all mathematics teaching. She supports the new curriculum orientations and says that the good teacher "puts in the classroom all the innovations... New technologies, group work, materials". And she indeed puts a lot of effort in selecting and preparing learning situations. In her classes, she uses investigations, games, puzzles, and activities related to students' out of school interests. In practice, problem solving is a means to introduce or apply different concepts and is only used when it fits within the curriculum:

> [Problems need to fit] not just the sequence of topics but the curriculum... [I find] it difficult to elaborate problems suitable for students and, finally, how to evaluate the students on this domain... From a theoretical point of view much is said about problem solving but from a practical point of view [not much]... How are we supposed to take problems to the classroom? How do we integrate them in the sequence of topics?
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> In terms of observation, I circulate among [students], I observe the difficulties but I cannot see the support that each student requires... And that makes me very confused... I begin thinking: Are they stuck? What kind of help shall I give to let them arrive at a given conclusion? Will this yield a great mess, each of us saying a different thing?... Sometimes the problems are not very well chosen.

Isaura has difficulty in finding good problems and in integrating problem solving activities within the sequence of topics. She prepares many materials but she is still uncertain about which are the really suitable ones. She also indicates difficulties in conducting her classes, especially at the level of interaction and discourse. Group work is frequent but the working climate is marked by many distractions and interruptions. Also, there is a noticeable pressure to rush up things to draw quickly conclusions without giving all the students the chance to think thoroughly the proposed tasks. Isaura's concern in "losing no time" is quite apparent in most of her activity.

There is a sharp discrepancy between Isaura's stated beliefs about mathematics teaching and her actual practices. However, when we look closer on the possible reasons why practice finds problems in matching those beliefs we also find issues that have to do with knowledge. We may understand Isaura a little better if we view our knowledge as broadly structured in different *worlds of experience*, each with its own "meaning-structure" leading us to operate with a particular "cognitive style" (Schutz and Luckmann, 1973).

There are several dimensions of these worlds of experience, all mutually interrelated. One is its coherence and self-sufficiency. The very intensive and practical nature of teaching demands this restricted world to be taken as a whole and complete. Therefore, the world of teaching, although overlapping the world of everyday life, has a distinctive coherence and distinctiveness (Elbaz, 1983).

Each world of experience is also characterized by a particular tension of consciousness and a form of spontaneity. The tension of consciousness bears on our level of inter-

est and attentiveness. Teachers need to be aware of many simultaneous phenomena, which make teaching a very stressing activity. The form of spontaneity of a given world of experience is different from that of other worlds. This explains why, to a "similar event" (to an outsider's perspective), one person may respond very differently in distinct worlds of experience. Teachers' forms of spontaneity in teaching may significantly differ regarding those of everyday life. They may even vary noticeably according to the specific professional context and the students they are working with.

This structure of our experience in different worlds makes quite comprehensible that a teacher as Isaura expresses (sincerely) some beliefs about the teaching of mathematics in an interview setting and acts in a rather different way in a classroom plenty of "difficult" students. There is no inconsistency but just a gap that can be bridged by research that looks more closely at teachers' actions in the classroom and their contextual and conceptual basis.

Our knowledge within a given world of experience may have different degrees of density and consistency. It conveys complex and detailed information relative to the domains with which we have to work with frequently. As our knowledge works in a satisfying way we suspend doubts about it. Only when our maxims fail in delivering what they promised in the world of experience in which they are supposed to work they may become seriously problematic (Berger and Luckmann, 1976).

In this framework, all our knowledge, including our beliefs and conceptions, has social roots in our activity and is shaped by our experience. Beliefs and conceptions cannot be viewed determining practice, since it is the nature of the social institutions in which we move — including schools — that mostly shapes them. In the long run, however, these conceptions are mostly framed by experience within social contexts. However, the interactionist perspective does not indicate the absolute domain of the social. There is a margin for the individual which may be widened by conscious reflection. Specific actions are framed by existing conceptions acting in a given world of experience which enable to make sense of situations and choose among alternatives.

Isaura has attended several workshops and discussions on problem solving and reads regularly the professional literature. However, she indicates that her personal relation with problem solving is not a very easy one:

> I do not always have persistence... For some [problems] I do not have much patience... I like to know how it is done... I go to the solutions...
>
> Many of the existing difficulties begin with the teachers and their lack of preparation in this field. In most of the cases, the teachers while they were students were not used to solving problems but just exercises... On the other hand, the word 'problem' has for us a very strong connotation, associated with the idea that was transmitted that who could not solve problems was stupid.

The activity of this teacher seems to be marked by some anxiety, which is probably a specific character trait. This results in some difficulty in managing a stable and productive working relationship with the students. As with Carolina, there are also some problems left to solve in her relation with mathematics. However, Isaura's beliefs supporting the new curriculum orientations seem to have a positive impact in her classes, enrichi students' learning experiences.

Júlia[4]. Júlia is a teacher of mathematics with 10 years of experience, who graduated from the Faculty of Sciences of Lisbon. She teaches 7th-11th grade students in a secondary school. She is a communicative and active person, who usually manages to accomplish what she plans and enjoys chatting and laughing. Júlia attended a couple of national meetings of the association of teachers of mathematics and was involved for several years in the national project MINERVA. She enjoys artistic activities, something she would now gladly consider for a profession. In teaching, she highly regards the possibility of designing and conducting what she views as "creative" classes.

Her classes have an enjoyable working climate, with most of the students involved in the proposed activities. Mathematical situations with a problematic character are the starting point for the activities that she carries with students. Most of these situations involve just mathematical notions and do not refer to real life contexts. She stresses interrelating graphical and analytical approaches, establishing relationships among concepts, generalizing and formalizing. All of this is carried encouraging communication among students and teacher and students.

During the interviews, she made in two different moments the following statements about problem solving:

> Look, I do not solve many problems myself... Perhaps because I am not a person that... That gets much interested about problem solving. It is not because I feel that as not important, you know?... But I think that it is important to propose problems to students. There, I miss some opportunities. As I do not solve many problems myself, I only know those most trivial ones and those that are on the books.

> I think it is important that [mathematics teaching] is done this way: putting the student in the position of the mathematician, of the mathematician that discovered, because he is also a person... [The student] also thinks and is able to discover the same as the [mathematician]. Or else, always in a [perspective of proposing] a problem that we have to solve... [I say], 'let us see how we can solve this' and here it comes the mathematics.

Júlia seems to have more than one conception of mathematics problem. On one hand, she regards a problem as a self-contained situation, referring to a mathematical or non-mathematical context, but not much related to the mathematics curriculum — something that she indicates to be important but that does not play a significant role in her teaching. On the other hand, by problem she means a strictly mathematical question that requires a non immediate response, implies to relate several concepts and yields the

discovery of new mathematics knowledge — a perspective that is consistent with her views and practices of mathematics teaching. She seems to refer to very different kinds of "problems", but the use of the same term to express distinct ideas generates what can be regarded as an inconsistency in her responses. Júlia is just referring to different worlds of experience: one concerns general talk about mathematics and mathematics education and another refers to actual classroom activity.

Situations of practice have unique characteristics of complexity, specificity, instability, disorder, and undetermination. Schön (1983) considers *knowing-in-action* as a kind of knowledge that is built into and reveals itself in action. In his view, knowing-in-action has an intuitive and tacit nature, is marked by spontaneity, and is learned through action and reflection in situations of practice. With a similar intent other authors speak of personal professional knowledge (Connelly and Clandinin, 1986) or teachers' craft knowledge (Grimmett and Mackinnon, 1992; Brown and McIntyre, 1993).

Educational theory is unable to direct practice taking into account all the myriad features underlying practical situations. If one is concerned with practice and with knowledge that evolves out of contextualized activity and informs intelligent action (such as that of the teacher in the classroom), we need to focus in a different kind of knowledge, which I will call professional knowledge.

A professional activity is characterized by the accumulation of practical experience in a given domain. It is not adequate to judge the professional knowledge of a practitioner by the standards of academic (scientific or philosophical) knowledge. Judgment, and on the case of teachers, judgment on the spot, plays an essential role in professional activity. This judgment may profit from academic knowledge but requires the use of other resources. It needs an intuitive apprehension of the situations, ability to articulate thinking and action, a sense of personal relationships, and self-confidence. That is, *professional knowledge* is essentially knowing in action, based both on theoretical knowledge and on experience and reflection on experience.

We may differentiate among academic, professional and common knowledge based on the ways the basic underlying beliefs are articulated with specific patterns of thinking (based on logical reasoning and experience). Experiential aspects are pervasive in more elaborated professional knowledge. Rational arguments predominate in academic knowledge. Academics and professionals (when they act in their quite circumscribed special domains) have a strong explicit or implicit concern for consistency and systematicity. Common people (and academics and professionals when they act outside their activity domains) have other priorities and do not worry too much about such matters.

If we want to recognize professional practice on its own right, we need to take it as the starting point for research and not just the place for application of theory. The concern becomes not just in applying theory to practice to improve it, but instead, to work with professionals to better understand practice and its constraints — to make it stronger. Or, as Connelly and Clandinin (1986, p. 294) put it, "to enhance its ongoing practicality".

Somehow disturbing is Júlia's little personal involvement in the practice of mathematics investigations and problem solving, suggesting a perplexing dissonance between what she considers a valuable learning experience for her students and what she values in her personal life. However, this teacher has the ability to make the mathematical content appear quite naturally in a problematic way. Her practices seem to fit what Stanic and Kilpatrick (1989) described as problem solving as context to develop mathematical ideas. One may wonder if her students get a sharp notion of what are mathematical problems, what are problem solving strategies and how mathematics is applied in real world situations. But Júlia certainly provides an example of great mathematics teaching, with plenty of opportunities for students to reason and communicate mathematically. Although not quite adjusted to all requirements of the new curriculum orientations, she gives us a superb case of professional knowledge.

The structure and development of teachers' professional knowledge

Structure. Elbaz (1983) suggests that teachers' professional knowledge is structured at three levels: images, practical principles, and practical rules. A *rule of practice* is a concise statement of what to do in a frequent practical situation. It may apply to rather specific or fairly general situations, but it always refers to their concrete aspects. Rules of practice concern means — the purposes of the action are taken for granted — and are quite idiosyncratic. They are formulated for the purpose of eliminating the need for unnecessary deliberative thought.

For Elbaz, *practical principles* are less explicit statements indicating purposes. They are more expressive of the personal dimension of professional practical knowledge. Principles may stand on theory, may develop out of experience, or both. Although they do not guarantee similar courses of action in similar situations, they still enable us to say that practice is principled. It is worth noting that such practical principles and rules of practice seem rather similar to actions and operations as assumed in activity theory (Crawford, 1992).

Images of how teaching should be are the less explicit and the most general level of teachers' practical knowledge. They are broad and metaphorical statements that express in a clear way some purpose and result from the combination of feelings, values, needs and beliefs. Images organize the teachers' knowledge in different areas. They capture some essential aspects of their perceptions of themselves, of their teaching, of their situation in the classroom, and of their subject matter. Summarizing her conclusions of a case study of a teacher, Elbaz (1983) writes: "Sarah's practical knowledge is structured around a small number of images that reflect the entire body of her knowledge and serve to hold together the principles and rules she uses in bringing her knowledge to bear on practice" (p. 144-5). The idea of image is also present in other authors. For example, Clandinin (1986) views images as "the coalescence of diverse experiences and their

expression in diverse practices" (p. 180) and Berliner et al. (1988) seem to regard images as reference frames of how things should go in a classroom.

Leinhardt (1988) uses two other constructs to study teachers' professional knowledge: script and agenda. A *script* refers to a specific lesson segment and concerns the goals and structured actions enacted to teach a particular topic. For each lesson the teacher has an *agenda*, which includes goals and actions, tests, and an overall strategy. Typically, in each lesson, the teacher sets an agenda and uses several scripts. A fundamental objective of teaching is to go on using a script, or, according to the intended agenda, to move to the next script. Modest adjustments are made when necessary in response to the needs of the students.

Leinhardt contends that teachers construct models, but not of individual students' knowledge. She found that teachers essentially diagnose what went well and what went wrong in their teaching and not the specific mental representations of their students. Therefore, the core of these models refers to the ways of teaching the subject matter and provides an orientation for the future — how each topic will be taught next time. This is a line of thinking that brings us close to Shulman's (1986) ideas about the centrality of subject matter pedagogical knowledge.

There are several differences between these authors. Elbaz is concerned with the general structure of the teacher's professional knowledge while Leinhardt, Berliner and their colleagues focus on the knowledge structures mostly bearing in the development of the lesson. Also, Elbaz regards teachers as having just a few organizing images while Berliner seems to extend this term to include practical principles, suggesting that teachers (especially experienced teachers) hold a great variety of images regarding classroom work. And thirdly, Elbaz strongly takes into account the personal and contextual factors bearing on teachers' knowledge while Leinhardt and Berliner seem to be essentially concerned with classroom performance.

Adopting Elbaz's terminology, we can say that Júlia seems to base all her teaching in a strong image of the working mathematician, as someone who has successes and failures in striving to relate concepts and solve problems. Her self-image as a creative and autonomous professional is also crucial and has a strong impact in her classroom practice. Her practical principles and rules of practice do not seem to be in conflict with her fundamental professional images.

Isaura also has a strong and clear (rather ambitious) image of what is an innovative teacher. However, when she comes to concrete action this image is overshadowed by concerns about the curriculum, the classroom dynamics, and her view of the students' capacities. She may face a problem of competing images. Alternatively, we may conjecture that she does not dispose of appropriate practical principles and rules of practice to enact her espoused teaching images. There is little doubt that if we just studied her stated beliefs and conceptions we would gather a very different picture of how she is as a teacher and what may be her most significant professional development needs.

Finally, Carolina distinguishes what may be all right in principle from what she feels confident in doing. She has a self-image of not being very well succeeded in mathematics nor in conducting classroom discussions. She organizes her professional practice around activities in which she feels secure and constructs her professional role through her personal relationship with the students.

Development. Experience is certainly one major factor contributing to the development of teachers' knowledge. As Berliner et al. (1998) indicate, experience changes the way we see the world around us, creating insensivity towards ordinary things and prompting us to notice atypical aspects. This is extremely helpful, especially in dealing with complex situations, since we do not need to look at usual features and can concentrate in a few selected issues.

Preservice teachers do not have a professional experience. But we may argue that their personal experience in elementary, secondary, and higher education yields the essential frame in which they organize their teaching images and tentative practical principles and rules of practice (Crawford, 1992). So, let us see in what ways experience may contribute to the development of teachers' professional knowledge.

Referring to their empirical research, Berliner et al. (1988) indicate that the responses of many of the experienced teachers reveal that they have rich images or prototypes for students and classroom events. These teachers also accumulate a great quantity of information about students, so that, in some sense, they seem to know their students even before they meet them. They also use routines in more areas of instruction more frequently and with more success. Supporting the view that professional knowledge is somehow distinct from common everyday knowledge, these authors say that experienced teachers show evidence of more reasoned thinking, referring to concrete evidence in their explanations and justifications of their actions.

Experienced teachers constantly monitor students and the class activity. If things are going as expected, there is no need to give too much attention to details and it is possible to follow the intended agendas and scripts. A similar view is proposed by Brown and McIntyre (1993), who claim that teachers are mostly concerned in maintaining some "desirable normal state of pupil activity" enabling progress towards the intended goals. But, if something unforeseen arises, the teacher needs to act in a different, deliberative mode, close to Schön's (1983) idea of reflection-in-action. Both through positive and negative instances, in an intuitive way, we subsume our experience in practical principles but we can also do it at a more conscious level, through deliberate and systematic reflection.

The influence of the context is a major concern of recent approaches to cognition. In these three teachers we note the influence of context through preservice and inservice opportunities. However this influence seems to be mediated by (a) their attitude towards the profession, (b) their personal relationships with mathematics, shaped by experience as elementary, secondary and university students, and (c) the way they personally relate to students, shaped by all their former experience of interpersonal relationships.

Júlia derived most of her images of the mathematics classroom from her preservice and inservice experiences, participation in projects, and other professional activities. Isaura, also took an interest in innovative approaches to mathematics education mostly from her intensive contacts with a professional association. Carolina, quite differently, did not accept the ideas proposed in her preservice education, showing how the effect of these professional learning contexts highly depends on a personal readiness factor. During her internship, under the influence of an enthusiastic and supportive supervisor and within a group context, she changed notably her general attitudes towards the current curriculum orientations and accepted to give a try to some of the new ideas. However, regarding the ways she presently organizes her classes, one may ask how strongly she changed her essential conceptions. All these personal processes take many years, showing that teachers' professional development has to be studied in a very different time scale of that usually adopted in teacher education programs.

Conclusion

This paper presented some theoretical concepts which may prove to be useful in studying teachers' professional knowledge. It also showed some evidence suggesting that a different view of the teacher's knowledge and professional activity may be fruitful in studying mathematics teaching.

Recent research has most emphasized teachers' conceptions and beliefs. We may also study teachers' images (a clearly less evaluative concept), practical principles and rules of practice (more directly related to teachers' actions), and how these relate to teachers' agendas and scripts in specific lessons. We should be looking for the internal integration of the different levels of the structure of teachers' knowledge as well as for their ability to guide actual practice in a variety of contexts.

The difficulty in integrating problem solving into the mathematics curriculum is a feature common to all these teachers. Júlia presents her students with situations with a problem solving flavour, but does not value specific problem solving activities or students' learning of problem solving strategies. Isaura proposes many problem solving tasks, but does not explore them to their full potential. She is aware that sometimes things do not go very well but has trouble in understanding the specific nature of the difficulties. Carolina agrees that problem solving activities would be desirable but simply does not feel comfortable in doing them and chooses to work in other directions.

Practice, an inherently complex and unpredictable realm, has its own specific characteristics that need to be valued on its own right. Teachers work within many constraints (of which we need a better understanding) but still create quite sensible solutions for their practical situations. Innovative curriculum orientations, such as problem solving, need to be studied more closely from the point of view of practitioners.

There is a need to keep discussing general models and concepts of teachers' professional knowledge as well as carrying specific studies on the external influences and on

the internal development processes. Such research may provide important guidance for the development of new professional development programs and promote a better account of the role of the teacher in curriculum development initiatives. To be successful, research in this field needs to include a strong participation of teachers where they are granted the role of active partners speaking on their own voice (Jaworski, 1992). This collaborative process may turn out to be a most valuable key for a better understanding and improvement of mathematics education.

Notes

[1] Although specific methodological aspects are not discussed here, important issues arise in this kind of research, such as the relative role of observation and interviewing and the relation of the researcher with participating teachers and students.

[2] This case is reported in detail in Ponte et al. (1993). In this and the following cases, I kept the pseudonyms used in the original research reports.

[3] This case is discussed in Delgado (1993).

[4] This case is taken from Canavarro (1993) and Ponte and Canavarro (1993).

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